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Symmetry: Culture and Science

Symmetry of Patterns, 2

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György Darvas and Dénes Nagy

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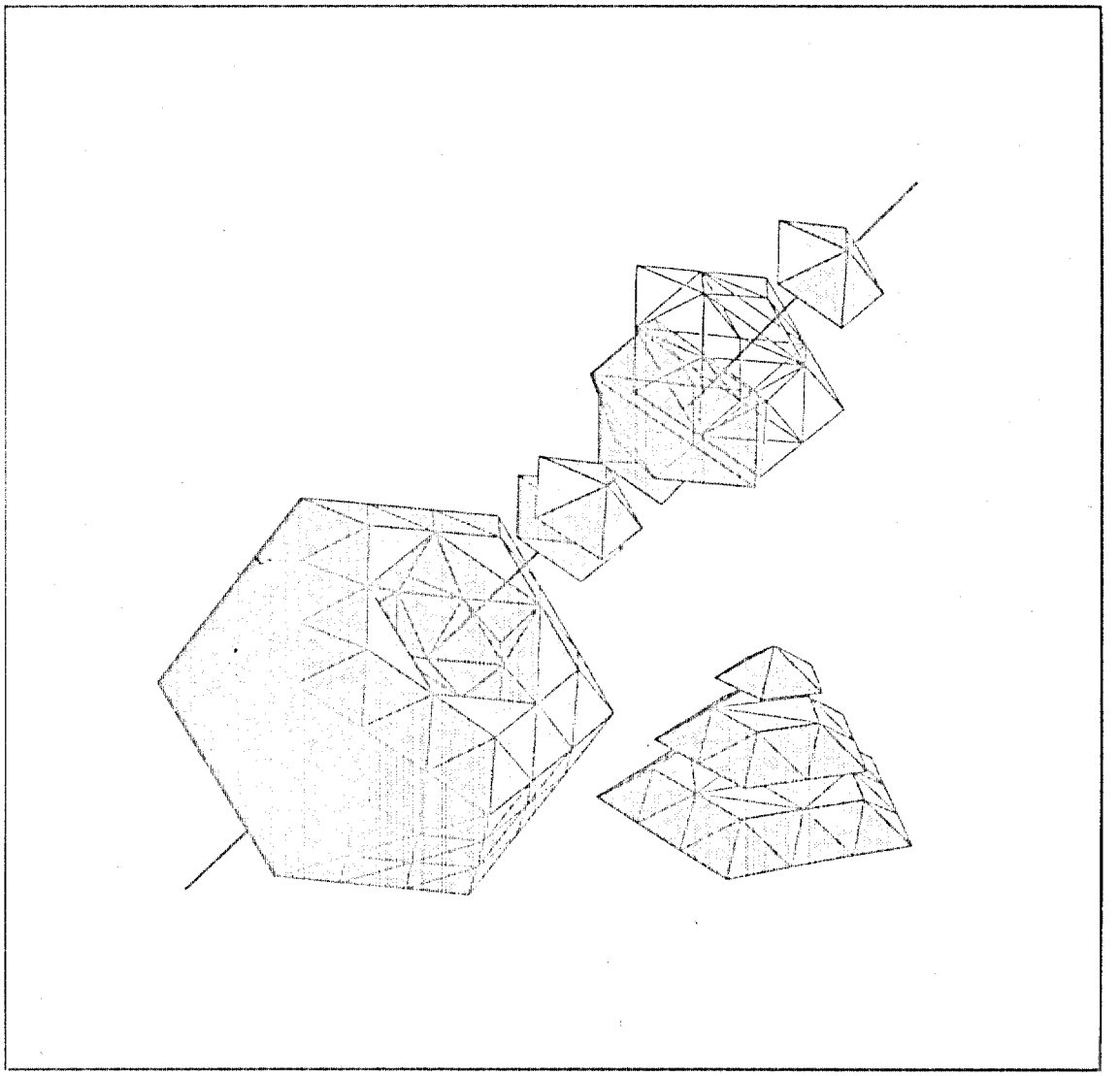

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PART 2

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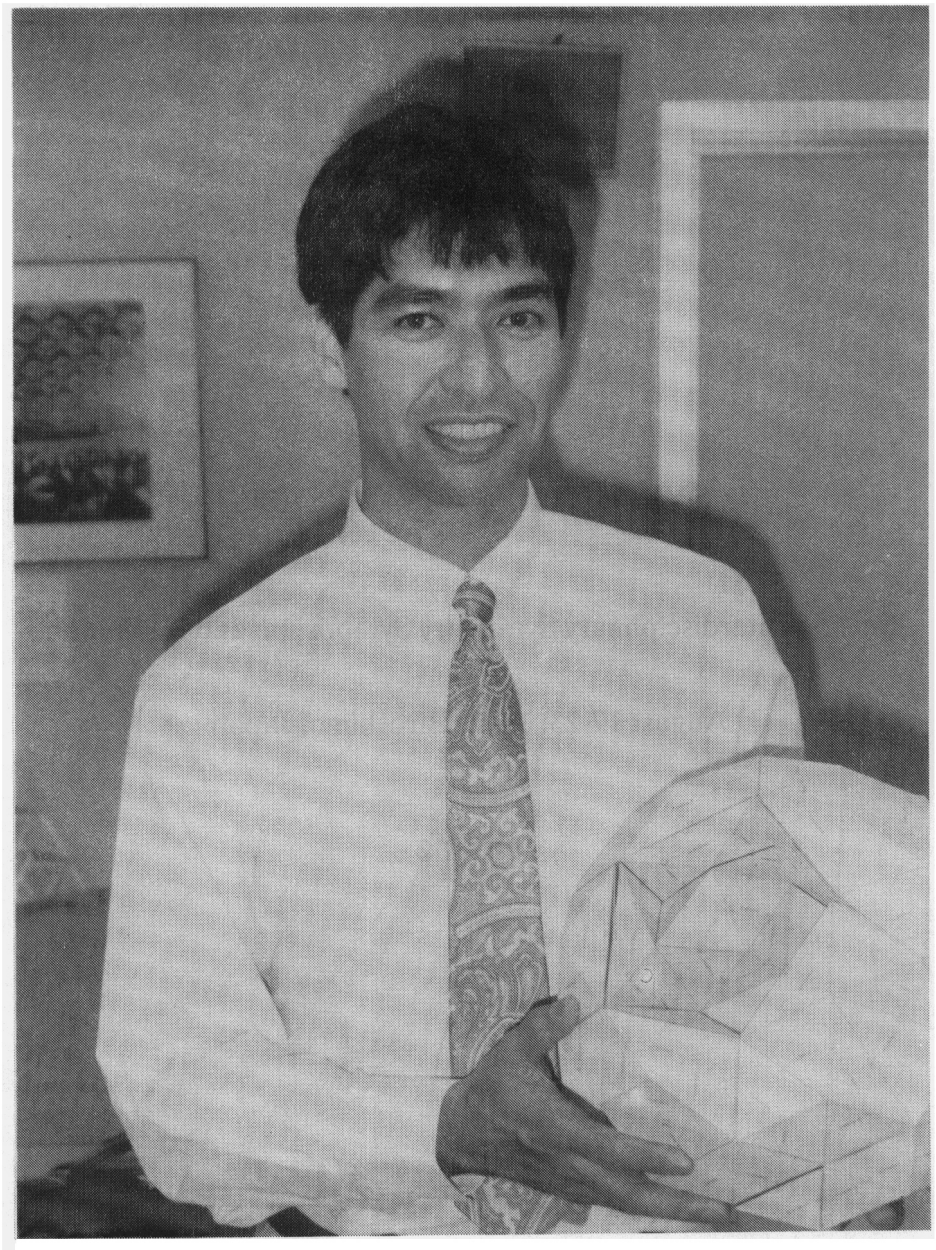

Yasushi Kajikawa with his 'grown' *icosahedron* model.

Photo: Caspar Schwabe

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GROWING ICOSAHEDRA

Yasushi Kajikawa

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It was discovered that by dividing an icosahedron into ten types of small modules, we can construct various 5-fold symmetrical polyhedra by recombining the modules again radially and symmetrically. This discovery suggests possible new paths of interaction between polyhedra, which are the subject of intense debate with respect to quasicrystals.

In crystallography, periodicity defines a structure that has both rotational and translational symmetry. Because of this restriction, 5-fold symmetry was ruled out in possible solutions to the question of the structure of crystals. However, 5-fold quasicrystals exist and have now been discovered. To cope with these inconsistencies, researchers are going to have to expand the framework of traditional crystallography.

In fact, all quasicrystals discovered so far have 5-fold symmetry. The Penrose lattice is a powerful means of coping with such structures, because it avoids too rigorous rotational and translational symmetry, and it provides us with an elegant order.

In geometry, there exist polyhedra with rigorous 5-fold symmetry. For example, if we project a dodecahedron or an icosahedron on a screen, we can see regular pentagons.

I first divided an icosahedron into ten types of small modules, then reconstructed various polyhedra by combining the modules radially and symmetrically. I found a beautiful hierarchical structure in the polyhedra, where in each layer in the reconstruction process there appear various 5-fold symmetrical polyhedra including the dodecahedron, icosahedron, and rhombic triacontahedron. Furthermore, it is possible to fill the entire space in these models while maintaining a hierarchical structure in the filling pattern.

Although this discovery was done outside the territory of translational symmetry in crystallography, it is quite suggestive of a solution to the structure of 5-fold symmetrical crystals that grow radially and symmetrically in the natural world.

Crystallography first began as a branch of geometry. From the 18th century until the early 20th century, crystallography researchers were developing an area of pure geometry of the structures of crystal lattices and the generalization of their patterns

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by investigating the symmetries which were revealed by measurements of the angles between adjacent faces of mineral crystals. They established the crystal lattice theory before the famous X-ray crystal structure analysis was invented by M. von Laue in 1912, which is deemed the monumental start of modern crystallography.

Crystal structure analysis is a method by which we can decide the atomic arrangement in a crystal by making a diffraction image of a crystal with an electron beam or X-ray. Note that the diffraction image does not show the edges and faces of a unit cell in the crystal directly. Instead, it only shows a transformed image which represents the relation between atoms in the crystal. Thus, in the image, the positions of the atoms are coded in an array of 2-dimensional dots.

Hence, information about distance between atoms (or bond length), their bond angles, and the space group is needed in advance in order to determine the shape and scale of the unit lattice from the diffraction image. In other words, the atomic arrangement cannot be determined without a minimal geometrical structure and the pattern of a 3-dimensional model.

Laue succeeded in adjusting the scale of independent geometrical concepts and the principles found in nature to allow them to coincide with each other and proved the correctness of the geometrical modeling method.

Now, we can visualize the atomic arrangement in a 3-dimensional form by using a geometrical approach with the three basic visual concepts of geometry (vertex, edge, and face) which allows us to reveal the relationship between dots in the diffraction image.

Technological advances in crystal structural analysis since Laue's invention have brought in a number of discoveries of quasicrystals with 5-fold symmetry which are crystals whose existence has never before been conjectured. The first quasicrystal Al-Li-Cu alloy was discovered by Hardy and Silcock in 1955, 30 years before the name 'quasicrystal' was first used for a newly discovered Schechtmanite Al-Mn alloy.

In 1986, a French researcher B. Dubost composed a very large quasicrystal of 1 millimeter in diameter, which contained 10^{20} atoms! It is an $Al_6Li_3Cu_1$ alloy whose shape is a complete rhombic triacontahedron. In 1988, A. P. Tsai and others of the Metal Material Laboratories of Tohoku University succeeded in composing a quasicrystal $Al_{65}Cu_{20}Fe_{15}$ of 2 millimeters in diameter whose shape is a complete dodecahedron.

GEOMETRICAL MODELING OF QUASICRYSTALS

A 3-dimensional geometrical model which can cope with 5-fold symmetry is necessary in order to investigate the structures of these new quasicrystals. There have been a number of proposed models in recent crystallography. Proposed models are, of course, decisively different from traditional ones. However, most of them are based on the traditional closest packing of spheres of the same diameter, and the all-space filling by two types of parallelepipeds called A_6 and O_6, respectively.

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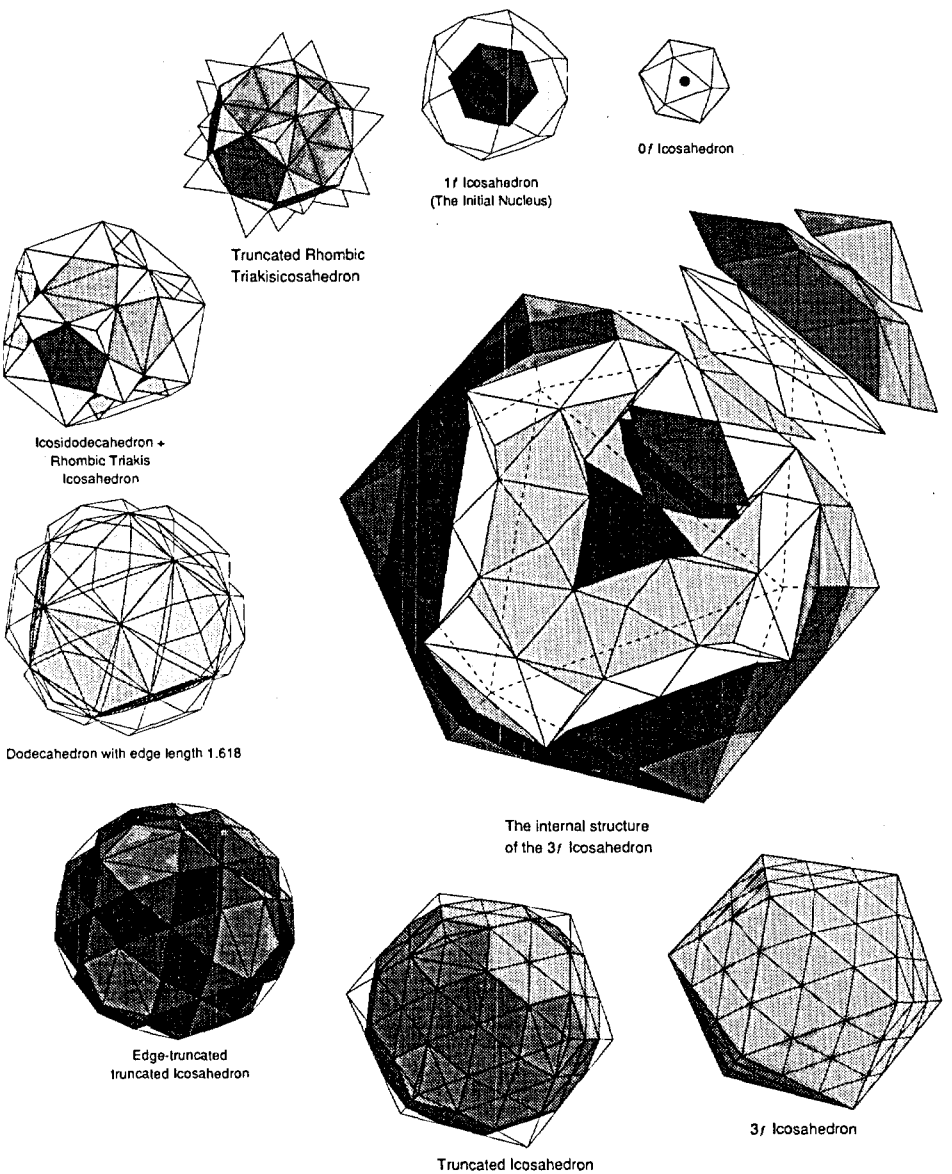

Figure 1: Hierarchical structure model

The model fills the space hierarchically starting from an icosahedron. The shape of each shell maintains the icosahedral 5-3-2 symmetry through the growing process. Some Platonic regular polyhedra and Archimedean semi-regular polyhedra which have 5-fold symmetry can be seen here. These polyhedra will fill space indefinitely, repeating a self-similar hierarchy every time their edge lengths become three times greater than before.

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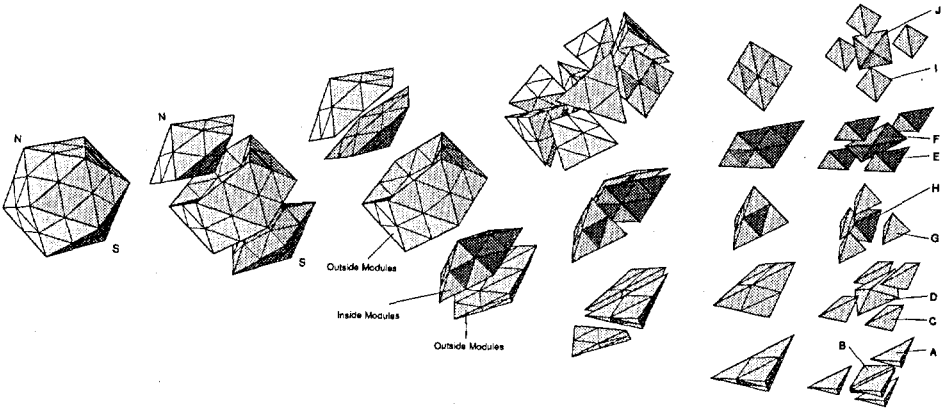

Figure 2: Synergetic modules

The model I discovered in which the space will be filled hierarchically on the basis of icosahedron consists of ten types of modules called the *synergetic modules*. These modules are obtained by dividing the icosahedron into two pentagonal bipyramids and one scooped out pentaprism and continuing to divide them at vertices, midpoints of edges, midpoints of diagonals and so on. The resulting ten types of modules are thought of as ultimate units in this division.

All of five tetrahedron modules A, C, G, E, I and five octahedron modules B, D, H, F, J are composed of only triangular faces. The volume of an octahedral module is four times as large as that of the corresponding tetrahedral module. (For example, the volume of F is four times as large as that of E.) The tetrahedron I is known to be able to constitute the whole icosahedron by itself. Let the volume of I be 1. Then, the volumes of the other tetrahedra G, A, and C are $1/\tau$, $2/\tau$, and $2/\tau^2$, respectively, where τ means the golden ratio.

The synergetic modules can be classified into the outside modules A, B, C, D, I, J whose face can be seen from outside, and the inside modules E, F, G, H which are hidden inside the icosahedron. Though icosahedra cannot fill the space without gaps, these ten types of modules can fill 5-3-2 symmetric space.

The closest packing of spheres of the same diameter has served as the traditional geometrical structural model to represent crystal structures. However, it cannot create polyhedra which have 5-fold symmetry, such as an icosahedron, a rhombic triacontahedron, or a dodecahedron composed of the same sized regular pentagons.

To construct an icosahedron by the closest packing of spheres, we have to reduce the size of the central sphere, around which 12 spheres of the same diameter can be arranged and if we want to enlarge the icosahedron by adding further spheres around it, it will soon turn out to be impossible because the bond angles and distances between spheres cannot be maintained exactly. Gaps in the outer shell will stop the growth.

This means that the closest packing sphere model cannot cope with even the simplest icosahedron and is thought to be inappropriate for the 3-dimensional geometrical model of quasicrystals which have 5-fold symmetry.

S. Baer is the first researcher who in 1970 discovered that A_6 and O_6 can yield 5-fold symmetry. He constructed a rhombic triacontahedron using ten parallelepipeds each of which is classified into either A_6 or O_6. The lengths of the diagonals of each parallelepiped equal the golden ratio. Then, for the first time, he

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developed a 3-dimensional space filling model, in which the internal construction of the units is non-periodic and the shape has 5-fold symmetry. He also succeeded in combining rhombic triacontahedra by overlapping adjacent ones without losing the 5-fold symmetry. He tried another all-space filling model using parallelepipeds whose diagonal ratios are different from those of A_6 and O_6, and found that 120 parallelepipeds of five types can fill a 5-fold symmetric rhombic enneacontahedron non-periodically.

In 1981, A. L. Mackay discovered 3-dimensional Penrose tiling and it became known that the parallelepipeds A_6 and O_6 are the units which can also fill a rhombic icosahedron and a rhombic dodecahedron along with the rhombic triacontahedron. However, it was impossible to fill polyhedra with 5-fold symmetry like an icosahedron or a dodecahedron.

Since the discovery of quasicrystals, it seems that only these approaches have been tried in order to prove the possibility of 5-fold symmetry in the 3-dimensional geometrical model. Former space filling models in 3-dimensional geometry aimed at filling the entire space.

However, I thought that the fundamental problem of these space filling models with respect to quasicrystals was how to construct a pure geometrical space filling model in which an asymmetrical or non-periodic internal structure could comprise a symmetrical outer shape. I believed that pursuing the units by which 5-fold symmetric polyhedra can be composed would lead to a new geometry of symmetry. This was the motivation that started me to investigate a generalized geometrical all-space filling model which shows the way to fill any closed 5-fold symmetrical polyhedron without gap or inconsistency.

HIERARCHICAL MODEL OF ICOSAHEDRON AND SHELL FILLING

In October 1989, I found that an icosahedron can be divided into a number of triangularized modules of ten types and that they can comprise more than one shell of different geometrical structures which grow concentrically and hierarchically to make up an icosahedron with 5-fold symmetry. As a result of my particular asymmetrical modular divisions, the outermost shell of the icosahedron shows a regular triangularized pattern in which the length of each edge is an integer multiple of the original edge length. This means that an icosahedron can grow by this multiplication.

My ten new types of modules are called *synergetic modules*. These modules can all-space fill higher frequency icosahedrons each of whose faces is a lattice of regular triangles. We can make an icosahedron grow by dividing its edges into a number of equally long parts and connecting these divisions in a triangular manner. The number of these divisions is called the frequency (f) of the icosahedron.

My icosahedron grows symmetrically in the radial direction. *Synergetic modules* have both growth ability and interchangeability in this hierarchy. All-space filling

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models with such characteristics have never been reported in the past history of geometry, physics or crystallography.

In the growth hierarchy from a $1f$ (one frequency) icosahedron with edge length 1.0 to a $3f$ (three frequency) icosahedron, there appear recognizable symmetrical polyhedra, including the truncated icosahedron with edge length 1.0, the icosidodecahedron with edge length 1.0 and the dodecahedron with edge length 1.618 . All polyhedra in this hierarchy have perfect 5-3-2 symmetry.

Note that the 5-3-2 symmetry is seen in the exterior shape of these polyhedra, not in the internal arrangement of modules nor in the triangularized pattern on their face. In fact, there is more than one combination or arrangement of the modules for each shell.

On the outside layer of the $3f$ icosahedron, the self-similar patterns of all the shell structures appearing in the hierarchy between the $1f$ and $3f$ icosahedron are replicated by a multiple of 3, because the number of divisions, or frequency, has to be a multiple of 3 in order to make each of the 20 vertices of the dodecahedron contact the center of the triangle lattice on the corresponding face of the outer icosahedron.

The $3f$ icosahedron can be considered as a minimum shell structure in the hierarchy in the sense that all the ten types of modules are used in it for the first time. In the $6f$ (six frequency) icosahedron, all the shell structures will appear again with their edge lengths doubled. However, in the $4f$ (four frequency) and $5f$ (five frequency) icosahedra, some of the layers are lost. That is, not all frequencies make icosahedra with complete hierarchical structures.

Super-high-frequency icosahedra are filled by iterating the hierarchy of the shell structure hierarchies. Thus, we can consider the concentrically expanding hierarchy of the polyhedra with 5-fold symmetry, a hierarchy which recurs periodically.

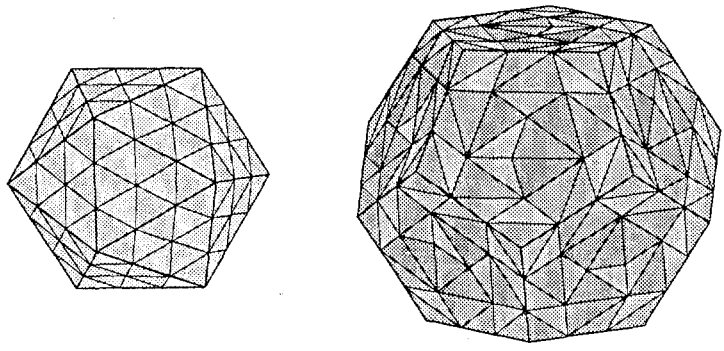

Figure 3: The $3f$ icosahedron and $3f$ dodecahedron
This figure illustrates the contrast between $3f$ (three frequency) icosahedra whose faces are composed of only outside modules and $3f$ (three frequency) dodecahedra whose faces are composed of only inside modules.

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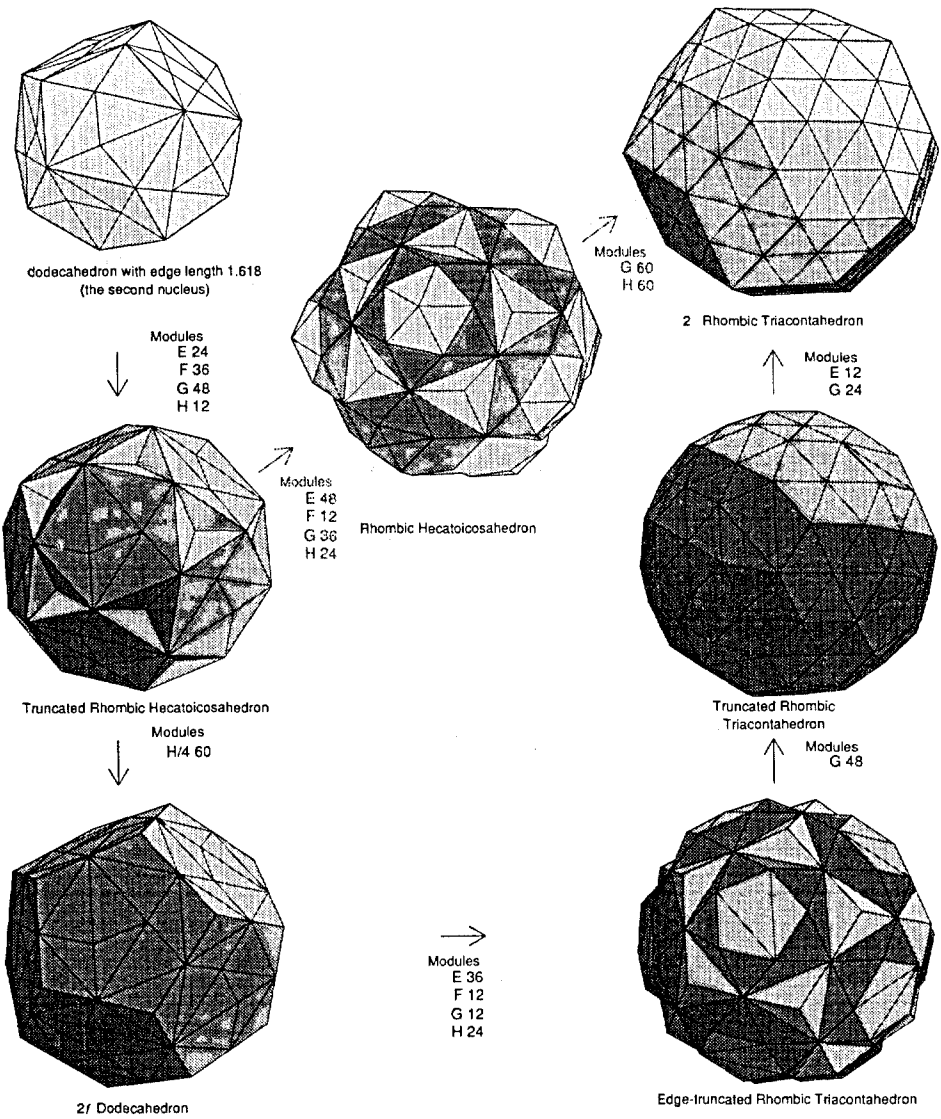

Figure 4: Rhombic triacontahedron

If we change the combination of modules from the first dodecahedron, we can obtain a rhombic triacontahedron which also has 5-3-2 symmetry. After that, the periodicity of the hierarchy will be 2. Beyond the first dodecahedron, only inside modules are used to make further shells. The face of the rhombic triacontahedron becomes a lattice of isosceles triangles. According to the model, two systems of the forms are possible. Quasicrystal alloys of millimeter size whose shapes are a rhombic triacontahedron and a dodecahedron have already been created. This choice from two may be explained by the hierarchical model for the super-high frequency rhombic triacontahedron. If this is the case, there may be an icosahedral quasicrystal alloy of three metal elements. The number immediately following a module name indicates the number of modules to be used in the growth step. Note that it is a multiple of 12.

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So far, icosahedra with 5-fold symmetry have been thought impossible to all-space fill without inconsistencies or gaps. However, it has now been proved that growing icosahedra can be all-space filled by using the *synergetic modules*.

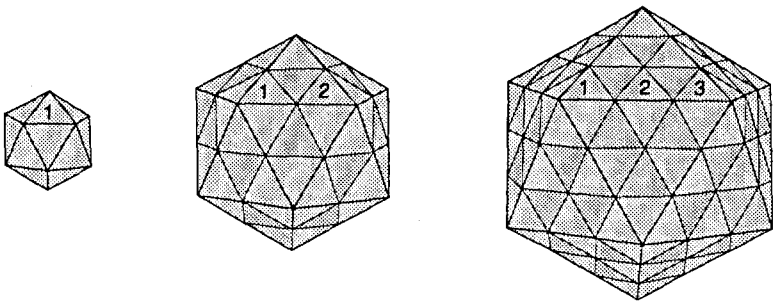

Figure 5: Icosahedral cluster structure with perfect 5-fold symmetry
The 1*f* icosahedron can be composed of only tetrahedral modules *I*. (Here, we use the same notation 1*f*, 2*f*, ...) The 2*f* and 3*f* icosahedra cannot be composed of *I* modules only. We have to use the corresponding octahedral module *J*. These icosahedra are formed by radially and symmetrically combining 20 tetrahedral subunits composed of modules *I* and *J*. There are no shell structures other than the icosahedron.

SYNERGETIC MODULES

The north pole and south pole among the 12 vertices of the icosahedron always function additionally to the system. In the formation process of the *synergetic modules*, the abstractness of the additive twoness is indeed replayed visually. These two poles are clearly distinguished from other vertices.

To obtain the *synergetic modules*, we have to divide an icosahedron into three basic parts first: the arctic part, the equator part, and the antarctic part, all of which are symmetrical with respect to an axis penetrating through the north and south poles. The arctic and antarctic parts are two identical pentagonal bi-pyramids and the equator part is a ring shape which is a pentaprism with both top and bottom scooped out. The volumes of the pentagonal bi-pyramid poles and the scooped out pentaprism are 5 and 10, respectively, if the volume of the icosahedron is assumed to be 20.

Next, we divide each pentagonal bi-pyramid into two (outer and inner) pentagonal pyramids by a plane perpendicular to its axis. This split is fundamentally related to the golden ratio. Further, we divide each pentagonal pyramid into three tetrahedra. We also divide the scooped out pentaprism into ten identical tetrahedra. Now, we have five types of tetrahedra, two from the outer pentagonal pyramids, two from the inner pentagonal pyramids and one from the equator scooped out pentaprism.

If we continue dividing each tetrahedron into four small similar tetrahedra and one small octahedron by splitting it at lines connecting its edge midpoints, we obtain five types of small tetrahedra and five types of small octahedra. Of course, the

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volume of each octahedron is four times as large as that of the corresponding tetrahedron.

All of the *synergetic modules* have triangular faces. However, there are only five types of triangular faces. Four of them are triangles with two equal sides, i.e, isosceles triangles with edge lengths 1.0 (the edge length of the original icosahedron), 0.95 (the distance between the center of the icosahedron and one of its vertices), or 1.618 (the length of a diagonal of the pentagonal face of the icosahedron). The other triangular face type is an equilateral triangle.

