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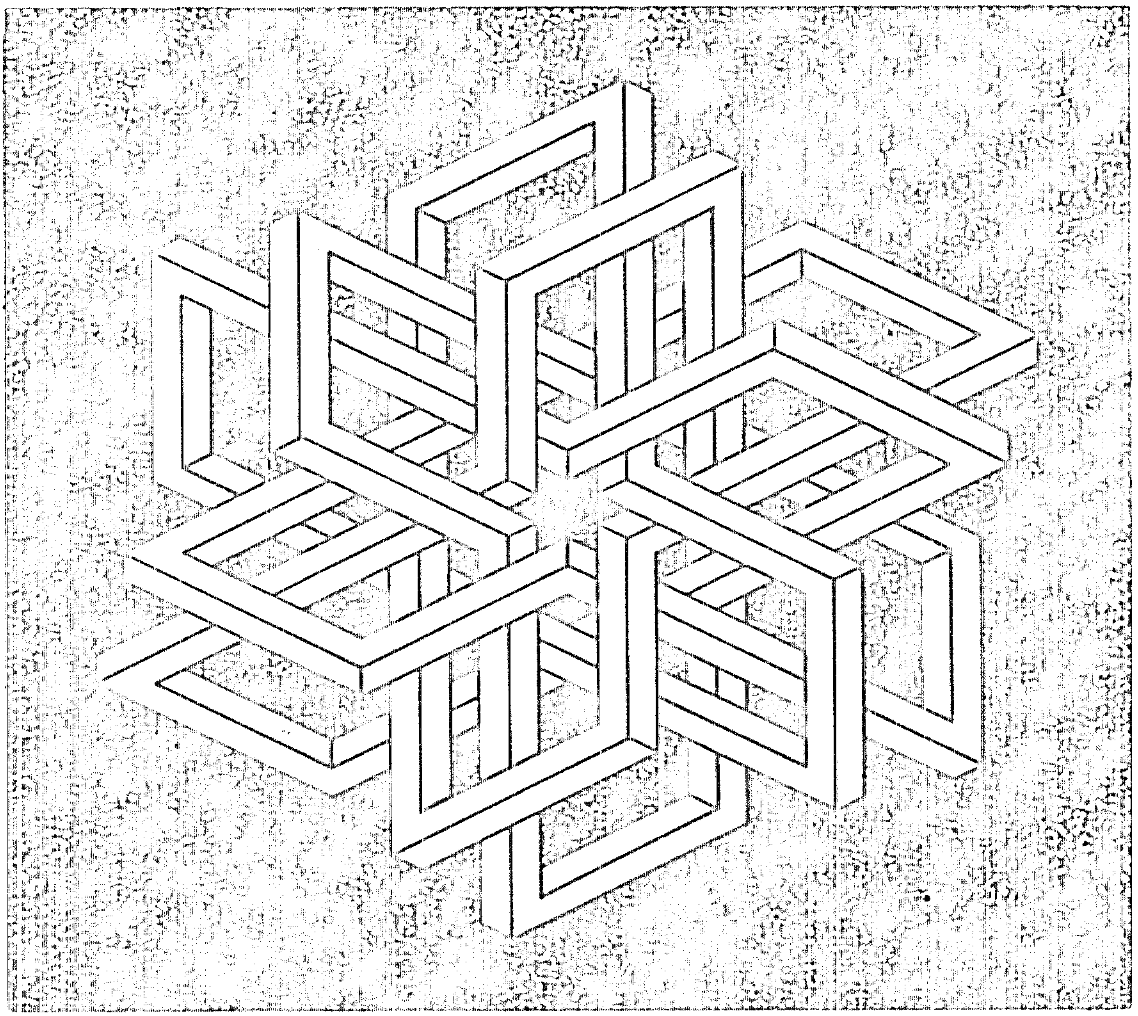

SEQUENCE OF JULIA-LIKE SETS PRODUCED BY A NUMERICAL RECURRENCE TECHNIQUE

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We look at a particular symmetry pattern that portrays smooth frontiers invading on a fractal support into a basin of attraction. The formerly connected basin is split up to a Feigenbaum sequence of 2, 4, 8, 16, etc. parts.

