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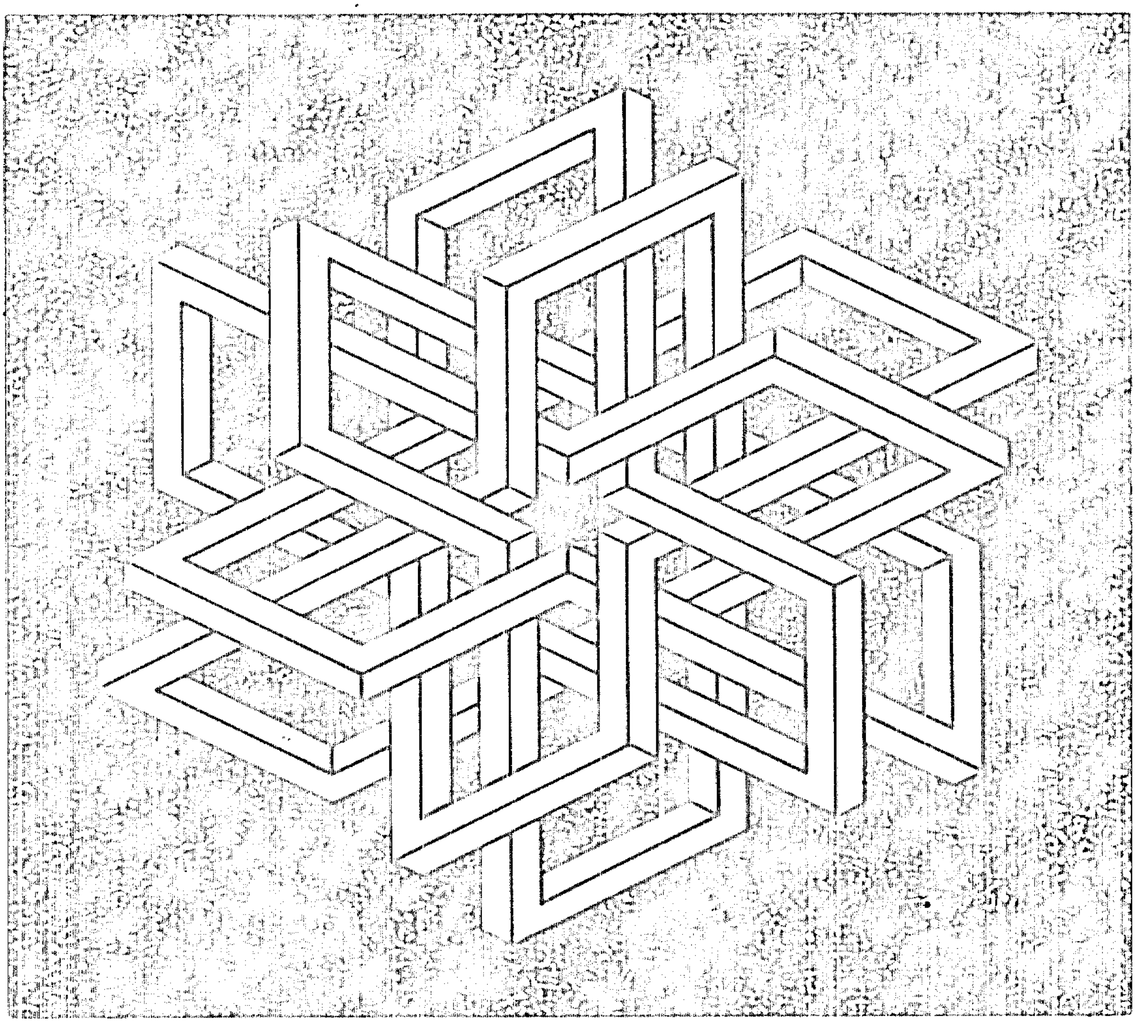


SEMEIOTICS, GOOD SYMMETRY, AND THE LOGIC OF PROPOSITIONS

Shea Zellweger © 1992
Mount Union College
Alliance, Ohio 44601 U.S.A.

Semeiotics is the study of all signs and sign processes. The logic of propositions is a fundamental part of symbolic logic. If one gives central emphasis to the role of SYMMETRY, when great care is put on SHAPE designing what it takes to construct a special set of sixteen ICONIC signs, then it is possible to bring to the logic of propositions an approach that not only SIMPLIFIES and CONSOLIDATES.

This approach, with its emphasis on symmetry, also receives major assistance from the ALGEBRA OF ABSTRACT GROUPS. It lends itself to the construction of as many as a tabletop of hand-held models. It brings elementary logic into the classroom of our secondary schools. It has practical implications for digital design, mirror logic, and optical computers. It bumps into and finds much in common with crystallography. What follows is both a small piece and a key piece from this approach.

