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Symmetry of Patterns

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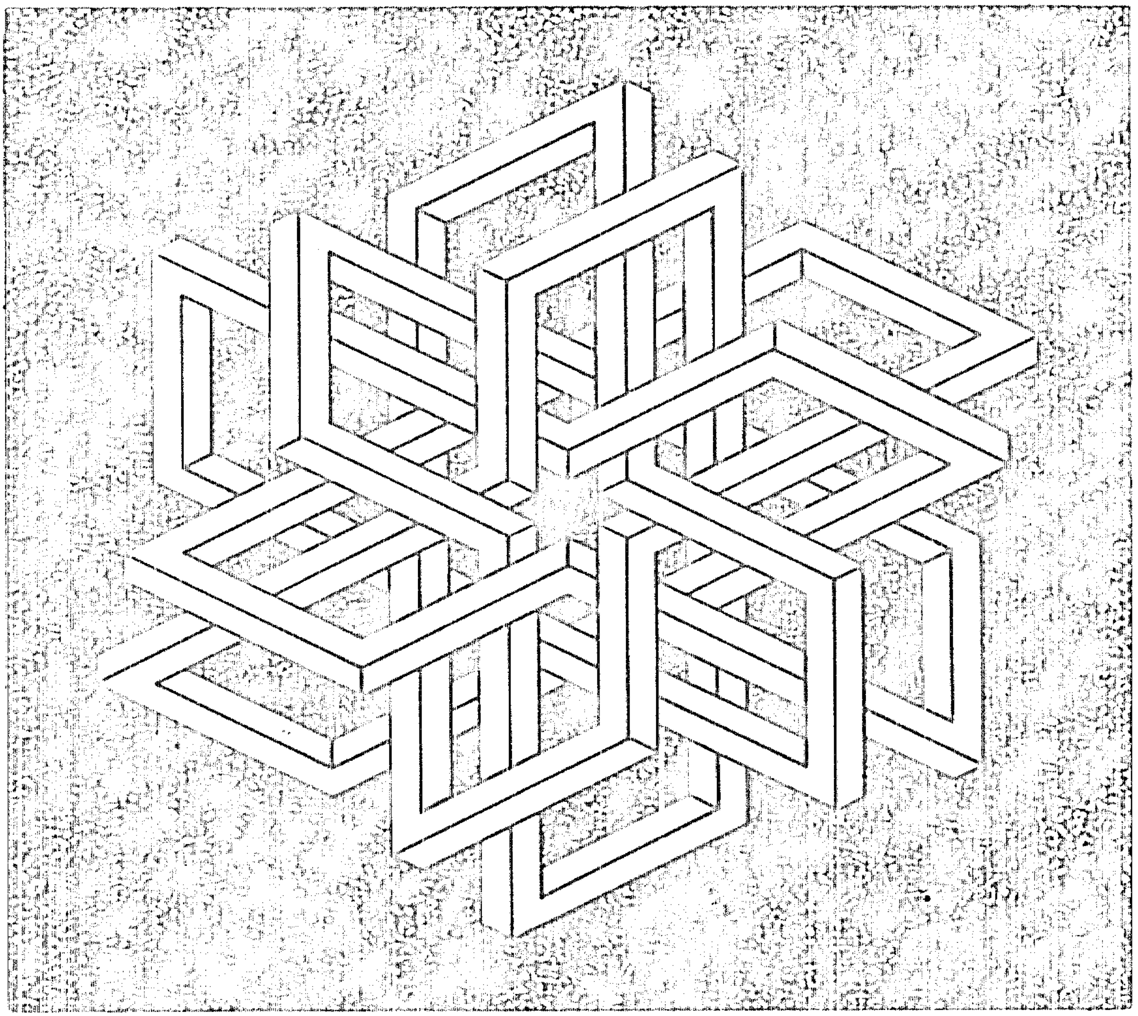

SYMMETRIES OF LAWS AND SYMMETRIES OF PATTERNS

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Symmetries of laws and patterns are related but not identical concepts. According to Hermann Weyl (1952) and in the words of Richard Feynman (1965) "a thing is symmetrical if there is something that you can do to it so that after you have finished doing it it looks the same as it did before". This symmetry concept directly applies to patterns and can be generalized to include laws. A symmetry transformation of the latter can also be characterized by the requirement that the *set* of patterns it allows remains unchanged under the transformation: a pattern the law allows is transformed into a pattern that it allows too. Examples firstly are the laws of physics and, as patterns, the orbits and processes they allow. Secondly, *problem* and *solutions* may be substituted for *law* and *patterns* in the definition of the symmetries of laws. A well-known example is the problem of Michel (1980) - see also Genz (1991) - to connect four cities located on the corners of a square by the shortest possible network of streets (Figure 1).

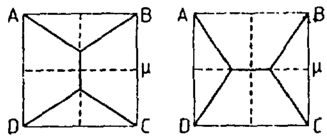

Figure 1
The shortest networks of streets connecting cities A, B, C, and D

The symmetries of the problem are that of a square, whereas each individual solution has the symmetries of a rectangle only. Thirdly, a symmetrical *purpose* (substituting for *law*) can be fulfilled by a machinery that is either itself symmetric (a fortress) or can *assume a symmetrical set of states* (the gun barrel of a tank). These points are discussed at length by Genz (1987) and (1991).

The symmetry transformations of suitably chosen patterns in space include translations, rotations, and mirror reflections. Translations and rotations are also symmetries of the laws of nature. The laws of nature are, however, *not* symmetrical under mirror reflections. This is well known from experiments on radioactive beta-decays of atomic nuclei originally performed 1957. Self-similar patterns, among them fractals, remain unchanged by scaling transformations - enlargements and diminuitions. Since the laws of nature allow the existence of atoms with their characteristic sizes, but not the existence of smaller or larger atoms, these laws are *not* scaling symmetric. At the most basic level, the laws of nature contain three quantities with independent dimensions: Plancks constant h, the velocity of light c, and the gravitational constant G.

They yield a length, a time, and a mass, which are therefore themselves fundamental, measurable quantities, named after Max Planck, and defined by the laws of nature alone. Taken together, the laws of nature can therefore be used to set the scale for any quantity having a dimension. This notwithstanding, the laws of nature for a *restricted class* of phenomena may be symmetrical under transformations that change the size, velocity and/or mass of the system and/or its state under consideration. Details are described in Genz (1987) and (1991).

As is well known, patterns cannot be symmetrical under arbitrary combinations of symmetry transformations of the plane. There are, in particular, only seventeen essentially different types of symmetries of wallpaper patterns (or plane crystals). A computer program of the author for the IBM-PC and compatibles generates symmetric plane patterns from the coordinates of arbitrarily given motives. Buchberger (1992) has recently written a new and much improved version of the part of the program that generates wallpaper patterns. It requires a video camera digitizing system as hardware and accepts arbitrary drawings, also in color, as input. Almost any input motive generates beautiful patterns as output.

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