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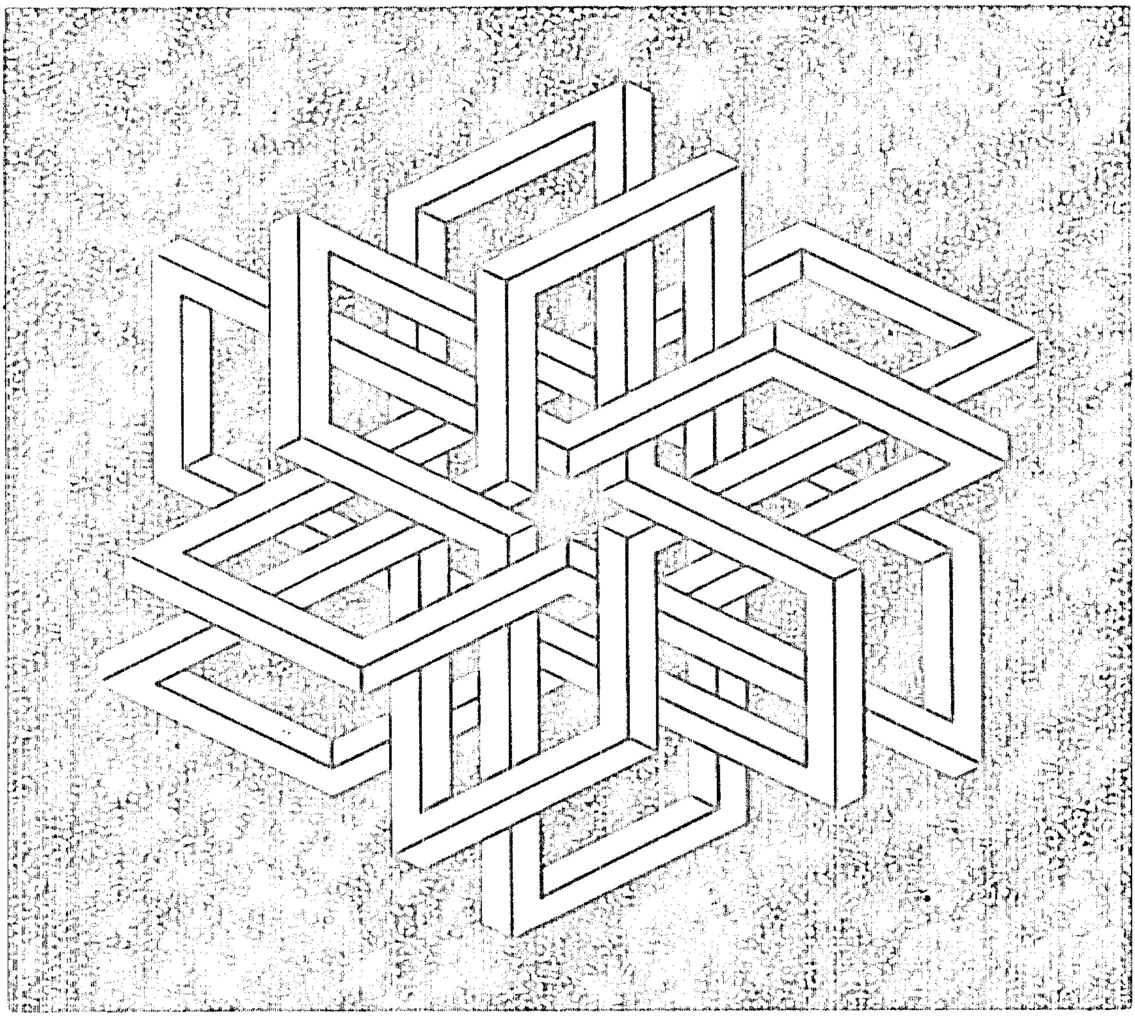

GROUP THEORY AND RECONSTRUCTION OF THE INTERNAL SPACE OF BIOLOGICAL OBJECTS

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Not-Eucledian manifestations of developing organism (S.V. Petukhov, 1981) point out that a biosystem exists and develops obeying its own natural laws and possessing its own topologic, algebraic and geometric structures. The structures are independent of the respective laws and structures in the perception space of the external observer. The observer perceives only projections of every object and event on the three-dimensional section in the universe of events. The reference system introduced by the observer for the description of macro-patterns in biological objects can be regarded as a purely external gauge field, that makes it possible to discern macro-details. But in a more profound sense, biological objects are indifferent to the reference systems of the external observer. A language that is adequate to the objects should employ other notions. The situation is similar to the electromagnetic field, that distinguishes charged particles in macrospace; however in a deeper structure of vacuum it loses any sense and other Yang-Mills gauge fields come to the foreground. Thus, I'd like to draw attention to a possibility of employing the language, which makes use of the lessons, taught by quantum physics and theory of symmetries. Because quantum theory apart from solving its own problems makes a contribution into the common pool of scientific worldoutlook as well.

