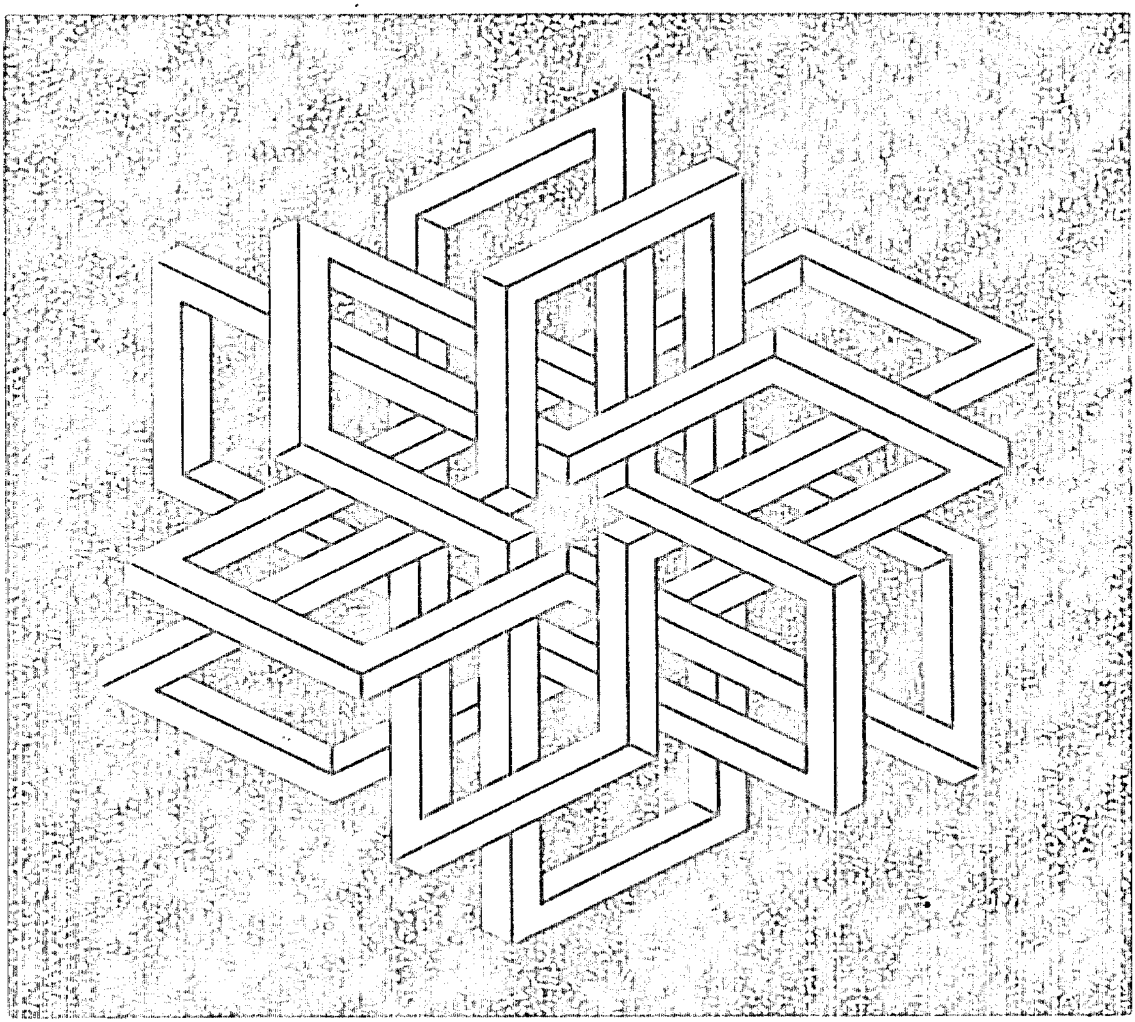
Symmetry: Culture and Science

SYMPOSIUM
Symmetry of Patterns

The Quarterly of the International Society for the Interdisciplinary Study of Symmetry (ISIS-Symmetry)

