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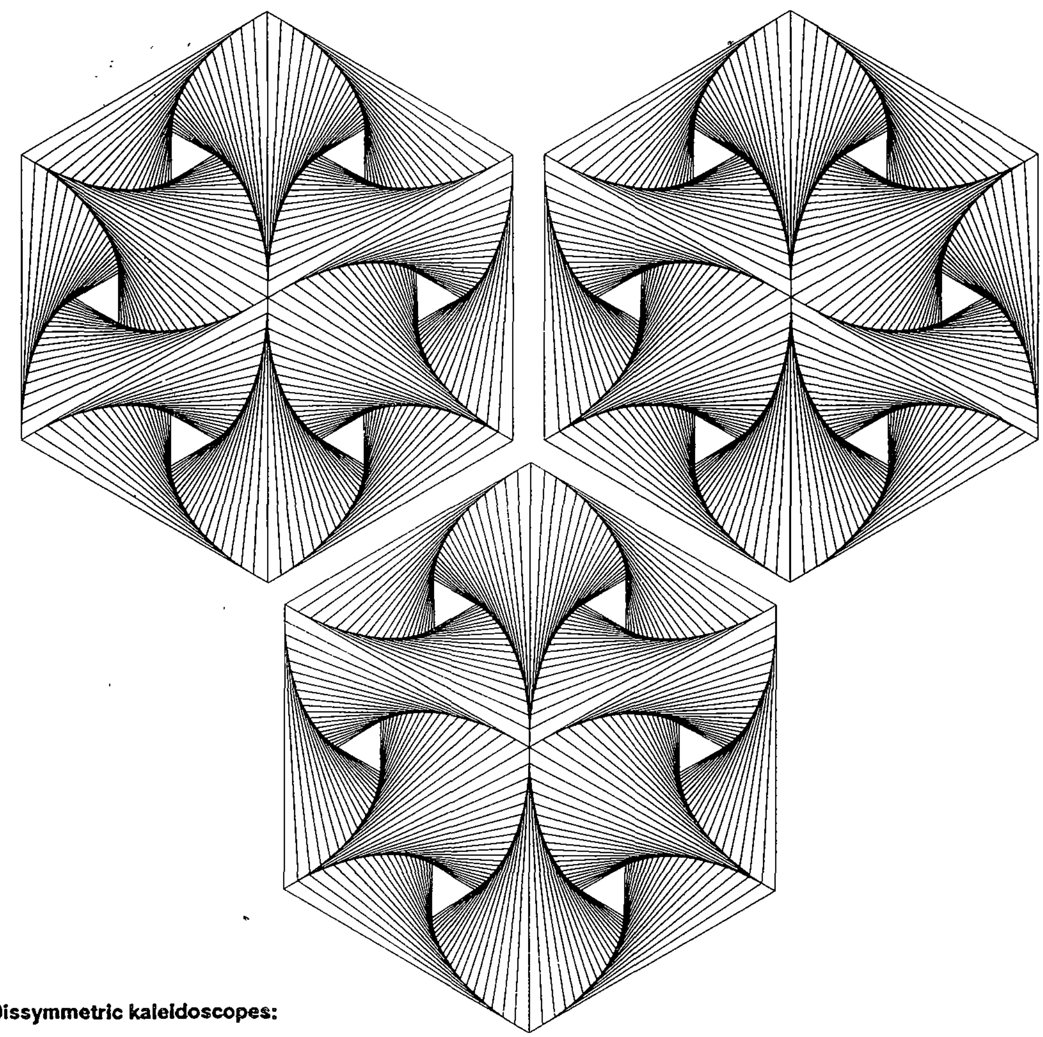

Dissymmetric kaleidoscopes:
Hommage à Pasteur

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SYMMETRY: SCIENCE & CULTURE

SYMMETRIC-ALGORITHMIC PROPERTIES OF REGULAR BIOSTRUCTURES

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QUESTION 1

what is symmetry?

The author researches systematically – with reference to modern scientific achievements – the problem of symmetry in macromolecular organic forms, which concerns not only biology and medicine but also technology (bionics, robotics, selforganizing cybernetic systems, technical design, etc.) and art (esthetics of proportions and configurations in architecture, pictorial art, music, etc.).

Symmetry in the configurations of biological bodies has forever attracted the attention of natural scientists as one of the most remarkable and mysterious natural phenomena. The very concept of symmetry emerged from ancient observations of the shapes of living bodies. Biosymmetries are devoted researches of many modern scientists and they were discussed by a special Nobel symposium in 1968. School curricula in biology include numerous instances of rotational, translational, mirror, and scale symmetries (that are similarity symmetries or Euclidean symmetries) in living organisms: metameric bodies of myriapods and annelids, fins of fishes, flowers, molluscs, shells, etc. A deeper biological insight resulted in the discovery of new facts of very different biological bodies (and their biological transformations) obeying the principles of symmetry and symmetrical algorithms.

It is well known that for biomorphology, symmetry plays a fundamental role in both the methodological and heuristic aspects. One can remember that the modern plant morphology has its origin in J. W. Goethe's biological researches. He paid great attention to the general tendency of plants to form spiral-symmetrical structures. Following Goethe, research of spiral-symmetrical biostructures became one of the first directions in mathematical biology; this direction produced, in particularly, many phyllotaxis theories and proved the existence of general biological laws (or properties) of morphogenesis.

Our investigations exposed the existence and the important biological role of the highest or non-Euclidean biosymmetries which were previously unknown. These symmetries are based on transformations from non-Euclidean transformation groups, which underlie the foundation of highest or non-Euclidean geometries (according to the well-known *Erlangen Program* of F. Klein and the generally accepted terminology). Following the *Erlangen Program*, the concept of transformation group found its principal application in the construction and classification of different geometries (and, respectively, symmetries): any geometry is comprehended as a science of invariants of the appropriate transformation groups. So the Euclidean geometry is based on the similarity transformation group. The affine geometry is founded on the affine transformation group; the conformal geometry is based on the Möbius (or circular) transformation group, while the projective geometry is founded on the projective transformation group.

