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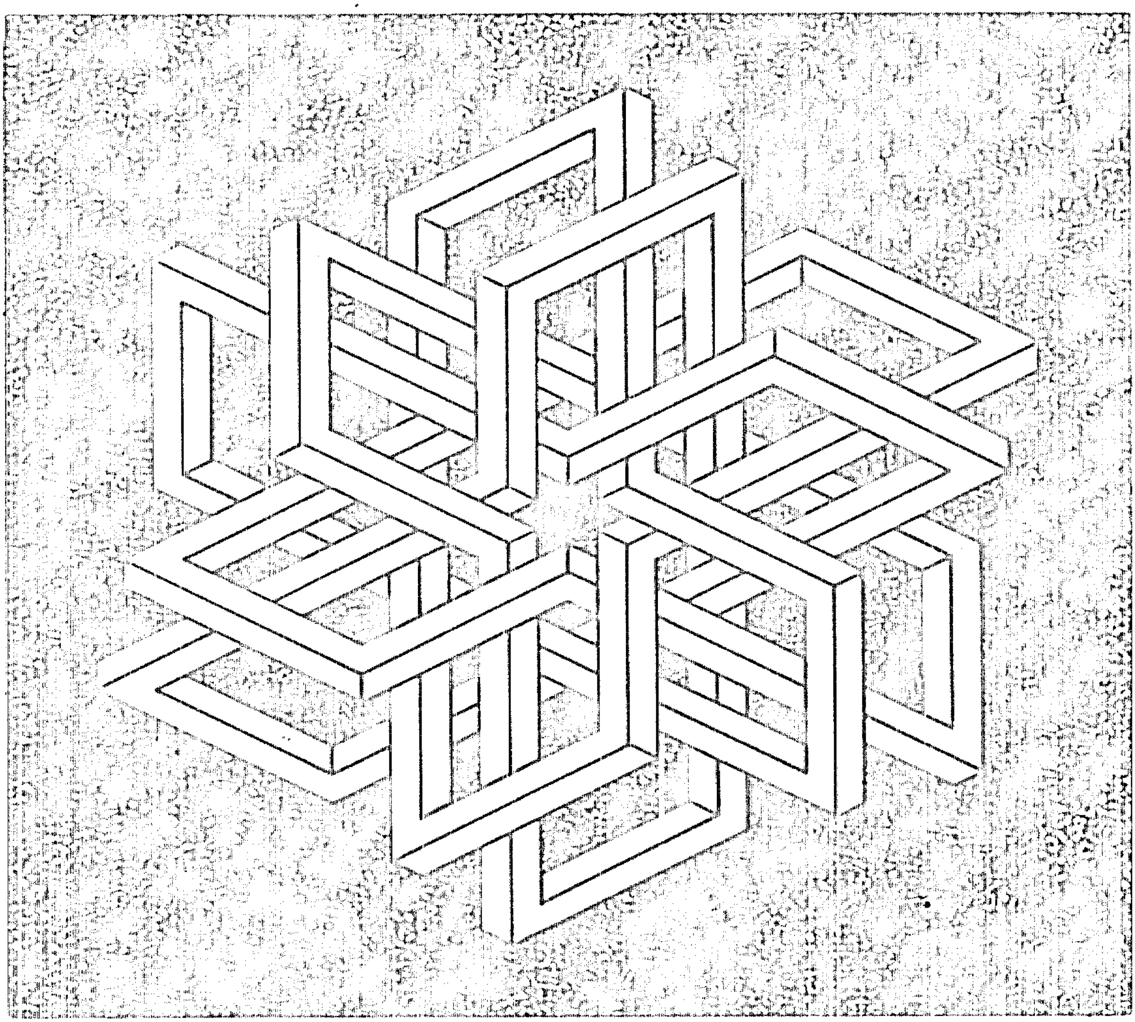
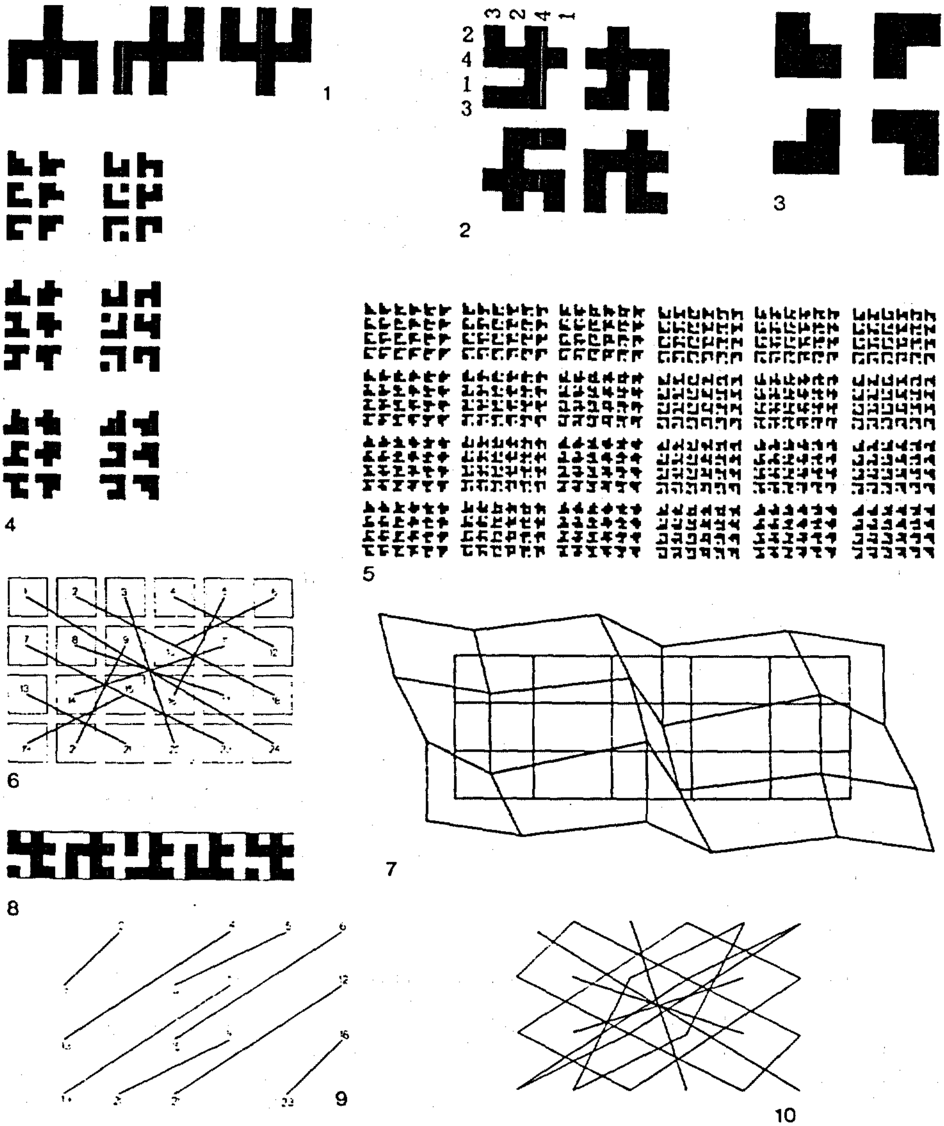