COMPLEMENTARINESS OF THE SYNERGETIC MODULES

The ten types of modules are classified into six outside modules and four inside modules, depending on where they are located in the original icosahedron. Modules can be joined to each other if they have the same mirror image face. However, the outermost face of a growing icosahedron can consist of only equilateral triangles with edge length 1.0, which is common to all outside modules.

The four types of inside modules can be arranged to form another shell inside the growing icosahedrons outer shell. Outside modules can in turn be arranged to form another inner shell to link with more inner shells. If faces of inside modules form some intermediate polyhedron, this polyhedron will not be an icosahedron. In other words, the formation of shells is intrinsically related to the complementariness of the outside and inside of the icosahedron.

Tetrahedral modules and octahedral modules are joined to each other in keeping with this complementary relationship. The first $1f$ icosahedron is composed by joining only tetrahedral modules with each other. However, in all other shell structures, each tetrahedral face must be joined to the face of an octahedron that can be joined to another tetrahedron on another face. That is, neither tetrahedral modules nor octahedral modules alone can fill these shells.

NUCLEUS FOR GROWTH

Historically, since they are dual to each other, we have not been able to determine which polyhedron is more fundamental, the icosahedron or the dodecahedron. However, in our growth system for the icosahedron, they differ in the hierarchy. We can distinguish them clearly by examining whether the pattern of the shell surface consists of only outside modules or only inside modules.

This leads to the important conclusion that the icosahedron belongs to a more fundamental hierarchy than the dodecahedron. The $1f$ icosahedron is a 5-fold symmetrical polyhedron which can be composed of the minimum number of modules. However, the very center of the $1f$ icosahedron, or its nucleus can be thought of as a $0f$ (zero frequency) icosahedron. In this sense, a single point is an initial polyhedron which has a hierarchy for the icosahedron already in it.

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In our hierarchy, there are axes which radiate from the center toward the 12 vertices of the icosahedron, axes which radiate from the center toward the centers of the 20 faces of the icosahedron, and axes which radiate from the center toward the 30 edge midpoints of the icosahedron. Thus, there are 31 axes of 5-3-2 rotation symmetry in total. In the hierarchy of growth, any point (including the kernel) where the vertices of modules meet contains the 31 rotation axes. The bond angle of the edges of modules at each point is a central angle made by some combination of these 31 axes. At the maximum in our hierarchy, 18 directions are selected among the 62 radial directions.

Both the local non-periodicity and the synergetic hierarchy emerge from the angular divisions made by these axes at the nucleus. The asymmetrical combinations of modules are caused primarily by the symmetry of the kernel where three types of rotational axes can co-exist.

The concentric polyhedral shell structures grow symmetrically with respect to the 5-3-2 rotational axes from the nucleus. The growth of the arrangement by the *synergetic modules* is governed by the rule that the bond angles must match each other at any point no matter how non-periodic and asymmetrical these angles are.

A point must contain the system in order to make sure the combination of *synergetic modules* will grow radially. This leads to the idea that the point represents a complex of realistic substances with some properties, rather than the idea in traditional Euclidean geometry that a point has no parts in it.

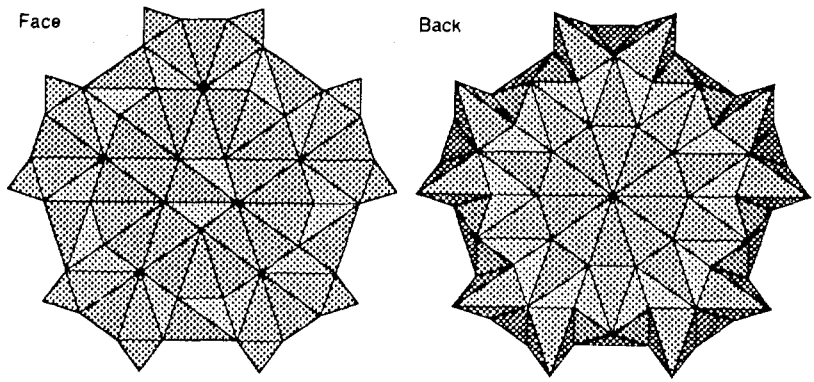

Figure 6: Tiling

In the concentric polyhedral hierarchy, the *synergetic modules* make possible such generalizations as 5-fold symmetry, periodicity of the hierarchy and radial growth. However, if we limit them to construct only a plane (not a mathematically rigorous plane since it has some thickness), we obtain non-periodic arrangements. This figure illustrates an arrangement composed of four kinds of inside modules E, F, G and H, which have a global orientation order and a global translation order. The patterns of the face (left) and back (right) are never identical. Any Penrose tiling pattern can be realized in this model. If we pile up symmetric layer on layer, and so on, we obtain a periodic structure with respect to the vertical axis. On the other hand, if we use the different modules A, B, C, and D, we obtain a layer with the same pattern but with a volume $2/\tau$ times as large as before.

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RADIAL GROWTH ON ROTATION AXIS

In the growing icosahedron, there are parallel layers expanding successively which are perpendicular to any 5-fold axis. The cross section perpendicular to the axis is always a regular pentagon. It is composed of outside modules and inside modules joined to each other. There are two kinds of thicknesses of the pentagonal layers which can be expressed in terms of the golden ratio τ if we assume the distance between the center of a $1f$ icosahedron and its surface equilateral triangles to be 1. Since the boundary between layers is also a boundary between shells, we can remove the layer from the hierarchy.

Parallel surfaces of regular pentagonal layers have intrinsically non-periodic patterns. These patterns are the result of the 3-dimensional combination of modules. Adjoining patterns are mirror images of each other. The combination of modules has 3-2 rotation symmetry. Furthermore, a pattern has either right-handedness or left-handedness. Each shell shows right-handedness or left-handedness alternately on its surface pattern, which is integrated into the inside surface of the next shell.

Perpendicular to the 3-fold axes, equilateral triangles expand out successively. These triangular layers cannot be removed since modules which compose the layers between two equilateral triangles are overlapping modules which compose the pentagonal layers perpendicular to the 5-fold axes. The thickness of a triangular layer is an integer multiple of the distance between the center of the $1f$ icosahedron and its face equilateral triangles. The patterns on these layer surfaces are periodic.

With respect to 2-fold axes, there is no module which has a surface perpendicular to these axes.

The ten types of *synergetic modules* can be classified into three classes by their height. Three kinds of layers appearing on the 5-fold and the 3-fold axes are truss structures of three thicknesses, each of which is composed of tetrahedral and octahedral modules of the same height. This is an economic and dynamically stable combination of modules.

SYNERGETICS TILING

I found a generalization which abstracts the method of module arrangement in the layers on the 5-fold axis in October 1989, and named it *Synergetics Tiling*. We can make tilings of two different thicknesses, which expand horizontally the layer of either the inside modules or the outside modules on the 5-fold axes. These tilings are non-periodic and do not have 5-fold symmetry.

The patterns appearing on both sides of these tilings can reproduce all of the non-periodic Penrose tilings. The patterns on both sides are never identical to each other. Moreover, one can arrange modules so that the pattern on the face is not periodic, while the pattern on the back has perfect 10-fold symmetry.

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If we pile up the layers of the *Synergetics Tiling* so as to make the adjoining patterns match each other, we have a periodic structure in the vertical direction which can fill the entire space. The reason why these modules can spread to infinity is the degree of freedom that exists when there is such a high level of possible combinations, 20 directions at maximum out of 62 radial total directions.

QUANTIZATION BY MODULES

Let the volume of the module I and E be 1. I found that the formula which tells the volume V of a growing icosahedron with frequency f is represented as:

$$V = 2 \times (2 \times 5) f^3$$

Although modules other than I and E have irrational volumes, they add up to an integer value when they compose an icosahedron, eventually canceling the irrationality of each other. Moreover, the volume of any shell structure is also an integer. Hence, the volumes of the shell structures are quantized by the *synergetic modules* and increase proportionally to the cube of the frequency.

Next, let f be the number of divisions or frequency of an edge, X be 1 for a regular tetrahedron, 2 for a regular octahedron, 5 for a cuboctahedron or an icosahedron. In 1960, R. Buckminster Fuller found that when a regular polyhedron is filled closely by spheres, the number of spheres on the faces, say N, can be represented as:

$$N = 2 \times f^2 + 2$$

He also found a similar formula for the number of points on the faces of an icosahedron which is divided into equilateral triangles.

Fuller's general formula also holds true for our hierarchical structural model of the icosahedron as follows:

$$N = 2 \times 5 f^2 + 2$$

where N means the number of points on the outer shell faces (points where edges of modules meet). That is, the number of points on the outer shell faces of an icosahedron is the square of the frequency times the particular prime number 5, multiplied by 2, and finally plus 2. Here we can notice another appearance of additive twoness.

Clusters of such rare gas atoms as argon and xenon are stable since they form icosahedral packing structures. Their magic numbers are said to be 13, 55, and 147. These values can be obtained by adding successively the numbers of the points of the $0f$ icosahedron, $1f$ icosahedron, $2f$ icosahedron, and $3f$ icosahedron, 1, 12, 42, 92, respectively. That is, 13=1+12, 55=13+42, and 147=55+92. The *synergetic modules* used here are only the tetrahedron I and the octahedron J. The filling has no internal dodecahedron structure but has perfect 5-fold symmetry.

This means that the icosahedron in a cluster form has no non-periodic combination and has a self-similar structure with perfect 5-fold symmetry. This geometrical

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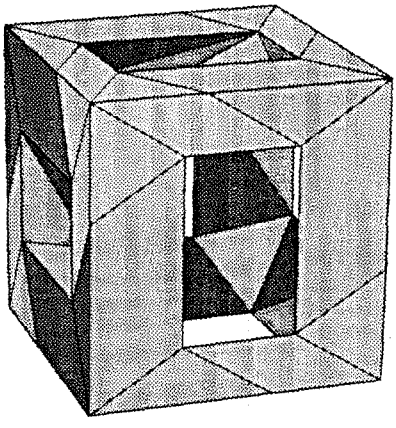
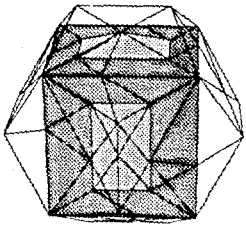
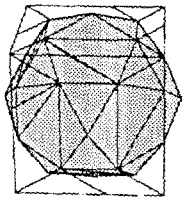
model is a good example to explain non-formative quantum leaps in physics. The fact that the magic numbers can be calculated this way, even though the edges of modules *I* and *J* are not equilateral, suggests that the closest packing by identical spheres is not an appropriate geometrical model for these clusters.

Synergetic modules with growth ability and interchangeability repeated in the hierarchical combination quantize the volume of 5-fold symmetric polyhedra. There exists a system which cannot be guessed only from the individual modules. Each *synergetic module* does not appear as a macroscopic shape in the hierarchy. In other words, *synergetic modules* form the structures and patterns as a whole by some system unpredictable from its parts.







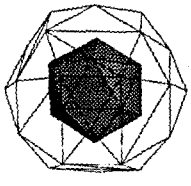


Figure 7: A regular hexahedron which inscribes a dodecahedron
We will ultimately obtain only tetrahedral modules if we divide the octahedral modules in the *synergetic modules* and decrease the degree of symmetry. If we use such 'minimum' tetrahedral modules, we can fill a regular hexahedron which inscribes a dodecahedron. In this case, modules are arranged symmetrically with respect to the kernel of the icosahedron which takes the regular hexahedral arrangement. Therefore, the 5-3-2 symmetry and the 4-3-2 symmetry can coexist in this space filling. In other words, if we use these tetrahedral modules, we can obtain both hierarchical systems of icosahedra and regular hexahedra. Note that even in this case, the golden ratio between volumes of modules is kept.

HIERARCHICAL MODEL OF RHOMBIC TRIACONTAHEDRON

My discovery of the hierarchical model for the rhombic triacontahedron was motivated by the discovery of a dodecahedron whose edge length is an integer multiple of 1.0 (the edge length of the 1*f* icosahedron) in the course of the combining of *synergetic modules*. Dodecahedra whose edge length is an integer multiple of 1.618 are necessary to grow icosahedra whose edge length is an integer multiple of 1.0, while dodecahedra whose edge length is an integer multiple of 1.0 are not needed at all.

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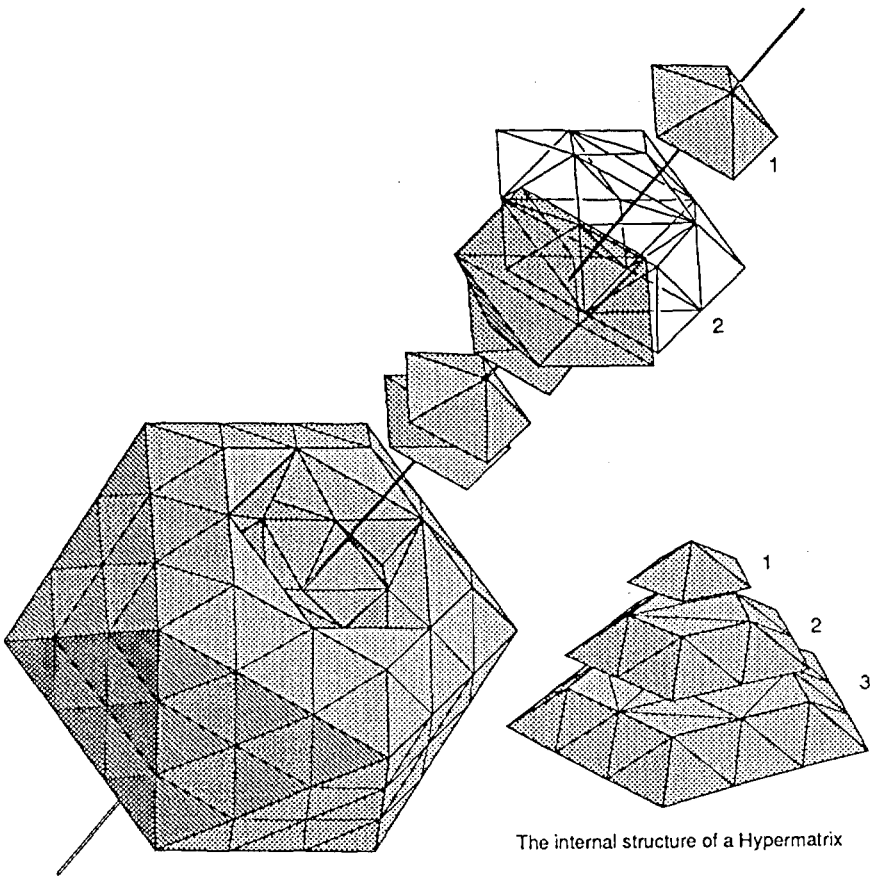

Figure 8: Hypermatrix

I call the wire-frame model of the hierarchy made by the *synergetic modules* the *hypermatrix*. The three kinds of vectors whose lengths are in the ratios 0.95, 1.0, & 1.618 and the angles of combinations of the 31 rotation axes make up a complicated shell structure system. On the 5-fold axes (one of them is shown in the figure) and the 3-fold axes, there are parallel and consecutive layers perpendicular to the axes. Each layer is a non-equilateral and non-periodic truss composed of tetrahedra and octahedra, which can be obtained at a small cost of energy and is dynamically stable. There must be a regular pentagon on the layers perpendicular to a 5-fold axis. The number 3 in the figure illustrates the layer which appears in the $4f$ icosahedron.

I thought that a dodecahedron whose edge length is an integer multiple of 1.0 belonged to another hierarchy and tried to find a rhombic triacontahedron which circumscribes this dodecahedron. I eventually succeeded in filling a rhombic triacontahedron that has 5-3-2 symmetry hierarchically with a 3-dimensional combination of irrational diagonals and non-integer angles as in the case of the icosahedron.

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In projective geometry, a rhombic triacontahedron can be constructed from the duality of the golden ratio between an icosahedron and a dodecahedron. However, it cannot be constructed in our hierarchical system although the icosahedron, dodecahedron, and rhombic triacontahedron with the same center have the same 5-3-2 symmetry. The reason is that a dodecahedron whose edge length is a multiple of 1.618 can inscribe an icosahedron whose edge length is a multiple of 1.0 while a dodecahedron whose edge length is a multiple of 1.0 can inscribe a rhombic triacontahedron whose edge length is a multiple of 0.95. Therefore dodecahedra, icosahedra and rhombic triacontahedra cannot constitute a single hierarchical structure. That is, icosahedra and rhombic triacontahedra constitute their own hierarchical structures, respectively.

On the other hand, these two hierarchical structures do share some first shells from the first $1f$ icosahedron with edge length 1.0 to the first dodecahedron with edge length 1.618. In the hierarchy of rhombic triacontahedra, however, only inside modules are used beyond the first dodecahedron.

Thus, if we call the $1f$ icosahedron the initial nucleus, the first dodecahedron common to the both hierarchies can be thought of as the second nucleus. Here emerges a hierarchy of nucleuses. The initial nucleus and the second nucleus are dual to each other. The duality in the hierarchy is based on the time axis of the frequency growth.

As the frequency of rhombic triacontahedra whose rhombic faces make a lattice of isosceles triangles increases, *synergetic modules* are able to completely fill them . Rhombic triacontahedra grow symmetrically, forming non-periodic parallel layers perpendicular to the radial direction from the center just as in icosahedra. The *synergetic modules* also maintain growth ability and interchangeability in this case.

The first rhombic triacontahedron appearing in this hierarchy is the $2f$ rhombic triacontahedron with edge lengths of 0.95. Before it appears, there appear a $2f$ dodecahedron with edge length 1.0, a $1f$ truncated rhombic hecatoicosahedron with edge length 1.618 and a $1f$ dodecahedron with edge length 1.618 which is common to the hierarchy of icosahedra. They all have 5-3-2 symmetry.

Beyond the $2f$ rhombic triacontahedron, similar shell patterns to those which have appeared so far are replicated every second time. That is, the period of the hierarchy is 2.

A rhombic triakisicosahedron inscribes the $1f$ dodecahedron with edge length 1.618. A rhombic icosahedron adjoins to each of its 12 cavities in the hierarchy of rhombic triacontahedra. A $1f$ rhombic triacontahedron can be formed by sharing structural parts of these two kinds of polyhedra.

Choose one 5-fold axis, then we can find a $1f$ rhombic triacontahedron whose center is on the axis and which contacts both the center and one of the vertices of the $2f$ rhombic triacontahedron. This $1f$ rhombic triacontahedron does not share the center with other shell structures, and its diameter is just half that of the $2f$ rhombic triacontahedron.

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Hence we have now two $1f$ rhombic triacontahedra which contact each other at the center of the hierarchy.

Rhombic triakisicosahedra, rhombic triacontahedra, and rhombic icosahedra can be composed of A_6 and O_6. However, there appear parallel layers with non-periodic patterns successively in our hierarchical system of *synergetic modules* unlike the construction by A_6 and O_6. There are 7 layers in the $1f$ rhombic triacontahedron and 14 layers in the $2f$ rhombic triacontahedron.

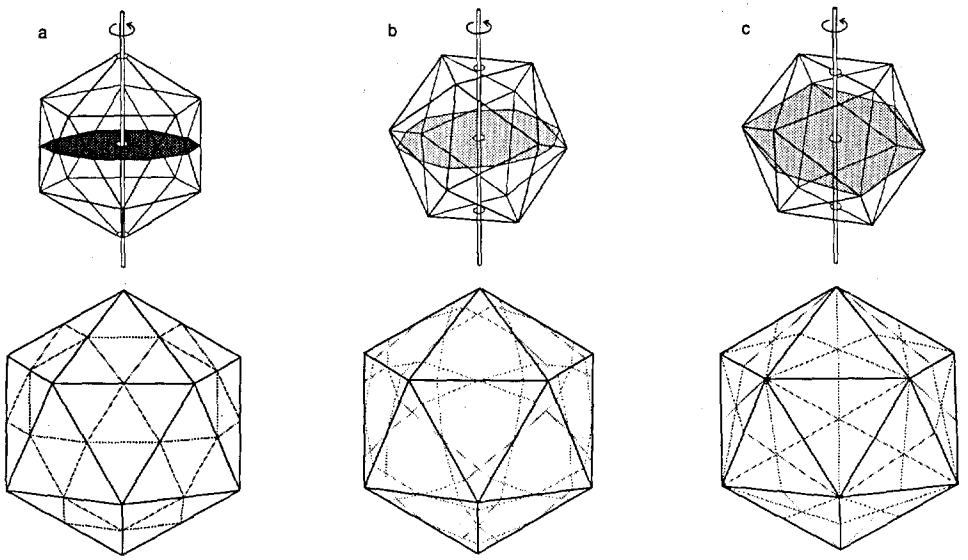

Figure 9: Rotation symmetry axes of the icosahedron
Axes which link two antipodal vertices are 5-fold axes (a). Since there are 12 vertices, there are six 5-fold axes. There are fifteen 2-fold axes, each of which links two antipodal midpoints of the edges (c). There are ten 3-fold axes, each of which links two antipodal centers of the faces (b). Hence we have thirty-one 5-3-2 symmetry axes in total. We call the cross section which cuts the icosahedron into two identical pieces at a plane perpendicular to a rotational axis a great circle. There are 31 great circles since there are 6 great circles (a) on 5-fold axes, 10 (b) on 3-fold axes, and 15 (c) on 2-fold axes. The 8 angles which appear in the *synergetic modules* are the central angles between these great circles: 31.717 degrees, 36 degrees, 58.283 degrees, 60 degrees, 63.435 degrees, 72 degrees, 108 degrees, and 116.565 degrees.

I found two new polyhedra in the growth process from a $1f$ to a $2f$ rhombic triacontahedron. One is the rhombic hecatoicosahedron composed of 120 identical rhombuses. This polyhedron can be obtained by joining 12 rhombic icosahedra around a rhombic triakisicosahedron. The other is a truncated rhombic triacontahedron composed of 12 regular pentagons and 30 regular hexagons.

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There is another hierarchy in which a Kepler's small stellated dodecahedron can be obtained by joining 12 pentagonal pyramids composed of modules *F* and *E* to a 2*f* rhombic triacontahedron. In the course of filling the vacant space among these pyramids, there appears Kepler's great dodecahedron. If we go further to fill the triangular pyramidal cavities in the great dodecahedron, we get another big rhombic triacontahedron.

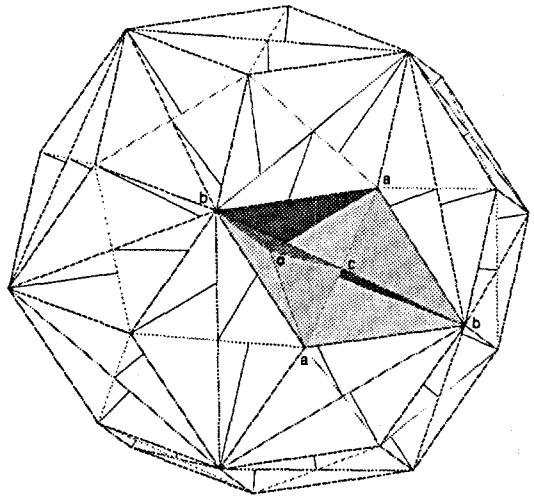

Figure 10: 5-3-2 Rotational symmetry axes
A rhombic triacontahedron can be divided into 120 identical tetrahedra (a-b-c-o in the figure). Once this division is done, we can integrate polyhedra with 5-3-2 rotation symmetry simultaneously. If we focus our attention on the rhombuses on the surface, we find a rhombic triacontahedron. If we focus our attention on the longer diagonals in the rhombuses, we find an icosahedron. If we focus our attention on the shorter diagonals in the rhombuses, we find a dodecahedron. In other words, 62 radial lines from the center amount to 31 rotational symmetry axes which are common to the rhombic triacontahedron, icosahedron and dodecahedron. All the combinations of edges at lattice points in the *hypermatrix* can be represented by the combination of the central angles made by these 31 rotational symmetry axes.

The possibility of radial symmetrical combinations of modules increases and 5-fold symmetry can be reproduced more abundantly as the frequency increases. Since only inside modules are used in the growth process, the volumes of polyhedra appearing in the hierarchy of rhombic triacontahedra are not integer valued except for the first few. Instead, they are intrinsically related to the golden ratio.

The formulas I discovered with respect to the hierarchy of rhombic triacontahedra are as follows :

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$$N = (2 \times 3 \times 5) f^2 + 2$$

$$V = (1+\sqrt{5}) \times (2 \times 3 \times 5) f^3$$

where N, V, & f mean the number of vertices, the volume and the frequency (which has to be even), respectively.

From another viewpoint, a rhombic triacontahedron is obtained by adding 12 pentagonal pyramids to a dodecahedron. Hence we can calculate the volume of a dodecahedron whose edge length is an integer multiple of 1.0 by the formula:

$$V = (1+\sqrt{5}) \times (2 \times 3 \times 5) f^3 - 12\sqrt{5} f^3$$

The volume of each pentagonal pyramid is $\sqrt{5}f^3$.

HYPERMATRIX

Synergetic modules can fill the two Platonic regular polyhedra, the dodecahedron, and the icosahedron, which were not filled by the unit cells in traditional crystallography, but they can also fill the rhombic triacontahedron, which was a basic element in traditional crystallography. This suggests that the hierarchies of icosahedra and rhombic triacontahedra bring in new lattices with different characteristics from those space lattices in traditional crystallography.

The space lattices which explain crystal structures in traditional crystallography are composed of identical unit lattices which are arranged periodically, each of which has the same peripheral arrangement. Each point can be obtained by translation and all points are equivalent. The form of the basic unit cell which determines the crystal can vary indefinitely.

The lattices formed by the hierarchy made from *synergetic modules* form a kind of wire frame of the corresponding shell structures. Unlike the lattices so far, they do not spread infinitely, but form a closed space within each shell structure. This growth limit results from 5-fold symmetry.

The lattices growing along their rotational symmetrical axes in the hierarchies of icosahedra and rhombic triacontahedra are called the *hypermatrix*.

The ratios of lengths for the three vectors in the *hypermatrix* are 0.95, 1.0, & 1.618, lengths which exist inherently in the icosahedron. The space group *hypermatrix* for concentric polyhedra is formed by these ratios and the 31 rotational axes; again they are the 12 radial directions toward the icosahedron vertices, 20 radial directions toward the icosahedron face centers, and 30 radial directions toward the icosahedron edge centers.

Any point has 5-3-2 symmetry and thereby has the possibility of being a nucleus. However, one of the points is selected as the nucleus and lattice points whose peripheral arrangement are mutually different are obtained by symmetrical radiation of the ten types of *synergetic modules*.

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As described above, there are two hierarchies beyond the 1*f* dodecahedron with edge length 1.618. However, the concepts of frequency, 5-3-2 symmetry, shell filling, quantization, 31 rotation axes, complementariness of tetrahedral and octahedral modules, non-periodicity, periodicity of the hierarchy, face and back, polarity, right-handedness and left-handedness, and radial growth are independent on the scale of the *hypermatrix*.

Five-fold symmetry is one of the generalized characteristics of the *hypermatrix* integrated radially in the concentric hierarchy.

FORMS AND MODELS

In 1985, L. Pauling pointed out that the quasicrystal alloy $Al_{86}Mn_{14}$ with 5-fold symmetry does not match any Bravais lattice and has a twin structure of more than one regular hexahedra. In 1986, T. Rajasekharan reported that the 5-3-2 symmetric units of the quasicrystal alloy $Mg_3(Al, Zn)_{49}$ are filled in the body-centered cubic lattice and that quasicrystals of Al Cu Fe family form the face-centered cubic lattice.

A. P. Tsai and others confirmed in their experiments that if three metal elements can be fused into a metal alloy by the liquid quenching method, the ratio of the radii of the solvent atom and the solute atom is 1 to between 0.85 and 0.95. The other ratio 1.618 can be detected as the distance between atoms by electron diffraction image.

This suggests an important analogy between the forms of quasicrystals and the formation and growth of the *synergetic modules*.

Each octahedron in the *synergetic modules* can be further divided symmetrically into two tetrahedra. Thus, we have now only tetrahedral modules. Therefore, these tetrahedral modules can be thought of as the minimum modules with the highest interchangeability which can be derived from the icosahedron. By using these minimum tetrahedral modules, we can obtain a shell in the shape of a regular hexahedron which has no 5-fold symmetry in the hierarchy of icosahedra. This is a regular hexahedron with edge length 2.618 which inscribes the 1*f* dodecahedron with edge length 1.618. Here, the 4-fold axis of the hexahedron and 2-fold axis of the icosahedron coincide with each other, as well as the 3-fold axis of the hexahedron and the 3-fold axis of the icosahedron.

The fact that some of the 31 rotational axes of the icosahedron and the rotational axes of the hexahedron are common in the *hypermatrix* suggests that the 4-3-2 symmetry of the hexahedron and the 5-3-2 symmetry of the icosahedron are fused physically.

Moreover, the fact that there are 5-fold symmetric shells inside the hexahedron (in fact, there is an icosahedron and an icosidodecahedron inside) suggests that the discrimination between crystals and quasicrystals by the concept of periodicity is not essential. This fact seems to fade out the contrast between quasicrystals and crystals smoothly and naturally.

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Form is the spatial arrangement of constituents in a material. Positions of individual atoms have come to be detectable by the rapid development of electron diffraction and X-ray diffraction technology. However, the structural patterns behind the form appear to be more important than the information of individual constituents.

In other words, the arrangement of the modules is far more important than the forms of the modules themselves.

The *hypermatrix* displays the internal interaction between real substances and concepts, but is not an illustrative reproduction of the spatial arrangement of the constituents of real materials. Hierarchical models made with the *synergetic modules* do not require that there really exist ten new types of unit cells corresponding to the modules. Instead, it is a visualization of a closed abstract system of relations between atoms.

The *hypermatrix* woven by the angles of the three kinds of rotational axes of the icosahedron and the three kinds of vectors of the *synergetic modules* represents a symmetry with very high structural stability, as in nature, where all things are structured in triangles.

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A REPORT ON PROGRESS TOWARD AN ISIS-SYMMETRY INTERTAXONOMY

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INTRODUCTION

Among the first tasks of a new professional society is to define what it is all about. This must be focused at a level of detail to which each individual member-worker can relate. The definition of the work of the society is especially important in the interdisciplinary case, where the membership shares no common disciplinary vocabulary. In this case, a common skeleton must exist or be created, one which allows members to relate their work to one another. In fact, the very reason for forming ISIS-Symmetry (ISIS-S) was to take advantage of the wonderfully symbiotic contributions from work from disparate areas related through *symmetry*.

The first order in providing this skeleton for interdisciplinary sharing is the creation (and support) of a taxonomy for the society[*1]. At minimum, such a taxonomy will be useful for indexing papers, works, etc. in a bibliographic sense. It may well also provide a basis for a common vocabulary to examine the relationships among different individual disciplines. In that case, interdisciplinary value would be added to the dialogue about and the rationale for ISIS-S . The benefits of participation in ISIS-S would be greatly increased. New perspectives may emerge.

The need for such a taxonomy was raised in Budapest in 1989. 'A project area' was proposed, and a board position established related to the scientific questions surrounding the problem. The project was announced in the Journal (Goranson, 1990).

Some parallel efforts are underway in a similar international, interdisciplinary forum, so some initial thinking has been done. These initial thoughts were presented in Hiroshima as a proposed starting point for the project (Goranson, 1992). They are summarized in this report.

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NATURE OF TAXONOMIES

Normally, a taxonomy for a discipline is not so much a project involving ideas as it is an accounting process. This is so because each individual discipline develops a fundamental set of concepts, uses a language (including shorthands, often mathematical) to express those concepts, and adopts certain elements of a worldview. In such a case, taxonomic controversies are few as the system evolves from the same consensus that forms the basis of the discipline.

Such taxonomies are normally simple in structure for three reasons:
– no formal process was considered in their development;
– they reflect the historical development of categories, for example, a subdiscipline is classed under a 'parent' discipline if its history derived from that parent.
– the underlying 'world view' is sufficiently robust to allow the taxonomy itself to be weak*2.