We will mainly consider aspects of the researched non-Euclidean biosymmetries. Classical Euclidean biosymmetries are special cases of these non-Euclidean biosymmetries.

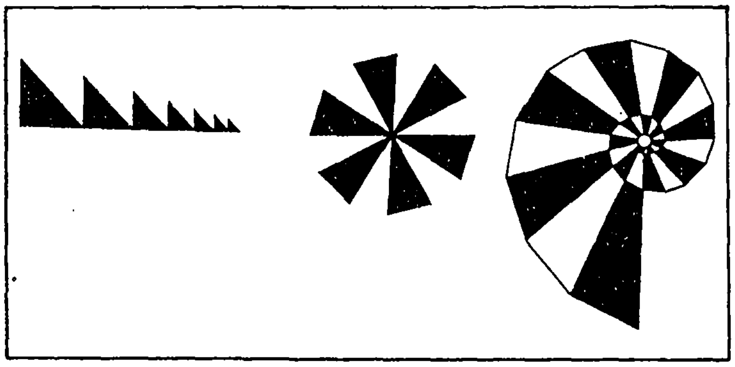


Figure 1: Geometric examples of similarity cyclomerisms or multiblock configurations with cyclic groups of similarity automorphisms (according to Shubnikov, 1960).

Within the existing variety of geometrically legitimate organic forms, we concentrate on structures whose components are integrated into an entirety in compliance with certain rules or algorithms which are the same along various lines and on various levels of biological evolution. These structures, which may be referred to as algorithmic, are of special interest for theoretical morphology and related sciences such as biomechanics, biotechnology, bionics, synergetics, etc. What important is that, in addition to regularly shaped biological objects, there are some in which the conjugation of components is less regular, if existing at all. This paper will consider algorithmic supramolecular biostructures which are chains or manifolds decomposable into commensurable and regularly positioned elements (or motive units S_k). Figure 1 shows such manifolds as discussed in the literature on biological symmetries.

The general rule of representing decomposable manifolds is that the preceding motive unit is transferred into the succeeding one by a certain fixed similarity transformation g. In other words, the neighboring motive units S_k are mutually conjugated by an iterative algorithm

$$S_{k+1} = g * S_k \tag{1}$$

Consequently, by reapplying the generating g transformation m times to a motive unit S_k, a component S_{k+m} is obtained; mathematically speaking, in the set S_k, a cyclic (semi-) group of transformations, G, is active which contains elements $g^0, g^1, g^2, ..., g^m, ...$ (a finite number of motive units in a biological object is neglected where necessary). In other words, this decomposition of the manifold, thus organized, includes a cyclic group of automorphisms and their motive units are aligned along the orbit of the appropriate cyclic group. For brevity, such configurations will be referred to as cyclomerisms, a term known in biology, no matter whether g is Euclidean or not in (1).

Similarity transformations in biomorphology are also known with reference to the scale of three-dimensional growth which is fairly frequently observed in animals and plants over extensive periods of individual development and is accompanied by mutually coordinated growth behavior of small zones distributed in the volume of the body, a behavior which is geometrically described as a scale transformation. With the transformation of as few as three points of the growing configuration known, the transformation of the continuum of its points may be assessed. Note that the growth-related ability of living organisms of most various species to exist in morphologically identical modifications of various scales is probably a morphological property of the living matter, which man has been aware of over a very long time and has been reflected in scientific, mythological, and fictional literature in which the dwarf—giant relationships are variously described.

Do the similarity symmetries and the scale of the volume growth exhaust all geometrically legitimate kinds of mutual conjugation of parts in a structure and ontogenetic transformations in living bodies? Or do they act in biomorphology as very particular cases of those kinds which are built according to non-Euclidean groups of transformations containing similarity subgroups? Our research has provided a positive answer to this latter question. As Sophus Lie (1893, p. 139), a pioneer of group theory in mathematics, noted, there are two basic ways to extend the similarity transformation group, either to Möbius transformation group or to projective transformation group. Both these ways have a biological value according to our research.

Here we especially emphasize the fundamental methodological and heuristic roles played by symmetry throughout our research. Now let us proceed to new findings on non-Euclidean symmetries in mutual integrations of individual biological bodies.

The biological value of similarity cyclomerisms (Fig. 1) seems to be clear. Which configurations do form an affine transformation if the generating transformation g of (1) is, for example, a Möbius' one? And do such non-Euclidean cyclomerisms have biological analogs? Our analysis reveals that the manifold of cyclomeric configurations noticeably expands in this case and includes, in addition to cylindrical, conic, and helical forms associated with similarity cyclomerism, more complex configurations such as lyre-, sickle-, bud-shaped, etc. (Fig. 2).

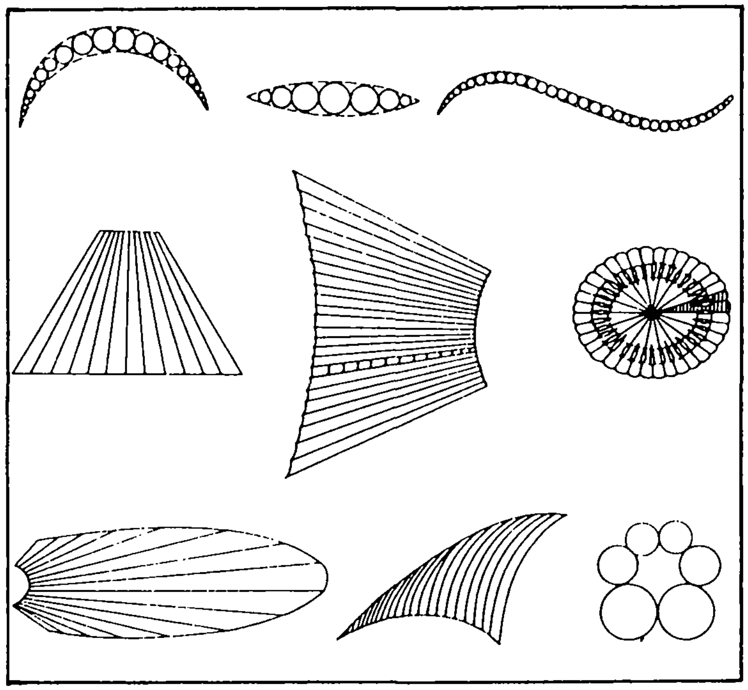


Figure 2: Examples of Möbius, affine, and projective cyclomerisms.