Editors:
György Darvas and Dénes Nagy

Volume 3, Number 1, 1992



COLORED ANTISYMMETRY

Slavik V. Jablan
The Mathematical Institute
11001 Belgrade, Knez Mihailova 35, pp. 367
Yugoslavia

The concept of P-symmetry (permutation symmetry) introduced by A.M.Zamorzaev is defined as follows. If P is a subgroup of the symmetric permutation group of p indices, and G is a discrete symmetry group, every transformation $C=cS=Sc$, $c\in P$, $S\in G$ is a P-symmetry transformation. Every group G^P derived from G by such a substituton of symmetries by P-symmetries is a P-symmetry group. If the substitutions included in G^P exhaust the group P, G^P is a complete P-symmetry group. Every complete P-symmetry group G^P can be derived from its generating group G by means of searching in G and P for normal subgroups H and Q for which the isomorphism $G/H\cong P/Q$ holds, by paired multiplication of the cosets corresponding in this isomorphism and by the unification of the products obtained. The groups of complete P-symmetry fall into senior ($G=H$ and $G^P=G\times P$), junior ($G/H\cong P$ and $G^P\cong G$) and middle groups for $Q=P$, $Q=I$ and $I\subset Q\subset P$, respectively.

In the case of Belov (p)-symmetry, the group $P=C_p$ is generated by the permutation $c_1=(12...p)$ satisfying the relations:

$$c_1{}^p=I \quad c_1S=Sc_1, \quad S\in G.$$

In the case of Pawley (p')-symmetry, the group $P=D_{p(2p)}$ is the regular dihedral permutation group generated by the permutations c_1 and $e_1=(11')$ satisfying the relations:

$$c_1{}^p=e_1{}^2=(c_1e_1)^2=I \quad c_1S=Sc_1 \quad e_1S=Se_1, \quad S\in G.$$

In the case of the ($p2$)-symmetry, the group $P=D_p$ is the irregular dihedral permutation group generated by the permutations c_1 and $e_1=(12)$ satisfying the relations:

$$c_1{}^p=e_1{}^2=(c_1e_1)^2=I \quad c_1S=Sc_1 \quad e_1S=Se_1, \quad S\in G.$$

By the mathematicians of Kishinev school (Zamorzaev, Palistrant, Galyarskii, Karpova...) the crystallographic space P-symmetry grups $G_3{}^P$ are derived for $P=C_p, D_{p(2p)}$ and D_p (p=3,4,6). After the small corrections made by the author for (62)-symmetry groups, their numbers are: 817 $G_3{}^p$ (111 $G_3{}^3$ + 327 $G_3{}^4$ + 379 $G_3{}^6$), 3610 $G_3{}^{p2}$ (309 $G_3{}^{32}$ + 1667 $G_3{}^{42}$ + 1634 $G_3{}^{62}$) and 2212 $G_3{}^{p'}$ (309 $G_3{}^{3'}$ + 950 $G_3{}^{4'}$ + 953 $G_3{}^{6'}$) (p=3,4,6).

If $P=C_2{}^l$ we have the simple (l=1) and multiple ($l\geq 2$) antisymmetry groups, where $P=C_2{}^l$ is generated by the antiidentities $e_1, e_2, \ldots, e_l$ of the first, second,..., lth kind satisfying the relations:

$$e_i^2=I, \quad e_i e_j=e_j e_i, \quad (i,j=1,2,\ldots,l)$$

and commuting with the symmetries of the generating symmetry group G.

The derivation of l-multiple antisymmetry groups is realized by Zamorzaev & Palistrant, and completed by the author. There are $N_1=1191$, $N_2=9511$ [3], $N_3=109139$, $N_4=1640955$, $N_5=28331520$ [1] and 419973120 [3] junior l-multiple antisymmetry groups of the M^m-type $G_3{}^l$.

The next problem: derivation of crystallographic colored l-multiple antisymmetry groups $G_3{}^{l,p}$, $G_3{}^{l,p'}$ and $G_3{}^{l,p2}$ (p=3,4,6) is solved by the use of generalized antisymmetric characteristic method (AC-method):

Definition 1. Let all the products of P-symmetry generators of a group $G^{(P)}$, within which every generator participates once at the most, be formed, and then the subsets of transformations equivalent with regard to P-symmetry, be separated. The resulting system is called the antisymmetric characteristic of the group $G^{(P)}$.

