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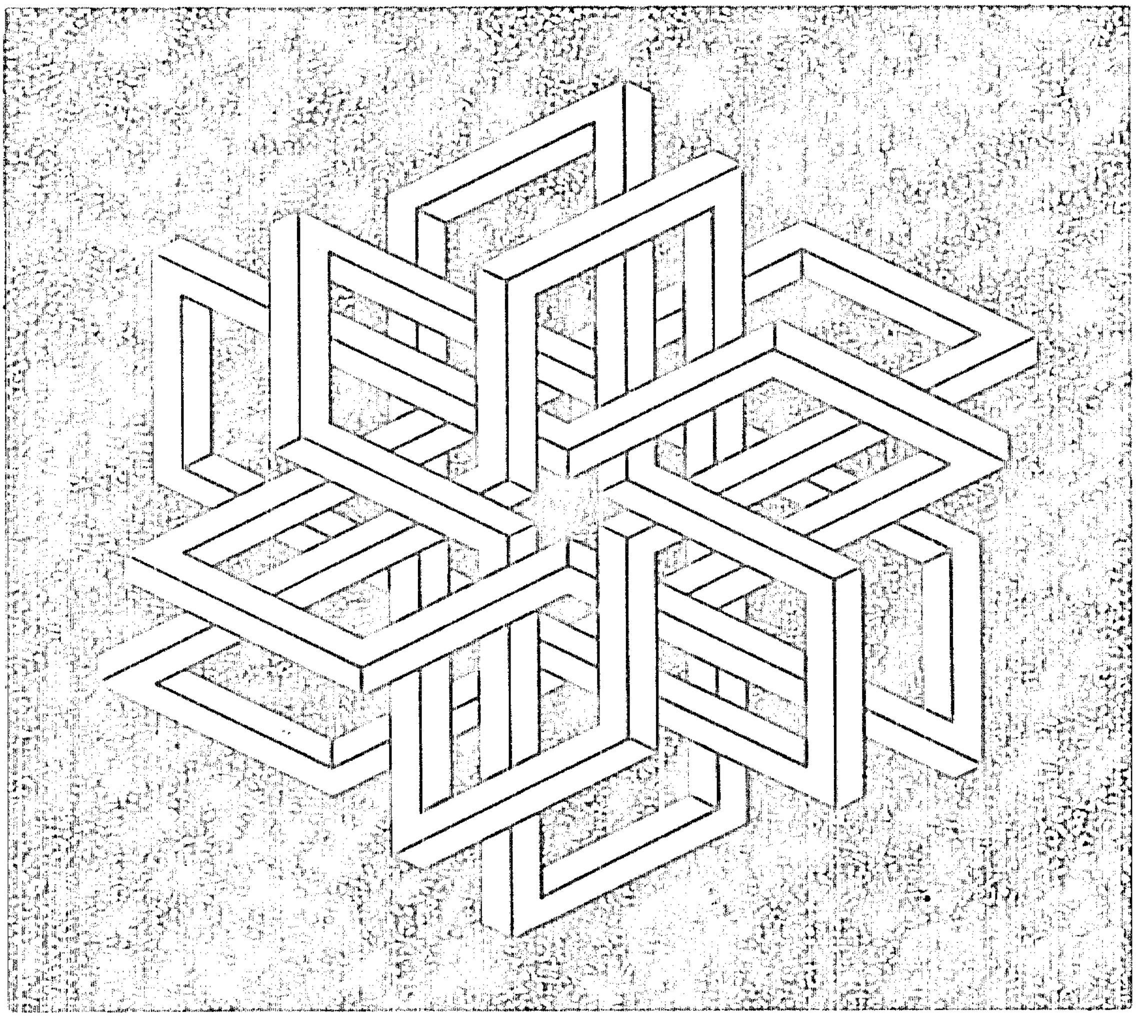

GENERATING TUBULAR POLYHEDRA

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I. pattern recognition

In our daily experiences, we passively encounter a myriad of visual, tactile and audible patterns. The people who search for and actively study patterns, such as a sculptor like myself, encounter an even greater number of them. Artists and scientists alike are consumed with the search for elemental relationships within and between patterns.

2. tiling operations

The most elegant and the most awesome pattern of symmetry conservation for me is Euler's discovery that, within any multifaceted polyhedron, the number of edges plus two is always equal to the number of vertices plus the number of faces (E+2=V+F). This invariance means that one must seek another scale of resolution for an explanation of the generation of polygon structures and their aggregation and separation patterns. Polygons must generate at a level below their own scale of reality. Heretofore, the polygon has been the given fundamental focus in the tiling of the two-dimensional plane and also in the construction of three-dimensional polyhedra. In the initial conditions of my scale of resolution, polygons do not exist. The polygon generating operation begins with one of the dynamic elements of lines and junctions called "foundation sutures." The elemental foundation suture interactions generate the polygons by replication. When wrapped into rings and capped, they generate families of periodic polyhedra. When the foundation suture elements are fitted together to fill the plane, they generate patterns of trigons, tetragons and hexagons. I have discovered five foundation suture elements that generate patterns of single polygons or patterns of combined polygons according to the kind of element under replication. A replicated string of one of these foundation suture elements (Fig. 1) generates a pattern of trigons and hexagons.

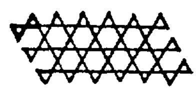

Fig. I: The bold outline identifies the foundation element that is under replication .

3. generating 3-D polyhedra

In existing preformed polyhedra, the foundation sutures are seen as zig-zagging equatorial rings or pathways that split the polyhedra into symmetrical hemispheres. When the hemispheres are fitted back together, they can be positioned so that they are twisted or untwisted in relation to each other. This sets up a whole system of new dual relationships among polyhedra in general. There is a glide reflection symmetry operation in the twisting and untwisting action between the hemispheres of spherical polyhedra.

Fig. 2: In the process of generating a 3-D polyhedron from the 2-D pattern (Fig. I), remove a string of three suture elements, wrap them into a ring, cap the ring with antipodal circumferential trigons and the three fold cuboctahedron is formed.

4. tubular polyhedra

I was introduced to buckytubes by Sumio Iijima (Nature, 1991) and realized that these same forms could easily be generated with my foundation suture elements. A sheet of suture generated polygon tessellations can be wrapped into tubes and capped to form 3-D tubular polyhedra (Fig. 3). Like Iijima's "coaxial tubes," more than one sheet can be wrapped together to form concentric polyhedral tubes.

Fig. 3: Wrap a sheet of hexagons (left) into a cylinder or free the foundation suture ring from the tetrakaidecahedron, for example, and stack it (right).

With this method, any spherical polyhedron can be decapped and the remaining foundation suture ring can be replicated, stacked and recapped to form a tubular polyhedron. Rings of different diameters are stacked inside each other to form concentric tubular polyhedra. The foundation suture rings are the prime elements in the generation and periodicity of conventional and tubular polyhedral lattices.

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