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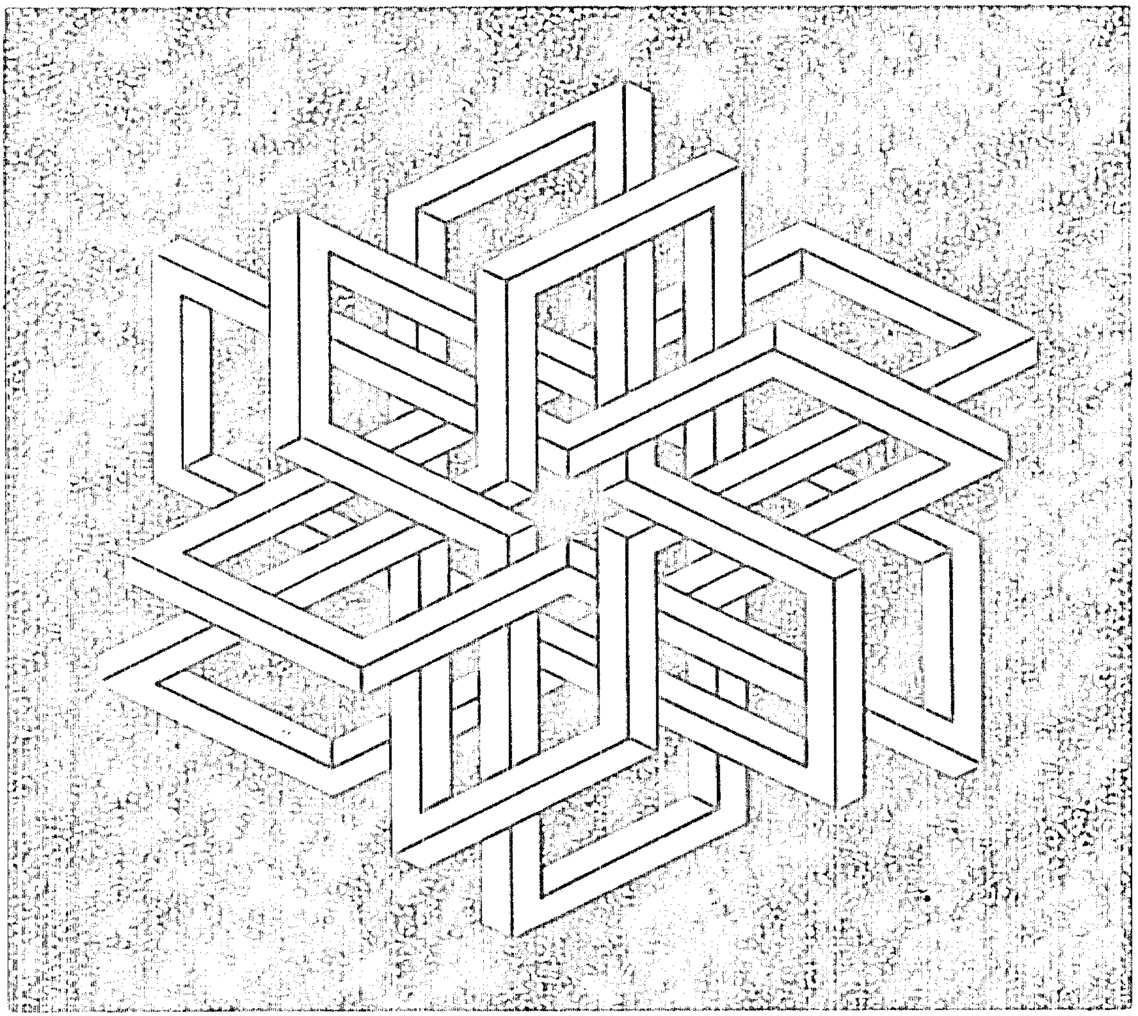

ARTISTIC CHALLENGES FROM GEOMETRIC SYMMETRY

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We shall consider three different situations in which the problem of illustrating a simple geometrical idea in a visually pleasing way poses an artistic challenge. The basic geometrical ideas referred to below are all described by Grünbaum and Shephard (1987).

1. Square tiles used as fundamental regions for plane patterns

Identical copies of a square tile containing an asymmetrical design, together with reflections of the tile if necessary, can be put together in 36 ways to form 'wallpaper patterns' in which each tile is a fundamental region. Patterns illustrating 11 of the 17 symmetry types can be obtained in this way. There are only 16 methods of assembling the tiles, but within a method the tiles can sometimes be rotated to give two or four distinct patterns; two different methods may give patterns with the same symmetry type. For instance, Figures 1 and 2 are produced by the same method of assembling the tiles; they have the type of symmetry known as *pmg*, with mirror lines in one direction and centres of 2-fold rotational symmetry in between them. Figure 3 uses a different method; the symmetry type is still *pmg* but the mirror lines are now twice as far apart. The symmetry type of Figure 4 is *p4g*, with mirror lines in two directions and centres of 4-fold rotational symmetry.

