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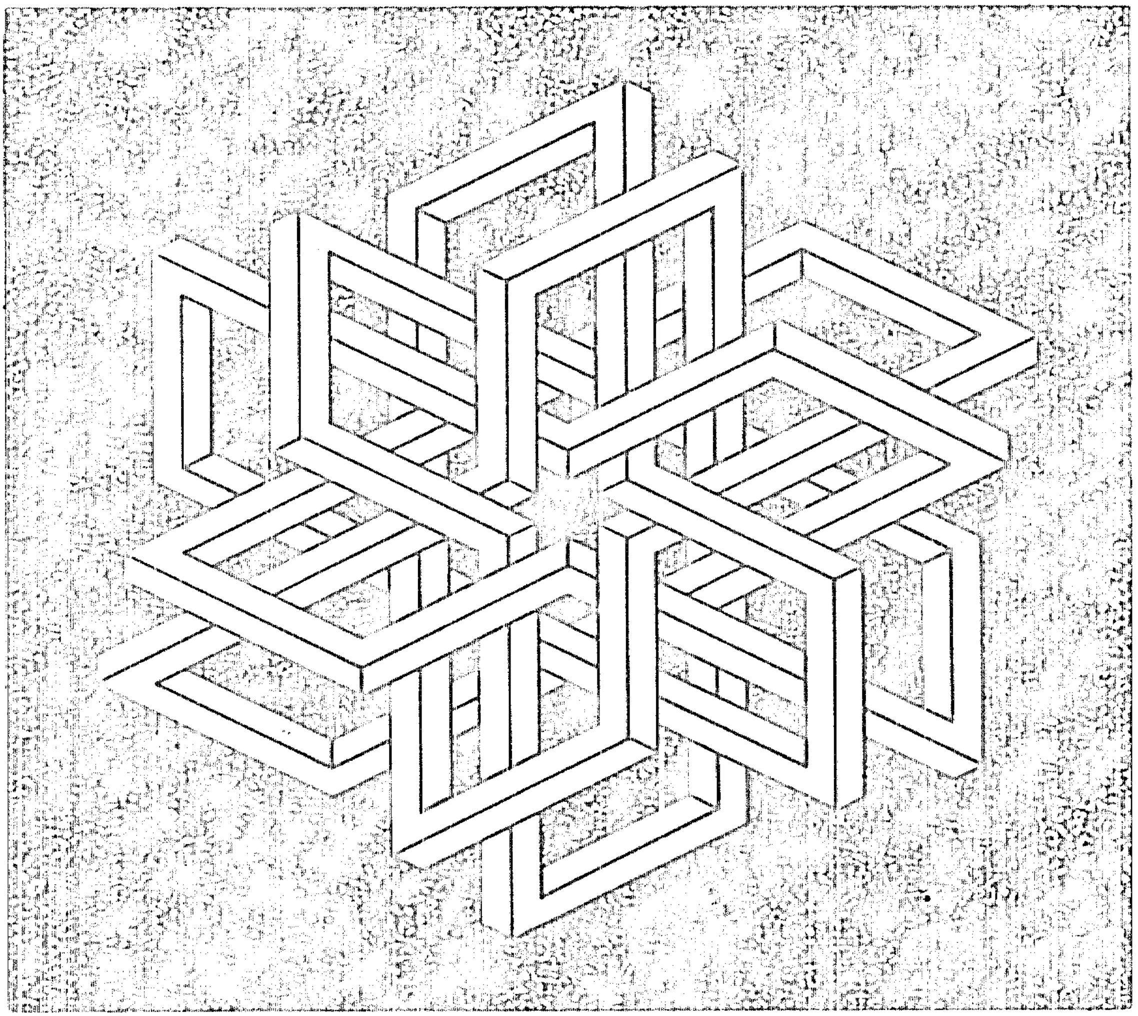

SQUARE-ONE (SQ-1): A NEW PUZZLE AND A NEW ALGEBRA

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While the title reminds of the classical question: *"which came first, the chicken or the egg?"*, in this case we know the answer. Pursuing the analogy a bit further, though not as far as to chaos or Big Bang, we can say: *"First there was the Rubik's cube"*. During the Rubik's craze time there further appeared an inconspicuous toy on the pages of French journal *"La Recherche" (1981)*. You may easily create this toy/puzzle yourself by gluing together certain pieces of Rubik's cube as shown in Figure 1. The unseen pieces are to be also glued together to form a 2 x 2 x 2 cubic block which prevents the rotations of the three unseen faces - the left, back, and the lower faces. Thus, only the three faces - the front, upper, and right can rotate and it is easy to show that there are restrictions even on these rotations, due to constraints exerted by blocks into which the original pieces merge in this arrangement. The idea belongs to French mathematician Raba and the puzzle is known as *"Raba's bicube"*. If you glue the pieces together in a mirror-like manner, you will create a second type of Raba's bicube. Crystallographers would call them *"enantiomorphic"* (mirror-like) partners.