The notion of "cell" within this approach is substituted by the concept of "space of every possible conditions of the cell" and this space can be simulated by a finite-dimensional vector space $\mathcal{H}_1^r$ of dimension r . Each state $|\varphi\rangle = \sum \langle i|\varphi\rangle |i\rangle$ can be identified completely by its projections $\langle i|\varphi\rangle$ onto the basic set $|i\rangle$ of states, expressing the independent potentials of the cell genome. If $|\varphi_0\rangle$ is spread uniformly over all $|i\rangle$, the cell itself is, in effect, an initial zygote and is not differentiated at all. In space $\mathcal{H}_1^r$ transformation group SU(r) is active. Developing its every potential uniformly in state $|\varphi_0\rangle$, the zygote can reproduce itself.

Let us consider the cell set of the organism during its onthogenesis from the zygote. Since the cells interact with each other, the state space of the whole organism $\mathcal{H}^{tot}$ is tensor product: $\mathcal{H}^{tot} = \mathcal{H}_1^r \otimes \mathcal{H}_1^r \otimes \cdots \otimes \mathcal{H}_1^r$

Any linear space can be decomposed in different ways into direct sums of subspaces: $\mathcal{H}^{tot} = \int \mathcal{H}_x dx$, $x \in X$. Any of the decompositions generates the fiber bundle space with base X and fibers $\mathcal{H}_x$, attached to points $x \in X$.

The space of organism's states $\mathcal{H}^{tot}$ can be performed as a fiber bundle space on different physically interpreted bases. For example we may get the bundling $\mathcal{H}^{tot}$ into a base of cells material, marked by an external observer with a system of macro-coordinates. But this base is inessential for the subject-matter and it can only be utilized while putting in the experimental data into the theory and putting out ready macroscopic results.

A state of the organism is vector $\Psi \in \mathcal{H}^{tot}$, which from the viewpoint of decomposition of $\mathcal{H}^{tot}$ into any base X is a cross-section of the fiber bundle: $x \mapsto \Psi(x)$. Changing the base can always be regarded as rebundling of fiber bundle $\mathcal{H}^{tot} \to X$ onto another base $\mathcal{H}^{tot} \to Y$ and a change of point of view on the cross-section:

$$\langle x | \Psi \rangle = \int_y \langle x | y \rangle \langle y | \Psi \rangle$$

That is a particular example of generalized harmonic analysis. More convenient base on which it is feasible to start constructing the theory is base $[SU(r)]^\wedge$, where $\hat{G}$ is understood as a multitude of irreducible representations of group G. Objects $y \in [SU(r)]^\wedge$ with its state space $\mathcal{H}_y$ are irreducible representations SU(r) and possess the same individuality as cell x, although in terms of functioning rather than of the "anatomical" sense.

It is necessary to present Hamiltonian H of the organism as an operator in $\mathcal{H}^{tot}$. And in terms of quantum numbers y it is easier to do. However, the most essential point for our analysis of onthogenesis is that identification of a Hamiltonian identifies its symmetry group G. Natural development of the organism is a quasi-stationary process and probability distribution P over $\mathcal{H}^{tot}$ is a Gibbs distribution. For an adult organism the distribution is focused on configurations $\Psi \in \mathcal{H}^{tot}$, where H reaches its minimum, i.e. on the ground states of the Hamiltonian. And it's essential that in our case one Hamiltonian corresponds to a number of the various ground states of the Hamiltonian. These ground states of adult organism describe pure thermodynamic phases, which are tissues of an organism.

Symmetry group G and set of its irreducible representations makes it possible for us to find a "quasi-corpuscular representation" for the organism, i.e. to find the independent objects of which the organism "is constituted" rather than "is made", because their interaction with each other is much weaker than their own internal energy. In our case this is nothing else but a set of different macro-tissues (organs) that "survived" and materialized owing to constructive interference of internal activities of the cells and interactions between them at number of cells approaching infinity.

And finally, group G gives us a chance to lay a bridge to perception Eucledian space and to calculate the phase equilibrium diagram i.e. distribution of macro-patterns for the survived proteins in the perception space of external observer.

So, outward appearance of an organism is nothing else but a diagram of the phase equilibrium.

References

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