Even a simple tile like the one used here can produce some striking patterns, but the author has striven to produce a tile depicting flowers, leaves and branches that produces a pleasing flowing design, no matter how the individual tiles are fitted together.

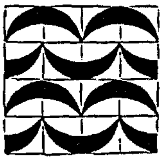

Figure 1

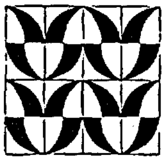

Figure 2

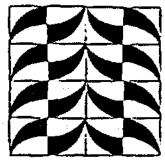

Figure 3

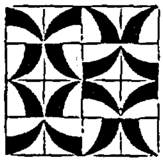

Figure 4

2. The use of Penrose tiles to produce Islamic-style patterns

Wallpaper patterns in the (Euclidean) plane can never have complete 5-fold rotational symmetry. Islamic patterns, however, often contain many motifs with 5-fold symmetry in a restricted region. Penrose's 'kite-and-dart' tessellations have the same property; in fact Penrose tessellations contain an infinite number of regions as large as you like with 5-fold symmetry. The problem of designing a motif on the kite, and on the dart, so that the Penrose tessellations will then produce an acceptable Islamic-style pattern, is a challenging one. Having produced a suitable motif, we can use a reducing photocopier to build up the pattern, in a manner that shows at the same time how the Penrose tessellations are produced. Lack of space precludes an illustration here.

3. Combining patterns of different pattern types

Figure 5 has symmetry type *p6*. It is composed of white flowers forming a pattern of type PP45, black motifs forming a pattern of type PP44, and motifs forming a pattern of type PP43. The three different motifs exhibit the different types of rotational symmetry occurring in any pattern of symmetry type *p6*. Asymmetric motifs forming a pattern of type PP42 (also within the symmetry type *p6*) have been excluded here because they do not focus our attention on any of the symmetries occurring in *p6*. Figure 6 has symmetry type *pmg*. The white motifs, with their mirror symmetry, form a pattern of type PP13 and emphasize the mirror lines of the figure. The other two motifs both form patterns of type PP12; they emphasize that centres of 2-fold rotational symmetry occur at two different sorts of position in any pattern of type *pmg*. It is important that individual motifs in the same pattern type should not coalesce, so the motifs in Figure 8 should be regarded as tiles separated by cement.

The author is in the process of producing similar designs for most of the 17 symmetry types of wallpaper pattern. The requirements are that the motifs should exhibit no symmetries in addition to those required of them, that different motifs with the same type of symmetry should be clearly distinguishable, and that the resulting design should be pleasing to the eye. Whether he has fulfilled the requirements will be for others to judge. Some of these designs have already been published (Rigby, 1990). Such designs in a sense complement the periodic drawings of Escher, whose motifs are mainly asymmetric but sometimes have mirror symmetry (Schattschneider, 1990).

References

Grünbaum, B. and Shephard, G.C. (1987) *Tilings and Patterns*. New York: Freeman, ix + 700 pp.

Rigby, J.F. (1990) Symmetry in Geometry. *Symmetry: Culture and Science*, 1, 63-76.

Schattschneider, D. (1990) *Visions of Symmetry: Notebooks, Periodic Drawings, and Related Work of M.C. Escher*. New York: Freeman, xiii + 354.

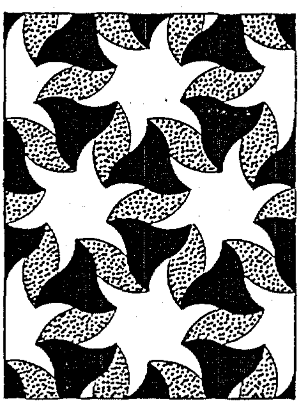

Figure 5

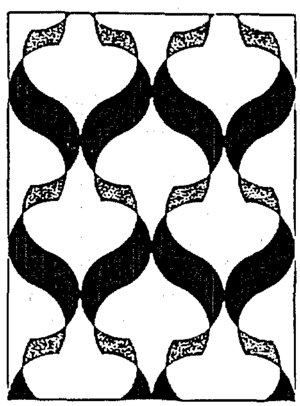

Figure 6