For these reasons, the simplest taxonomic structure is also the most common. Everywhere one turns, one finds hierarchical taxonomic structures. In textual form, these are 'indented' lists, often using tabs and/or some labeling scheme to show which is the parent entry and which is the child. In graphical form, these take the form of 'tree' structures which more clearly show levels and dominance. The indexing system used by Mathematical Reviews (which indexes this journal) is of the indented type, shown here generically in tree form.

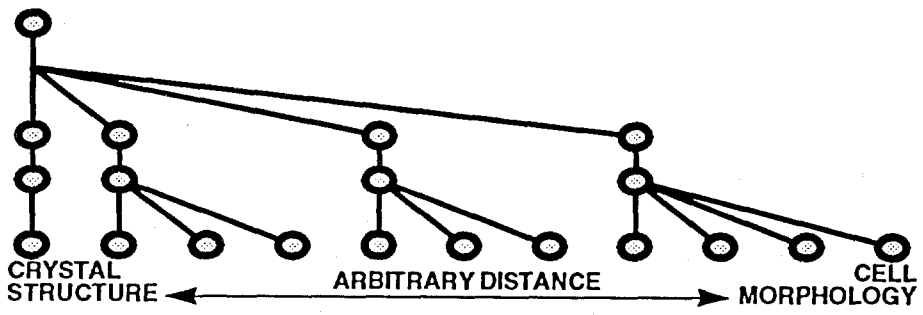

Figure 1

When one computerizes indexes and their taxonomies, the mechanics must be made more formal if they are not already sufficiently so. For example, the need to digitally store hierarchical indices has spawned a set of formal mechanisms of the 'indentured' type (similar to 'indented'). In this family, specific rules are developed as to what properties (or components of the 'world view') are inherited by the child, when the influence stops, how deep it goes and so on.

Such formal considerations are made necessary by the recent progress by the information science community into taxonomic considerations. As a result, new insights about how concepts are represented have emerged. These principles can be

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loosely considered as a new science of taxonomies, or more properly: 'Category Theory'. (See Asperti, 1991 for an introduction). The taxonomic principles depend heavily on concepts of symmetry and incidentally may be of significant utility in helping ISIS-S members understand one another.

THE SPECIAL NEEDS OF ISIS-S

ISIS-S is an interesting case. The disciplines represented within the society do not share a common parent science (at least not in historical times). There is no default consensus world view or shared language. The opposite is the case; maintaining the diversity of worldviews is the point. But this places the burden of providing a lowest common denominator for intercourse on the taxonomy.

A hierarchical breakdown of disciplines will not do for ISIS-S . Any one researcher may feel comfortable breaking disciplines into certain parents and children. But the distance between any two entries is somewhat arbitrary and the result will be seen as unacceptable to others. Figure 1 shows an example. It may be common practice to classify some crystallographic work deep within the physical sciences and similarly to place an example activity of cell morphology under the life sciences. Yet the analytical tools of certain workers may be much closer than the distance infers.

Is there a taxonomic methodology which would capture the similarity of the two disciplines? Could it indicate the linkages from individual workers which may share a common theoretical basis while maintaining differing applications? Could the many linkages contained therein be united under one common set of principles? Could those principles be the principles of symmetry? Finally, would it be possible for future researchers (or philosophers, theoreticians, indeed artists) to glean new insights into the fundamental (i.e. symmetric) principles which underlie the taxonomy?

ISIS-S CAN LEVERAGE NEW RESULTS

I believe that all of these questions can be answered in the affirmative. My confidence comes from taxonomic studies sponsored under international aegis on a similar problem. (Petrie, 1992, especially in the nine contributions of Goranson, define the problem and project.) In that case, the problem deals with the problem of common indexing, via computer, of the thousands of disciplines which must be coordinated in a large, complex industrial enterprise. Examples are semiconductor or aerospace enterprises.

This interdisciplinary work is well resourced and involves many bright minds and institutions. New results have emerged, some in fundamental new practical techniques. It may be no surprise to ISIS-S that a fundamental principle of such efforts involves symmetry. It appears that those techniques are applicable to the ISIS-S problem, resulting in a remarkable proposal: to use principles *of* symmetry to index work related *to* symmetry.

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In particular, three techniques from this project can be put into service for the ISIS-S taxonomy. Each of these is briefly discussed below.

FIRST TECHNIQUE: A FEDERATED TAXONOMY

A guiding principle of the ISIS-S taxonomy project is the eschewing of creating yet another discipline, with its unique taxonomy. Instead, the ISIS-S taxonomy should allow the taxonomies of each individual discipline to be eminent. In such a case, the ISIS-S taxonomy would *federate* the taxonomies of the individual disciplines involved.

'Federation' is understood here as a special kind of synthesis, where the individual sources are preserved and are unconstrained. The federation mechanism is a unifier which allows concepts to be expressed in a neutral form. The most useful characteristic of a fully federated index is that each user can fully express the whole index in his/her own system.

As an example, our previously mentioned crystallographer has a set of indices and related tools to express the world; the molecular biologist another set, perhaps quite different. An unacceptable approach to enhancing the interdisciplinary dialogue is to force each to use a third, 'lowest common denominator' system.

A much better way is to provide a federating mechanism which maps or transforms perspectives, even concepts from one system into another. In this case, each individual uses the worldviews which are natural and the tools which are efficacious to the discipline.

Federated systems are much more challenging to create than traditional trees. It appears that two underlying mechanisms are required: A shared, higher unifying principle and a 'technology'[3] to support the transformation process of one system to another. The ISIS-S project has the former in *SYMMETRY*, and the latter in the condition of proliferating personal computing.

The first use for the federating, or *inter*taxonomy will probably be to index journal contributions. Nearly all of the disciplines involved in ISIS-S which have such indices, have automated tools for reference management. So the 'source' indices are already to a large extent computerized.

It is also presumably the case that most persons likely to find an intertaxonomy useful will have access to a computer. It is the in the nature of the society to be distributed globally. One would expect electronic media, such as the email journal, to be more preferred for many ISIS-S members.

SECOND TECHNIQUE: METASTRUCTURE

Methods of *abstraction* are fundamental to understanding. When one abstracts according to a structured system, he/she can reason in the abstraction in order to gain insights into the primary situation. It appears that all useful abstract systems

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have an internal structure. One could term this a 'physics', a 'grammar' or a 'mathematics', depending on one's background. The challenge of *federation* therefore depends on discovering an underlying, global structure of these individual structures.

This structure of structure, or *METASTRUCTURE*, would form the basis of the federation mechanism of an intertaxonomy, so is of especial interest. A simple example of metastructure is indicated in the figure 3. A (US) national laboratory has crystallographers and molecular biologists collaborating on a project. They were in need of common computerized models that both could use.

Among the problems encountered were profound differences in as fundamental a concept as the periodic table of elements. Each discipline had specific properties of the periodicity which were emphasized. But this difference in emphasis was sufficiently substantial to make the models distinct in structure. Resulting models from the system could not be related and the collaboration was threatened. Figure 2 schematically shows the two views[*4]. Note the fundamental differences in structure.

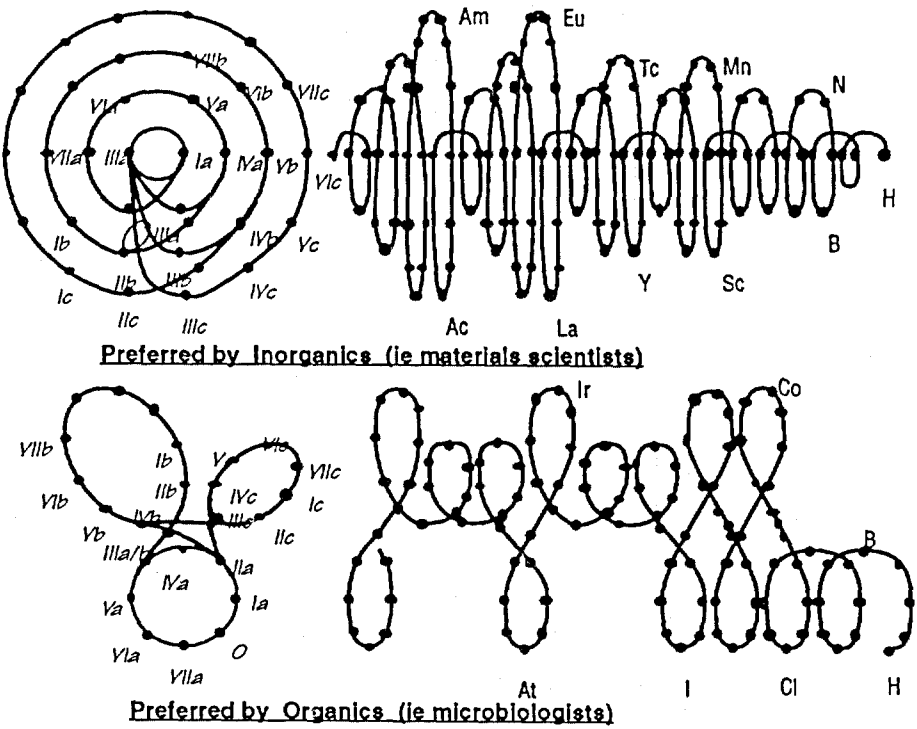

Figure 2: Two periodic tables.

There were also many other discipline-specific twists in this area, each peculiar to a different group of researchers and resulting in a 'different' periodic table. The solution was to devise a structure of these structures. (The method used is indicated

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by the number, types and relationships of the 'loops' shown on the left.) In a sense, this metastructure was a periodic table of periodic tables. At the same time, it contained all of the information of the various views.

Symmetry appears to be the only practical mechanism for creating and understanding these metastructures. (von Fraassen, 1989 is representative of the best presentations in this vein.) However, in considerations of symmetry, the application of metastructure is more straightforward than in the general case of metaphysics. This is because the *form* of the sources emphasizes the symmetric principles which can be leveraged for abstracting to a higher structural level. The most accessible use of this principle is Haresh Lalvani's work where polyhedral structures are used as an organizing mechanism for *types* of polyhedral forms. (Lalvani, 1982 and 1992 are representative of his work in this area.)

THIRD PRINCIPLE: LATTICES

The final basic technique proposed for use in the taxonomy project is the *CONCEPT LATTICE*. The idea originated in the set and group theoretical branch of mathematics (Wille, 1987); formally speaking, the technique defines the structure of relationships which themselves can be structures of relationships. As such, it can provide a theoretical foundation for a metastructure to federate individual taxonomies.

But the technique is useful for another reason: apart from the mathematics, it provides an intuitively accessible way of visualizing relationships, much like the 'tree' diagram of Figure 1, or the richer type of Figure 2. The approach has special power when the symmetry of the lattice is regulated. If this is done, all of the important properties of the lattice can be described *and easily seen* in terms of symmetry characteristics.

Figure 3 shows a simple concept lattice as an illustrative example. This type of lattice is directional, though many other, more powerful, types exist. Each of the four directions imparts a property, as shown by the four arrows. The lattice shows the same relationships as the table shown in Figure 4, a simple characterization of ten (fictitious) source taxonomies.

The ten rows of the table denote ten example disciplines. Row one might be a library discipline. The note indicates that this may be an ISIS-S contribution related to the properties of 'classifying classification'. This paper is an example. Row #7 is noted as a contribution on the symmetric properties of stained glass design (a personal interest of this reporter).

The first three columns show whether a property is held by one of the ten or not. A marker indicates yes; for example the study of stained glass work (row #7) emphasizes the use of intuitive methods, whereas row #6 does not. (It may be a more pedantic discipline or contribution.) The fourth column is more sophisticated. It shows, in crude sense, degree. An entry of 1 indicates some degree of the property – 2 shows twice that degree. So that the taxonomy paper (row #1) emphasizes intuitive access to the internal mechanics of the work to twice the extent that the fictitious contribution of row #7.

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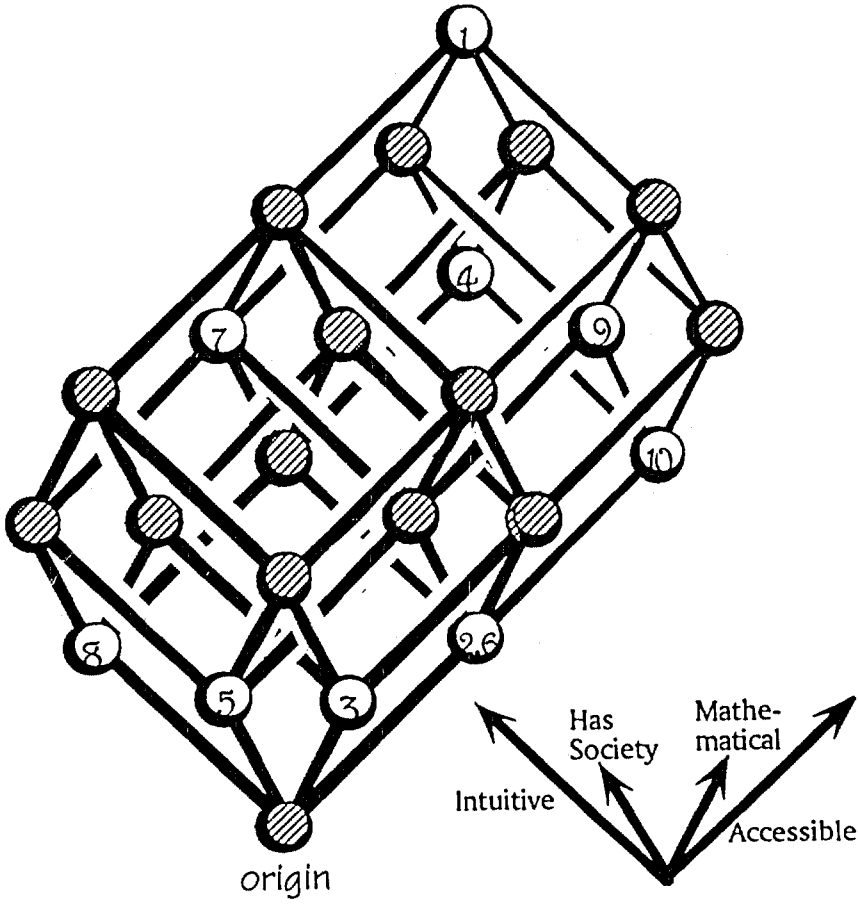

Figure 3: A simple concept lattice.

Turning to the lattice, one begins at the bottom. Moving in a direction bestows the property of that direction (shown by the four 'arrows'). So moving one unit in the direction of 1:00 denotes the property of 'using mathematical techniques', the same as the third column in the table. An entry of '3' at this node, shows that row #3 has that property and no other. Note that the notion of two levels of degree from column four shows in the lattice as a doubling of the basic cell.

In this simple example, the lattice appears to be a more complex system than the table. But in meaningful situations, the number of rows and columns is very large. And the table would require many dimensions. For example, there is a relationship (in our example) between whether the discipline *uses* intuitive methods (as in designing stained glass), and whether the discipline (as in understanding stained glass effects) emphasizes intuitive access. As a result, tables would have to be many dimensioned with additional tables to show column-to-column relationships.

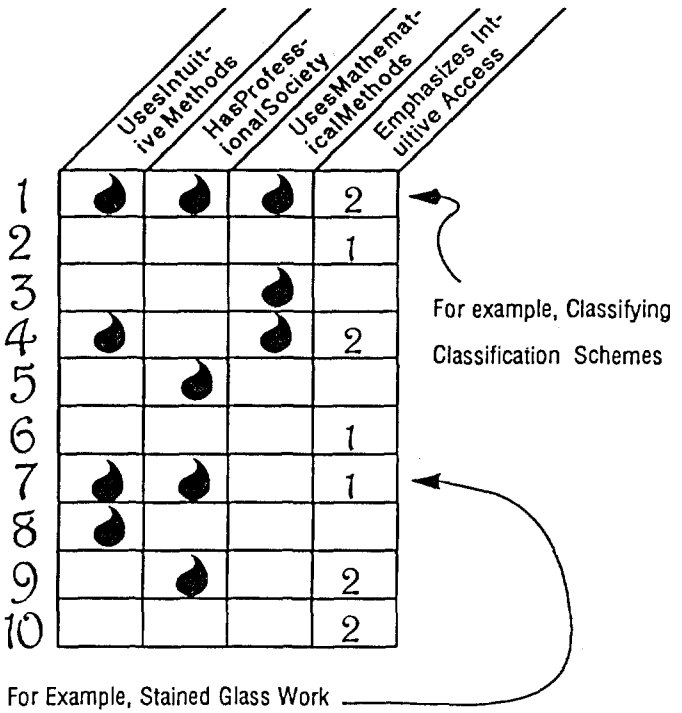

Figure 4: A table view of the lattice relationships.

As the situation represented grows more complex, the lattice does not, and therein lies its power. Every entry and property can be formally characterized by a few, fundamental symmetry properties. It is also the case that tree representations, tables and other conventional representations can be extracted from the lattice. In a practical sense, this means that the stained glass artist could have the entire lattice represented in whatever simple tree structure he or she is used to. This satisfies the requirement for federation.

Lattices used by Sirius-Beta for similar representation applications use techniques not shown in the example: the lattices are infinite and have multiple dimensions (internal to the machine); the lattices are dynamically linked among differing symmetry types; and new 'intersymmetries' are extracted for special shortcuts.

NEXT ACTIONS

These three techniques are proposed as a beginning dialogue toward an ISIS-S intertaxonomy. It is hoped that reaction to the proposal will prompt some response, and initiate the project as an activity within ISIS-S . In a future report, we will provide an example drawn from recent ISIS-S contributions.

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REMARKS

*1 Another required task is to coherently recount the intellectual history behind the shared ideas. Note Nagy careful surveys (Nagy , 1990).

*2 Another perspective is that the formally considered elements of the taxonomy are kept vital in the world view, and the discipline's indexing systems is a mere artifact (as are the papers, etc.). In either case, attention is not paid to the explicit formal basis of the taxanomic system.

*3 The term is used here in the most general sense to include: spoken language, the printed word, diagrams and mathematical tools as well as more conventionally described technologies.

*4 The schematic representations are derived from Mazurs (1957), a study in alternative representations of the periodic table.

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EXTENDED ABSTRACTS

SYMMETRY OF PATTERNS

Part 2

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FORM: BY SCRIPT AND/OR IMAGE

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Preface

I am an artist who uses the computer as a tool or environment for expression. In preparation for creating with a computer, one must devise an order consisting of commands and data. What results are algorithms which are formed by the ideas and procedure involved, expressed in explicit detail. Combined, they comprise a script for creating graphic images. Because scripts require so much thinking about the steps involved, I have been prompted to rethink the process of creation.

Only the objects which can be translated into numbers are visible on the screen. At that moment, the image and its script are placed as a composition of symmetry.

1: Image and Script are in Symmetry

Since I first encountered computer graphics techniques in the early 80's, the prevailing interface with the computer has been the "command line interface." This approach requires a "command" (function) and a "target" (parameter) to define the purposes of the action. Both components are crafted into a textual script, which in the case of 3-dimensional images, defines the image. The user, the artist, must deal with numbers and commands in the context of such a script. Clarifying the image necessarily involves manipulating the script. If any part of the script changes, the resultant image will change.

My day-to-day experience with this process suggests that the script and the image are standing on alternative sides of a mirror. But the reflection is one-way only,from the script to the image.

(In the future the reflection may be more two-way, allowing more interesting metaphors of the world in the way the user sees the "script." Interactivity is may be a key to open two-way reflection. Already most 2- dimensional software allows the artist to interface with and manipulate the image interactively, directly on the screen. In fact the script is still there, but is hidden from the user for convenience. The numbers are still acting and running inside the computer.)

2: The Computer is a Type of Modeler

Computer programs are formed from our sequences of thinking and acting. The computer acts by following and tracing our process of thoughts through manipulat- ing programs and data. For example, algorithms included in computer-graphics programs are transformed from Newtonian models of physics, optics and geometry. It is a simulation of the model of our viewing-system in the 19th Century's style.

We can use computer technology as a modeler, and we can model a part of our world inside the computer. Then we can see our modeled world from the outside, in the image. The reflection of the world in the model re-attracts me to the notion of a symmetrical composition. It is important that this notion comes directly from daily experience with this process.

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3: The Pre-formative stage

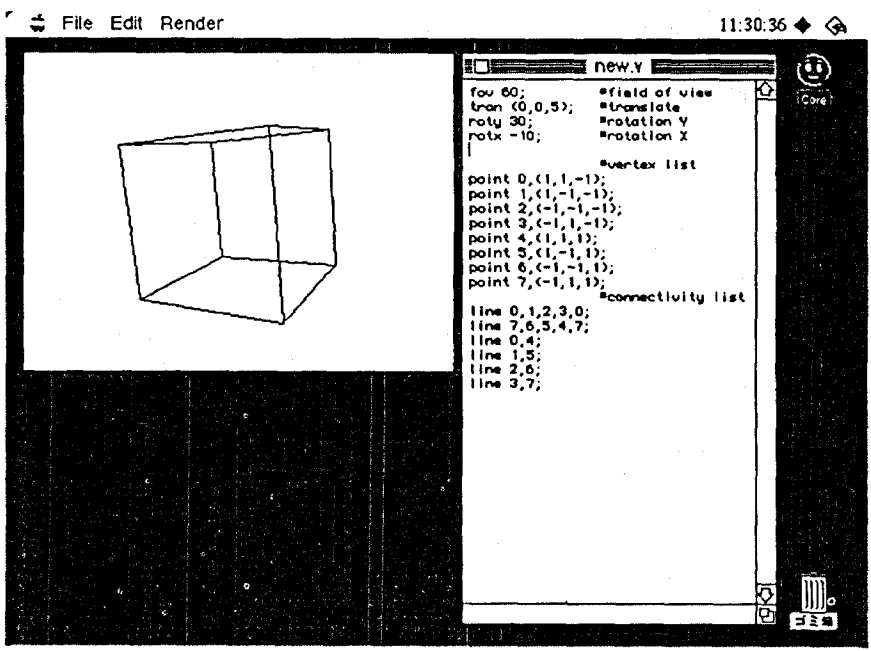
A fascinating element of the symmetrical reflection arises when one considers that the image produced is not physically realized and is an illusion. The "original" image exists only in the mind of the artist and needs to be transformed into a form in order to be expressed. Here, the real creation involves both the power of realization techniques and the power of imagination.

The poet uses words as elements for creating a poem to express his "form". When we read a good poem, we get the form in our minds. The painter uses pigments as elements, canvas and brush as tools for fixing an image of his "form". This painted 2- dimensional image projects the form into the viewer's mind. The viewer can understand the "form" and communicate his impressions, perhaps into words.

The point is simple that we, as expression-ists need to make the objects from our imagination.

4: A Conclusion?

Scripts for computer image generation and the final image are the same object. For the human, image is intuitively understandable, but scripts for computers are not. In the case of the computer, scripts are a poem forming the "form." Here we are standing in a strange
place. The modeling power of the computer gives us the ability to simulate on screen this strange environment. Thus, the reflection between scripts and images brings us to a place of "self-reflective-symmetry."



Bitmap dump from Macintosh DeskTop screen.
The image in the left window was drawn from the script in the right window.
This software was programmed by the auther, for the purpose of demonstration.

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POLYHEDRAL MODELING AS A PREPARATION FOR THE CREATION OF VISUAL MUSIC

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As used here the term 'visual music' will be used to denote a sequence of images that transform from one to another with the passage of time. It may or may not be accompanied by conventional aural music and the performing musician may be enjoying aural or visual feedback, or both from his essentially kinesthetic input. Computer graphics is the medium that currently could facilitate its unfoldment. To enable this to take place in real time, the system of image generation will need to be user friendly to the performer, and economical in the information used to describe aesthetically acceptable images in order to be affordable. What is required is an organizing principle comparable to the function of scale selection in aural music. Useful scales are in fact subsets of all possible frequencies, as selected by particular organizing principles. The imagery that would appear most suitable from the twin viewpoints of simplicity, and efficiency of generation, could be selected from those shapes that are both highly symmetrical, and not removed by too many steps from simplicity itself, as represented by the undifferentiated sphere and its most symmetrical progeny, the Platonic solids, followed by Archimedeans and their duals.

These thirty-one shapes together with the prisms and antiprisms could be extended by the addition of the ninety-two convex polyhedra made up of regular polygons, and the uniform polyhedra, and the possibility of further additions from the transpolyhedra of included shapes. Another group of shapes worthy of inclusion could be the holohedrally symmetrical convex polyhedra of constant edge length. These retain 2, 3, 4 or 5 fold symmetry at their familiar positions but as 4, 6, 8 or 10; or 6, 9, 12 or 15; or 8, 12, 16 or 20 sided polygons, do not necessarily have to be regular polygons. many of these polygons can be illustrated or constructed at the workshop, but I do not yet know how many polyhedra qualify for this category. They can be broken down to subgroups of polyhedra sharing the same kind of faces which can have different shapes depending on where they meet each other. Such members of subgroup are all connected to each other through an operation I term *edge insertion* or *edge subtraction.*

When we take all the members of the families of shapes referred to above, the vertices, and points where faces are tangential to a sphere, consist of a surprisingly small number. It should be possible to compute them all, and with the help of a simple color coding system ascribe a unique name to each in a language and vocabulary with which we are already familiar. Furthermore the name itself would immediately designate the approximate locality that we should expect to locate the point, whether it represented a vertex, the center of a face, or a mid-edge. We can

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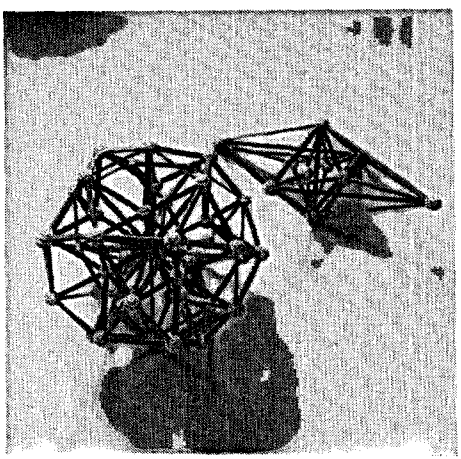
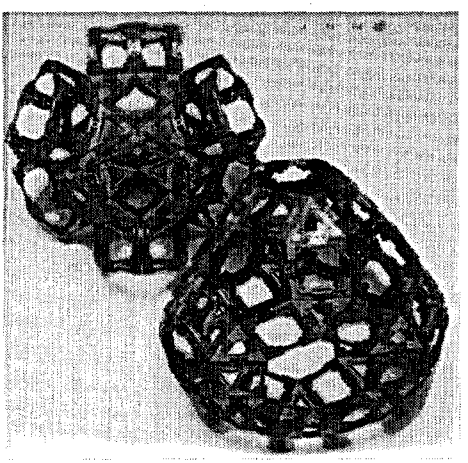
accomplish this by locating these, landmark as it were, within the context of what Buckminster Fuller describes as the lowest common denominator triangles of the tetrahedron, octahedron and icosahedron. A variety of coordinate systems could accomplish this including Cartesian coordinates applied to axes through the center point of a rhombus (square in the case of the tetrahedron), polar coordinates from the same point, or the specification of the two angles. By ascribing a primary color to each vertex of the L.C.D. triangle, we can then assign a unique mix of primary colors to each point within the L.C.D. triangle. At this point we can adapt our terminology from the available names used to describe shades of the color spectrum. The perimeter of the triangle would consist of the colors of the spectrum, while all points within the triangle would have some of all three colors and consequently a component of white. What is proposed is that we designate two fold axes in blue. three fold axes in green, and any third kind of axis whether 3, 4, or 5 fold in red. This largely corresponds to existing usage by two of the major manufacturers of polyhedral models, Googolplex and BioCrystal, with the variation that green would need to substitute for yellow to be compatible with R.G.B. system suitable for computers. Perhaps if this symposium could endorse these proposals, it might encourage manufacturers to make available the necessary components.

Colors would apply to directions parallel to radii emanating from the center of an object, and also to the planes of the great circles derived from such radii. Points on the surface of a sphere would derive their color from the intercepting radius, and would extend their influence to planes tangential to such points. This is going to create substantially different appearances between the a face or plane defined system, such as Googolplex, and an edge and vertex defined system such as BioCrystal whose node system makes available only three types of edge direction, but whose nodes could occupy an infinite variety of directions relative to the center of a symmetrical polyhedron. Each of these directions within the L.C.D. triangle could command its own color, although in practice yellow, magenta, cyan, and white would allow us to designate the vertices of the Archimedeans with the exception of the snub polyhedra.





Models by John Gibbon

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USING ASYMMETRICAL PROBABILITY DENSITY FUNCTIONS (PDFS) IN SIMULATIONS

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Simulations, even where valid models exist, are frequently limited by the availability of good data. This is particularly true in an area of current global concern, namely, models for forecasting the future incidence of HIV/AIDS.

Epidemiological models and their corresponding computer simulations can be used to determine the effects of varying assumptions concerning the mechanisms for the spread and containment of the AIDS pandemic. Populations susceptible to AIDS, defined as sub-populations of the general population, may be considered to be distinct classes within which the number of AIDS cases propagate. The use of transmission categories allows definition of distinct susceptible populations for which independent growth curves can be calculated. Since crossover among these populations occurs, it is also necessary to provide for dynamic interactions among them in the calculation of total incidence rates.

Data which are only partially quantified play a major complicating role in the significant problem of forecasting the future course of the AIDS pandemic. For example, the likelihood of HIV infection is a function of both the risk of infection per exposure and the frequency of exposure. Factors such as these can best be described in terms of probabilities, and these probabilities typically are difficult to estimate.

Current modeling and computer simulation tools do not effectively incorporate partial information. Modelers are therefore forced to choose arbitrary numbers in order to use existing modeling and simulation technology. In these cases, the reliability of the scenarios generated becomes questionable. Never-the-less, simulation is a promising tool in predicting the future course of the AIDS pandemic particularly in studies of the relative impacts of alternative public health strategies.

The the focus of the work described here relates to logistic population growth in one mode of HIV transmission, specifically transmission through intravenous drug abuse, in an HIV/AIDS epidemiological model. A typical resulting logisitc forecast is given in Figure 1. Simulations based on this model use "poorly quantified" data derived from historical sources and estimations of an upper bound (maximum size of the susceptible population).

Recent research has provided insight into the implications of using asymmetrical probability density functions (pdfs) as contrasted to uniform or normally (symmetrically) distributed data. This is an important consideration since many of the critical determining factors in HIV/AIDS epidemiology are known to be skewed even if of uncertain mean values.

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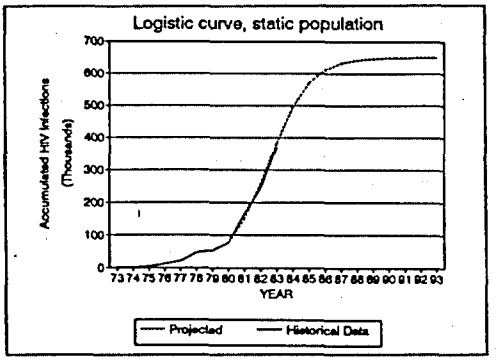

Figure 1. Logistic growth of accumulated infections within a static population of suceptibles.

The products of an exact value and a symmetrical probability density function (pdf) or of two symmetrical pdfs are symmetrical. The iterated products of asymmetrical pdfs with either other asymmetrical or symmetrical pdfs may exhibit unexpected properties. These properties may have profound effects on the behavior of complex models. In the domain under consideration, population and epidemiological modeling, the underlying mechanisms dictate that asymmetrical pdfs be used (e.g. determinants of mortality and natality across age cohorts or the rate at which a population undergoes certain demographic changes).