In these configurations the motive units may be different in shape, and the variation of these units along the cyclomerism may be essentially non-monotone. What important is that non-Euclidean cyclomerisms are as widespread in biological bodies along most varied lines and at various levels of evolution as the similarity cyclomerisms, but in the field of biosymmetry they have not been studied. Remarkably, non-Euclidean and Euclidean cyclomerisms are observed in analogous multi-component biological bodies simultaneously.

In particular, the horns of numerous animals are helically or rectilinearly conical and thus can be described as similarity cyclomerisms. In other animals, the horns are essentially different and configured as non-Euclidean cyclomerisms. Thus

Figure 3 shows the horns of a *Pantholops hodgsoni* which are described by a cyclomerism obtained by a Möbius generating transformation (of the so-called loxodromic type).

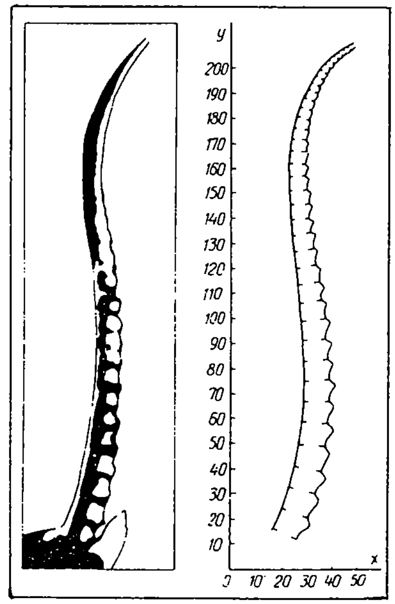


Figure 3: A segmented horn of the *Pantholops hodgsoni orongo* (according to Zenkevich, 1976) and its model as a cyclomerism with a Möbius generating transformation.

This example illustrates the general morphological procedure which reproduces the conventional procedure in which similarity cyclomerisms are analyzed. To begin with, the manifold of basic geometrical configurations is obtained, for instance by computer graphics. These configurations may be obtained by applying the iterative algorithm (1) to some motive unit, in particular, a point, with g from the group G of, say, Möbius or projective transformations. These abstract configurations and actual biological structures are compared. In establishing visual kinship, the coefficients of g for this biostructure are updated by the following procedure: the general analytical form of transformation from G is described and its coefficients are computed by substituting into this general equation the coordinates of those associated points of biological motive units which are transformed into one another by this transformation. Applying the transformation of g thus specified to the motive unit the required number of times, the desired cyclomerism is obtained which models the biostructure as a whole.

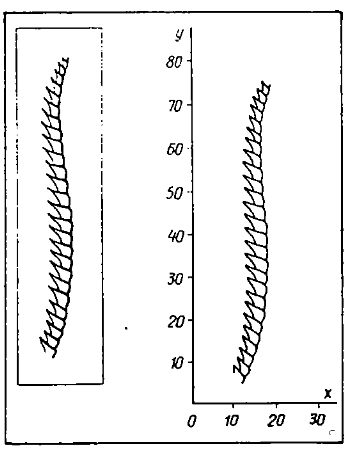


Figure 4: A comb-like antenna of an insect and its model as a Möbius cyclomerism.

Non-Euclidean cyclomerisms are also observed in the antennas of insects (Fig. 4).

Figure 5 shows that the sequence of vertebral disks in the human dorsal vertebra is reproduced by Möbius cyclomerisms in contrast to the configuration of the spine of numerous animals, in particular lizards, which is described by a classical similarity cyclomerism. By modifying the generating transformation and thus bending the "normal" configuration of the spinal cord cyclomerism, models of morphogenetic anomalies of the spine can be obtained. This is consistent with our data on Euclidean and non-Euclidean cyclomerisms, in men, animals, and plants for both normal and pathological shapes. This also agrees with the known formula for the morphology of plants that ugliness is a version of the norm.