Let us look at the 2-valued logic of two sentences, no more than that. Notice that (1) is a MASTER EQUIVALENCE, one that is sufficiently abstract, one that has enough algebra in it, so that

$$(1)\qquad (\underset{\smile}{\dot{A} \,\dot{*}\, \dot{B}}) \equiv (\underset{\smile}{\dot{A} \,\dot{*}\, \dot{B}})$$

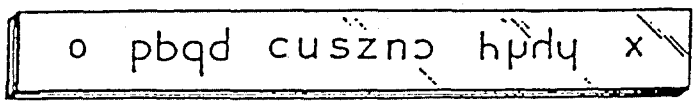
it covers 4096 ATOMIC EQUIVALENCES. Notice also that (1) contains fourteen nodes of substitution, seven on each side of the equivalence sign, and that (1) brings three subsystems into the same system, itself able to receive one by one all of the 4096 unique substitutions in this CUSTOM-CONSTRUCTED larger system.

There are four nodes in the first subsystem, namely. two A's and two B's. These nodes cover the two PROPOSITIONAL variables.

There are two nodes in the second subsystem, namely, the two asterisks. These nodes cover the two CONNECTIVE variables. More specifically, each one stands for all of, and exactly any of, the sixteen binary connectives (and, or, if then, etc.).

There are eight nodes in the third subsystem, namely, six over-dots and two under-arcs. These nodes cover the eight OPERATION variables. Each dot stands for the PRESENCE or ABSENCE of (N)egation, and each arc stands for the PRESENCE or ABSENCE of (C)onversion [(A,B) or (B,A)]. Each side allows any of the sixteen combinations of dot-dot-dot-arc to operate on (A * B).

The figure at the top of the next page shows a ruler-sized FLIPSTICK, one that has been patented and one that has another set of letter-shapes (LSs) on its back side, in see-through orientation. It can be flip-flip-rotated and across the center mated. All of the LSs belong to a special set of sixteen TOPOLOGICAL ICONS that possess some carefully determined



ALGEBRAIC-GEOMETRIC PROPERTIES. The LSs stand for the sixteen binary connectives. These signs will be substituted for the asterisks in (1), as needed, and in keeping with any of the 4096 atomic equivalences, in order to honor the equivalence sign.

The LSs carry all combinations of four STEMS, from zero (o) to four (x). Put the truth table for (A,B) at the quadrants (TT, TF, FT, FF), in the same positions and just like the plus-minus signs in (x,y) coordinates. For example, because the d-letter has one T-stem in the upper-right TT-quadrant, N(A d B) stands for Not(A and B). (A h B) stands for the Sheffer stroke (FTTT).

Now, along the lines of what happens in crystallography (by way of the symmetries of a square), introduce ALL COMBINATIONS OF FOUR SYMMETRY RULES, along with the algebra of abstract groups. The four symmetry rules, called FLIP-MATE-FLIP AND FLIP (f-m-f and f), are in a class very much by themselves. They treat all of the LSs in the same way. NA FLIPS from left to right, N* always MATES with the LS symmetrically across the center of the flipstick, NB FLIPS from top to bottom, and C FLIPS diagonally from upper-right to lower-left. N(A an-d B) mates to (A h B).

NA, N*, NB, and C, acting alone, activate 2-groups; (NA,NB), a Klein 4-group; (NA,N*,NB), the 8-group (C2 x C2 x C2); (NA,NB, C), the octic group (D4); (NA,N*,NB,C), the 16-group (D4 x C2).

This approach lends itself to the construction of a whole family of hand-held symmetry models. In effect, (1) is a master equivalence that subordinates 4096 atomic equivalences, which is ALL AND EXACTLY ALL of the WELL-FORMED EQUIVALENCES that can be written under this algebraic form, including De Morgan's laws and many of the standard tautologies, when the asterisk on the right is determined by all of the subsets of substitutions that can be made at the other thirteen nodes. The marvel is that it takes only four symmetry rules (f-m-f and f) to cover all of these determinations, when THE RULES FIT THE CALCULATIONS.

References (See next abstract for more diagrams of the models).

Clark, Glenn (In Press). New Light on Peirce's Iconic Notation for the Sixteen Binary Connectives. In Studies in the Logic of Charles S. Peirce, Nathan Houser, Don D. Roberts, and James Van Evra (eds.), Bloomington: Indiana University Press.

Zellweger, Shea (In Press). Untapped Potential in Peirce's Iconic Notation for the Sixteen Binary Connectives. In Studies in the Logic of Charles S. Peirce, Nathan Houser, Don D. Roberts, and James Van Evra (eds.), Bloomington: Indiana University Press.

SYMMETRY MODELS THAT LEAD INTO ELEMENTARY LOGIC

Shea Zellweger © 1992
Mount Union College
Alliance, Ohio 44601 U.S.A.

A careful use of the concept of symmetry makes it much easier to present logic to beginners. This becomes especially evident when symmetry is built into a system of educational toys. What follows is a short report that describes how cards, mirrors, and hand-held models can be used to lead into elementary logic.