A model and simulation that accommodates a dynamically changing population of susceptibles, with asymmetrically distributed "refresh" rates (R), yields qualitatively different growth curves as shown in Figure 2. Note the shifting minima and maxima

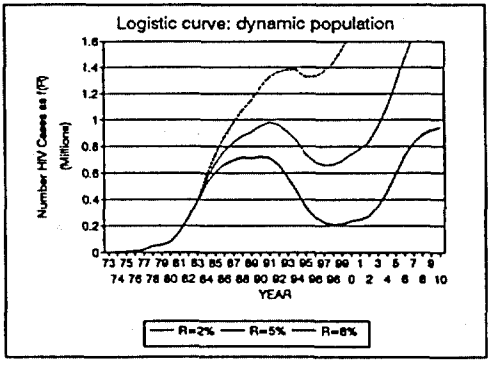

Figure 2. Logisitic growth within a range of dynamically changing populations of suceptibles.

Since data about the HIV/AIDS pandemic often is incompletely quantifiable, and cannot be expressed as simple numerical values, the use of other formalisms for expressing partially quantified knowledge is critical. As these modes are added to a simulation, novel risks may be introduced. These risks may be associated with limited understanding of the proper use of semi-quantitative data, the issue of data independence or the presence of correlated variables, and the implications of alternative mathematical operations on variables representing asymmetrically distributed values.

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A FEW THOUGHTS AROUND THE INSTALLATION JAPANESE GRAFFITI

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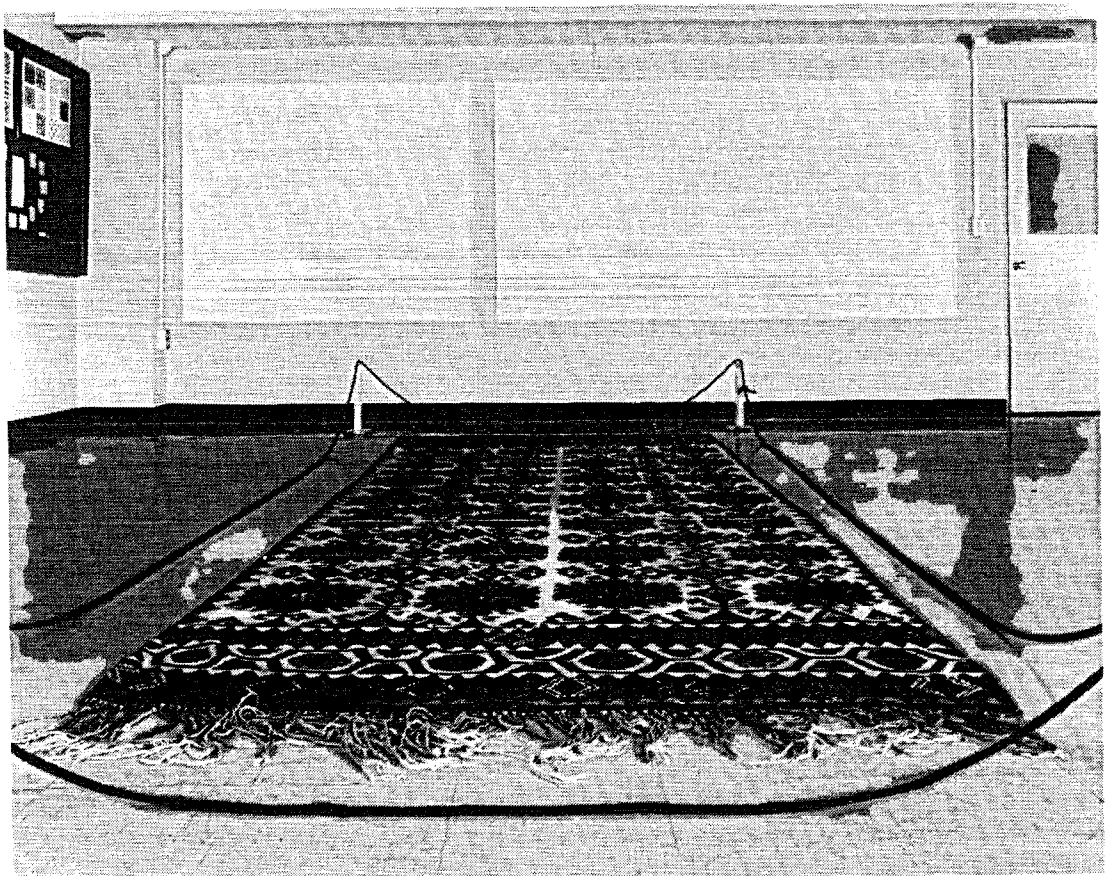
My artistic work revolves around the concept of the body/individual and it´s relation to social and political power structures. In my site specific installations I often use medical science as a startingpoint to trigger philosphical reasonings around western democracy and it´s relation to the media, ethics (religious leftovers) and science.

I often use the concept of quantum mechanichs metamorphosed in many different forms in my work. To have two contradictory concepts parallelly that both can be true depending on the viewing point of the beholder, creates an extremely interesting symmetry. This symmetry is a lot more powerful than that created by sheer abolition of absolute truth or the concept of the ideal, which has been the focus of a lot of attention in contemporay art in recent years.

When one accepts this plurality of truths one becomes free to juxtapose contradictory statements or to mix elements with no apparent connotation to previously unseen contextual units. This gives the artist and the beholder keys to read, or visualize patterns that never before have been seen in human cultural behavior. The best example of this so far is the midi-technics and what it has meant to contemporary black music.

So in one respect one can regard my installation "Japanese Graffiti" as a Hip Hop version, or more pointedly a remix, of a problem that has been a central theme in my work for a long time - the relationship between rational and emotional. These two enteties could be represented by a large number of other opposites or simply by the two hemisperes of the brain. In the installation I´ve chosen the latter to express this duality. I ´ve copied excerpts from a paper by Professor S.F. Witelson of the Mc Master University, Hamilton, Canada on the functional asymmetries in the human brain. I have copied these texts in for direction to create a pattern. This pattern turned to be very similar to that of details in Turkish rugs. I evolved this to not simply copying the texts on top of details from various rugs but also to include the actual rug as an integrated part of the installation. The prescence of a 19th century rug together with the visual distortion of the texts challanges our conception of cultural conventions. Not only does it force us to question cultural borders and the eurocentricity of these conception, but it is also questioning our preconceptions about signs outside the written language as a means of communication.

The primary reason for using science as a metaphor and as an integrated part in my work is to try and understand our cultural inhibitions. We have taken aboard a great deal of misconceptions about nature, which seems to create a problem in our relation to our bodies. Christian ethics clearly states that we´re above nature. This seems to be refering to the mind and we´re stuck with our bodies that is still on the other side. It is a problem that´s been dicussed since the concept of mind was brought alive, but maybe if we can reestablish the symmetry between mind and body if we learn to take aboard more contradictory information. And not constantly be refering to our mind but sometimes give our bodies a chance.



Japanese Graffiti, 1992, approx. 5x2x5 m, ink on paper and 19th century Central Anatolian Kilim rug, Installed at Synergetics Institute, Hiroshima, Japan. The rug by kind permission of J. P. Willborg AB, Stockholm.

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ORIGAMI ARCHITECTURE

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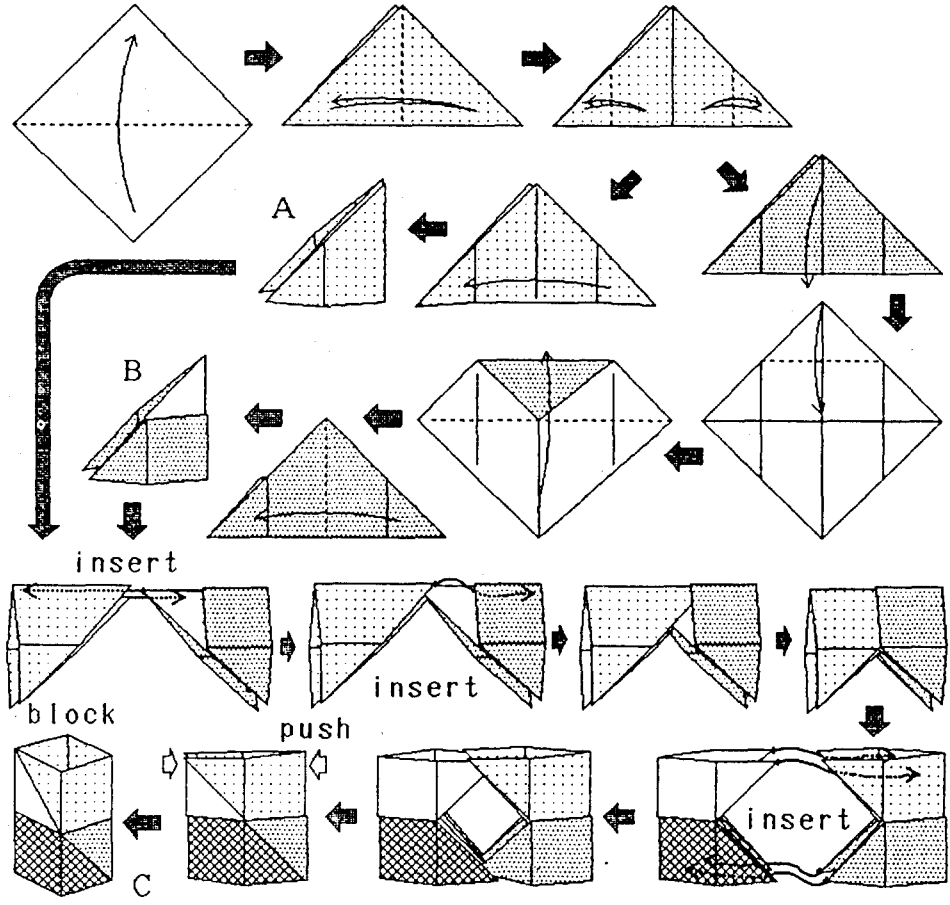
Unit Origami is a new way of folding paper and joining the parts to form objects without using scissor or paste.Mitsunobu Sonobe thought of a 6-unit modular cube which is the origin of Unit Origami. In recent years, much work has done on the development of Unit Origami by Tomoko Fuse.
Block C is a 4-unit modular square cylinder. It is combined with other blocks by using joints D, F and etc like E and G.You can construct complex architecture by combining blocks. A block is very simple and has some symmetry. Its symmetry group is a free abelian group with order 4 generated by a half rotation and a reflection. When you observe combined blocks, you may find new symmetry.



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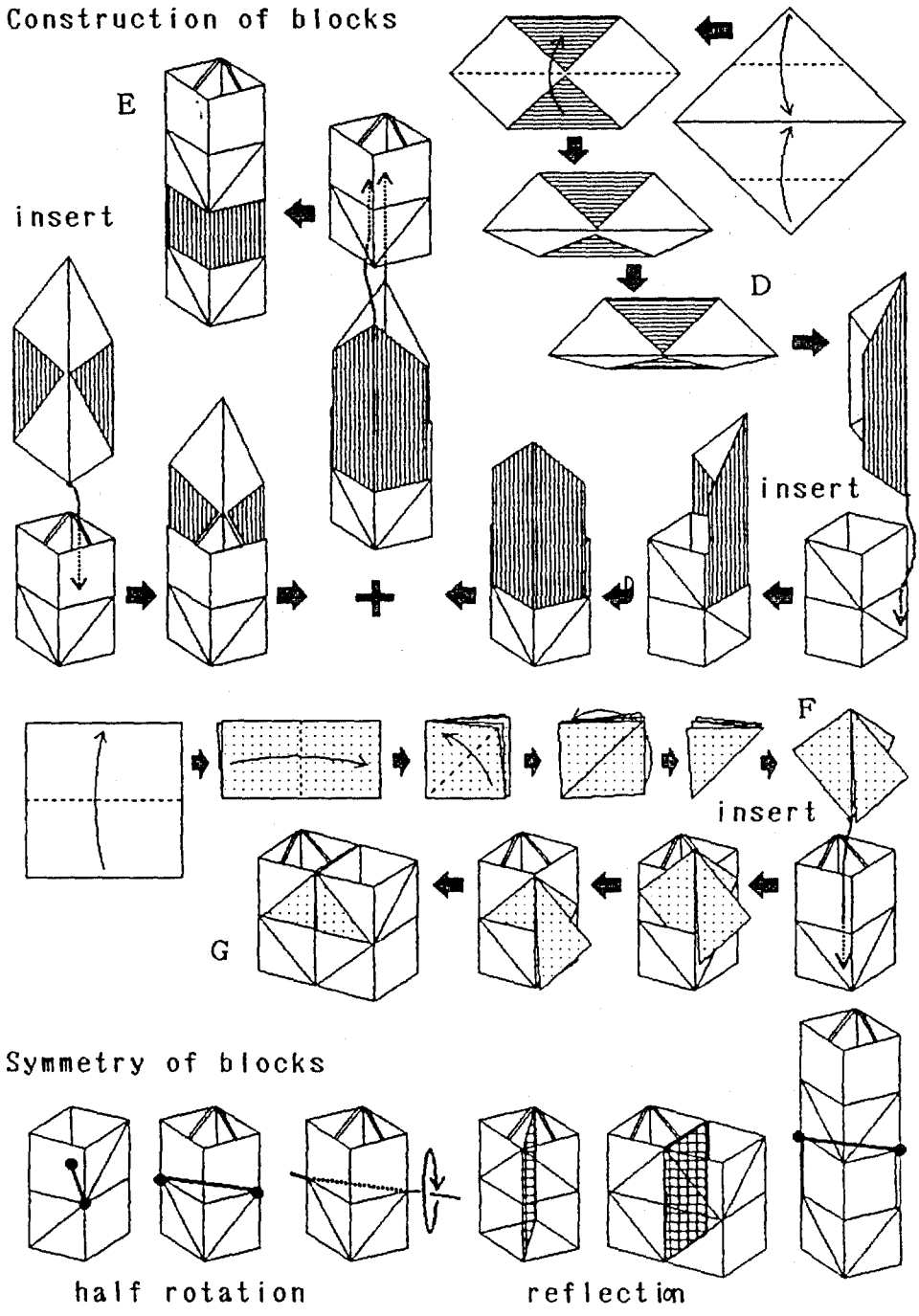

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EXPERIMENTAL STUDY OF WAVES IN FIBONACCI AND PENROSE LATTICES

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Recently, wave propagation in modulated structure has been receiving considerable attention. In particular, the wave propagation in quasiperiodic systems is of great importance, since the phenomena have close relation to the physical properties of real quasicrystals. The essence of the phenomena is lying in the interference between multiply scattered waves in the modulated structure. In order to observe these effects experimentally, it is inevitable to preserve the phase coherency during the multiple scattering. In this sense the sound waves with little attenuation are favorable for the study of these phenomena. Third sound is a wave propagating in superfluid helium films, of which the attenuation is small. Since it propagates in the adsorbed helium film, the modulation of the solid surface can be used as the modulated structure in which third sound propagates. Recently, we have developed an experimental method to study the transmission spectra of third sound in one-dimensional lattices, which are discrete variations of the modulated structure. The modulation of the surface was done with aluminum strips (width $d \sim 80\mu m$). Periodic [1], Fibonacci [2], Thue-Morse [3], and random [4] lattices were studied so far.

The Fibonacci sequence is generated by the following recurrence formula:

$$S_{n+1} = \{S_{n-1}, S_n\}, \text{ with } S_1 = \{B\} \text{ and } S_0 = \{A\}.$$

For example, $S_2 = \{AB\}$, $S_3 = \{BAB\}$, and $S_4 = \{ABBAB\}$. We have fabricated the Fibonacci lattice by mapping A to an aluminum strip on a glass substrate, whereas B to the bare glass surface of the same width. The Fibonacci lattice is a prototype of the *one-dimensional* quasiperiodic lattice. The Penrose lattice can be produced by putting the aluminum pads on a glass surface according to the rule which generates Penrose tiling. The Penrose lattice is a prototype of the *two-dimensional* quasiperiodic lattice.

Figure 1 shows the transmission spectrum of third sound in the Fibonacci lattice, where k is the wave number of third sound. Two large transmission gaps, where the transmitting wave power is small, are observed. These transmission gaps locate at $kd \sim 1/\tau^2$ and $kd \sim 1/\tau$, where τ is golden ratio ($\tau = (1 + \sqrt{5})/2$). The bands, where the wave is transmitted fairly well, are eroded away further by smaller gaps in a nested way. Figure 2 shows the spectrum of third sound in the Penrose lattice. The inset shows how the aluminum pads were distributed on the Penrose tiling. The length of the basis vector is expressed by a. The structure in the spectrum is weaker than the Fibonacci case. The similarity is noticed, however, between the spectrum in the Fibonacci lattice and that in the Penrose lattice. The spectra in the other directions have to be measured, since the lattice is two-dimensional.

The author is grateful to the Mitsubishi Foundation.

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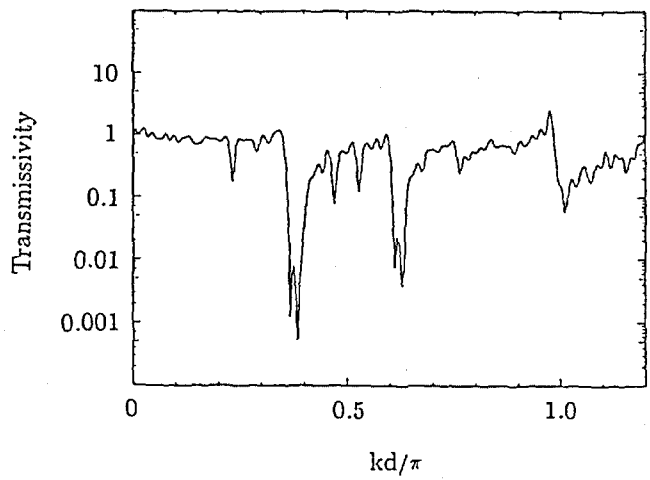

Figure 1: Transmission spectrum of third sound in the Fibonacci lattice.

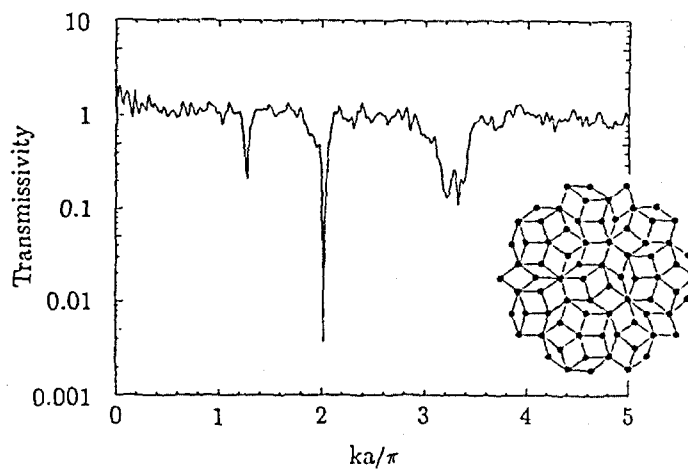

Figure 2: Transmission spectrum of third sound in the Penrose lattice.

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EVOLUTION OF ORIGAMI MODELS

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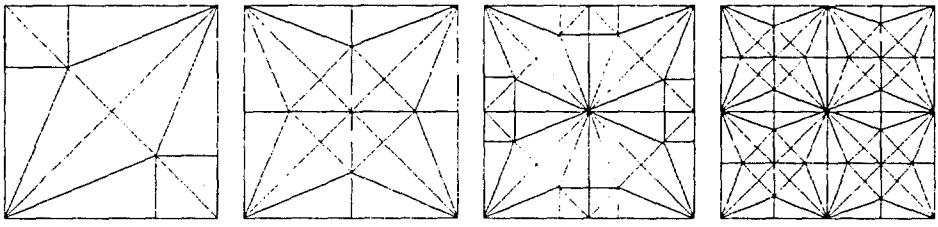
1. Orientation

At all times, my greatest interest in origami(paper-folding) is to create original models. Fold lines analysis, theorems about them, taxonomy,... These are entangled topics on the way to create new models. This report is one of such rutted roads.

2. Origin

Nobody knows who created traditional origami models. Traditional models are results of historical selection over 1000 years. We can compare it to biological selection. In this analogy, most important question is expressed as follows. What is selection pressure?
In order to answer this question, I think another analogy between origami and biology. That is anatomy. We can find "clane series" as typical case of anatomical approach. "Clane" is representative traditional model. We have many "clane" based models.
Marrow pattern of "clane" is folded up to flat form under stable (little stress) state. And, this pattern has sphere phase as a result that edge of paper is fitted to another edge.
"Clane base" is a member of "clane series" like binary system.



Fish base Clane base Flog base Beetle base

(Fig. 1) Clane series

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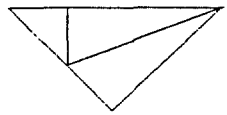
3. Organ

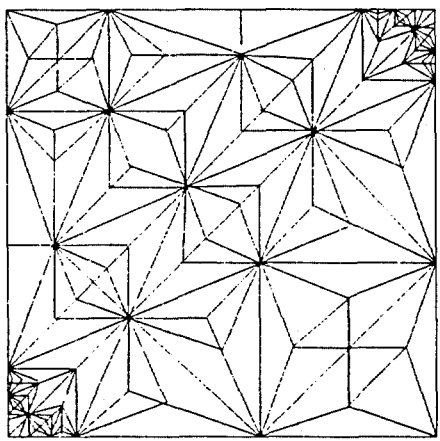
Fundamental region of "clane series" is a right-angled isoscales triangle. We can subdivide this triangle into two type triangles. These two triangles are elementary units of "clane type origami".



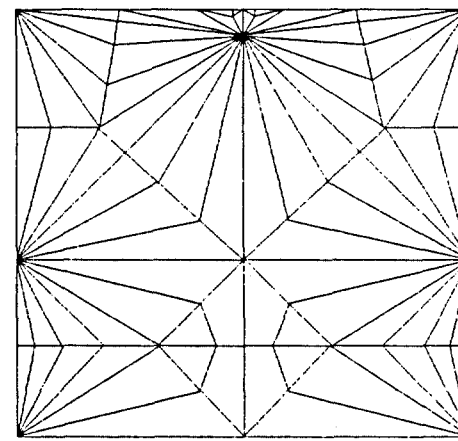
(Fig. 2) Fudamental region of clane series Elementary units

4. Origination

We can regard "clane type origami" as a conditional tiling work using the elementary units. I can get many models by the tiling work (Example:Fig. 3) These elementary units are 90/4 degree base. We can extend them to 90/3, 90/5, and 90/6 degree based pattern. (Example: 90/6 degree pattern:Fig. 4)



(Fig. 3) Devil



(Fig. 4) Mantis

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AN ORIGIN OF SYMMETRY BREAKING: IRREVERSIBLE THERMODYNAMICS FROM AN INTERNALIST PERSPECTIVE

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Symmetry and its preservation have long been taken as the fundamental ingredients of any dynamics (Wigner, 1964) . Identification of symmetry presumes the dichotomy of the operation of preserving a symmetry property and the state to which such operation applies. This formulation of dynamics leaves the state to which the operation of symmetry preservation applies as being a mere passive entity. Dynamics of the quantum mechanical state as embodied in Schroedinger's equation of motion is just a successful example of showing the separation between the symmetry preserving dynamic operator and the quantum state to be operated. However, the present scheme of depriving the quantum state of its dynamic activity raises a fundamental question on how could the initial quantum state to be acted on subsequently be prepared in the first place. This question in fact centers around the problem of measurement in quantum mechanics. Although it is possible to predetermine the value of a quantum mechanical observable if the initial quantum state happens to be available by whatever means, it is not the symmetry preserving quantum operator itself which could prepare and determine the initial quantum state.

Availability of state on a nonlocal scale which is prerequisite to dynamics of the quantum mechanical state is, however, a theoretical artifact at its best no matter how successful it has been in countless physical experiments. Initial preparation of the quantum state does assume an active operation on the part of the agency in charge, or experimenters. State dynamics of preserving a symmetry property could be permissible only in the limit case that the operation of preparing the initial state would not interfere with another operation of preserving the symmetry property latent in the once prepared state. This potential difficulty with any type of state dynamics preserving its symmetry property would become most keen when one comes to face with thermodynamics or irreversible thermodynamics in particular, since in the latter of which the idea of state variables has been questioned. For instance, whether the idea of entropy as a state variable could firmly be established even in irreversible thermodynamics still remains to be seen.

Irreversible thermodynamics unquestionably allows in itself a set of local observables such as local energy flows, but they cannot be equated with local state variables in their own right. For the insistence on local state variables and on the resulting state dynamics preserving a certain symmetry would have to claim to be prepared with the initial state without having any interference with the subsequent dynamic development. On the other hand, local observables assume measurement dynamics of their own identification because they are identifiable in the record, though not accompanied by state dynamics. The measured record of local observables cannot substitute for the state to be driven by state dynamics, because the record is already a consequence of dynamics that has made measurement possible. Irreversible thermodynamics lacking its local state variables still, however, rests itself upon the process of measurement proceeding internally (Matsuno, 1989).

Internal measurement of local observables is intrinsically irreversible and breaks

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temporal symmetry because it lacks the state to be driven by symmetry preserving state dynamics. This fact simply implies that there is no means to predetermine what will be measured beforehand and that there is an apparent asymmetry between the prior indefiniteness yet to be actualized and the posterior definiteness in the record. Temporal symmetry breaking latent in internal measurement will become more evident in relation to the manner of how conservation laws such as energy flow continuity could materialize. Although state dynamics asking a complete identification of its local state variables takes conservation laws to be no more than a form of truism because the identification cannot be separated from observing these laws, internal measurement comes to fulfill them only a posteriori. How conservation laws will be fulfilled remains indefinite for internal measurement yet to come. Internal measurement as a local process assume only the limited access to the complete situation on a global scale. Only the record can tell how these conservation laws have been fulfilled. This leads to the fact that even the first law of thermodynamics on energy conservation, let alone the second law, is irreversible in its operation within the scheme of internal measurement.
Internal measurement in irreversible thermodynamics can serve as a generator of breaking temporal symmetry because of its incompatibility with the state to be driven by symmetry preserving state dynamics.

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WALLPAPERS PRECISELY 17: AN EYE-OPENING CONFIRMATION

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Early geometry was constructive and visual; in time, geometry came to be expressed by abstract notation. Undoubtedly, the venerable Pythagoras beheld his extraordinary right triangle theorem rather than formulated it. Subsequent algebraic equations merely restate what was visually evident. As mainstream geometry increasingly became the province of the mathematician, graphic geometry, construction without calculation (in the "Platonic" tradition), became the province of the designer. These parallel paths can but benefit from interdisciplinary bridges such as the *Scientific American* column (1956-81) of Gardner and *The World of Mathematics* (1956) of Newman, whose aim was to "present mathematics as a tool, a language and a map; as a work of art and an end in itself."

A classic topic, attracting attention of both mathematician and designer, is that of the 17 *infinitely repeating patterns with two independent translations*, commonly called the 17 *wallpapers*. Various derivations (generally deficient as proofs) routinely utilize geometric and algebraic formulations, including group theory. In the proliferation of mathematical and other scientific texts, with redundant iterations of the problem, certain deficiencies have been perpetuated. The devising of a more visual approach to this highly visual problem can not only make the wallpaper lore more accessible to designers, but perhaps provide new insight into its rudiments and insinuate a proof.

The 17 wallpapers were included in a basic design curriculum, instituted in 1960 by William Huff; a visual approach to their derivation (and proof)—to facilitate the instruction of design students—was explored in 1961 with mathematician Richard Durstine, who proposed a strategy that was not fully developed at the time. In 1990, Jack Holnbeck and I elected to work on different aspects of the 17 wallpapers as theses topics. A principal task of mine has been to pick up the threads of the Durstine proposal—while Holnbeck has scoured innumerable texts for their treatments of this persisting theme. As I worked out the Durstine strategy, a cumbersomely large chart resulted. In consultation with Dénes Nagy (1992), a somewhat different strategy was devised, based on preliminary work of Nagy's student, S. Prakash (1990). See TABLE.

A basic condition of the 17 wallpapers is that pattern coverage must be achievable through translation alone, along two different vectors. A second (symmetry defining) condition is that there is pattern coverage (pattern invariance), when *any* operation, inherent in any patterns, is effected. Prevailing literature implies a third condition: that upon the operation of any element of symmetry, inherent in the pattern, the lattice, as well as pattern, is left invariant. Weyl's *Symmetry* (1952) gives this impression: "Having found the 10 possible groups Γ of rotations and the Lattices L *left invariant* by each of them, one has to paste together a Γ with a corresponding L so as to obtain the full group of congruent mappings. . . While there are 10 possibilities for Γ, *there are exactly 17 essentially different possibilities for the full group of congruences* Δ [*italics* mine]."

Lockwood and Macmillan's *Geometric Symmetry* (1978) is the rare text, in apprising that *it cannot be said that the [lattice] is unchanged by a glide*. These authors, in their cautious, negatory statement, stop short of nailing it down for the outsider. Shubnikov and Koptsik, writing in *Symmetry in Science and Art* (1974) about 3-D space groups, bring to light Fedorov's recognition of *symmorphic* and *nonsymmorphic* groups, the latter, characterized by the presence of *glide* or *screw* symmetry. Wallpapers (2-D) cannot accommodate screw operations (3-D); they can and do incorporate glide.

Failing to pick up on Fedorov, the majority of texts either gloss over the thorny problem

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of the special nature of the glide operations by leaving the impression that all 17 patterns are achieved without event or they err in stating outright that all operations of a given pattern effect lattice coverage along with pattern coverage. To fail to instruct students about how the nonsymmorphic groups function in the 17 wallpapers is to fail to instruct them thoroughly about the analog that crystallographers have used to explain the 3-D space groups, not easily envisioned in the mind's eye.

A designer's simple device was concocted for the investigation of the concurrence of lattice coverage with pattern coverage. With duplicate copies of each pattern, one opaque, one transparent, any sort of move of the transparency over the opaque copy, which brings the pattern into coverage, effects a proper operation (translations, rotations), inherent in the pattern. It is discerned that both pattern coverage and lattice coverage occur in all instances. Improper operations (mirror reflections, glides) are simulated by flipping the acetate. It is revealed that in four cases involving glide (but not all) lattice coverage does not concur with pattern coverage. This investigation also challenges the crystallographer's dubious *centered rectangular lattice* (Buerger, et al.).

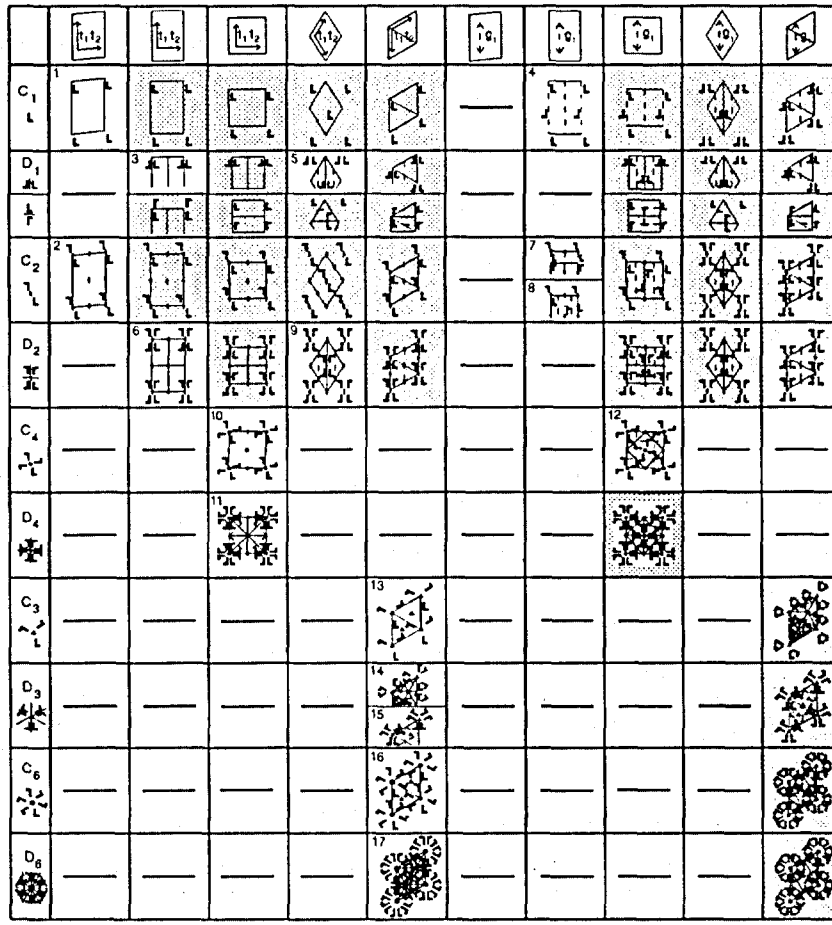

TABLE: The five lattices undergoing symmorphic and nonsymmorphic operations.