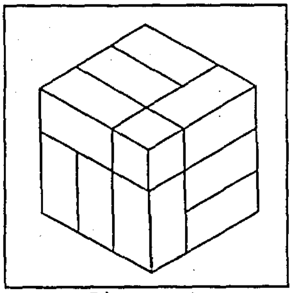

Figure 1.
Raba's bicube

Virtually on the same page there also appeared another creation of Raba, which he calls the *"taquenoscope"*. This flat puzzle, shown in Figure 2., consists of three intersecting circles, which you can cover by any picture you want, to start the usual game of *order→disorder→back-to-order*. Again it can appear in two mirror-like variations, which form an *"enantiomorphic pair"* in the language of crystallography. It is clear that the mirror image induces *"operator isomorphism"* of the two taquenoscopes as well as the two bicubes. It is far more interesting that the taquenoscopes are also *"operator isomorphic"* with the bicubes.

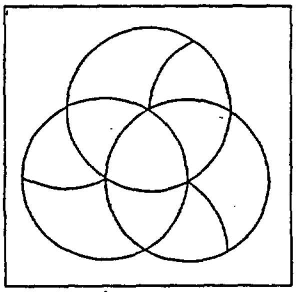

Figure 2.
The taquenoscope

In the third instance there appeared the *NEW ALGEBRA*. An algebra, suitable for the description of motions in systems like the bicubes or taquenoscopes. To feel its spirit, it is good to compare it with the solution of Rubik's cube. All shifts and rotations of parts of Rubik type puzzles form motion groups. Elements of the group of motions for Rubik's cube are (co)sets of words in six generators *R, L, U, D, F, B* (right, left, up, down, front, and back) and every set of words has a meaning of a motion. Every element of this group can be applied to any state.

The algebra of Raba's bicube is more complicated, though the number of states is much smaller (only 43 560). The states are to be described separately by geometry and color patterns. The motion from one state to another is to be described by a special graph construction which we call the *"graph amalgamated by a group"*. It consists of a *"basic graph"*, which describes transitions between geometries. An *"amalgamating group"*, which applies only to states with *"basic geometry"*, permutes the segments (colors).

This result inspired a search for a puzzle, more spectacular and inviting than Raba's bicubes or taquenoscopes. The story of the invention is described by V. Kopský in pilot issue of *"Puzzle World"* (1992) and the *NEW PUZZLE*, marketed now by *IRWIN TOY* under the name *SQUARE-1*, is shown in its pristine form in Figure 3. It is deliberately designed to evoke memories of Rubik's cube. After a few moves, however, you realize that this puzzle represents a major advance beyond Rubik's cube. The toy is player friendly. You can enjoy the shape changes before discovering the laws behind them. If Zassenhaus (1981) characterized the cube as *"group theory for everybody"*, then with SQ-1, we have the *"graph theory for everybody"* as well.

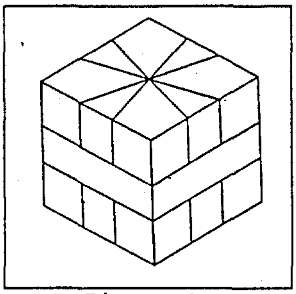

Figure 3.
SQUARE - 1

The graph of SQ-1 can be viewed as a large number (1 625 702 400) of sheets on each of which there is a copy of basic graph connecting the 340 geometric states (shapes). A set of algebraic tricks cuts the problem down to a graph with 65 vertices. Determination of the graph is an example of development of fine mathematics on a common sense basis. There are the two problems for connoisseurs: Find the path to the most distant shape from the original, so-called *"Antipode"* (Figure 4.). Secondly: Find all those picturesque shapes (called *"extra shapes"*) which appear when the horizontal rotation is performed only half-way.

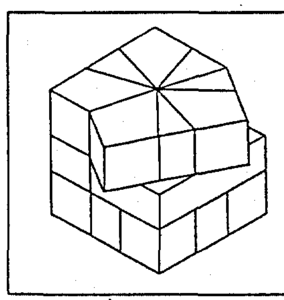

Figure 4.
The Antipode

The curious algebra which intertwines graphs with groups, can be so far applied to four types of known toys: *Raba's bicubes* and *taquenoscopes*, *Rubik's Magic*, and *Square - 1*. Perhaps the future will bring also a more serious use for this algebra.

References:

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