As the final result, for the complete crystallographic (P,l)-symmetry junior three-dimensional space groups of the M^m-type ($P=C_p$, $D_{p(2p)}$, D_p) the numbers $N_m{}^P$ (p=3,4,6) are the following:

$N_1{}^P = 379\ G_3{}^{1,3} + 1705\ G_3{}^{1,4} + 2050\ G_3{}^{1,6} = 4134\ G_3{}^{1,p}$
$N_2{}^P = 2050\ G_3{}^{2,3} + 11447\ G_3{}^{2,4} + 16234\ G_3{}^{2,6} = 29731\ G_3{}^{2,p}$;
$N_3{}^P = 16234\ G_3{}^{3,3} + 90160\ G_3{}^{3,42} + 153720\ G_3{}^{3,6} = 260114\ G_3{}^{3,p}$;
$N_4{}^P = 153720\ G_3{}^{4,3} + 645120\ G_3{}^{4,4} + 1249920\ G_3{}^{4,6} = 2048760\ G_3{}^{4,p}$;
$N_5{}^P = 1249920\ G_3{}^{5,3} = 1249920\ G_3{}^{5,p}$;

$N_1{}^{P'} = 1634\ G_3{}^{1,3'} + 6361\ G_3{}^{1,4'} + 7288\ G_3{}^{1,6'} = 15283\ G_3{}^{1,p'}$;
$N_2{}^{P'} = 13391\ G_3{}^{2,3'} + 53664\ G_3{}^{2,4'} + 78825\ G_3{}^{2,6'} = 145880\ G_3{}^{2,p'}$;
$N_3{}^{P'} = 150197\ G_3{}^{3,3'} + 441924\ G_3{}^{3,4'} + 967568\ G_3{}^{3,6'} = 1559689\ G_3{}^{3,p'}$;
$N_4{}^{P'} = 1888320\ G_3{}^{4,3'} + 2056320\ G_3{}^{4,4'} + 10321920\ G_3{}^{4,6'} = 14266560\ G_3{}^{4,p'}$;
$N_5{}^{P'} = 19998720^{5,3'} = 19998720\ G_3{}^{5,p'}$;

$N_1{}^{P2} = 1634\ G_3{}^{1,32} + 11365\ G_3{}^{1,42} + 13391\ G_3{}^{1,62} = 26390\ G_3{}^{1,p2}$
$N_2{}^{P2} = 13391\ G_3{}^{2,32} + 94281\ G_3{}^{2,42} + 150197\ G_3{}^{2,62} = 257869\ G_3{}^{2,p2}$;
$N_3{}^{P2} = 150197\ G_3{}^{3,32} + 809088\ G_3{}^{3,42} + 1888320\ G_3{}^{3,62} = 2847605\ G_3{}^{3,p2}$;
$N_4{}^{P2} = 1888320\ G_3{}^{4,32} + 3870720\ G_3{}^{4,42} + 19998720\ G_3{}^{4,62} = 25757760\ G_3{}^{4,p2}$;
$N_5{}^{P2} = 19998720\ G_3{}^{5,32} = 19998720\ G_3{}^{5,p2}$ [2].

References:

[1] JABLAN S.V. (1987). Acta Cryst., A43, 326-337.
[2] JABLAN S.V. (1992). Acta Crystall. A48 (to appear).
[3] ZAMORZAEV A.M. (1976). *Teoriya prostoj i kratnoj antisimmetrii*. Kishinev: Shtiintsa.
[4] ZAMORZAEV A.M., GALYARSKIJ E.I., PALISTRANT A.F. (1978). *Tsvetnaya simmetriya, eyo obobscheniya i prilozeniya*, Kishinev: Shtiintsa.
[5] ZAMORZAEV A.M., KARPOVA Yu.S., LUNGU A.P., PALISTRANT A.F. (1986). *P-simmetriya i eyo dal'nejshee razvitie*. Kishinev: Shtiintsa.