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SOME GEOMETRICAL ATTEMPTS
Quasicrystals, Fractal Tesselation, Ideal Critical Pattern

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Some of geometrical attempts are proposed. The author is originally a theoretician mainly physics of condensed matters and focuses his interest on geometrical problems since he organized an interdisciplinary meeting "Morphysics" (morph + physics) in Kyoto in 1980.

1. Quasicrystals. [1], [2], [3]

[A] The author discovered three-dimensional Penrose tiling in 1985. There are two kinds of rhombohedral unit cells, A_6 and O_6. An A_6, beeing an expanded A_6, is consists of 55 A_6's and 34 O_6's and an O_6*, being an expanded O_6, is consists of 34 A_6's and 21 O_6's, so to express as

$$A_6^* = 55\,A_6 + 34O_6,$$

$$O_6^* = 34\,A_6 + 21O_6.$$

It is equivalent to

$$(\tau^{-1}A_6^* + \tau^{-2}O_6^*) = \tau^9(\tau^{-1}A_6 + \tau^{-2}O_6)$$

corresponding to the fact that their relative composition is golden mean $\tau = (1+\sqrt{5})/2 = 1.618$. The arrangement of A_6 and O_6 in A_6* and O_6* was discovered and the same procedure can be repeated as many times as desired. It is noted that the structure and then the procedure has some freedom. So-called projection model is contained in the present model. The author solved it as a puzzle and then he knows the feature of the structure very well. While the projection method is so useful that some users have only little knowledge about the structure. The ball and stick model of this structure is exhibited. The procedure can be expressed in another way. Every vertex is transformed into the centre of a flower dodecahedron, exactly speaking, which is a 60-hedron with icosahedral symmetry and consists of 20 A_6's.

[B] The concept of graphic geodesic line was introduced by the author and R. Collins to characterize any two-dimensional network with triangular meshes. All the geodesics were completely traced in the special case of triangulated Penrose tiling which was obtained by drawing all the minor diagonals in Penrose tiling of rhombic version. There are five sets of parallel lines, some circular roops and two hierarchical sets of rather complicated closed geodesics. This analysis figures out that Penrose tiling has rather strong fluctuations which cancel out in a small area.

There seems to appear a chaotic behaviour in the corresponding problem of octagonal Penrose tiling.

[C] Recently, the author found that the allocation scheme of the seats in an election in the proportional representation[4]. Though the time will not be enough to explain it, some copies of the concerned paper will be brought. It was written for physicists investigating quasicrystals or generallized crystallography The next publication in more geometrical description is now in preparation.

2. Fractal tessellation of a plane and a spherical surface. [5],[6]

Koch curve is a typical artificial fractal curve. When Koch curves are arranged in some symmetrical way, a fractal tessellation of a plane is obtained. There are five-fold case and six-fold case. By a similar way, some fractal tessellation of a spherical surface can be obtained. There are two cubic cases and an icosahedral case. The concept of similarity was extended in these cases since there is no similarity on a spherical surface that is not a linear space.

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Peano curve, *covering* a cubic region, was extended to cover a spherical surface by a single curve as uniform as possible.

3. Ideal critical pattern [7].

You may regard a critical state as the state at the conceptual border of mixing and segrigating, though the concept is well defined in Physics. The motivation of this attempt is to combine intuitive sensibility and logic. The first aim is to realize some finite *ideal critical pattern* which can be the representative of the ensemble of all the critical configurations. The pattern should have both properties of mixing and segregating. One, finding some defects, tends to feel unsatisfactory in the pattern that he felt acceptable before.

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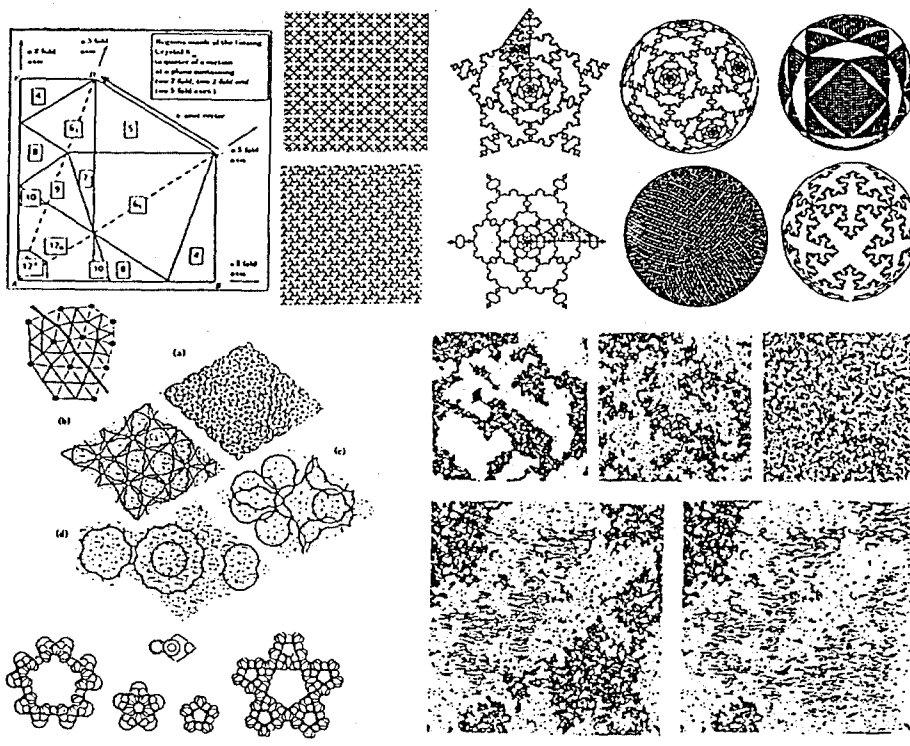

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FLEXING POLYHEDRA

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Flexing polyhedra are closed surfaces which are bordered by fixed, even polygons that can articulate along the edges. Such polyhedra permit of a deformation. Visualize a closed surface which is composed of flat pieces of cardboard and held together with adhesive tape along the edges. If the form of the polyhedron can change without tearing the tape or bending the cardboard, then we have a flexing polyhedron. The bellows of a camera, for example, function only with soft material; hence they are not genuine flexing polyhedra, being mathematically very impure.

In 1812 the well-known French mathematician Cauchy proved that convex polyhedra, i.e. those curved outwards, are immobile. In a generalisation of this principle, it was then postulated that concave polyhedra, i.e. those curved inwards, are also rigid. In 1897, however, a Belgian engineer named R. Bricard refuted this assumption. He discovered mobile octahedra strips, although they could not be completed as polyhedra because they showed some overlapping. Nevertheless it was regarded as impossible to construct a genuine flexing polyhedron.

Only in recent years did R. Connelly with his revolutionary 36-sided polyhedron, succeed in modifying Bricard's model in such a way as to produce the world's first genuine flexing polyhedron. This polyhedron was later modified by N.H. Kuiper and P. Deligne to only 18 faces. To top this, in 1977, K. Steffen found his famous flexing polyhedron with only 14 faces and 9 vertices. All those flexing polyhedra are based on the model of Bricard and their mobility is severely limited by parts which impede one another. The mathematically pure flexing polyhedra discovered so far have constant capacity. It is therefore generally assumed that the volume of every possible flexing polyhedron remains constant during flexure.

Primary examples of flexing polyhedra with a variable capacity -although these are not mathematically pure examples- are W. Blaschke's flexing octahedra and M. Goldberg's double pyramid, resembling Siamese twins. More recent models, such as the diverse infinitesimal flexing polyhedra of W. Wunderlich and the 16-sided so called Quadricorn designed in 1981 by myself, allow rather more precise, effortless movements. The Quadricorn is the first practically perfect flexing polyhedron with a mathematically pure middle position and two flat boundary forms, i.e. its volume can be reduced to zero. As stated, the movements of all these flexing polyhedra with a variable capacity are not mathematically pure, for when they are in motion, tiny deformations hardly measurable and invisible to the naked eye will occur on the edges and surfaces. But one day , someone may discover a mathematically pure flexing polyhedron with a variable volume and thereby disprove the assumed constancy of capacity -who can tell?

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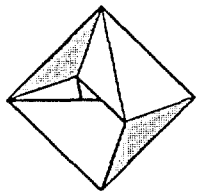
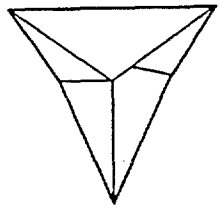
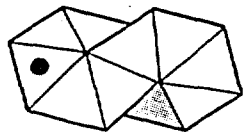
Bricard's
OCTAHEDRON



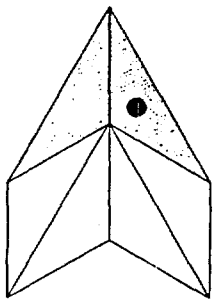
Steffen's
TETRADECAHEDRON



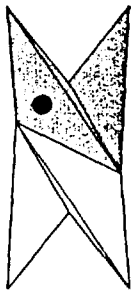
Goldberg's
SIAMESE TWINS



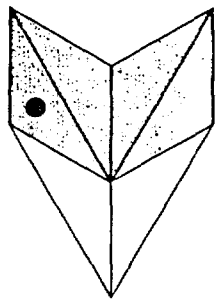
The QUADRICORN



flat
position x



middle
position



flat
position y

One question remains: What sort of shape would a mathematically pure flexing polyhedron with a variable volume have?
Would it need to be symmetrical, or could it be of asymmetrical shape, a ring shaped torus or something resembling a so-called UFO? Might there be a space-filling flexing polyhedra? If so, this could be a dynamic structure where a single flexing polyhedron controls and determines the motion of all the adjacent flexing polyhedra. I firmly believe that any such discovery would have an enormous impact on physics.

You will find on the pages 213-221 the nets of the four flexing polyhedra illustrated above.

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- R. Connelly: A flexible sphere. Mathematical Intelligencer 3, 130-131 (1978).
- Phänomena 1984. ISBN 3-909290-01-9. Page 81 (1984).
- W. Wunderlich, C. Schwabe: Eine Familie von geschlossenen gleichflächigen Polyedern die fast beweglich sind. Elemente der Mathematik. Vol. 41-4 (1986).
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GROWTH AND SHAPES IN QUASICRYSTALS

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The discovery of a quasicrystal[1] with icosahedral symmetry has attracted great interest not only for physicist or mathematician but also for architect and artist. Study on equilibrium morphology of non-periodic system is possible since the discoveries of a series of stable quasicrystals. A material revealing a atomic structure with icosahedral symmetry is expected to see a morphological shape reflecting its atomic structure. To date, three kinds of morphology with icosahedral symmetry have been observed in quasicrystalline alloys; a rhombic triacontahedron for an $Al_6Cu_1Li_3$[2], a pentagonal dodecahedron for an $Al_{65}Cu_{20}Fe_{15}$[3] and global shape for an $Al_{75}Cu_{15}V_{10}$[4]. The triacontahedron with 30 diamond faces, 32 vertices and 60 edges in a solidified Al-Cu-Li alloy, can be constructed by two kinds of rhombohedron; a prolate and an oblate, corresponding to the atomic cluster of quasicrystalline Al-Cu-Li structure. The triacontahedral atomic cluster with a size of few nm can be grown to the size as large as a mm order by an inflation operation of the rhombohedral units. There is a very reasonable relation between the atomic structure and morphology for this quasicrystalline alloy. On the other hand, although the Al-Cu-Fe alloy reveal a beautiful pentagonal dodecahedron, it is still unclear that how to grow a mm sized pentagonal dodecahedron from its atomic cluster. The atomic structures of the Al-Cu-Li and the Al-Cu-Fe quasicrystalline alloy are described by three dimensional Penrose tiling constructed by two kinds of rhombohedral unit with different atomic decoration which build up different fundamental atomic cluster; a triacontahedron for the former and a Mackay icosahedron for the latter. The quasicrystalline Al-Cu-V alloy seems to be described by the icosahedral glass model. The icosahedral glass model relies on local interaction to join clusters of atoms in a somewhat random way. In this model, all the clusters have the same orientation, but because of random growth the structure contains many defects. The icosahedral glass model is suitable for quasicrystal-

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line Al-Cu-V in two ways. First, it removes the necessity of arcane matching rules and gives a plausible explanation the growth of quasicrystal from the amorphous phase. Second, the disorder introduced through randomness closely mimics that evidenced by peak boardening of the diffraction peaks in Al-Cu-V quasicrystals. In growth process, it reveals a global shape without significant facet.

In view of the growth condition we note that quasicrystal could be formed from melt, amorphous and crystalline, respectively reveal outlooks of pentagonal dodecahedron, star polyhedron and global shape. The shape is very sensitive to the environmental factors such as temperature and composition fluctuation of the parent phase. We shall discuss the facets of quasicrystal by taking the structural model and the growth conditions into account.

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[1] D.Shecthamn, I.Blech, D.Gratias and J.W.Cahn, Phys.Rev.Lett. 53,1951(1984).

[2] P.Saindort, P.Dubost & A.Dubus, A.C.r.hebd. Seanc.Acad.Sci.Ser.II, Paris 301,689(1985).

[3] A.P. Tsai, A.Inoue and T.Masumoto, Jpn.J.Appl.Phys, 26,1505(1987).

[4] A.P. Tsai, A.Inoue, Y.Bizen and T. Masumoto, Acta. Metall. 37,1443(1989).

[5] D.Shechtman and I. Blech, Metall.Trans.A16, 1005 (1985).

[6] P.W.Stephens and A.I.Goldman, Phys.Rev.Lett.56,1168 (1986).

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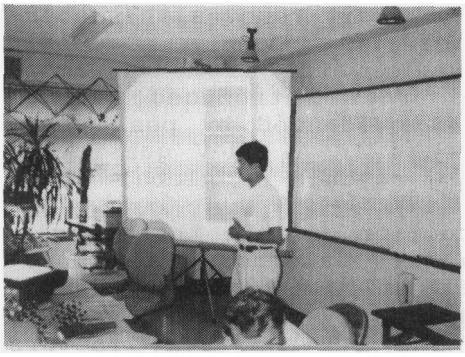
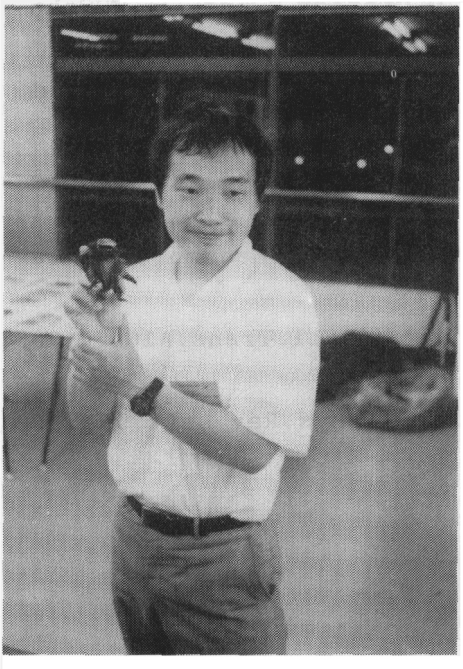


An-Pang Tsai ...

Photo: Yasushi Kajikawa

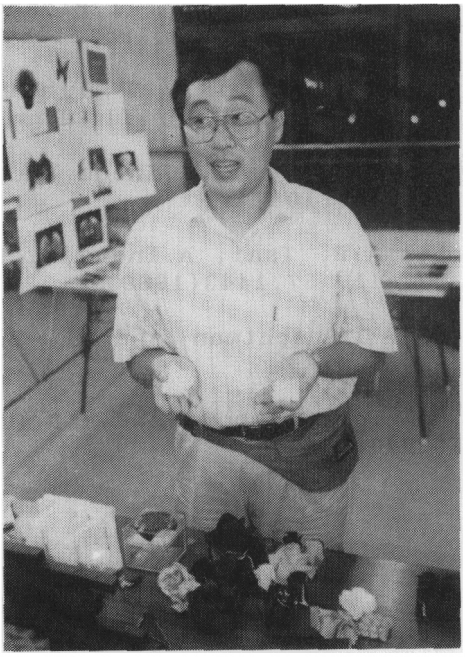
... and a big quasicrystal.

Photo: Yasushi Kajikawa



Jun Maekawa with one of his 'creatures'.

Photo: Yasushi Kajikawa



Toshikazu Kawasaki with some of his origami works.

Photo: Yasushi Kajikawa

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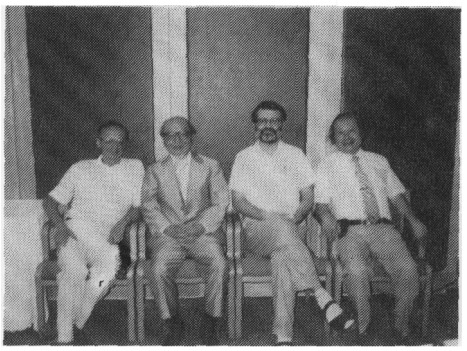
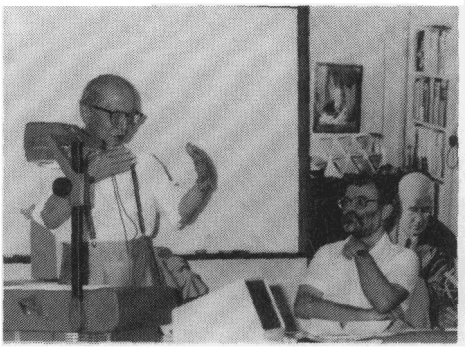


György Darvas, Kodi Husimi, Dénes Nagy, and Tohru Ogawa (from left to right). Husimi-sensei just asked György about the Hungarian mathematical competitions. He is interested in everything.

Photo: Yasushi Kajikawa

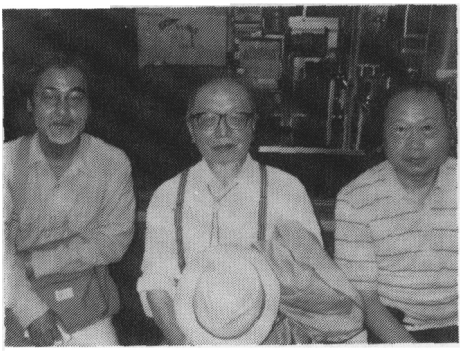


Kodi Husimi's inaugural lecture. Next to him Dénes Nagy, while Buckminster Fuller is keeping watch on them from a backside picture.

Photo: Caspar Schwabe

Kodi Husimi with Caspar Schwabe.

Photo: Caspar Schwabe



Hiroshi Tomura, Kodi Husimi, and Tohru Ogawa (from left to right).

Photo: Caspar Schwabe

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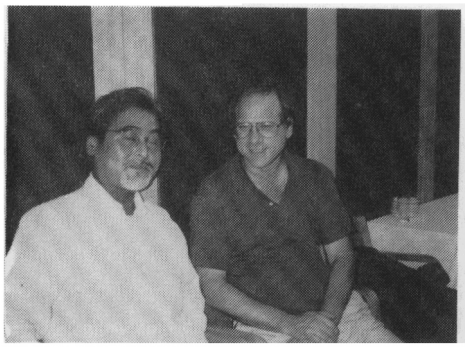
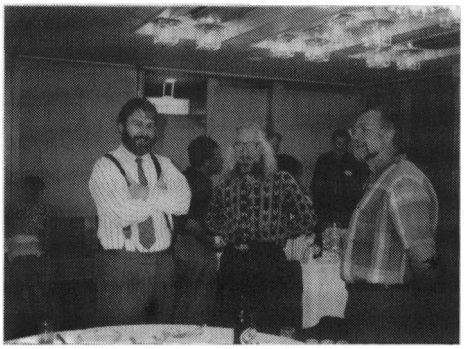
Peter Klein is all smiles with Katalin Fittler (left) and Biruta Kresling (center) at the reception. Obviously it is a forum of education, musicology, origami — and some food.

Photo: Yasushi Kajikawa



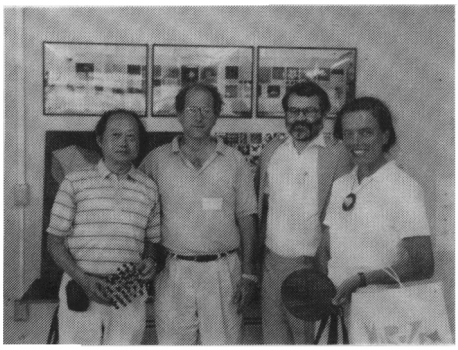
Hiroshi Tomura, alias 'Tom' (left) and Tony Robbin (right): Topology (or even 'tomology') and four dimensions

Photo: Yasushi Kajikawa



Ted Goranson, John Gibbon, and Richard Parker (from left to right).

Photo: Yasushi Kajikawa



Tohru Ogawa, Tony Robbin, Dénes Nagy, and Biruta Kresling in the exhibition hall.

Photo: Caspar Schwabe

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A group of symposium participants at a workshop, framed by the models of Yasushi Kajikawa.

Photo: Yasushi Kajikawa

Humiaki Huzita, Caspar Schwabe, Hiroshi Tomura, and William Huff in Yasushi Kajikawa's workshop in the Synergetics Institute.

Photo: Caspar Schwabe

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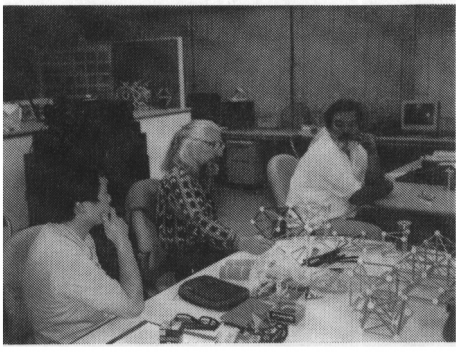
John Gibbon



... between Jun Maekawa (left) and Vojtech Kopský (right), plus John's 'simplest' polyhedra.

Photo: Yasushi Kajikawa

... with Daniel Huson.

Photo: Yasushi Kajikawa

... and with some of his models.

Photo: Yasushi Kajikawa

Daniel Huson, in an E-shirt (Escher shirt), rotates Gibbon's polyhedron; or the polyhedron rotates Daniel.

Photo: Yasushi Kajikawa

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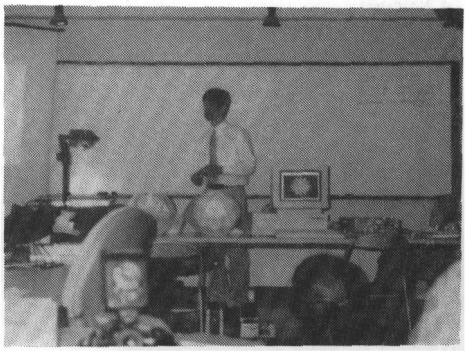
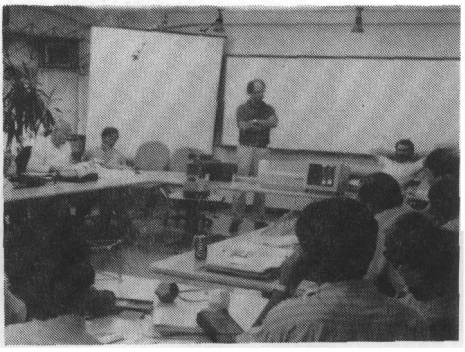
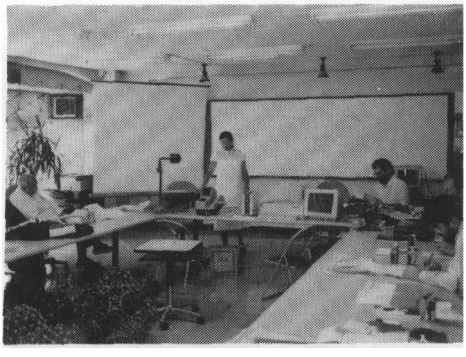


Yasushi Kajikawa in the focus.
Photo: Yasushi Kajikawa



Tony Robbin in action
in a workshop.
Photo: Yasushi Kajikawa



Katalin Fittler provides musical
symmetries, while the floor is covered by
John Gibbon's polyhedral symmetries.
(N.B. Kepler connected these two topics.)
Photo: Yasushi Kajikawa

Katalin Fittler and William Huff listening
attentively to György Darvas.
Photo: Dénes Nagy

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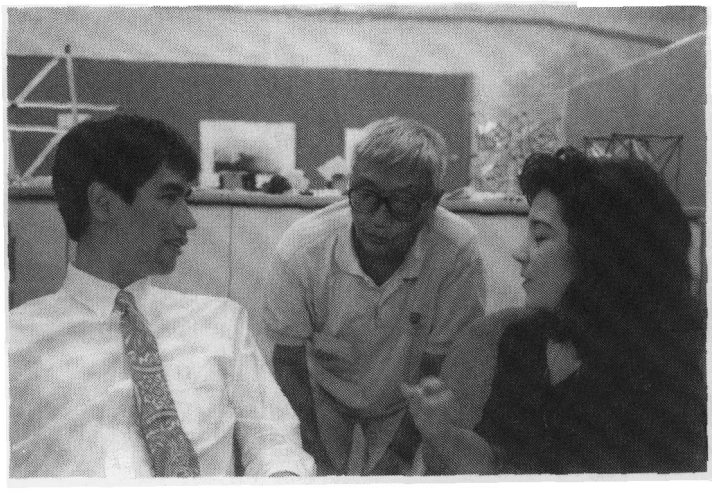
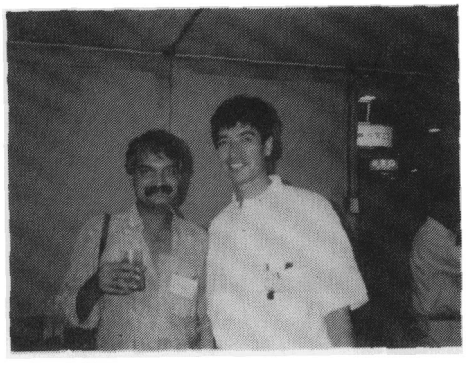


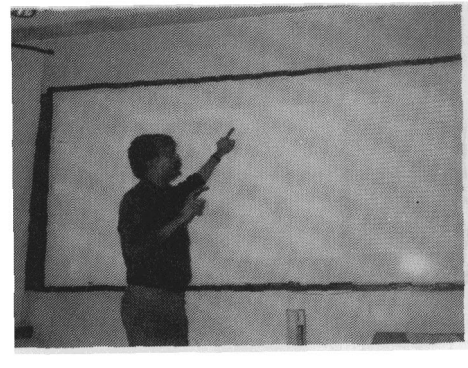
Humiaki Huzita between Tamara and Yasushi Kajikawa.

Photo: Caspar Schwabe



Kirti Trivedi and Yasushi Kajikawa.

Photo: Yasushi Kajikawa



Kirti Trivedi accompanied by *Chitra-kavya (picture-poetry)*.

Photo: Yasushi Kajikawa

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Vol. 3, No. 2, 1992, 179-186

SYMMETRIC GALLERY

Second Interdisciplinary Symmetry Symposium and Exhibition

SYMMETRY OF PATTERNS

August 17-23, 1992

Synergetics Institute, Hiroshima, Japan

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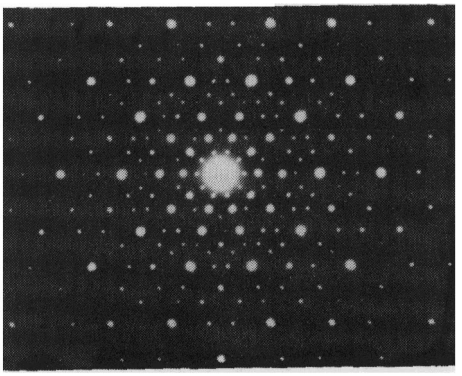
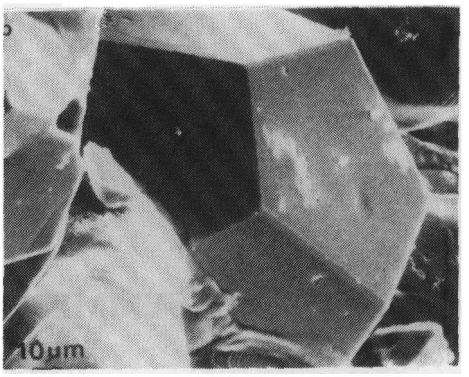
Quasicrystals



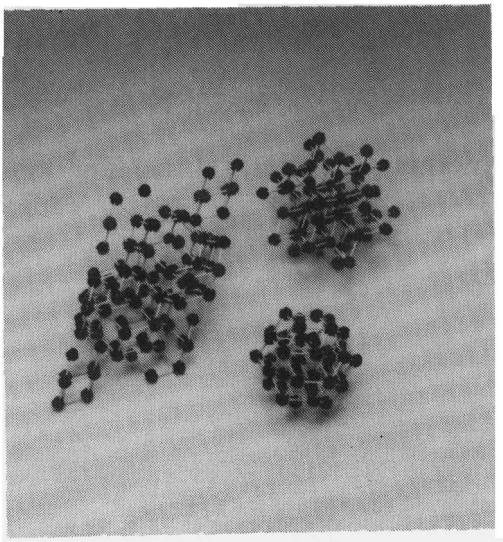
An-Pang Tsai: A five-fold diffraction pattern of $Al_{65}Cu_{20}Fe_{15}$ quasicrystal.

Photo: An-Pang Tsai



An-Pang Tsai: Morphological shape of pentagonal dodecahedron in $Al_{65}Cu_{20}Fe_{15}$ quasicrystal.

Photo: An-Pang Tsai



Tohru Ogawa's quasicrystal models.