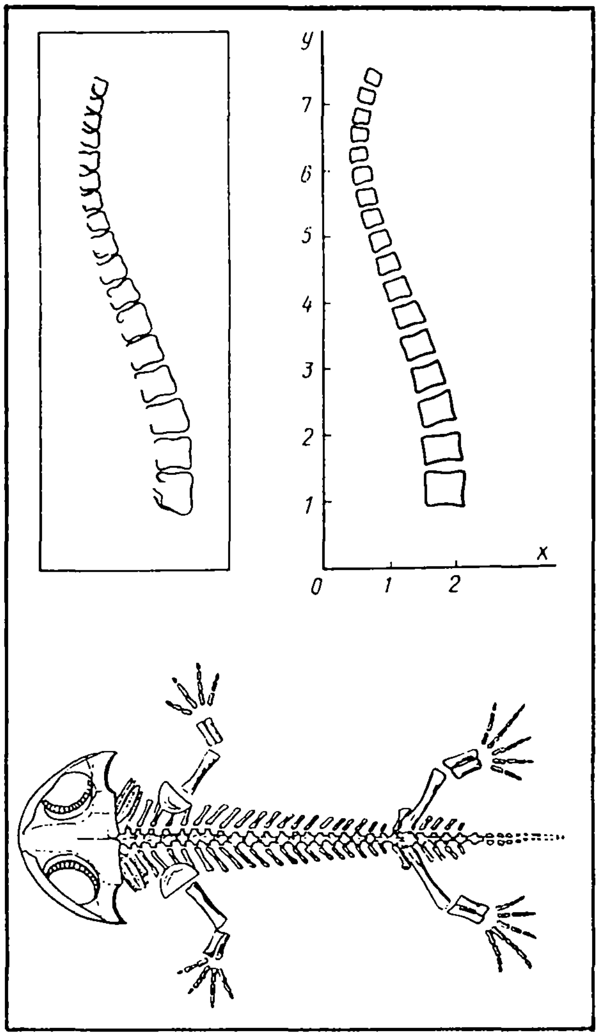


Figure 5: Vertebra of the torso part of the human spine and its model as a Möbius cyclomerism; for comparison, a similarity cyclomerism in the structure of the torso part of the spine is shown (according to Haeckel, 1898).

Möbius, as well as Euclidean, cyclomerisms are realized in the configurations of bird feathers, for, in addition to straight or helical shapes, there are lyre-shaped feathers as is the case of lyrebirds, so named because of the shape of their tails, and the tail of the Caucasian heathcock. Similarity cyclomerisms are most vivid in the shells of protozoa, but non-Euclidean cyclomerisms are also visible there (Fig. 6).

Both kinds of cyclomerisms are also observed in the structure and functioning of the vestibular organ and the eye muscles; the structure of the nervous system, bone tissue, vessels, and muscles; the positioning of biologically active points in the human body; buildings instinctively made by protozoa and social insects; formation of psychophysical delusions; the esthetics of proportions and shapes in architecture and art; etc. These are manifested in unicellular as well as multicellular organisms; consequently, the cell is not a morphogenetic unit in the general case. It is important that biological structures are frequently created not as a single cyclomerism, but as a cyclomerism hierarchy. (For more details see Petukhov, 1988, pp. 14-19.) Furthermore, the cyclomeric structuring is realized in the coloring series, weight parameters, and numerous other characteristics of living organisms as well as in the series of body units.

Nonlinearity of the generating Möbius and projective transformations makes possible for the sizes of individual cyclomerisms and their relations to vary in the same cyclomerism series; the geometrical invariants over the entire series are the only characteristics that are invariant with projective (or Möbius) transformations; for example, the values of cross ratios, or wurfs. Because Möbius and projective transformation groups underlie the conformal and projective geometries, these abstract geometries are materialized in biological structures.

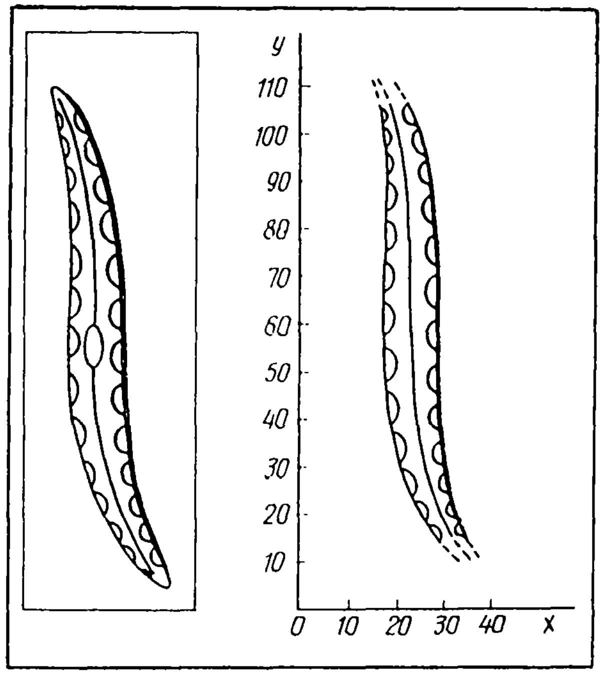


Figure 6: The shell (according to Haeckel, 1904) and its model as a Möbius cyclomerism.

Until recently, the desirability and adequacy of non-Euclidean transformations in the morphology of living bodies remained an open question. Furthermore, numerous scientists believed that the Euclidean transformations were quite sufficient for biomorphology. In particular, J. Bernal, a well-known British researcher, and his co-author, S. Carlyle, used the Euclidean group of movements in their concept of generalized crystallography which was to cover, in particular, the symmetries of living bodies (Bernal and Carlyle, 1968).

The desire to use generalized geometric concepts in biology was manifested as long ago as in the papers by D'Arcy W. Thompson (1917), D.V. Nalivkin (1951), and V.I. Vernadskii (1965). The research reported in this article has certain features which make it different from the foregoing articles. D'Arcy Thompson tried to find transformations which change one another into bodily shapes of various organisms such as the perch and pike. He obtained occasionally very complicated curvilinear transformations which reflected, in particular, the relative autonomy of the morphological development of body organs; he did not use the *Erlangen Program* as the basis in this comparative analysis. Nalivkin introduced curvilinear symmetries also without reference to the *Erlangen Program*; the specific rules that he proposed for the construction of "symmetrical" transformations resulted in transformation which did not belong to groups of point transformations because they disturbed the group principle of the one-to-one correspondence of points and were not conventional in other fields of natural sciences, as well. Vernadskii did not make his views on the non-Euclidean geometry of living matter explicitly, nor did he rely on the *Erlangen Program*. None of these papers considered special non-

Euclidean iterative algorithms and configurations to which they lead with cyclic groups of non-Euclidean automorphisms. Unlike those papers, this article analyses, above all, the rules for mutual conjugation of natural components in the individual biological body; this analysis proceeds along the lines of the *Erlangen Program*, which is important for mathematical natural sciences, in terms of specific non-Euclidean groups of transformations which include the similarity subgroup. Iterative algorithms which give rise to discontinuous configurations with a cyclic (semi)group of non-Euclidean automorphisms are given special attention.