In order to end up with such a structural form, we start with an iterative mapping somewhat similar to the complex logistic one,

$$x_{n+1} = x_n^2 - y_n^2 + ax_n + b \; ; \quad y_{n+1} = 2x_n y_n + c \; . \qquad (1)$$

x_n and y_n denote the variables with index n = 0,1,2,..., while a, b, and c represent constants. Due to the term ax_n, equation (1) is not analytic any more. On the basis of that map, basins of attraction, i.e., filled-in Julia-like sets, as well as Mandelbrot-like sets can be calculated, reflecting self-similarity and further types of fractality (Rössler et al., 1986, 1990; Röhricht et al., 1987; Peinke et al., 1987, 1988, 1991; Parisi et al., 1991; Klein et al., 1991).

Figure 1 is obtained from a series of Julia-like sets. At first, the initial conditions were chosen as

$$-1.5 \leq x_0 \leq 1.5 \; ; \quad -1.5 \leq y_0 \leq 1.5 \qquad (2)$$

and divided up into 600 x 1200 points. Each of those in the plane of the picture stands for a pair of initial conditions (x_0, y_0) and was iterated 500 times with double precision (FORTRAN). Only such points that fulfill the condition

$$x_n^2 + y_n^2 \leq 10 \qquad (3)$$

were marked by dots. This numerical procedure was performed for the parameter values a = 0.3, b = 0.1225390, and c = 0. The next step implies the application of the same procedure with b = 0.1225392 (a and c remain constant). But, this time, non-divergent points marked before were now erased. That process was then repeated three times, always increasing the last digit of the value of b by 2 and inverting the color of the previous

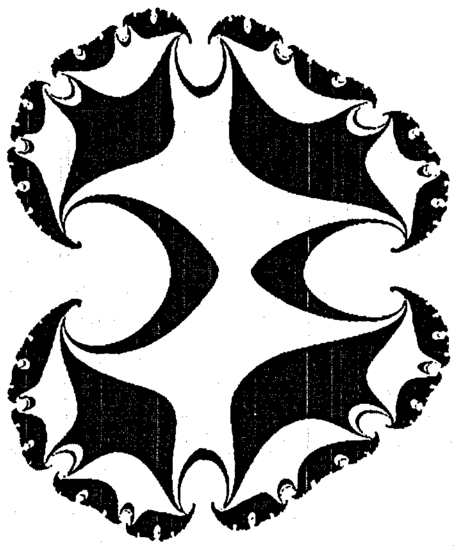

Fig. 1. Feigenbaum sequence of Julia-like sets. For details, see text.

step. With the help of this technique, it is possible to visualize how a Julia-like set vanishes under variation of a control parameter (here, b), corresponding to a transition over a borderline of an appropriately chosen Mandelbrot set.

References

Klein, M., Parisi, J., Peinke, J., Baier, G., Kahlert, C., Hudson, J.L., and Rössler, O.E., Comput. Graphics 15, 583 (1991).

Parisi, J., Peinke, J., Klein, M., Rössler, O.E., and Kahlert, C., in: Fractals in the Fundamental and Applied Sciences, eds. Peitgen, H.-O., Henriques, J.M., and Penedo, L.F. (North-Holland, Amsterdam, 1991) p. 329.

Peinke, J., Parisi, J., Röhricht, B., and Rössler, O.E., Z. Naturforsch. 42 a, 263 (1987).

Peinke, J., Parisi, J., Röhricht, B., Rössler, O.E., and Metzler, W., Z. Naturforsch. 43 a, 14 (1988); Z. Naturforsch. 43 a, 287 (1988).

Peinke, J., Klein, M., Kittel, A., Baier, G., Parisi, J., Stoop, R., Hudson, J.L., and Rössler, O.E., Europhys. Lett. 14, 615 (1991).

Röhricht, B., Metzler, W., Parisi, J., Peinke, J., Beau, W., and Rössler, O.E., in: The Physics of Structure Formation, eds. Güttinger, W., and Dangelmayr, G. (Springer, Berlin, 1987) p. 275.

Rössler, O.E., Kahlert, C., Parisi, J., Peinke, J., and Röhricht, B., Z. Naturforsch. 41 a, 819 (1986).

Rössler, O.E., Baier, G., Parisi, J., Klein, M., Peinke, J., Krug, M., and Hudson, J.L., Comput. Phys. 4, 494 (1990).