Photo: Yasushi Kajikawa

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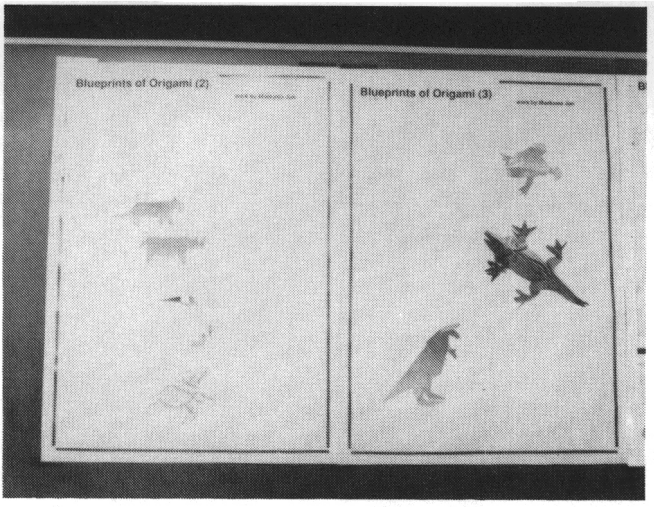
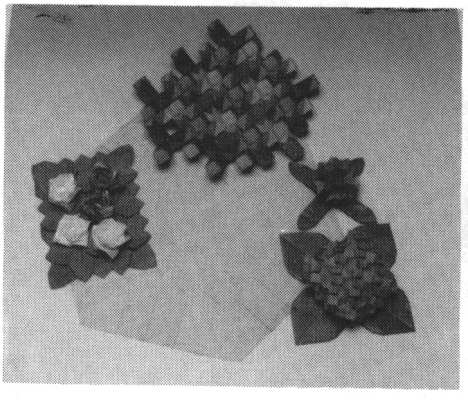
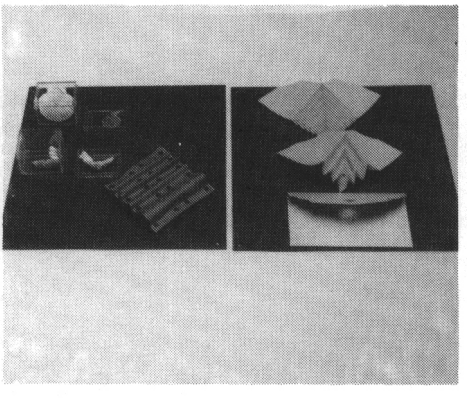
Origami festival



Jun Maekawa's folded pets.
Photo: Yasushi Kajikawa



Toshikazu Kawasaki's composition.
Photo: Yasushi Kajikawa



From nature to origami:
Some items by Biruta Kresling.
Photo: Yasushi Kajikawa

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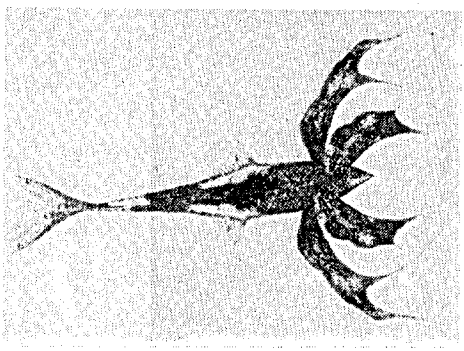
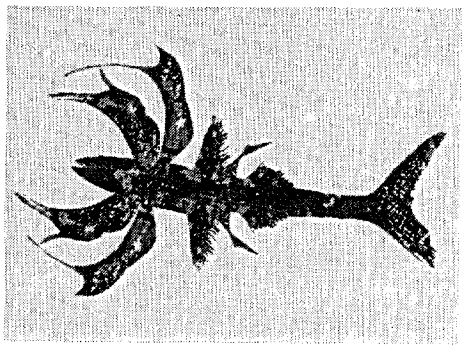
Aldo d'Angelo



Imaginary fish.



Imaginary crab.

Soapbubbles;frames.

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Aldo d'Angelo

Monroe's original image

Monroe's symmetrical image processed

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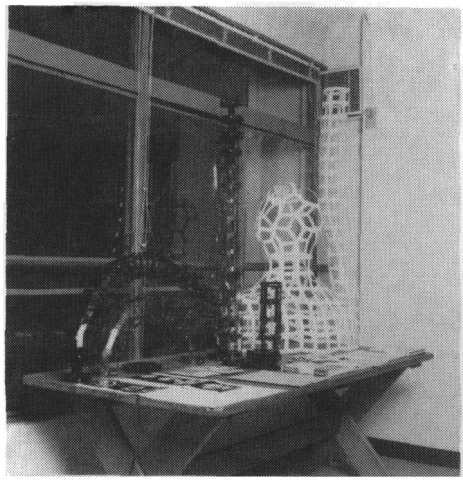
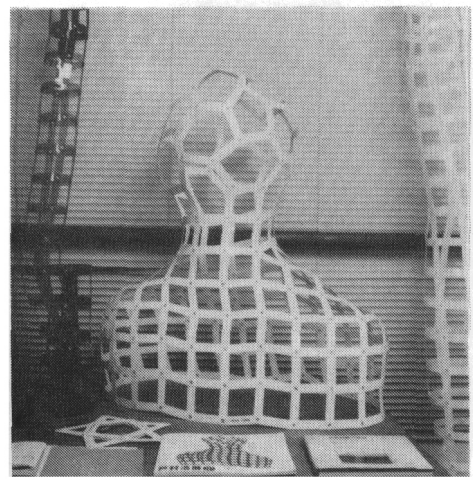
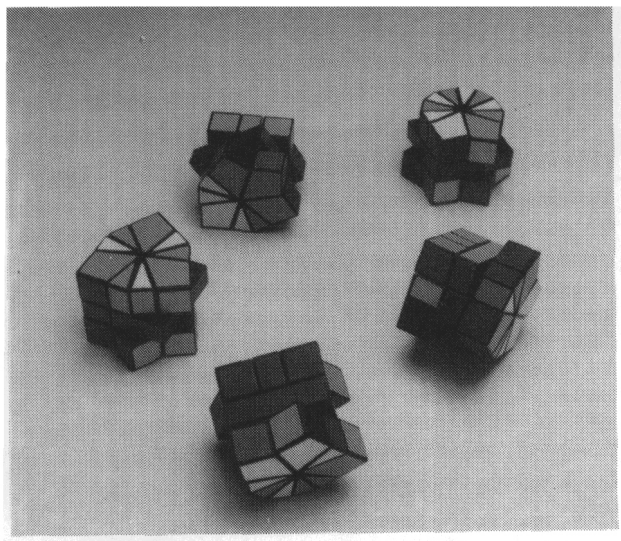




Hiroshi Tomura's exhibition.
Photo: Yasushi Kajikawa



'Square-it', a new puzzle co-invented by Vojtech Kopský.
Photo: Yasushi Kajikawa

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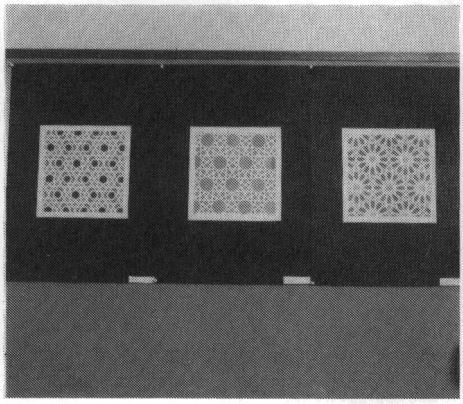
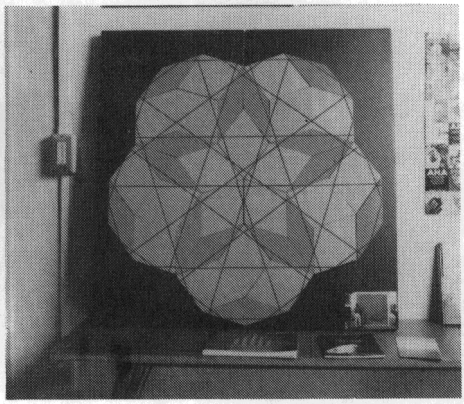
Tiling



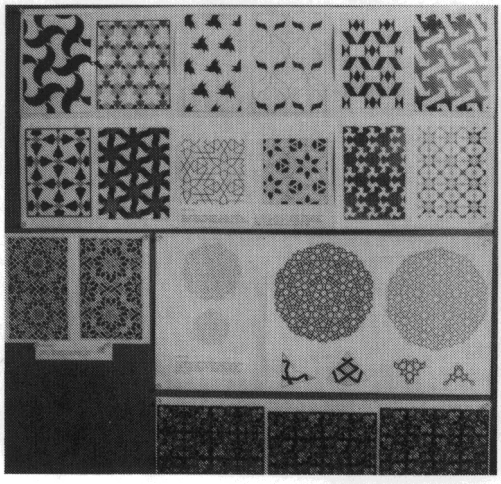
Daniel Huson's exhibition, together with Olaf Delgado and Andreas Dress.

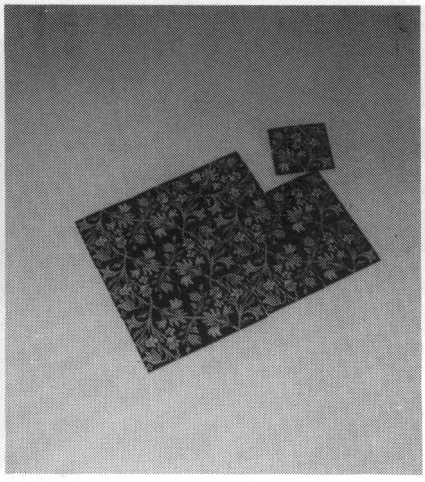
Photo: Yasushi Kajikawa



AHA, it is by Caspar Schwabe: Magnetic version of Penrose-tiling with Ammann-line.

Photo: Yasushi Kajikawa





John Rigby's symmetric tilings.

Photo: Yasushi Kajikawa

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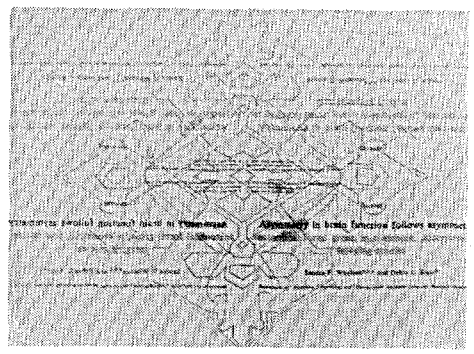
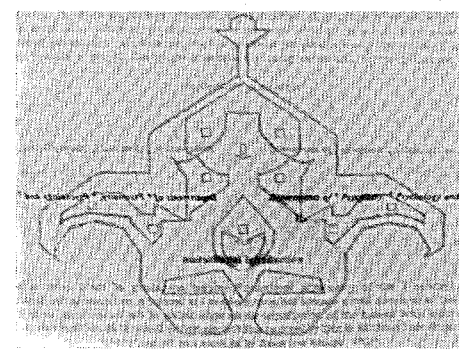
Per Hüttner



Sketch for Japanese Graffiti, 1992,
62x45 cm, graphite on paper.



Sketch for Japanese Graffiti, 1992,
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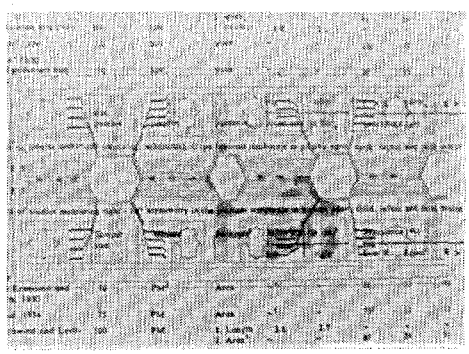
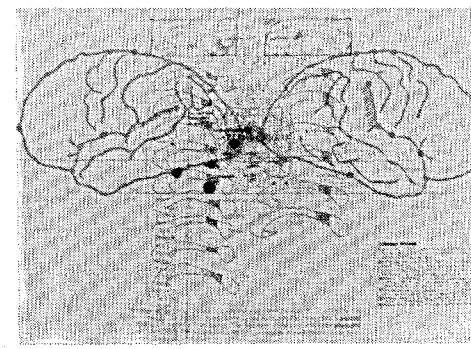


Sketch for Japanese Graffiti, 1992,
62x45 cm, graphite on paper.



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SYMMETRY: CULTURE & SCIENCE

SYMMETRY AND IRREVERSIBILITY IN THE MUSICAL LANGUAGE(S) OF THE TWENTIETH CENTURY

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Much more than their nineteenth-century precursors, composers of our times seem to strive for order, in the sense of conscious and deliberate organization. This is not to say that order substitutes beauty or emotion. But while it seemed acceptable in earlier times to create new facets of beauty and unheard-of depths of feeling within structural patterns as conventional as the ternary form and its many derivatives, there is a striking – and increasing – need in our century to establish order on levels and in parameters unique to our age.

Arguably the most significant concept in this pursuit is the concept of symmetry. Apparently opposed to it, or at the very least in stark conflict with it, there is the concept of irreversible progression. Both are, as a large body of research into the fields, particularly that of symmetry, shows, omnipresent in nature. Symmetry occurs in relation with space. It is the single strongest building principle in the physical realm – be it the lateral symmetry of man and most other creatures or the more complex symmetry in crystals, minerals and many other chemical elements, in the course of atoms and the orbit of stars. On the man-made side of the physical realm, symmetry reigns supreme in architecture, geometry and ornamental art, to name just a few. Irreversible progresses, by contrast, are connected with time. From the development of

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an individual life to world history, from the appearance and subsequent decay of mountains and flowers, cultures and ideas to the evolution of species and the expansion of the universe, there appears the same vectorial one-directedness.

This paper aims to investigate some of the basic aspects under which symmetry manifests itself in twentieth-century music. Examples are taken from three compositions for piano solo written around the middle of our century, by composers coming from the German-speaking (and -thinking) tradition: Anton Webern's Variations op. 27 *(1935/6), Paul Hindemith's* Ludus Tonalis *(1942), and Wolfgang Fortner's* Sieben Elegien *(1950).*

CHAPTER 1

To argue for symmetry in Hindemith's *Ludus Tonalis* (1943/1968) seems, at face value, almost trivial. The cycle consists of twelve fugues – one on each of the semitones – linked by interludes and wrapped by a prelude at the beginning and a postlude at the end. If this were all there is, it would hardly be worth mentioning. The fact, however, that the postlude is a visual retrograde inversion of the prelude – one in which the score of the prelude can literally be turned upside down and read backwards – should alert musicians; there is bound to be more to it.

As it turns out, Hindemith has conceived the twelve fugues and the eleven interludes in such a way that they form a strikingly symmetrical cosmos (this despite the fact that the obvious purpose of each interlude is modulation, the transition from the key of the preceding fugue to that of the subsequent one: a clearly linear process). Let me explain a few details on a transparency (Fig. 1).

The interlude which forms the centre of the cycle, connecting Fugue 6 with Fugue 7, is a March. With its strong sense of tonality and slightly rambunctious mood it represents a character of its own which is not repeated in any other part of the cycle.

The fifth and seventh interludes, those before and after the March, can both be identified as Romantic piano miniatures – romantic in their aesthetics, not, of course, with regard to the tonal language employed. One of them, with intensely emotional treble lines and elegant accompaniment patterns, seems reminiscent of Chopin's style, while the other, in thicker homophonic texture and a heavier, in some instances brooding character, recalls Brahms' expressive language.

The fourth and eighths interludes recall Baroque patterns; one is composed in a style similar to that found in many of Bach's preludes, the other appears as a toccata. The third and second-from-last interludes – note the dissymmetry! – are conceived as folk dances. While their melodic and rhythmic idioms are basically timeless, their metric organization links them to two well-known dance forms known from early music: the gavotte and the courante. Complementing the dissymmetry, the second and third-from-last interludes both represent pastorales: melodies reminiscent of a solo flute or recorder, floating in languid mood above a simple accompaniment*[1]. Finally, the first and the last interludes are held once more in the style of Romantic piano pieces. The former is an improvisation which, although written in triple time throughout, is so full of intricate metric shifts that it

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appears much more 'impromptu' than any of the nineteenth-century pieces formally carrying that name; the latter is an elegant waltz.

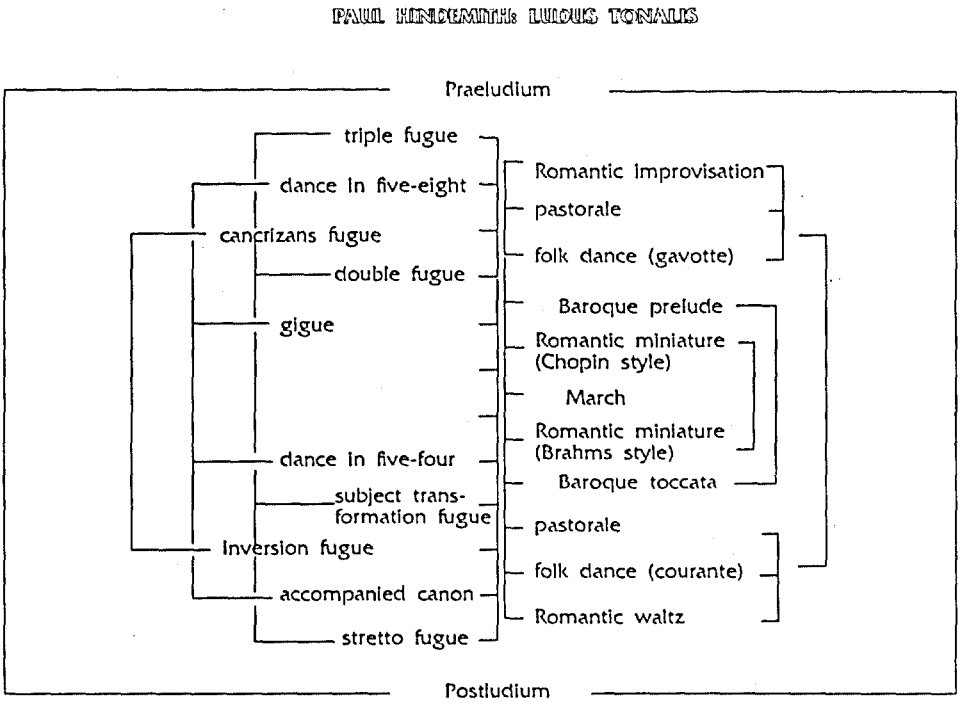

Figure 1

The fugues, too, profess a strikingly symmetrical layout, with possibly the only exception in the centre of the cycle. Fugue 3 and Fugue 10, in actual playing time almost equidistant between the prelude and the postlude, each reflect one of the compositional principles governing the framing pieces: in Fugue 3, the second half retraces the first half in retrograde; in Fugue 10, the second half is the exact inversion of the first.

Next, there are four symmetrically located fugues – the first and the last as well as the fourth and the fourth-from-last (i.e. Fugue 9) – which build on strict contrapuntal technique. At the beginning of the cycle there are a triple fugue (Fugue 1) and a double fugue (Fugue 4); towards the end, the subject appears in

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almost constant stretto in Fugue 12, and developed section by section through all possible transformations in Fugue 9.

The remaining four fugues, while certainly true fugues in both texture and structural layout, actually represent character pieces. There is a dance in five-eight time (Fugue 2), a dance in the rhythmic pattern of a gigue (Fugue 5), a dance-like form building on a subject in five-four organization, and a two-part canon supported by a very metric bass accompaniment.

PAUL HINDEMITH: Series I

"gradually lessening relationship to the centre"

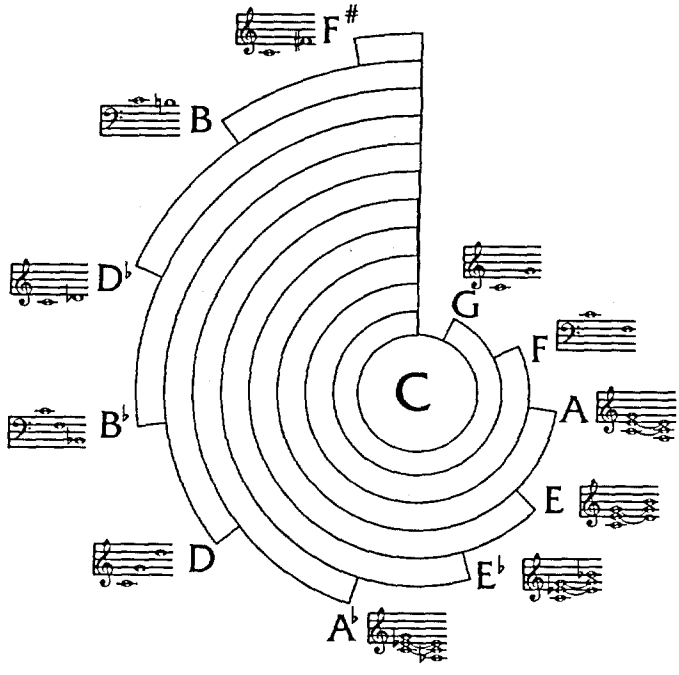

Figure 2

CHAPTER 2

The opposing concept, that of irreversible progression, proves to be equally present in the *Ludus Tonalis*. Although the fact that Hindemith writes twelve fugues on the twelve semitones of the scale has earned his work the nick-name of the 'Well-Tempered Clavier of the Twentieth Century', his tonal organization is by no means that of Bach who, as we all know, progresses chromatically: on each semitone one

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prelude & fugue in the major and one in the minor mode. Nor does Hindemith follow the tonal layout employed e.g. by Chopin in his twenty-four Préludes who proceeds through the circle of fifths, pairing each major-mode piece with one in the relative minor key. These two types of tonal organization are of the symmetrical kind insofar as the distance between all major-mode pieces on the one hand, all minor-key works on the other hand is equal, and the initial work in C major would in fact constitute a logical continuation of the final work in each collection.

Not so in the *Ludus Tonalis*. While Hindemith, like Bach, Chopin and many others before him, begins in C, he interprets this C as the tonal centre of the cycle from which is triggered a progression of tonal areas in an order of gradually lessening relationship. G, the fifth above C, is most closely related, followed by F, the fifth below the centre. Slightly weaker in their relationship to the centre are those tonal areas which draw their relationship to C from presumed triads with shared notes: A, as the keynote of the relative of C major, is thus more closely related to the centre than E (which, as E minor, also shares two notes with the C major triad); $E^{\flat}$, as root of the relative of C minor, follows next in line, preceding $A^{\flat}$ (which, in the form of the $A^{\flat}$ major triad, also shares two notes with the C minor chord). More remote but still related to the centre are D (two fifths above C) and $B^{\flat}$ (two fifths below C). No relationship of natural frequencies but only spatial proximity links the semitones above ($D^{\flat}$) and below ($B^{\natural}$) to the central C. Finally, the irreversible progression away from the centre is concluded with the interval that was regarded as offensive through much of music history: the tritone ($F^{\sharp}$). The tonal organization of the fugues in the *Ludus Tonalis* thus resembles an open spiral (Fig. 2).

CHAPTER 3

Wolfgang Fortner's *Sieben Elegien* (Seven Elegies) date from 1950, a period during which the composer is known to have worked towards his own approach to twelve-tone music. All seven pieces are built on a single dodecaphonic row, and musicologists compare the cycle to Schoenberg's famous *Suite op. 25*. While such a relationship with the exemplary work of the great master of the Second Vienna School may sound fascinating to students of musicology, we must realistically admit that it is not likely to inspire confidence in musicians; Schoenberg's cycle is notoriously difficult for fingers, ear and mind. Equally, the information that the work is strictly serial in its pitch organization will frighten rather than attract most prospective performers – not to speak of their potential audiences.

While the factual information about Fortner's Elegies can thus be expected to cause a shrinking from the work, rather than an interested curiosity towards it, investigations into the use of symmetry in this work can be shown to contribute essentially to an adequate understanding of the tonal vocabulary and grammar. Let me demonstrate this with the help of the first of the *Seven Elegies*.

The elegy is short (44 bars), metrically regular (four-four time throughout), and fairly easy to overlook in its structural layout. A four-bar 'main theme', introduced with a one-bar anticipation of its accompaniment pattern, leads into two short developmental phrases before it recurs in variation. A contrasting secondary

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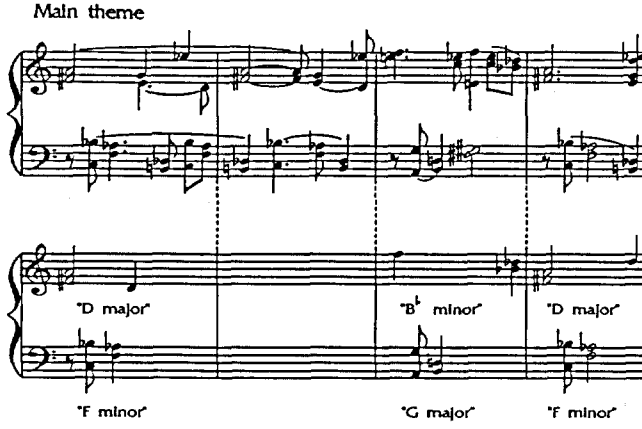
theme, consisting of a four-bar phrase and its sequence in inverted hands, is followed by a transposition of the main-theme variation. After a very melodious closing theme in monodic texture, the elegy is rounded off by a four-bar coda. All thematic material is easy to recognize, containing none of Schoenberg's often highly complicated rhythmic modification. The texture is organized in such a way that all passages allow to clearly distinguish leading voices from secondary lines. So far for the general, quite encouraging details.

Assistance with the musical language can be provided by a number of observations for which the following may serve as an example (Fig. 3).

WOLFGANG FORTNER

SEVEN ELEGIES, NO. I



primary "keys" on polar axis D - f,
contrast on "subdominant" axis G - b♭

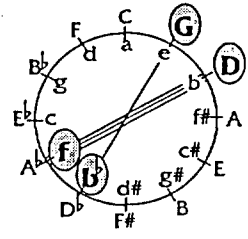


Figure 3

1.a

In the first, second and fourth bars of the main theme, Fortner uses the first half of his twelve-tone row (the pitches C B♭ F A♭ D♭ B♮) to create an accompaniment. Due to the particular interval structure, the first four notes of the row are heard as

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D^7 – T of F minor. The remaining two pitches appear, particularly in the position where the composer places them, as a two-fold leading-note to the dominant ($D^\flat$ $B^\sharp$ being semitones above and below C). The left-hand part would thus sound a fairly tonal F minor. In the thematic right-hand part, the second half of the twelve-tone row is employed to create the impression of D major, with the scale segment D E $F^\sharp$ G A only coloured by an additional – yet spatially detached – $E^\flat$.

1.b

The third bar of the main theme provides a contrast with regard to both the melodic components of the right-hand part and the tonal organization. The left-hand pattern, recognizably related to that of the surrounding bars, evokes G major (with $F^\sharp$ and $G^\sharp$ as double leading-note to the root of the triad), while the right-hand part, particularly towards the end of the bar, tonally refers to $B^\flat$ minor.

1.c

As the graph below the score excerpt shows, the keys paired in each of these two bitonal combinations represent the opposite poles of an axis through the circle of fifths. The main axis D major/F minor is interpolated in the third bar by a second axis G major/$B^\flat$ minor. As the keys in the second pair embody the subdominant of the corresponding keys in the main pair (G = IV/D, $b^\flat$ = iv/f), the main theme of Fortner's *First Elegy* can be interpreted as a 20th century equivalent of the I-IV-I plagal progression.

2.a

The main-theme variation (Fig. 4) is drawn from the original by way of several inversions. The most obvious are the inversion of hands and the mirroring of the pitch lines. Other inversions require closer inspection: the melodic part of the first, second and fourth bars sounds now in minor mode while its accompaniment is in major, and the 'V ^{7} - i' impression is here created in the contrasting third bar (not, as before, in bars 1, 2, and 4).

2.b

These inversions result in an interesting modification of the tonal relationships: the combination of the two polar key-pairs $B^\flat$ major/$C^\sharp$ minor and G major/$B^\flat$ minor creates a perfect axis symmetry*2. The same holds true for the transposition of the main-theme variation, the keys of which are given in the graph at the bottom right corner of the transparency.

2.c

The tonal relationship between the main theme and its variation is rooted in the 'subdominant': both share the secondary axis G major/$B^\flat$ minor. Having observed this, it may hardly come as a surprise that Fortner conceives the transposition of the main-theme variation as a further step in the subdominant progression (see e.g. main tonal axis $E^\flat$ major/$F^\sharp$ minor = subdominant of $B^\flat$ major/$C^\sharp$ minor, etc.).

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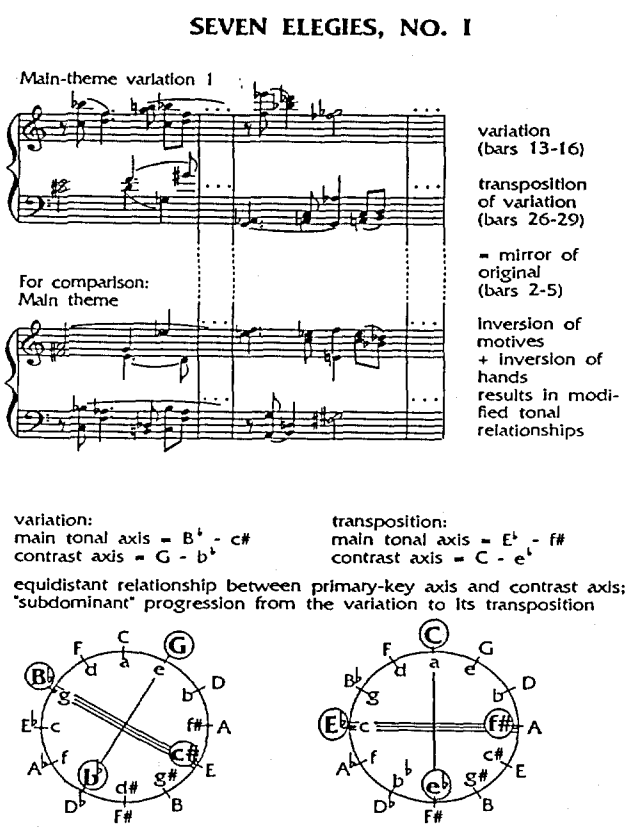

Figure 4

3

Many more details could be mentioned. May it suffice here to add the following brief remarks regarding the remainder of the piece.

3.a

The secondary theme and its sequence, the two phrases in the closing theme, and the two segments within the coda, although each entirely different from the main theme in material, structure, texture etc., are equally each built on one polar axis.

3.b

Secondary theme, closing theme and coda share one axis (F major/G# minor) which represents a further step in the subdominant progression (compare this axis with

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the secondary axis C major/E^{b} minor in the transposition of the main-theme variation).

3.c

Finally, the two axes of the coda complement the continuous subdominant progression with a plagal close (IV-I).

Thus, while the very idea of axis tonalities provides a perfect example for the use of symmetry as a building principle, the various elements and their bi-tonal frames are clearly organized along a single progressive line.

CHAPTER 4

While the two preceding examples have shown the use of symmetry once in the field of musical structure (Hindemith) and once in that of 'musical grammar' (Fortner), Anton Webern's *Variations op. 27* contribute several new aspects to the same concept. I wish to comment on the short piece op. 27/II which represents a particularly intriguing example for symmetry in 'musical vocabulary'. Moreover, this piece demonstrates that without an understanding for the tonal vocabulary, neither phrasing nor emotional content are truly accessible'[3]. Here is a simplified 'dictionary' listing the musical vocabulary employed in this piece (Fig. 5).

The tonal material of this piece consists exclusively of note-pairs. A note repetition on the tuning-fork A serves as a mirror in which are reflected the six intervals from the semitone to the tritone. The note-pairs, however, do not appear in this simple format of closed-position intervals. Webern adds extra flavour (and considerable technical difficulty) to these simple note-pairs by the devices of octave displacement and inversion. He uses only two intervals in their closed position (minor third and tritone), one interval in simple inversion (major third becoming minor sixth), one interval in simple octave displacement (increasing the semitone to a minor ninth), and two intervals inverted and displaced (whole-tone and perfect fourth sounding as compound minor seventh and compound fifth respectively). This is all there is in musical 'syllables'; any nuances stem from either 'pronunciation' or emphasis (i.e. articulation or dynamics).

Deciphering this tonal vocabulary in its symmetrical location around the central A may already have a merit of its own. It would, however, be a task half solved to abandon endeavors at this point, before decoding the 'grammar' as well as the emotional content employed by Webern through and with these musical syllables. In the absence of melodic or rhythmic features, with no structural clues other indicating the repeat of the two halves, the internal layout of this piece appears almost impossible to disclose. Only through an understanding of the syntactical function of the 'syllables' will the phrase structure become transparent.