The above discussion dealt with the structure of static organic forms. But Euclidean and non-Euclidean cyclomerisms have also a direct bearing on the kinematics of a broad range of biological movements which can be on numerous occasions interpreted as a process in which cyclomerisms replace one another. When non-Euclidean cyclomerisms are brought into the picture, a better insight is obtained into the relation of development of organisms or their parts with the ability of the living matter to remain in different cyclomerism states.

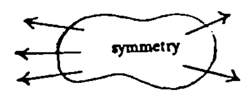
In analogy with the polymorphism of crystals, whose lattices can under certain conditions be restructured with change of their symmetry groups, the ability of living bodies to restructure their Euclidean and non-Euclidean cyclomerisms may be referred to as cyclomeric polymorphism. The concept of cyclomeric polymorphism sheds additional light on the fact that in individual development of multi-component structures of the cyclomeric type, the transition from one cyclomerism to another proceeds as a relay race in the series of motive units (for example, opening of cones as the scales ripen in case of composite flowers such as daisies, etc.) and may be referred to as the cyclomerism change wave. The natural morphogenetic movements in numerous multi-component biological bodies such as the so-called excurvature in development of the *Volvox* colony may be modeled as the simultaneous propagation of two or more cyclomeric change waves in the series with a certain interval. (For more details see Petukhov, 1988, pp. 17-21.)

Many living bodies of cyclomeric and non-cyclomeric constructions are capable of integral three-dimensional growth of geometrically regular kinds. Three-dimensional growth of living bodies is a challenging and mysterious case of orderly cooperative behavior of numerous elements. This growth is essentially different from the surface growth of crystals which occurs by accumulation of matter on the surface and does not involve the internal areas. The three-dimensional growth of living bodies entails change in the dimensions and frequently in the shape of the internal areas. The process is cooperative in the sense that, from a knowledge of the transformation of some points in the body figure, the transformation of the entire set of points in that figure may be determined. This is the case of plants and animals (growth of leaves and flowers of some plants, larvae of insects, adult fishes, etc.) and is expressed in the proportional growth of all parts of the body. With the transformation of the position of three points in a body, undergoing a known scale transformation, the transformation of the entire set of points in the body can be determined.

Our research revealed the existence of non-Euclidean kinds of three-dimensional growth, notably Möbius and affine, which had not been known before. It is also important that the three-dimensional growth of living bodies can be interpreted in terms of cyclomeric polymorphism. (For more details see Petukhov, 1988, pp. 26-31.)

Shubnikov and Koptsik (1972, p. 26) define symmetry as the broadest maximal group of automorphisms of the object. The author's own finding is that by disregarding non-Euclidean automorphisms, biology overlooks symmetries and the very fact of symmetric-algorithmic constructions of the majority of symmetric organic forms. Bearing this in mind, morphology would have to expand dramatically its geometric basis by using non-Euclidean rather than classical similarity transformation groups.

QUESTION 2



The interdisciplinary impact of this symmetry research on other scientific and cultural spheres is very noticeable. This concerns roots and the modern meaning-family of symmetry as described by D. Nagy (1988). [Cf., p. 228 in this issue.]

Studies of the laws and algorithms of organic shaping is a major direction in the biology of development which is expected by many scholars to yield significant basic discoveries and important applications. Morphology has forever been all-important for biology, but at the current stage, scientists in various disciplines pay special attention to morphological self-organizations, properties of biological structures, and their evolutionary transformations. Unlike, say, crystallography, mathematical biology does not rely on a generally acclaimed formal theory of morphogenesis, although numerous attempts have been made at creating such a theory with the aid of various initial models of control engineering, diffusion-reaction, etc. Development of such a theory is difficult, largely because of the shortage of data on the common biological properties of morphogenesis which are capable of being formalized and theoretically interpreted. R. Thom (1975, p. 4), a French researcher, was right when he said "... a geometrical attack on the morphogenesis problem is not merely justified, it is essential". The status of the mathematical biology of development is such that this field has yet to cover the evolutionary path from accumulation of knowledge on key morphological properties and adequate geometric classifications to the development of the desired theory, a path which has been covered by crystallography and other natural sciences dealing with objects much simpler than those of biology.

Biological symmetry is embodied to a lesser or greater degree in numerous biological theories, some highly controversial, such as N.I. Vavilov's law of homological series; A.G. Gurvich's theory of morphogenetic field; Vernadskii's theory of non-Euclidean geometry of living matter; the biological significance of the diffusion-reaction model of morphogenesis developed by A.M. Turing and of self-organizing growing automata whose theory is being developed by J. von Neumann's followers; fractal theories in biology; morphogenetic mechanisms underlying numerous psychophysical phenomena including the esthetic preference of the morphogenetically significant golden section which is expressed by Fibonacci numbers; etc.

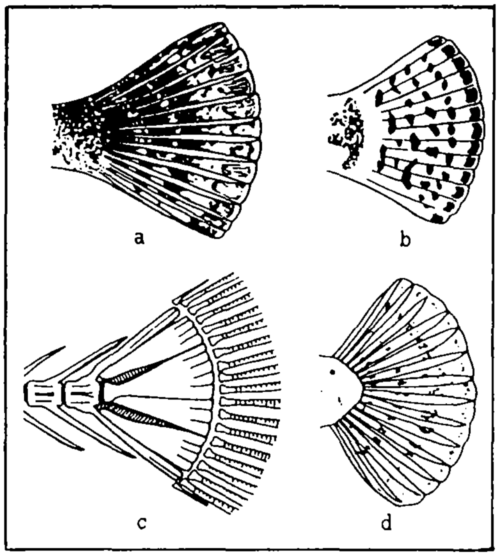


Figure 7: Caudal fins in *Scorpaena porcus Linne* (a), *Siniperca chua-tsu* (b), *Platessa platessa* (c), *Rhombus maeoticus* (d) (according to Berg, 1961) as cases of similarity cyclomerism.

