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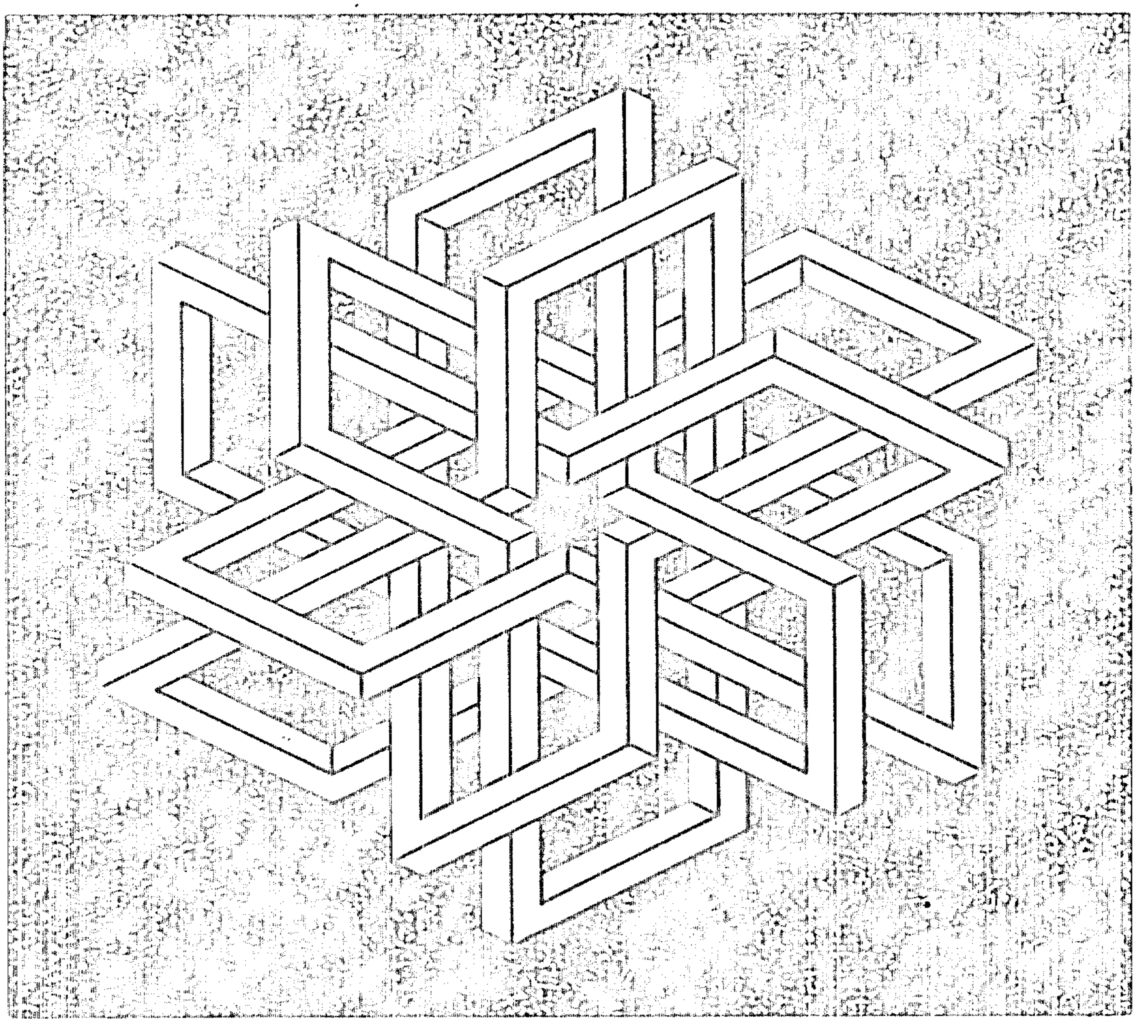

THE SHAPE OF HARMONIC RELATIONSHIPS IN NOISE

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Many disciplines attend to the *harmonic relationships* among the design elements of their domain. These harmonic relationships are *metric ratios* related to the extensive dimensions of various sensory channels: space, time, pitch, hue, posture, etcetera. Disciplines that are concerned with the *construction* of ratios must also attend to their *measurement*: valid construction must be verified. For example, a piece of music must be played within an acceptable tolerance of the score's denotations to satisfy a knowledgeable listener.

In the other direction, disciplines concerned with measurement often use *found ratios* to infer design intent or construction principles. But, measurement and construction always occur with some "random" error. Surprisingly, all ratios are not equally likely, when generated from random data. This has interesting implications for both measurement and construction of harmonic ratios.

Construction error can play a significant role in *illusion making*. The following simple ratios generate angles that are significant in modular structures.

RATIO	DEGREES	ERROR ON RANGE
acos(-1/3)	109+	less than 1%
3 pi / 5	108	
2 pi / 5	72	less than 1%
acos(1/3)	70+	
atan(2)	63+	less than 3%
2 atan(0.618..)	63+	
acos(1/2)	60	
atan (1.618...)	59-	

We might do well to look for these ratios when measuring a thing that is composed of many similar parts. Unfortunately, the untrained eye, as well as crude measurement or reasoning techniques will not detect the differences between members of the same groups. Some of these ratios arise from incommensurate geometries. Boundaries containing such angles combined with *construction error* allow for *constructive aliasing* and transitions between incommensurate geometrical forms.

Measurements are inaccurate on the order of the smallest unit used. This *measurement error* and the promotion of suspected ratios (such as those identified above) can lead to aliasing when the suspects occur within the construction and/or measurement tolerances. To be more accurate in expressing found ratios then, they should be expressed with both the ratio and its associated error. Unfortunately this is not enough.

Indeed, a number of other forces can conspire to create an illusion. Comparing ratios derived by measurement with those generated by chance will give some indication of their significance. Since all ratios do not occur with equal frequency, the problem of identifying *significant ratios* is compounded. The curve describing the likelihood of different ratios (when generated from random data,) must be identified. This curve encodes "the shape of harmonic relationships in noise." A methodology for identifying significant ratios is derived by relating this curve to found data.

It is odd that this curve is not flat and has a maximal value. We are left with some intriguing questions. Is this a "design principal" that promotes certain ratios? Is *random luck* a good designer? Can this "design principal" account for occurrence of certain ratios in our observations or are other design principals at work? Could other "design principals" be designed in terms of this one? When we examine mathematically defined constructs such as regular polyhedra, semi-regular polyhedra and modular arrays, are the found ratios significant using this methodology? When are found ratios illusory? When are they meaningful?