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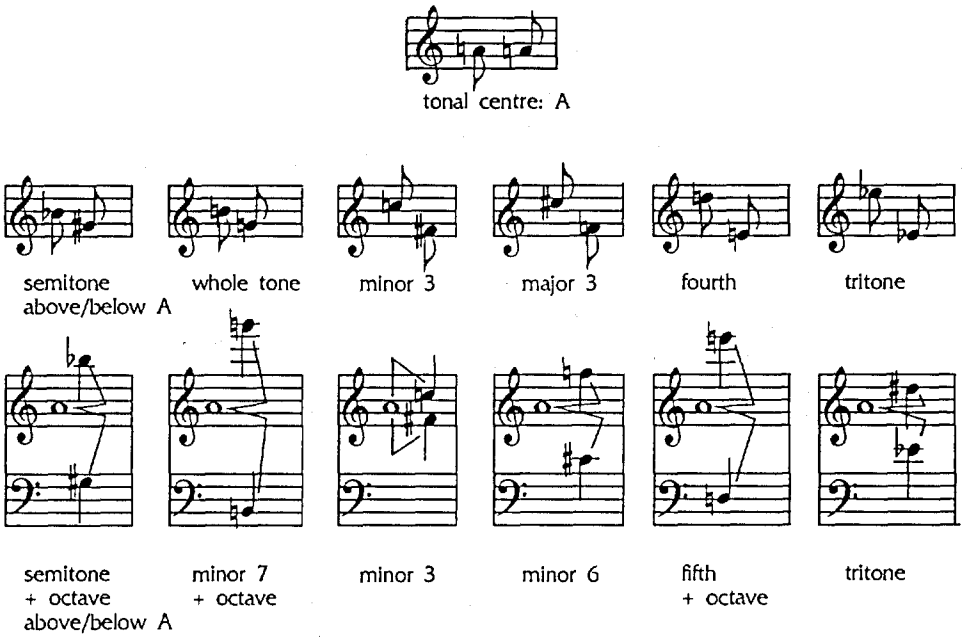
ANTON WEBERN: VARIATIONS OP. 27, II

TONAL MATERIAL



STRUCTURAL USAGE OF PITCH PAIRS

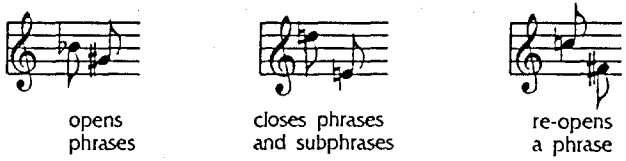


Figure 5

The piece opens with B♭-G♯, the note-pair composed of the semitones above and below the central A. The fact that the same B♭-G♯ also opens the second half of the piece encourages the hypothesis that this pair might serve an initiating function. This is confirmed by the consistent syntactical usage of another 'syllable'. The note-pair concluding the final complete sentence of the piece, D-E, can be found to precede all but one of the assumed phrase-beginning pairs (the exception being the phrase ending before the repeat sign). This observation invites the conjecture of a fixed closing particle. Interestingly, the fact that this phrase-closing pair is based on the perfect fifth interval above and below the central note is strikingly reminiscent of the use of subdominant and dominant in traditional closing formulas.

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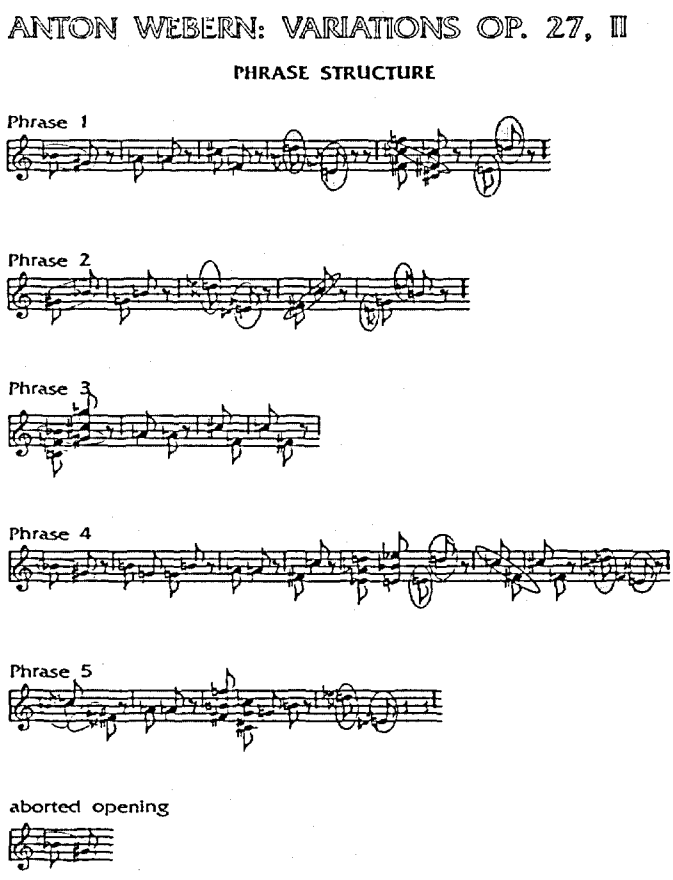

Figure 6

Understanding the syntactical function of these two note-pairs enables the interpreter to distinguish five complete phrases in Webern's op. 27/II (Fig. 6). One step further on in the analysis it can be discovered that phrases 1, 2 and 4 are divided into 'main clause' and 'subordinate clause'. This must be concluded from the fact that the composer uses the closing note-pair occasionally in the middle of a sentence, and that in all cases, this 'half-close' is followed by the same note-pair C-F♯. This syllable C-F♯ can thus be identified as opening subphrases.

One peculiarity in the structure of this piece is worth mentioning, particularly since it seems to stress the aspect of progression vs. that of symmetry, and in a very unique way. The only moments in the course of the composition which, at first glance, would seem to require no interpretation with respect to their structural relevance, the note-pairs before the repeat of the first half and at the very end of the piece, actually both present significant exceptions from the otherwise orderly phrase structure. The final phrase of the first section ends with C-F♯, the note-pair which, in the two preceding phrases, opens the subordinate clauses. Even more drastically are the final notes of the piece. Separated from the fifth phrase by a

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striking four-beat rest, the composition ends with the 'opening' note-pair $B^{\flat}$-G#. Both halves thus break off with grammatically incomplete sentences – thus pointing towards a 'progression' beyond the notes?

CHAPTER 5

Perceptions such as the one regarding what I call the 'aborted openings' are of course of supreme importance for the performer and, indirectly, for the listener. This brings me to the emotional content of the piece. (Fig. 7) Webern takes greatest care in determining the colour and intensity of each of his musical syllables, combining the seven symmetrical pitch-pairs with five kinds of articulation (*legato, staccato, legatissimo, staccato* preceded by *acciaccaturas, marcato*) and three degrees of loudness (*p, f, ff*). Furthermore, he creates agogic tension by means of rhythmic contraction (omitting the rest in his basic quaver pattern of note/note/rest). The use of these parameters allows the following observations:

The note-pair A-A is the only one to retain its colour and intensity (*staccato/p*) in each recurrence. Already low in tension due to its position at the axis of the symmetrical system and to its note repetition, the consistent colouring specifies the expressive function of this note-pair as relaxed or introverted. At the other end of the spectrum, marcato occurs exclusively in superimposed note-pairs (i.e. chords), in either *f* or *ff*. These combinations must thus be identified as extroverted. Both 'introverted' and 'extroverted' pairs are introduced in phrase 1.

Phrase 2 begins with the agogic enhancement mentioned above. The omitted rest between two note-pairs creates tension not through quantity of loudness and notes as in the 'extroverted' *marcato* chords, but through intensification on the plane of time. The assumption that this 'two-syllable word' constitutes a distinct item in the vocabulary of this work is confirmed by the fact that its recurrences, towards the end of phrases 3 and 4, also combine chromatically adjacent pitch-pairs in the articulation *legato/staccato* and the dynamic setting *f/p*.

As a development of this 'two-syllable' feature there appears, right after the opening of phrase 4, the rhythmic contraction of two identical pairs in the form of a horizontal symmetry (B-G/G-B). Highest intensification is achieved where, in phrases 4 and 5, all enhancing features coincide. The 'extroverted' *marcato/ff* chords are mirrored in rhythmic contraction by one of the note-pairs.

As a conclusion, it is intriguing to observe how the understanding of Webern's musical language on the basis of his use of symmetry as a building principle can even help interpreters determine the emotional content and the grades of intensity in the composition. This is particularly important since Webern's repeated use of only three dynamic degrees might otherwise give the erroneous impression of a music static in terms of its expression.

– Phrase 1 contains both the 'introverted' and the 'extroverted' pairs, one in each of its subphrases. It is thus balanced in its expressive content.

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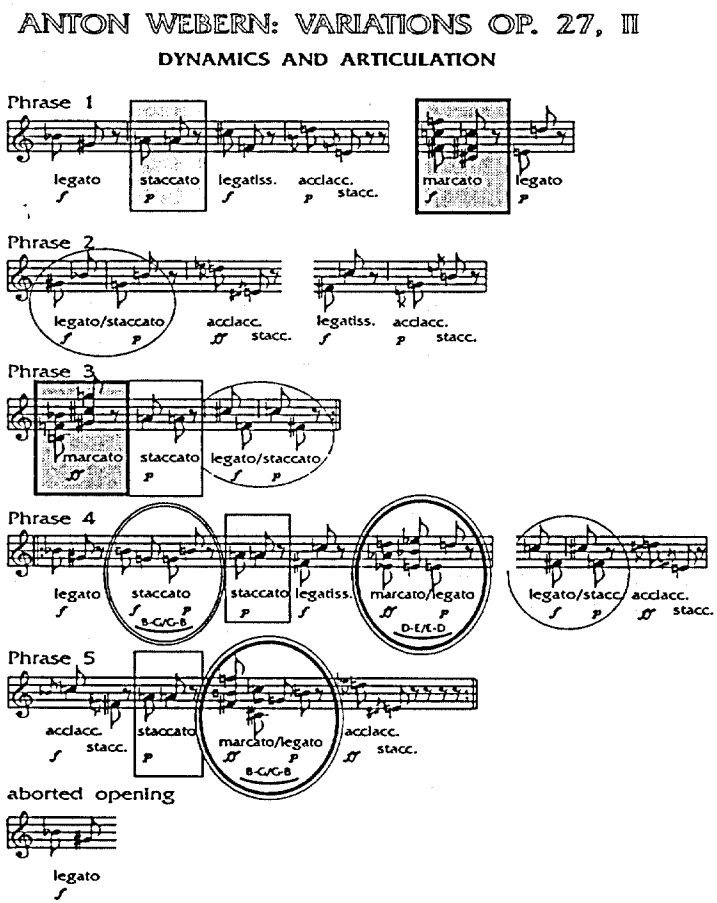

Figure 7

– Phrase 2 by contrast does not feature either of these special pairs. The dramatic tension of its first subphrase, stemming from the rhythmic contraction at the outset and the first use of *ff*, abates in the subordinate clause.

– Phrase 3 presents, at its very beginning, an immediate contrast of the extroverted with the introverted pair. This is followed by the intensifying feature of rhythmic contraction. The open phrase-ending (see the absence of the 'closing' pair D-E) gives this highly-strung phrase a strong quality of restlessness.

– Phrase 4 is not only the longest but also clearly the most dramatic, containing all of the emotionally designated features. In its first subphrase the opening pair, followed by the rhythmically contracted mirror and the 'introverted' pair leads through the 'subphrase-opener' directly into the first multi-feature climax. The shorter second subphrase incorporates the 'secondary opener' into another rhythmic contraction which is then rounded off with the closing pair.

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– Phrase 5 sets out with a combination of the pitch pairs identified as primary and secondary phrase-opener ($B^{\flat}$-$G^{\sharp}$ with C-$F^{\sharp}$). Tension is suspended in the subsequent introverted pair but then bursts out into the second climax of the piece. By comparison with the preceding phrase 4 which contains the other main climax, phrase 5 is conceived as more concise.

CHAPTER 6

It has been attempted to show, in a few examples out of what could be many more, how a mode of expression relying as vitally on the course of time as music does, can nevertheless be open to symmetric organization. To be sure, we are not expecting audiences to become intellectually aware of a symmetry – no more than we expect them to grasp the details of a modulation in nineteenth-century music, or to understand the poetic depth of the lyrics in a song recital. But we can – and should – request performers to thoroughly understand whatever language they are reciting.

The fact that none of these symmetries are likely to be perceived as such by all addressees (i.e. music listeners) and most mediators (i.e. performers) should not discourage anybody, as the same holds true for such intricately beautiful symmetries as those photographed in snowflakes. The futility of the art work obviously does nothing to prevent nature from repeating it in ever new designs.

REMARKS

*1 I owe thanks to Prof. Dénes Nagy, president of the International Society for the Interdisciplinary Study of Symmetry, who pointed out to me the importance of a dissymmetric element in every basically symmetric organism. I intend to investigate further whether or in which respect his statement that such dissymmetry often constitutes the life force of an organism (see the heart in its off-centre location) applies in music.

*2 Although the axis system manifest in Fortner's piece is not identical with that used e.g. by Bartók, my understanding of these tonal relationships is greatly indebted to Ernő Lendvai's study of tonality (1979).

*3 To appreciate what is at stake: this is an extremely fast piece of about half-minute duration, employing exclusively one rhythmic value (the quaver or eight note), and featuring pitches distributed without any melodic connection over four octaves. Played by an uncomprehending performer, it is most likely to sound like a madam's nightmare.

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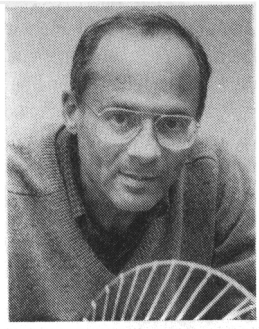
REDUCTION AS SYMMETRY

Joe Rosen



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Publications: *Symmetry Discovered: Concepts and Applications in Nature and Science*, Cambridge: Cambridge University Press, 1975, xi + 138 pp.; Resource Letter SP-2 (ed.): Symmetry and group theory in physics, *American Journal of Physics*, 49 (1981), 304-319; Reprinted in: Rosen, J., ed. *Symmetry in Physics*, College Park MD: American Association of Physics Teachers, 1982, 153 pp., 1-16; Also reprinted in: *AAPT Resource Letters*, Book Four, College Park MD: American Association of Physics Teachers, 1983; *A Symmetry Primer for Scientists*, New York: Wiley, 1983, xiv + 192 pp.; Fundamental manifestations of symmetry in physics, *Foundations of Physics*, 20 (1991), 283-307.

Science operates by reduction, whereby we separate nature into simpler parts that we attempt to understand individually under the assumption that the whole is understandable as the sum of its parts. Reduction implies symmetry, according to the conceptual formulation of symmetry as immunity to a possible change. That is so because, if a part of nature can be understood individually, then it exhibits order and law regardless of what is going on in the rest of nature, which is immunity (of aspects of that part of nature) to possible changes (in the rest of nature). Three ways science commonly reduces nature – observer and observed, quasi-isolated system and surroundings, and initial state and law of evolution – are considered and the symmetry implied by each is examined.

1. REDUCTION

For the purpose of our discussion we take *nature* to mean *the material universe with which we can, or can conceivably, interact*. The material universe is everything having a purely material character. To interact with something is to act upon it and be

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acted upon by it. That implies the possibility of performing observations and measurements on it and of receiving data from it, which is what we are actually interested in. To be able conceivably to interact means that, although we might not be able to interact at present, interaction is not precluded by any principle known to us and is considered attainable through further technological research and development. Thus nature, as the material universe with which we can, or can conceivably, interact, is everything of purely material character that we can, or can conceivably, observe and measure.

That is my own conception of nature. If your conception is different from mine, please set it aside for the time being, as the following discussion is based on the definition of nature just presented.

We live in nature, observe it, and are intrigued. We try to understand nature in order both to improve our lives by better satisfying our material needs and desires and to satisfy our curiosity. And what we observe in nature is a complex of phenomena, including ourselves, where we are related to all of nature, as is implied by our definition of nature as the material universe with which *we can, or can conceivably, interact.* This possibility of interaction is what relates us to all of nature and, due to the mutuality of interaction and of the relation it brings about, relates all of nature to us. It then follows that all aspects and phenomena of nature are actually interrelated, whether they appear to be so or not; whether they are interrelated independently of us or not, they are certainly interrelated through our mediation. Thus all of nature, including *Homo sapiens*, is interrelated and integrated.

Science is our attempt to understand the reproducible and predictable aspects of nature. But how are we to grasp this wholeness, this integrity? When we approach nature in its completeness, it appears so awesomely complicated, due to the interrelation of all its aspects and phenomena, that it might seem utterly beyond hope to understand anything about it at all. True, some obvious simplicity stands out, such as day-night periodicity, the annual cycle of the seasons, and the fact that fire consumes. And subtler simplicity can be discerned, such as the term of pregnancy, the relation between clouds and rain, and that between the tide and the phase of the moon. Yet, on the whole, complexity seems to be the norm, and even simplicity, when considered in more detail, reveals wealths of complexity. But, again due to nature's unity, any attempt to analyze nature into simpler component parts cannot but leave something out of the picture.

Holism is the world view that nature can be understood only in its wholeness or not at all. And that includes human beings as part of nature. As long as nature is not yet understood, there is no reason *a priori* to consider any aspect or phenomenon of it as being intrinsically more or less important than any other. Thus it is not meaningful to pick out some part of nature as being more 'worthy' of investigation than other parts. Neither is it meaningful, according to the holist position, to investigate an aspect or phenomenon of nature as if it were isolated from the rest of nature. The result of such an effort would not reflect the normal behavior of that aspect or phenomenon, since in reality it is not isolated at all, but is interrelated with all of nature, including ourselves.

On the other hand lies the world view called *reductionism*, which is that nature is indeed understandable as the sum of its parts. According to the reductionist posi-

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tion nature should be studied by analysis, should be 'chopped up' (mostly conceptually, of course) into simpler component parts that can be individually understood. (By 'parts' we do not necessarily mean actual physical parts; the term might be used metaphorically. An example of that is presented in section 5.) A successful analysis should then be followed by synthesis, whereby the understanding of the parts is used to help attain understanding of larger parts compounded of understood parts. If necessary, that should then be followed by further synthesis, further compounding of the compound parts to obtain even larger parts and attaining understanding of the latter with the help of the understanding achieved so far. And so on to the understanding of ever larger parts, until we reach an understanding of all of nature.

Now, each of the poles of holism and reductionism has a valid point to make. Nature is certainly interrelated and integrated, at least in principle, and we should not lose sight of this fact. But if we hold fast to extreme holism, everything will seem so fearsomely complicated that it is doubtful if we will be able to do much science. Separating nature into parts seems to be the only way to search for simplicity within nature's complexity. But a position of extreme reductionism might also not allow much science progress, since nature might not be as amenable to reduction as this position claims. So science is forced to the pragmatic mode of operating as *if* reductionism were valid and adhering to that for as long as it works. But it should be kept in mind that the inherent integrity of nature can raise its head at any time and indeed does so. The most well known aspect of nature's irreducibility is nature's quantum character (Davies and Brown, 1986; Davies 1980).

2. SYMMETRY

Symmetry at its most fundamental is the possibility of making a change that leaves some aspect of the situation unchanged, or, most succinctly, *symmetry is immunity to a possible change* (Rosen, 1990a, 1990b). This is the conceptual formulation of symmetry. It might also be called the qualitative formulation of symmetry, in contrast to the group-theoretical formulation, which might be called the quantitative formulation of symmetry. The latter is expressed in terms of transformations (or operations), transformation groups, equivalence relations, equivalence classes, symmetry transformations (or symmetry operations), and symmetry groups, and can be developed from the conceptual formulation (Rosen, 1983). For the description and treatment of many derivative applications of symmetry in science the group-theoretical formulation is the appropriate one. But for the account of the fundamental manifestations of symmetry in science it is the conceptual formulation that is the more suitable (Rosen, 1990b).

As an example of symmetry, consider a uniform metal plate in the shape of an equilateral triangle. There are many changes that might be imposed on this system, and among the possible changes there are those that indeed leave some aspect of the system unchanged. For example, rotating the triangle by 120° or 240° about its center within its plane is a change, but does not affect the appearance or macroscopic physical properties of the system. Thus the piece of metal of this example possesses symmetry under these rotations with respect to appearance and macroscopic physical properties. If the metal were not uniform or the triangle were not equilateral or had a corner chopped off, the system would not possess this symme-

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try. It must be emphasized that there are aspects of the system that are *not* left unchanged by these rotations. For example, molecular positions are certainly altered.

Reduction in science, the separation of nature into parts that can be individually understood, implies symmetry. The point is that if a reduction separates out a part that can be understood individually, then that part exhibits order and law regardless of what is going on in the rest of nature. In other words, that part possesses aspects that are immune to possible changes in the rest of nature. And that is symmetry, by the conceptual formulation of symmetry presented above. It is then possible to make a change (in the rest of nature) that leaves some aspect of the situation (some aspect of that part of nature) unchanged.

Reduction of nature can be carried out in many different ways. As the old saying goes, there's more than one way to slice a salami. We will now consider three ways reduction is commonly applied in science, three ways that nature is commonly 'sliced up,' and will examine the symmetry implied by each.

3. OBSERVER AND OBSERVED

The most common way of reducing nature is to separate it into two parts: the *observer* - us - and the *observed* - the rest of nature. This reduction is so obvious that it is often overlooked. It is so obvious because in doing science we *must* observe nature to find out what is going on and what needs to be understood. Now what is happening is this: Observation is interaction, so we and the rest of nature are in interaction, are interrelated, as was pointed out above. Thus anything we observe inherently involves ourselves too. The full phenomenon is thus at least as complicated as *Homo sapiens*. Every observation must include the reception of information by our senses, its transmission to our brain, its processing there, its becoming part of our awareness, its comprehension by our consciousness, etc. We appear to ourselves to be so frightfully complicated, that we should then renounce all hope of understanding anything at all.

So we reduce nature into us, on the one hand, and the rest of nature, on the other. The rest of nature, as complicated as it might be, is much less complicated than all of nature, since we have been taken out of the picture. We then concentrate on attempting to understand the rest of nature. (We also might, and indeed do, try to understand ourselves. But that is another story.) However, as we saw above, since nature with us is not the same as nature without us, what right have we to think that any understanding we achieve by our observations is at all relevant to what is going on in nature when we are not observing? The answer is that in principle we simply have no such right *a priori*. What we are doing is *assuming*, or adopting the working hypothesis, that the effect of our observations on what we observe is sufficiently weak or can be made so, that what we actually observe well reflects what would occur without our observation, and that the understanding we reach under this assumption is well relevant to the actual situation. This assumption might be a good one or it might not, its suitability possibly depending on the aspect of nature that is being investigated. It is ultimately assessed by its success or failure in allowing us to understand nature.

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As is well known, the observer-observed analysis of nature is very successful in many realms of science. One example is Newton's explanation of Kepler's laws of planetary motion. This excellent understanding of an aspect of nature was achieved under the assumption that observation of the planets does not affect their motion substantially. In general, the reduction of nature into observer and observed seems to work very well from astronomical phenomena down through everyday-size phenomena and on down in size to microscopic phenomena. However, at the microscopic level, such as in the biological investigation of individual cells, extraordinary effort must be invested to achieve a good separation. The ever-present danger of the observation's distorting the observed phenomena, so that the observed behavior does not well reflect the behavior that would occur without observation, must be constantly circumvented.

At the molecular, atomic, and nuclear levels and at the subnuclear level, that of the so-called elementary particles and their structure, the observer- observed analysis of nature does not work. Here it is not merely a matter of lack of ingenuity or insufficient technical proficiency in designing devices that minimize the effect of the observation on the observed phenomena. Here it seems that the observer-observed interrelation cannot be disentangled *in principle*, that nature holistically absolutely forbids our separating ourselves from the rest of itself. Quantum theory is the branch of science that successfully deals with such matters (Davies and Brown, 1986; Davies, 1980). From it we learn that nature's observer-observed disentanglement veto is actually valid for *all* phenomena of *all* sizes. Nevertheless, the *amount* of residual observer-observed involvement, after all efforts have been made to separate, can be more or less characterized by something like atom size. Thus an atom-size discrepancy in the observation of a planet, a house or even a cell is negligible, while such a discrepancy in the observation of an atom or an elementary particle is of cardinal significance.

One aspect of the symmetry implied by the observer-observed reduction, when the latter is valid, is that the behavior of the rest of nature (i.e., nature without us) is unaffected by and independent of our observing and measuring. That behavior is thus an aspect of nature that is immune to certain possible changes, the changes being changes in our observational activities. It is just this symmetry that allows the compilation of objective, observer-independent data about nature that is a *sine qua non* for the very existence of science. It is intimately related to *reproducibility*, which was also shown to be a symmetry (Rosen, 1989a, 1989b).

Inversely, another aspect of this symmetry is that our observational activity is unaffected by and independent of the behavior of the rest of nature, at least in certain respects and to a certain degree. For example, if we had an ideal thermometer, we would make exactly the same temperature measurement regardless of the system whose temperature is being taken. (In practice, of course, things are not so simple.) The symmetry here is that our observational activity is an aspect of nature that is (at least ideally) immune to changes in what is being observed. This symmetry, as limited as it might be in practice, allows the setting up of measurement standards and thus allows the meaningful comparison of observational results for different systems. For instance, we can meaningfully compare the temperature of the sea with that of the atmosphere.

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4. QUASI-ISOLATED SYSTEM AND SURROUNDINGS

Whenever we reduce nature to observer and the rest of nature, we achieve simplification of what is being observed, because, instead of observing all of nature, we are then observing only what is left of nature after we ourselves are removed from the picture. Yet even the rest of nature is frightfully complicated. That might be overcome by the further slicing of nature, by separating out from the rest of nature just that aspect or phenomenon that especially interests us. For example, in order to study liver cells we might remove a cell from a liver and examine it, still living, under a microscope.

But what right have we have to think that by separating out a part of nature and confining our investigation to it, while completely ignoring the rest, we will gain meaningful understanding? We have in principle no right at all *a priori*. Ignoring everything going on outside the object of our investigation will be meaningful if the object of our investigation is not affected by what is going on around it, so that it really does not matter what is going on around it. That will be the case if there is no interaction between it and the rest of nature, i.e., if the object of our investigation is an *isolated system*.

Now, an isolated system is an idealization. By its very definition we cannot interact with an isolated system, so no such animal can exist in nature, where nature is, we recall, the material universe with which *we can, or can conceivably, interact*. The state of our present understanding of nature, as incomplete as it might be, is still sufficient to preclude the existence of systems that are somehow observable yet are isolated from the rest of nature. The known anti-isolatory factors include the various forces of nature, which can either be effectively screened out or can be attenuated by spatial separation (Davies, 1986). Additional anti-isolatory factors involve quantum effects and inertia, which can be neither screened out nor attenuated. Thus even the most nearly perfectly isolated natural system is simply not isolated, and I therefore prefer the term *quasi-isolated system* for a system that is as nearly isolated as possible.

The separation of nature into quasi-isolated system and surroundings will be a reduction, if, in spite of the system's lack of perfect isolation, there are aspects of the system that are nevertheless unaffected by its surroundings. And the fact of the matter is that the investigation of quasi-isolated systems does yield considerable understanding, thus proving quasi-isolation to be a reduction of nature. Indeed, science successfully operates and progresses by the double reduction of nature into observer and observed and the observed into quasi-isolated system and its surroundings.

One side of the symmetry implied by this reduction is that those aspects of quasi-isolated systems that are not affected by their surroundings are aspects of nature that are immune to possible changes, the changes being changes in the state of the surroundings. This symmetry is intimately related to *predictability*, which was also shown to be a symmetry (Rosen, 1989a, 1989b). Inversely, due to the mutuality of interaction or of lack of interaction, there are also aspects of the surroundings of quasi-isolated systems that are immune to certain changes in the state of the quasi-isolated systems. That is another side of the symmetry implied by this reduction.

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5. INITIAL STATE AND LAW OF EVOLUTION

The previous two ways of reducing nature – separation into observer and the rest of nature and separation into quasi-isolated system and its surroundings – are literal applications of the reductionist position. The present way of reducing is a metaphorical application, or a broadening of the idea of a part of nature. Rather than a separation that can usually be envisioned spatially – observer here, observed there, or quasi-isolated system here, its surroundings around it – the present reduction is a conceptual separation, the separation of natural processes into initial state and law of evolution.

Things happen. Events occur. Changes take place. *Nature evolves.* That is the relentless march of time. The process of nature's evolution (where 'evolution' is intended in the general sense of temporal development) is of special interest to scientists, since predictability, one of the cornerstones of science, has to do with telling what will be in the future, what will evolve in time. Nature's evolution is certainly a complicated process. Yet order and law can be found in it, when it is properly sliced. First the observer should separate him- or herself from the rest of nature. Then he or she should narrow the scope of investigation from all of the rest of nature to quasi-isolated systems and investigate the natural evolution of such systems only. Actually, it is only for quasi-isolated systems that order and law are found. (This statement is really more flexible than it might sound. The demand of quasi-isolation can be relaxed, along with a softening of what is considered order and law.)

Finally, and this is the present point, the natural evolution of quasi-isolated systems should be analyzed in the following manner. The evolution process of a system should be considered as a (continuous or discrete) sequence of states in time, where a state is the condition of the system at any time. For example, the solar system evolves, as the planets revolve around the Sun and the moons revolve around their respective planets. Now imagine that some duration of this evolution is recorded on a reel of photographic film or on a videocassette. Such a recording is actually a sequence of still pictures. Each still picture can be considered to represent a state of the solar system, the positions of the planets and moons at any time. The full recording, the reel or cassette, represents a segment of the evolution process.

Then the state of the system at every time should be considered as an *initial state*, a precursor state, from which the following remainder of the sequence develops, from which the subsequent process evolves. For the solar system, for instance, the positions of the planets and moons at every single time, such as when it is twelve o'clock noon in Tel-Aviv on 20 October 1989, say, or any other time, should be considered as an initial state from which the subsequent evolution of the solar system follows.

When that is done, when natural evolution processes of quasi-isolated systems are viewed as sequences of states, where every state is considered as an initial state initiating the system's subsequent evolution, then it turns out to be possible to find order and law. What turns out is that, with a good choice of what is to be taken as a state for any quasi-isolated system, one can discover a law that, given *any* initial

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state, gives the state that evolves from it at *any* subsequent time. Such a law, since it is specifically concerned with evolution, is referred to as a *law of evolution.*

For an example let us return to the solar system. It turns out that the specification of the positions of all the planets, moons, etc. at any single time is insufficient for the prediction of their positions at later times. Thus the specification of states solely in terms of position is not a good one for the purpose of finding lawful behavior. However, the description of states by both the positions and the velocities of the planets, moons, etc. at any single time does allow the prediction of the state evolving from any initial state at any subsequent time. The law of evolution in this case consists of Newton's three universal laws of motion and law of universal gravitation.

So the reduction needed to enable the discovery of order and law in the natural evolution of quasi-isolated systems is the conceptual splitting of the evolution process into initial state and law of evolution. The usefulness of such a separation depends on the independence of the two 'parts', on whether for a given system the same law of evolution is applicable equally to any initial state and whether initial states can be set up with no regard for what will subsequently evolve from them. Stated in other words, the analysis of the evolution process into initial state and law of evolution will be a reduction, if, on the one hand, nature indeed allows us (at least in principle) complete freedom in setting up the initial state, i.e., if nature is not at all concerned with initial states, while, on the other hand, what evolves from an initial state is entirely beyond our control.

This reduction of evolution processes into initial states and laws of evolution has proved to be admirably successful for everyday-size quasi-isolated systems and has served science faithfully for ages. Its extension to the very small seems quite satisfactory, although when quantum theory becomes relevant, the character of an initial state becomes quite different from what we are familiar with in larger systems. Its extension to the large, where we cannot actually set up initial states, is also successful. But we run into trouble when we consider the universe as a whole. One reason for this is that the concept of law is irrelevant to the universe as a whole (Rosen, 1981, 1991). Another reason is that it is not at all clear whether the concept of initial state is meaningful for the universe as a whole; I do not think it is (Rosen, 1987).

