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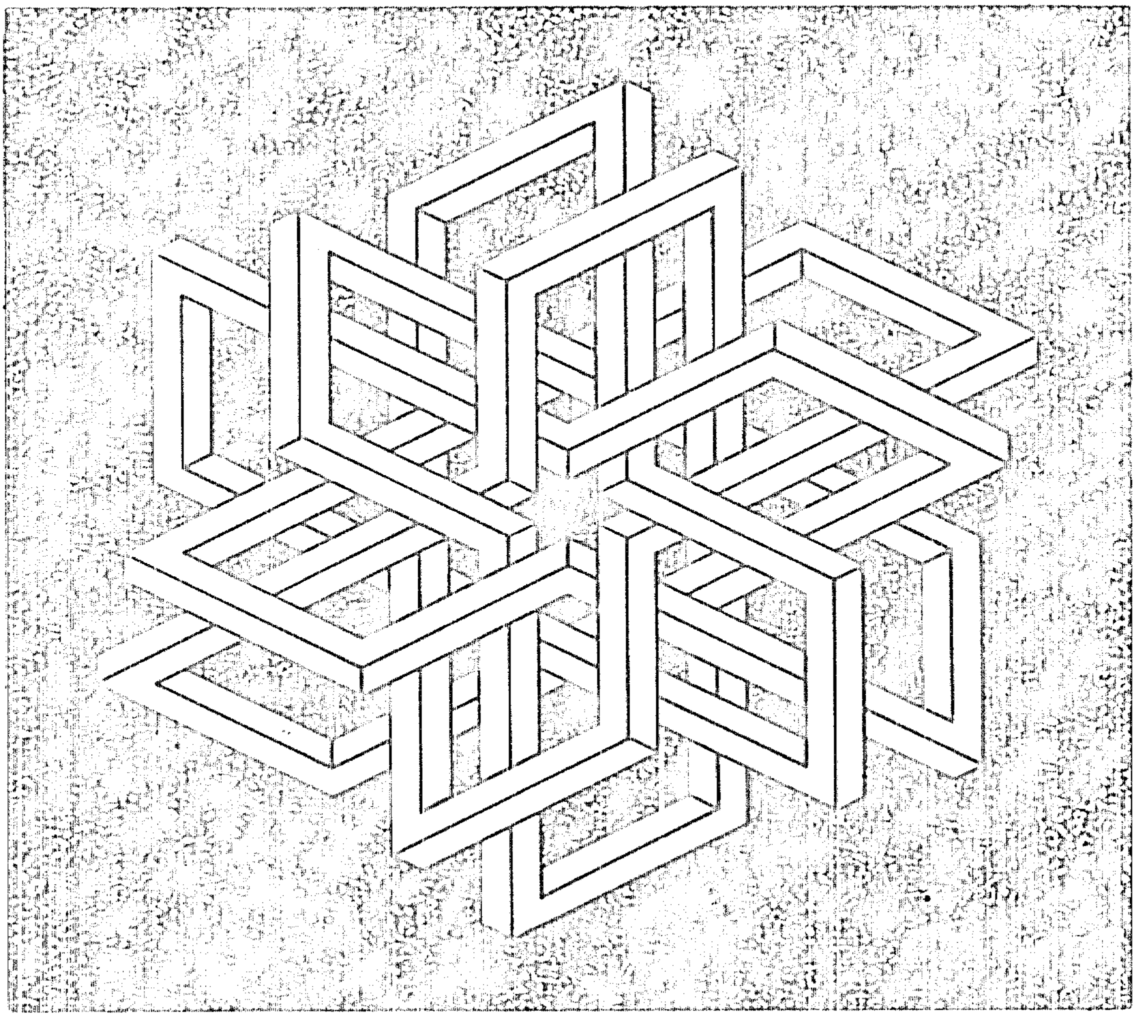

SYMMETRICAL QUASIPERIODIC TILINGS

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Tilings of arbitrary point symmetry are readily generated from multigrids by the dual method (de Bruijn, 1981) in which overlapping grids of a given spacing are oriented perpendicular to regularly spaced directions in the plane. The dual transformation then maps the intersections of grid lines into tiles and open spaces into tile vertices. If the multigrid is "singular" (i.e. possesses multiple-line intersections) the tiling contains polygons. For instance a 3-, 4- or 5-line intersection maps into a hexagon, octagon or decagon. etc. A non-singular multigrid leads to a tiling consisting only of rhombs. Of special interest are those aperiodic patterns generated when the grid-spacing consists of two different spacings. Moreover, if the tiling is amenable to self-similar decomposition (inflation or deflation) it may readily be continued to infinity. In particular such a tiling may be obtained if the grid spacing is quasiperiodic and is itself amenable to decomposition into quasiperiodic sequences consisting of smaller or larger spacings. The latter may or may not be self-similar with respect to to the starting sequence. However, the spacings must be suitably related to the point-symmetry associated with the grid directions. The most significant sequences are those that reoccur after several inflations or deflations. Familiar examples are the rhombic versions of Penrose's infinite-sun, -star or -cartwheel pentagonal patterns which may be obtained from the Fibonacci sequence (Grünbaum and Shepard, 1986). The cartwheel pattern results because the Fibonacci sequence has a point of pseudo-mirror symmetry. However, the Fibonacci sequence and, in general, other quasiperiodic sequences possess points of perfect mirror symmetry. When these points are located at the center of the resulting multigrid, striking patterns of even point symmetry are produced. Generally, such a tiling may be decorated with any of its deflated tilings as well as the associated multigrids. The various decorations of the tiles yield ways of distinguishing inequivalent tiles of the same shape. By giving inequivalent tiles different colors, especially striking patterns may be obtained.

Here we describe recent work on decagonal and octagonal tilings. Interesting decagonal tilings may be generated by taking advantage of the three mains points of mirror symmetry in the Fibonacci sequence (Ingalls, 1992). Two of the resulting pentagrids generate different patterns, or more precisely, patterns not locally isomorphic (LI) to each other, while the third, which is singular, generates a tilings of polygons. These three tilings are also of different LI classes from that of the Penrose patterns. One is the basic pattern for windows of the University of Washington's new Physics Building. There are also points in these 10-fold tilings that lead to infinite 5- or 2-fold tilings of the same LI class. Octagonal tilings, on the other hand, may be generated from tetragrids based on several completely different quasiperiodic sequences compatible with 8-fold symmetry. Each of the latter also possesses three points of mirror symmetry and one point of pseudo-symmetry. As in the decagonal case, singular as well as non-singular multigrids are obtained. The resulting tilings therefore may be rhombic or polygonal, and may possess not only points of 8-fold symmetry but also points of cartwheel-, 4- and 2-fold symmetry as well. The figures show two tetragrids of 8-fold symmetry, based on different grid sequences, and the tilings superposed, resulting from these tetragrids.

References

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