We should not overlook the fact that from the geometric (i.e. group invariant) viewpoint, the entire classical biomorphology is essentially an extension of the group of similarity transformations. This is the case for morphological studies and theories of mirror symmetry and asymmetry of biological bodies, multi-component biological forms which embody similarity symmetry, scaled three-dimensional growth of biological objects, dwarfs and giants among organisms of the same species, etc. On the other had, there was no good reason to believe that the geometric fundamentals of morphology were confined to the similarity group; V.I. Vernadskii's assumption (1965) on the important biomorphological significance of non-Euclidean geometry has for a long time been awaiting verification.

More light must be shed on the geometric fundamentals of morphology, because they will dictate the geometric specifics of morphological studies and facts and their interpretation. A change of geometrical fundamentals would entail changes in all higher levels of this field and dictate new requirements and approaches to the development of formal theories which should be consistent with these fundamentals. Biomorphology is related to many fields of biology such as the biomechanics of postures, biomechanics of growing and motoric movements, psychophysics of perception, etc. The updating of its geometric fundamentals may give rise to new research approaches and encourage discovery of new properties of biological self-organization in these fields.

Let us discuss, in particular, the impact of morphogenetic symmetry research on the biomechanics of motoric movements. Euclidean and non-Euclidean cyclomerisms are involved in the apparatus of motoric movements in numerous animal organisms, in particular in multi-needle fish fins. Classical cases of similarity cyclomerisms in the tails of some fish are shown in Figure 7, and cases of affine and projective cyclomerism in the tails of others in Figure 8.

Numerous specifics in the structure of fins as multi-component parts may be described and explained in terms of symmetric morphogenesis algorithms, leaving aside the locomotoric functions of these organs. Incidentally, the superficial but widespread view is that the fins are intended for swimming and evolution made their structure optimal for this function, and so the specific of the structure may and must be derived from hydrodynamics only. In other words, knowledge of the hydrodynamic equations is supposed to be sufficient for understanding the structure of fish fins. The above findings of group-invariant analysis refute this view and draws attention to the multitude of fin functions which include, in addition to the locomotoric function, the general biological function of contributing to the morphogenetic processes of inheriting body form with algorithmic mutual conjugation of components.

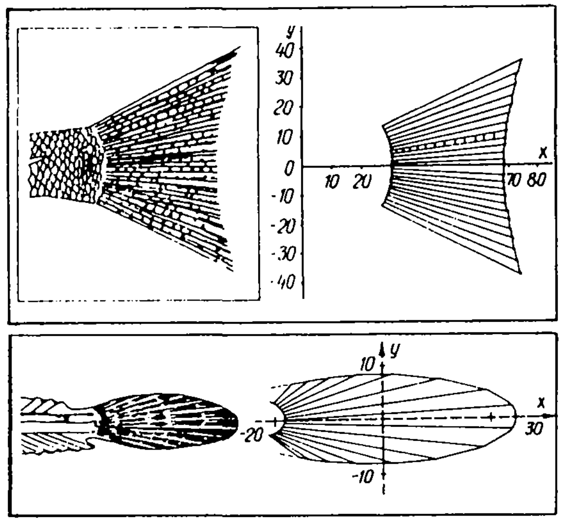


Figure 8: The multi-block structures of the caudal fins in *Acanthurus sp.* (upper part) and in *Lumpenus lampetraeformis* (according to Berg, 1961) and their models as cyclomerisms with projective generating transformations.

This remark leads to a question: to what extent may the structure of a living organism and the performance of some function in the environment be taken by machine designers as a model to be imitated, bearing in mind the millions of years of evolution and natural selection. Studies of symmetry mechanisms in biological bodies reveal that the structure of organs is dictated not only by optimal adaptation to specific functions in the environment, but the laws of biological morphogenesis are also very important. In effect, the machine designer can concentrate on optimal functioning in the environment and does not have to adapt to either the needs of the internal requirements of the organism or purely biological requirements (volume growth in individual development, inheritance of biological properties, etc.). Still, the achievements of living nature in the performance of functions must not be dismissed out of hand. Living nature has long solved numerous problems in functioning in the environment that designers face today. These solutions are not necessarily optimal in the usual sense, but they are invaluable because they draw attention to the existence of challenges and demonstrate ways to meet them; they stimulate human imagination and have acted as catalysts for technological progress throughout the entire course of human history.

Let us consider the physiologically normal posture, a concept widely used in biology. In the entire set of postures or positions of the body parts, such as the tail, the trunk, etc. some positions of the body components *vis-à-vis* one another are inherited. They are instinctively taken in a stereotype way in the cases of fear, weariness, rest, etc. These include the posture of rest of the starfish with a symmetrical position of the rays which is described as rotational cyclomerisms, mutual position of components of the fin at rest, the metameric posture of a resting caterpillar, etc.

Research into such in-born postures and positions of groups of mobile components ("segmentary" postures) of the support and motor apparatus is also important because it appears to constitute the basis for the construction of motoric movements and for the complex system of muscular drives for which genetically dictated characteristics and conditioned reflexes are developed, physiological analyzers are coordinated, etc. A better insight into their relations to the functioning of various systems of organisms such as neuromuscular, vestibular, etc. will be useful in optimizing human postures and movements; reduction of muscular weariness in human operators; reduction of detrimental effects of unfavorable factors such as vibrations and overloads; increasing the vestibular stability; choosing optimal postures in the cases of protracted immobility (for instance in treating broken extremities); forecasting and explaining human senso-motoric reactions under unusual conditions such as zero gravity in space missions; better coordination of space suits and exoskeletons with the specifics of the human support and motor apparatus; improved movements of athletes; improving sporting equipments; and development of senso-motoric systems in zoomorphous robots.