The example that follows will be limited to the logic of any two things, also called the logic of two propositions, no more than that. Although it may not be evident at first, this example calls on, and draws out, a special set of relations, namely, all of, and not one less than, the sixteen relations between the two things (A,B), such as 'A and B,' 'A or B,' 'if A, then B,' etc. These relations are often called the sixteen binary connectives.

The sixteen DOT-SQUARES in Fig. 1 show all combinations of enlarged corners at the four corners of a front-back square card. There is one dot-square card for each letter-shape (LS) on the FLIPSTICK that appears in the preceding abstract in this volume. These DOTS are coded the same as the STEMS in that abstract.

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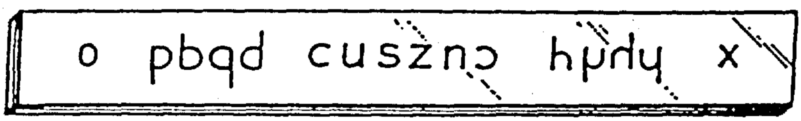
2 

The FLIPSTICK in Fig. 2 also has another set of letter-shapes on its back side, in see-through orientation. The CLOCK-COMPASS in Fig. 3 shows all of the LSs when they have been placed inside of the corresponding dot-squares. The BINOMIAL SUBTOWERS in Fig. 4 have LSs on all four faces: (H)orizonatal flips to the left, (V)ertical flips to the right, and HV-rotates on the back side. The LOGIC BUG in Fig. 5 can also be subjected to the same system of H-flips, V-flips, HV-rotates, and across the center (M)ates.

Fig. 6 shows the EIGHT SYMMETRIES OF A SQUARE: the four flips (horizontal, vertical, two diagonal) and the four rotations (90, 180, 270, and 360 degrees) that are generated when any dot-square is placed between two mirrors at 45 degrees (heavy lines), in this case one darkened in the upper-right quadrant.

The LOGICAL GARNET in Fig. 7 shows what happens when the LSs are placed at the vertices of a rhombic dodecahedron; two vertices (sz) are fused at the co-center. The 8-CELL OF LOGICAL GARNETS in Fig. 8 is generated when three mutually perpendicular mirrors act on an identity garnet (A * B). Three dots in (1) become all combinations of Negation (N,N,N) that act on A, on *, and on B.

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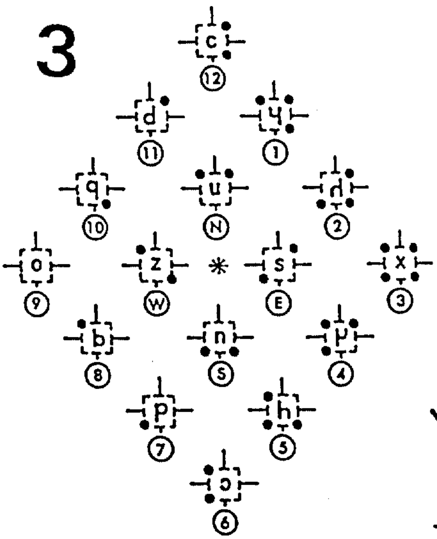

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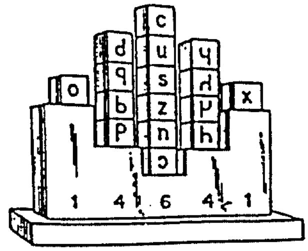

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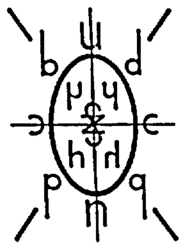

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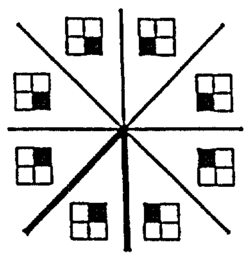

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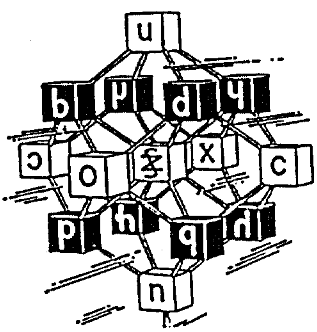

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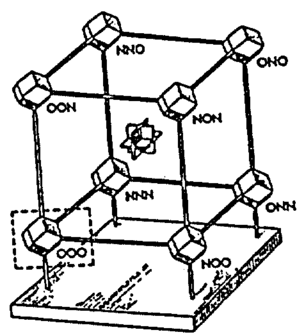

References (See preceding abstract for more details)

Zellweger, Shea (1981). Devices for displaying or performing operations in a two-valued system. Patents in the United States (1981, 1983, 1985), Canada (1983), and Japan (1991).

_______________ (1982). Sign-creation and man-sign engineering, Semiotica 38:17-51. Figs. 3, 7, and 8 are from this paper.

_______________ (1992). Cards, mirrors, and hand-held models that lead into elementary logic. Displayed at the Sixteenth Meeting of the North American Region of the International Group for the Psychology of Mathematics Education, 6th-11th August, University of New Hampshire (Durham, N.H.).