SYMMETRIES IN PERMUTATION–GENERATED PATTERNS

George G. Lugosi
Howard Florey Institute of Experimental Physiology and Medicine
University of Melbourne
Parkville, 3052, Victoria, Australia
E-mail : george@hfivax.hfi.unimelb.edu.au

This amazing sign-system was born about twenty years ago. A friend of mine, an artist (József V. Molnár, painter), pointed out that certain signs were existing in different cultures, far from each other in space and time. Investigating the most frequently repeated elements of them, he filtered out a nice collection of basic signs, formed groups of them on the basis of the formal similarity, and recreated them using connected grid of squares (fig. 1). This gave the opportunity to analyse their form, logic, and symmetries. Then we started to work in the opposite direction : how to generate signs, and how to create a full system, where all the possible elements of signs are represented.
On the basis of the analysis we used the 'world' of $n \times n$ matrix of squares. It turned out that to generate the highest variety of signs it is enough to use one rule: both in the columns and in the rows of the matrix there has to be represented from one to n every number (fig. 2). The next step was to create all the possible signs on the basis of permutation in the 2×2, 3×3, $4 \times 4, \ldots, n \times n$ 'worlds' as explained in the article of Otto Mezei in the Müvészet 77/3 and 78/6. In the block of the signs of one 'world' we started a new table as the column vector changed, and started a new line when the first element of the row vector changed (fig. 3- 5). For example the creation of the second table of the 4×4 'world' : we started to use the new column vector $4,3,1,2$ (after we had swept through the $4,3,2,1$ column vector of the first table with the row vectors from $1,2,3,4$ to $4,3,2,1$) and in the first row we used the row vectors from $1,2,3,4$ to $1,4,3,2$; we started in the second row with the row vector $2,1,3,4$ and so on.
On this basis in the $n \times n$ 'world' we can create $n!$ tables arranged in an $(n-1)! \times n$ matrix, and each of the tables will consist of $(n-1)! \times n$ signs.
Observing the 'behaviour' of the signs and their arrangement we can find almost unfathomable richness of symmetries. (I can mention here only some of them.) It is visible that in every table there are not 24 signs, but twelve pairs of signs which are up–down mirroring each other. If we connect these pairs we will get a set of symmetrical lines. The same set of lines describes the symmetries between tables, where the signs of the two connected tables are symmetrically left-right with each other, so the $1-24, 2-18, 3-22, 4-12, 5-16, 6-10, 7-23, 8-17, 9-20, 11-14, 13-21$, and the $15-19$ signs and tables are forming connected pairs (fig. 6).
Because of the limited length of this paper, I can tell only some words about the importance of the diagonally symmetrical (d.s.) signs. It is obvious that there are two (and only two) d.s. signs in each table. Looking the full block, we can see that one set of these signs, where the column vector equal to the row vector, positioned according to a rigid rectangular matrix, the other set (where the column vectors and the row vectors are inverse of each other) follows an irregular pattern (fig. 7). The signs of the matrix arrangement show the position of the table in the block, using the inverse central-symmetric position. If we connect the two d.s. signs in the tables, the lines (compare to the fig.6 symmetry structure) show the type of the 'structural unit'.
Another important feature which we can see (observing the d.s. signs) is the so called 'logical place' and the 'negative space'. As we put the signs beside each other, leaving a little space (the size of the unit square) between them, we can get a sign, formed between and by the two bordering signs in the so called negative space. See the white space between the black signs (fig. 8). The logical place is determined by the column and row vectors, describing a sign (or its inverse in the negative space).
The above mentioned matrix arrangement shows (mechanically) the serial number of the tables in central-symmetrical reverse order (24 for one, 23 for two and so on), the other arrangement (according to the logical place) is standing either on the place of the real serial number (1.,3.,8.,11.,14.,17.,22. and 24. table; these are the inactive logical places), or on the place where we can find the symmetrical pair of the inverse table (these are the active logical places). We can create the inverse table in a simple way : start the block and the tables in the lower right corner instead of the upper left, and continue both the lines and the tables from right to left instead of left to right. Looking the original and the inverse table, we can see the changing pairs (fig. 9). If we project this on the top of the symmetry structures (fig. 6),

we can get another interesting visual form (fig. 10).
There are several well researched, and (it is sure) lots of not yet recognised symmetries in this really endless system, where the number of tables and the signs in them growing with the n according to factorials. It is worth while to 'dig in' and find your own 'golden symmetries'.



Figures 1. – 10.