Our research (Petukhov, 1988, pp. 20-26) reveals that the set of inherited postures includes, in addition to (segmentary) postures described by similarity cyclomerisms, postures which are described as non-Euclidean cyclomerisms, which significantly expands the data on the relationship of such postures to symmetrical morphogenesis algorithms. This corroborates the fact that the kinematics of biological (morphogenetic and motoric) movements is often a transition from one cyclomerism type to another, or involves cyclomeric polymorphism. We believe that the morphogenetic significance of iterative algorithms in biology is attributable to the mechanisms of interaction in biological layers of tissues and replication of supramolecular structures; in this connection the concept of replicational morphogenesis was formulated.

Our symmetry analysis of spatial biostructures is also useful for still another scientific field – temporal biorhythmology. Researchers in various countries have been for a long time studying biorhythms, and this scientific field can boast of its own traditions, terminology, and challenging findings. Still, it has concentrated attention on periodic rhythms of physiological processes such as breathing and walking, repeating processes occurring simultaneously with periodic diurnal and seasonal changes, etc. The reader of the literature on biorhythms may think that no biorhythms other than periodic are significant or possible. In point of fact, however, the range of biologically significant rhythms is broader, and the periodic one is but a particular, albeit important, sub-class.

The periodic rhythms of any process may be interpreted as structuring by a transitive iterative algorithm (1) whose generating transformation is a parallel shift along the time axis by the period.

But are there less trivial iterative algorithms whose time structuring is dictated by more complicated generating transformations not implemented in biological processes? This very important question, which is answered in the affirmative, draws attention to specific biorhythms and regularities in time-structuring that are usually neglected, but seem to be worthy of most serious attention and are capable of significantly enriching the study of biological rhythms.

An illustrative example of an iterative algorithm whose generating transformation is scale similarity is the well-known moulting of *Crustacea* (see Waterman, 1960, p. 88). The time between two moults is seen to increase monotonically with a constant scale during the entire lifetime over which moults occur. Not only the total time between moults is scaled but, also and with the same factors the duration of every stage in the preparation for a moult. Besides, every moult is followed by a proportional increase in the size and mass of the organism. In other words, this biological transformation is a case of a remarkable organization of a most complicated biological process in space–time which demonstrates the cyclomeric properties and adds legitimacy to a search for symmetrical-structuring algorithms in time biorhythms that would be identical to such algorithms in biological series in space.

Nontrivial Euclidean and non-Euclidean iterative algorithms are obviously at work in certain rhythmic processes by *Arenicola marina*, *Bonasa umbellus*, some kinds of cardiac arrhythmias, etc. (For more details see Petukhov, 1988, pp. 34-36). The analysis of non-Euclidean symmetries in biological processes such as complex structured communication signals between living organisms, rhythmical change of state parameters of organisms in normal and pathological functioning, etc. must undoubtedly be continued and systematized. A search for such symmetries in morphogenetic processes such as moulting is especially important and challenging for the field of general biology.

Let us return to spatial biological cyclomerisms. The recognition of non-Euclidean symmetries in algorithmic mutual conjugation of parts of organic forms has dramatically extended the range of biological objects, the spatial behavior of whose parts may be quite legitimately and stringently described in geometrical terms. The existence in most diverse biological bodies of the same geometrical types of cooperative structure and behavior is seen as an important feature of biological evolution and suggests the existence and general biological significance of a morphogenetic regulatory system which is responsible for this coordinated behavior of body parts in the growth and development of the organism.

This morphogenetic system of integrating regulation seems to exist concurrently with the nervous and humoral systems which also perform certain functions in integrating parts of the organism. The system seems to be a protosystem, above all in that sense that it emerged earlier in the evolutionary process (for the ability to grow is the most ancient property of living matter; morphogenesis and growth of geometrically regular and nontrivial forms is observed in protozoa, in particular, and in unicellular organisms which have no nervous system). In other words, other systems are included in this one by their structural formations and also emerge in philo- and ontogenesis against this background.

Note also that three-dimensional growth which is heavily dependent on the cyclomeric (or cyclogenetic) properties of living matter integrates the numerous parts of the body into a growing ensemble which functions so as to prevent the body transformations occurring with aging from invalidating all the senso-motoric habits that were acquired at the preceding ontogenetic stages when the body was "different". In this context, the geometrical properties of three-dimensional growth and the biochemical media which make this growth possible have, in our opinion, a direct bearing on the formation in man of an inborn idea of the structure of his body, an idea which has for a long time been regarded as an important element in the spatial perception and coordination of movements. The morphogenetic regulatory system, which largely functions by cyclogenesis, performs, among other things, the function of coordinated adjustment of numerous muscular, joint, and other proprioceptors which contribute to man's awareness of the structure of his body. The morphogenetic regulatory system, which looks after three-dimensional growth, acts for diverse proprioceptors and muscles as a distributed cooperation enforcing unit, that is, a tuning-fork. In other words, the various elements of the senso-motoric system act in unison not only by virtue of their direct mutual links, but also because they are merged into an orderly growing environment which influences their operation and aids the brain in composing generalizing adequate images out of innumerable reports from the set of receptors. Without studying the morphogenetic regulatory protosystem and its general biological significance, one cannot understand to the full the more recent organism regulatory systems, in particular the nervous system.