The symmetry that is implied by reduction into initial state and law of evolution follows immediately from the independence of the two 'parts', as described in the paragraph before last. On the one hand, laws of evolution are an aspect of nature that is immune to possible changes, the changes being changes in initial states. On the other hand, initial states are an aspect of nature that is immune to possible changes, where the changes are hypothetical changes in laws of evolution, in the sense that initial states can be set up with no regard for what will subsequently evolve from them. This symmetry, too, is intimately related to *predictability* (Rosen, 1989a, 1989b).

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Symmetry: Culture and Science
Vol. 3, No. 2, 1992, 211-212

SFS: SYMMETRIC FORUM OF THE SOCIETY (BULLETIN BOARD)

All correspondence should be sent to the editors. For the list of publications of members (and non-members) refer to the 'Symmetric Reviews' in Symmetro-graphy.

SYMMETRIC NEWS

The Second Interdisciplinary Symmetry Symposium and Exhibition of the Society: *Symmetry of Patterns* was held at the Synergetics Institute, in Hiroshima, Japan, August 17-23, 1992.

Participants from 14 countries of four continents gave lectures and/or exhibited their works during the six days' program, or organized workshops, panel discussion or brain storming at the evenings.

One of the most exciting events was the *minisymposium on quasicrystals*. A special *origami festival* was devoted to the Japanese arts and crafts, with workshops at the exhibition.

Several *board meetings* were included in the program, however, many board members couldn't take part at the Symposium, so formal decisions were not made. The President and the Executive Secretary gave account on the three years' activity as well as the financial affairs of the Society. They reported the establishment of the International Symmetry Foundation, which should facilitate the better financial conditions (tax advantages) of the Society; as well as the reforming of the Society's Budapest Office into *Symmetrion – The Institute for Advanced Symmetry Studies,* emphasizing the shift from administrative functions to professional activity: organization of lectures, courses, exhibitions etc. In the lack of the majority of board members, the mandate of ISIS-Symmetry's officials was prolonged for the next three years. The board should be revised. The Symposium participants remembered two board members: *Andrew Duff-Cooper* and *Jarek Woloszyn*, who passed away last year. The invitation of some new members to the Board was decided.

ISIS-Symmetry plans its 3rd International Interdisciplinary Symmetry Symposium and Exhibition in 1995. There are invitations to the following places to organize forthcoming events: Germany (Ulm and North German spots), Hungary (Szeged), India (Bombay), the Netherlands (Delft), Portugal (Lisbon), USA (Seattle, Washington and Tempe, Arizona). The logistical and financial conditions will motivate the final decisions.

Specialized regional meetings are planned between the Society's three years' symposia, in 1993 and 1994.

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According to a decision, made in Hiroshima, an *E-mail journal* (Bulletin Board) is under organization by George Lugosi (Howard Florey Institute, University of Melbourne, Parkville, Vic. 3052 Australia, E-mail: *george@hfivax.hfi.unimelb.edu.au* or *x9902975@ucsvc.ucs.unimelb.edu.au*) which will be available for everybody who joins the 'Club' and has an access to use E-mail. The E-mail journal will be updated continuously and appear on your screen whenever you contact the E-mail address which will be identified soon.

Decision was also made on a special issue on *Origami*, during the Hiroshima Symposium.

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Symmetry: Culture and Science
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RECREATIONAL SYMMETRY

This is a non-regular section for symmetry-related problems and puzzles (symmetr-enigmas), as well as games, computer programs, descriptions of scientific toys, and other topics which are connected with both recreation and education.

FLEXING POLYHEDRA

Nets by Caspar Schwabe

On the following four sheets you will find the nets of the four flexing polyhedra illustrated in C. Schwabe's abstract on pp. 168-169 of this issue.

Before cutting out the sheets, please enlarge them by Xerox to 175 per cent on A3 paper size, if you wish colored paper of about 170 grams per m^2.

The nets are the courtesy of Caspar Schwabe.

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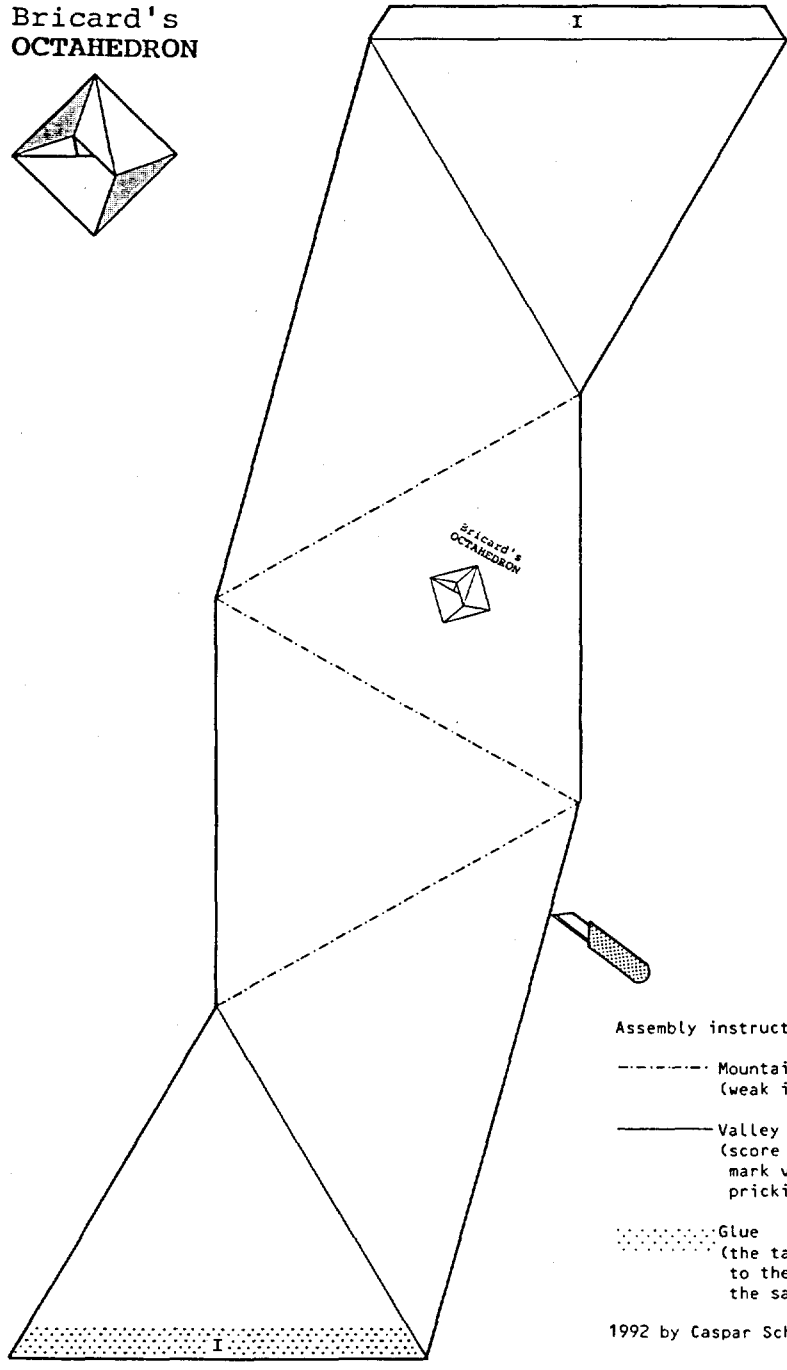
Bricard's
OCTAHEDRON



Assembly instruction:

Mountain folds
(weak incision)

Valley folds
(score on the back,
mark vertices by
pricking with pin)

Glue
(the tap is joined
to the edge with
the same number)

1992 by Caspar Schwabe, Zürich

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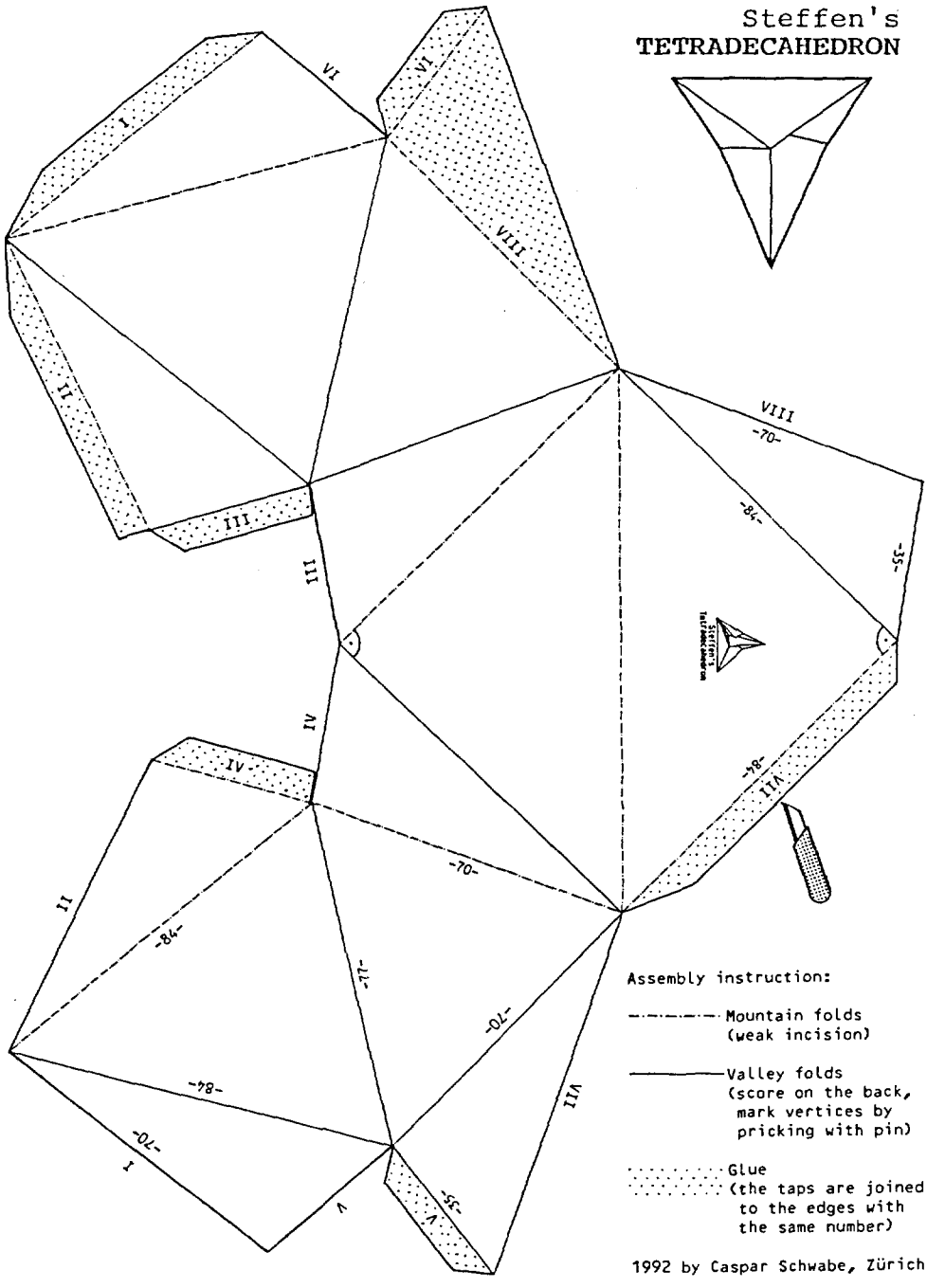

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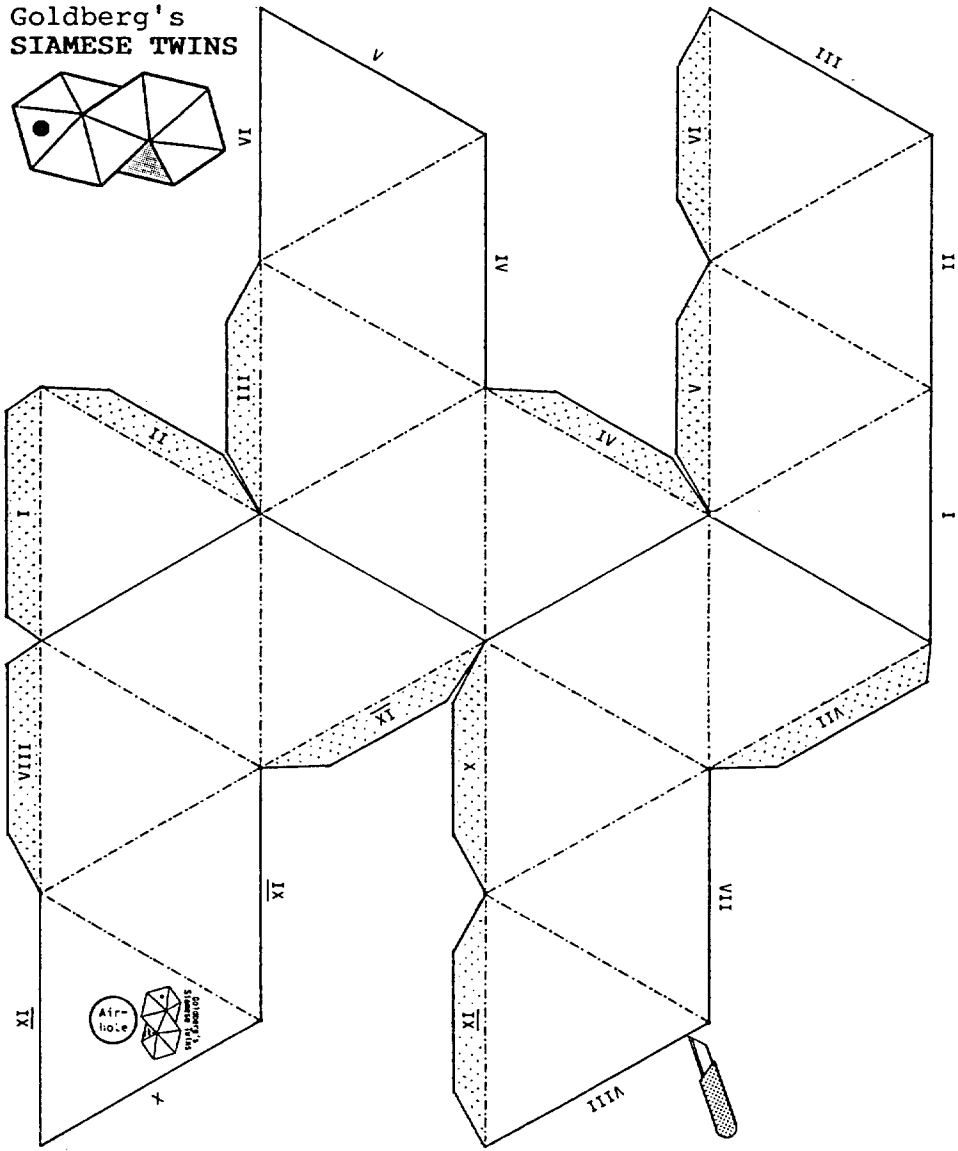

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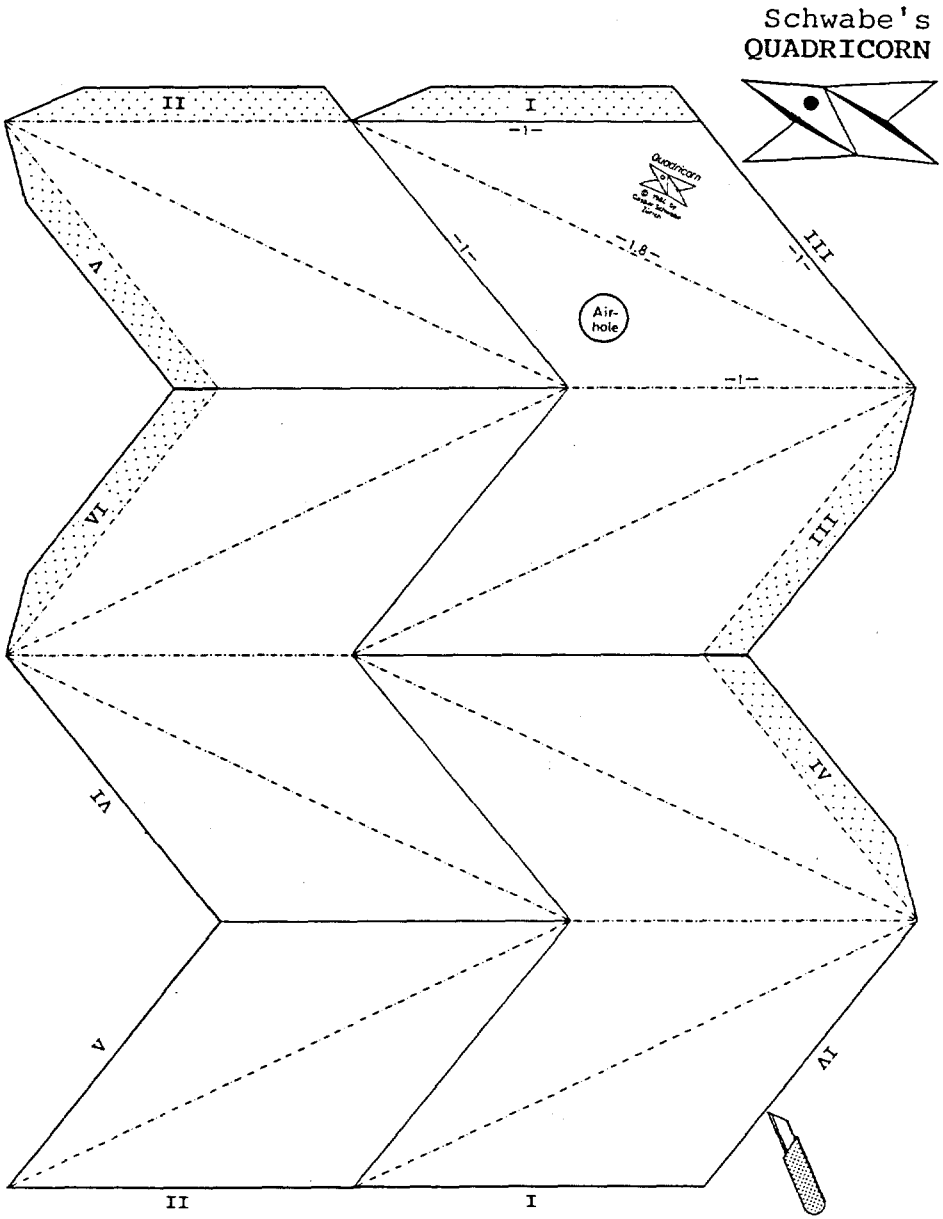

1992 by Caspar Schwabe, Zürich

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Peter Klein (left) plays on Caspar Schwabe's 'accordion'; it is a flexible polyhedron (see on the opposite page) that actually provides music by pressing out the air through a whistle. Vojtěch Kopský (right) enjoys the performance. Luckily his umbrella was not required

Photo: Caspar Schwabe

Shutaro Mukai with Caspar Schwabe's *quadricorn*.

Photo: Caspar Schwabe

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Contributions to *SYMMETRY: CULTURE AND SCIENCE* are welcomed from the broadest international circles and from representatives of all scholarly and artistic fields where symmetry considerations play an important role. The papers should have an interdisciplinary character, dealing with symmetry in a concrete (not only metaphorical!) sense, as discussed in 'Aims and Scope' on p. 224. The quarterly has a special interest in how distant fields of art, science, and technology may influence each other in the framework of symmetry (symmetrology). The papers should be addressed to a broad non-specialist public in a form which would encourage the dialogue between disciplines.

Manuscripts may be submitted directly to the editors, or through members of the Board of ISIS-Symmetry.

Contributors should note the following:

- All papers and notes are published in English and they should be submitted in that language. The quarterly reviews and annotates, however, non-English publications as well.

- In the case of complicated scientific concepts or theories, the intuitive approach is recommended, thereby minimizing the technical details. New associations and speculative remarks can be included, but their tentative nature should be emphasized. The use of well-known quotations and illustrations should be limited, while rarely mentioned sources, new connections, and hidden dimensions are welcomed.

- The papers should be submitted either by electronic mail to both editors, or on computer diskettes (5 ¼" or 3.5") to György Darvas as text files (IBM PC compatible or Apple Macintosh); that is, conventional characters should be used (ASCII) without italics or other formatting commands. Of course typewritten texts will not be rejected, but the preparation of these items takes longer. For any method of submission (e-mail, diskette, or typescript), four hard-copies of the text are also required, where all the necessary editing is marked in red (inserting non-ASCII characters, underlining words to be italicized, etc.). Three hard-copies, including the master copy and the original illustrations, should be forwarded to György Darvas, while the fourth copy should be sent to Dénes Nagy. No manuscripts, diskettes, or figures will be returned, unless by special arrangement.

- The papers are accepted for publication on the understanding that the copyright is assigned to ISIS-Symmetry. The Society, however, aiming to encourage the cooperation, will allow all reasonable requests to photocopy articles or to reuse published materials. Each author will receive a complimentary copy of the issue where his/her article appeared.

- Papers should begin with the title, the proposed running head (abbreviated form of the title of less than 35 characters), the proposed section of the quarterly where the article should appear (see the list in the note 'Aims and Scope'), the name of the author(s), the mailing address (office or home), the electronic mail address (if any), and an abstract of between 10 and 15 lines. A recent black-and-white photo, the biographic data, and the list of symmetry-related publications of (each) author should be enclosed; see the sample at the end.

- Only black-and-white, camera-ready illustrations (photos or drawings) can be used. The required (approximate) location of the figures and tables should be indicated in the main text by typing their numbers and captions (Figure 1: [text], Figure 2: [text], Table 1: [text], etc.), as new paragraphs. The figures, which will be slightly reduced in printing, should be enclosed on separate sheets. The tables may be given inside the text or enclosed separately.

- It is the author's responsibility to obtain written permission to reproduce copyright materials.

- Either the British or the American spelling may be used, but the same convention should be followed throughout the paper. *The Chicago Manual of Style* is recommended in case of any stylistic problem.

- Subtitles (numbered as 1, 2, 3, etc.) and subsidiary subtitles (1.1, 1.1.1, 1.1.2, 1.2, etc.) can be used, without over-organizing the text. Footnotes should be avoided; parenthetic inserts within the text are preferred.

- The use of references is recommended. The citations in the text should give the name, year, and, if necessary, page, chapter, or other number(s) in one of the following forms: ... Weyl (1952, pp. 10-12) has shown...; or ... as shown by some authors (Coxeter et al., 1986, p. 9; Shubnikov and Koptsik 1974, chap. 2; Smith, 1981a, chaps. 3-4; Smith, 1981b, sec. 2.12; Smith, forthcoming). The full bibliographic description of the references should be collected at the end of the paper in alphabetical order by authors' names; see the sample. This section should be entitled *References*.

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Sample of heading (Apologies for the strange names and addresses)

SYMMETRY IN AFRICAN ORNAMENTAL ART
BLACK-AND-WHITE PATTERNS IN CENTRAL AFRICA
Running head: Symmetry in African Art
Section: Symmetry: Culture & Science

Susanne Z. Dissymmetrist and Warren M. Symmetrist
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E-mail: symmetrist@symmetry.edu

Abstract

The ornamental art of Africa is famous ...

Sample of references

In the following, note punctuation, capitalization, the use of square brackets (and the remarks in parentheses). There is always a period at the very end of a bibliographic entry (but never at other places, except in abbreviations). Brackets are used to enclose supplementary data. Those parts which should be italicized – titles of books, names of journals, etc. – should be underlined in red on the hard-copies. In the case of non-English publications both the original and the translated titles should be given (cf., Dissymmetrist, 1990).

Asymmetrist, A. Z. (or corporate author) (1981) *Book Title: Subtitle*, Series Title, No. 27, 2nd ed., City (only the first one): Publisher, vii + 619 pp.; (further data can be added, e.g.) 3rd ed., 2 vols., ibid., 1985, viii + 444 + 484 pp. with 2 computer diskettes; Reprint, ibid., 1988; German trans., *German Title*, 2 vols., City: Publisher, 1990, 986 pp.; Hungarian trans.

Asymmetrist, A. Z., Dissymmetrist, S. Z., and Symmetrist, W. M. (1980-81) Article or e-mail article title: Subtitle, Parts 1-Z, *Journal Name Without Abbreviation*, [E-Journal or Discussion Group address: journal@node (if applicable)], B22 (volume number), No. 6 (issue number if each one restarts pagination), 110-119 (page numbers); B23, No. 1, 117-132 and 148 (for e-journals any appropriate data).

Dissymmetrist, S. Z. (1989a) Chapter, article, symposium paper, or abstract title, [Abstract (if applicable)], In: Editorologist, A.B. and Editorologist, C.D., eds., *Book, Special Issue, Proceedings, or Abstract Volume Title*, [Special Issue (or) Symposium organized by the Dissymmetry Society, University of Symmetry, San Symmetrino, Calif., December 11-22, 1971 (those data which are not available from the title, if applicable)], Vol. 2, City: Publisher, 19-20 (for special issues the data of the journal).

Dissymmetrist, S. Z. (1989b) *Dissertation Title*, [Ph.D. Dissertation], City: Institution, 248 pp. (Exhibition Catalogs, Manuscripts, Master's Theses, Mimeographs, Patents, Preprints, Working Papers, etc. in a similar way; Audiocassettes, Audiotapes, Compact Disks, Computer Diskettes, Computer Software, Films, Microfiches, Microfilms, Slides, Sound Disks, Videocasettes, etc. with necessary modifications, adding the appropriate technical data).

Dissymmetrist, S. Z., ed. (1990) *Dissimmetriya v nauke* (title in original, or transliterated, form), [Dissymmetry in science, in Russian with German summary], Trans. from English by Antisymmetrist, B. W., etc.

Phyllotaxist, F. B. (1899/1972) *Title of the 1972 Edition*, [Reprint, or Translation, of the 1899 ed.], etc.

[Symmetrist, W. M.] (1989) Review of *Title of the Reviewed Work*, by S. Z. Dissymmetrist, etc. (if the review has an additional title, then it should appear first; if the authorship of a work is not revealed in the publication, but known from other sources, the name should be enclosed in brackets).

In the case of lists of publications, or bibliographies submitted to *Symmetro-graphy*, the same convention should be used. The items may be annotated, beginning in a new paragraph. The annotation, a maximum of five lines, should emphasize those symmetry-related aspects and conclusions of the work which are not obvious from the title. For books, the list of (important) reviews, can also be added.

Sample of biographic entry

Name: Warren M. Symmetrist, Educator, mathematician, (b. Boston, Mass., U.S.A., 1938).

Address: Department of Dissymmetry, University of Symmetry, 69 Harmony Street, San Symmetrino, Calif. 69869, U.S.A. *E-mail:* symmetrist@symmetry.edu

Fields of interest: Geometry, mathematical crystallography (also ornamental arts, anthropology – non-professional interests in parentheses).

Awards: Symmetry Award, 1987; Dissymmetry Medal, 1989.

Publications and/or *Exhibitions*: List all the symmetry-related publications/exhibitions in chronological order, following the conventions of the references and annotations. Please mark the most important publications, not more than five items, by asterisks. This shorter list will be published together with the article, while the full list will be included in the computerized data bank of ISIS-Symmetry.

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There are many disciplinary periodicals and symposia in various fields of art, science, and technology, but broad interdisciplinary forums for the connections between distant fields are very rare. Consequently, the interdisciplinary papers are dispersed in very different journals and proceedings. This fact makes the cooperation of the authors difficult, and even affects the ability to locate their papers.

In our 'split culture', there is an obvious need for interdisciplinary journals that have the basic goal of building bridges ('symmetries') between various fields of the arts and sciences. Because of the variety of topics available, the concrete, but general, concept of symmetry was selected as the focus of the journal, since it has roots in both science and art.

SYMMETRY: CULTURE AND SCIENCE is the quarterly of the *INTERNATIONAL SOCIETY FOR THE INTERDISCIPLINARY STUDY OF SYMMETRY* (abbreviation: *ISIS-Symmetry*, shorter name: *Symmetry Society*). ISIS-Symmetry was founded during the symposium *Symmetry of Structure* (*First Interdisciplinary Symmetry Symposium and Exhibition*), Budapest, August 13-19, 1989. The focus of ISIS-Symmetry is not only on the concept of symmetry, but also its associates (asymmetry, dissymmetry, antisymmetry, etc.) and related concepts (proportion, rhythm, invariance, etc.) in an interdisciplinary and intercultural context. We may refer to this broad approach to the concept as *symmetrology*. The suffix *-logy* can be associated not only with knowledge of concrete fields (cf., biology, geology, philology, psychology, sociology, etc.) and discourse or treatise (cf., methodology, chronology, etc.), but also with the Greek terminology of proportion (cf., *logos*, *analogia*, and their Latin translations *ratio*, *proportio*).

The basic goals of the *Society* are
(1) to bring together artists and scientists, educators and students devoted to, or interested in, the research and understanding of the concept and application of symmetry (asymmetry, dissymmetry);
(2) to provide regular information to the general public about events in symmetrology;
(3) to ensure a regular forum (including the organization of symposia, and the publication of a periodical) for all those interested in symmetrology.

The Society organizes the triennial *Interdisciplinary Symmetry Symposium and Exhibition* (starting with the symposium of 1989) and other workshops, meetings, and exhibitions. The forums of the Society are *informal* ones, which do not substitute for the disciplinary conferences, only supplement them with a broader perspective.

The Quarterly — a non-commercial scholarly journal, as well as the forum of ISIS-Symmetry — publishes original papers on symmetry and related questions which present new results or new connections between known results. The papers are addressed to a broad non-specialist public, without becoming too general, and have an interdisciplinary character in one of the following senses:
(1) they describe concrete interdisciplinary 'bridges' between different fields of art, science, and technology using the concept of symmetry;
(2) they survey the importance of symmetry in a concrete field with an emphasis on possible 'bridges' to other fields.

The Quarterly also has a special interest in historic and educational questions, as well as in symmetry-related recreations, games, and computer programs.

The regular sections of the Quarterly:
- **Symmetry: Culture & Science** (papers classified as humanities, but also connected with scientific questions)
- **Symmetry: Science & Culture** (papers classified as science, but also connected with the humanities)
- **Symmetry in Education** (articles on the theory and practice of education, reports on interdisciplinary projects)
- **Mosaic of Symmetry** (short papers within a discipline, but appealing to broader interest)
- **SFS: Symmetric Forum of the Society** (calendar of events, announcements of ISIS-Symmetry, news from members, announcements of projects and publications)
- **Symmetro-graphy** (biblio/disco/software/ludo/historio-graphies, reviews of books and papers, notes on anniversaries)
- **Reflections: Letters to the Editors** (comments on papers, letters of general interest)

Additional non-regular sections:
- **Symmetrospective: A Historic View** (survey articles, recollections, reprints or English translations of basic papers)
- **Symmetry: A Special Focus on ...** (round table discussions or survey articles with comments on topics of special interest)
- **Symmetry: An Interview with ...** (discussions with scholars and artists, also introducing the Honorary Members of ISIS-Symmetry)
- **Symmetry: The Interface of Art & Science** (works of both artistic and scientific interest)
- **Recreational Symmetry** (problems, puzzles, games, computer programs, descriptions of scientific toys; for example, tilings, polyhedra, and origami)

Both the lack of seasonal references and the centrosymmetric spine design emphasize the international character of the Society; to accept one or another convention would be a 'symmetry violation'. In the first part of the abbreviation *ISIS-Symmetry* all the letters are capitalized, while the centrosymmetric image iSIS! on the spine is flanked by 'Symmetry' from both directions. This convention emphasizes that ISIS-Symmetry and its quarterly have no direct connection with other organizations or journals which also use the word *Isis* or *ISIS*. There are more than twenty identical acronyms and more than ten such periodicals, many of which have already ceased to exist, representing various fields, including the history of science, mythology, natural philosophy, and oriental studies. ISIS-Symmetry has, however, some interest in the symmetry-related questions of many of these fields.

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continued from inside front cover

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