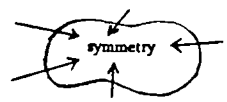
Now let us take up non-Euclidean symmetries in psychophysical phenomena. The idea of close linkage between the specifics of spatial perception and morphogenesis principles is deeply rooted. Since the times of J. Kepler, the esthetic quality of the golden section has been related in the literature to its morphogenetic materialization in living bodies. Bertrand Russell (1962, p. viii), who also worked on the geometrization of psychology wrote: "Il faut construire un pont en commençant à la fois par ses deux extrémités: c'est-à-dire, d'une part, en rapprochant les assomptions de la physique des données psychologiques et, de l'autre, en manipulant les données psychologiques de manière à édifier des constructions logiques satisfaisant de plus près aux axiomes de la géométrie physique." Today's field of psychology has developed significant experience in using higher geometry in simulating observed psychological phenomena. The latest data allow us (Petukhov, 1981, pp. 77-85; 1988, pp. 38-41) to suppose that groups of non-Euclidean transformations and cyclomerism principles contribute to the genesis of spatial representations in the individual. In this connection, new light has been shed on the Helmholtz-Sechenov-Poincaré's idea which propounds the leading role of kinematic organization of human body for the genesis of spatial representations in the individual.

The additional significance of morphogenetic structures for spatial perception and active ordering of the environment by the organism is demonstrated by constructions made by insects and some other organisms. Usually instinctive and following certain shape standards, this activity is an enigma and a scientific challenge. In numerous cases, these constructions of individual organisms and joints of numerous individuals appear to embody Euclidean and non-Euclidean cyclomerisms.

It is interesting to note that from early ancient times, on the basis of their great esthetic feeling and observations of living nature, artists and architects have used not only Euclidean but also non-Euclidean cyclomerisms in their compositions (ornaments, decorations of musical instruments, architectural elements, etc.), as our investigations have revealed.

Sensor perception, as in the above cases of biological kinematics, is largely structurally related to morphogenesis and may be treated as a structural extension of morphogenesis rather than something entirely new that nature devised at some evolutionary stage. In "mastering space" (Le Corbusier's expression) the organism seems to use the same structuring principles and algorithms, be they morphogenetic, kinematic or of psychophysical mastering. Important advantages are offered to living organism by this unity of structuring principles and algorithms for various systems and sophistication levels which are coordinated and add up to a living organism that "mastering space". We are confident that the biological significance of iterative algorithms and non-Euclidean symmetry will be detected in a wide diversity of biological fields, in particular in the encoding of biological information, including within the nervous system.

QUESTION 3



In this twentieth century, symmetry is a most important and thoroughly explored methodological principle to be observed in the formulation of a scientific theory. This fact makes symmetrological studies of living matter with its striking properties of self-organizations and information inheritance still more challenging.

For biosymmetry studies, the well-known process of "geometrization of physics" has a stimulating meaning. Recall that the revolution in geometry brought about by the *Erlangen Program* was later extended to the physical and to philosophical views of space. The very fundamentals of human perception of space have changed, and have been ever since related to the concept of a mathematical group of transformations. Henri Poincaré formulated this relationship in very simple terms: space is a group. The advent of the special theory of relativity gave birth to a new term, "geometrization of physics", which stood for the fact that formally this theory was a theory of invariants of some group of transformations (Poincaré-Lorentz group), or of a geometry. The ideas of geometrization of physics and the representation and description of its theories in the language of invariants of transformation groups were extended to quantum mechanics, the theory of conservation laws, the theory of elementary particles, and other physical subdisciplines. The group invariant approach and symmetry concepts became a cornerstone of today's group-theoretic thinking. Groups became a primary and most profound element in a physical description of nature. Also, in the words of H. Weyl (1952), the symmetry method is the guiding principle of today's mathematics and its applications.

To understand basic biological phenomena in terms of mathematical natural sciences is to interpret these phenomena and their laws in a language of more profound concepts which are characteristic of the mathematical natural sciences. The mainstream by which biology may bridge the gap to the exact sciences is by the penetration of group invariant concepts and methods into biology through the study of biological symmetries. Typical titles of modern papers on mathematical and theoretical biology are: "The Concept of a Group and Perception Theory", "Biological Similarity and Group Theory", "Research in Non-Euclidean Biomechanics", etc. In other words, new daring attempts have been made to build theoretical models of specific biological phenomena in the fields of morphogenesis, psychophysics, etc. as formal theories of invariants of certain groups of transformations. In effect, the geometrization of physics goes hand-in-hand with attempts to geometrize biology. The theoretical biology of the future seems to be bound to become largely a group invariant biology. Natural sciences will then make a step to A. Eddington's ideal of combining whatever we know of the physical world into one science whose laws could be expressed in geometric or quasi-geometric terms.

The discovery of non-Euclidean biological symmetries provides valuable data for the development of a formal theory of morphogenesis, the ability to explain the existence of these symmetries being an illustrative indicator of the adequacy of this theory.

One of the promising ways to model the phenomena of symmetric biological shaping is in the application of the theory of growing automata to the modeling of morphogeneses. This control engineering modeling is legitimized by the fact that in biological shaping a feedback control system is involved which is distributed throughout the developing body. Cellular automata, uniform arrays devised by von Neumann and his followers, and the ideas of Lindenmayer (1978) on parallel grammars are widely used for this purpose.

In light of the data stated above on the important morphological value of iterative algorithms, an improved modeling approach can be developed, in which the concept of an autonomously growing automation (or networks of such automata) would be useful.

Recall that a finite automaton is a dynamic system whose behavior at specified times (clock times) $1, 2, ..., p$ is described by the equation

$$X(p) = f[X(p\text{-}1), U(p\text{-}1)], \tag{2}$$

where $X(p)$ and $U(p)$ are variables which take on values from specified finite alphabets, $X(p)$ representing the internal state of the automaton at time p, and $U(p\text{-}1)$ the state of the automaton input at the preceding time which reflects the impact of the "environment" on the automaton. A special case is an automaton whose behavior does not depend on the environment at all. Such an automaton is referred to as autonomous; for it a change in the state is obviously dictated (in a way similar to algorithms of biological cyclomerisms) by the iterative algorithm

$$X(p) = f[X(p\text{-}1)] \tag{3}$$

But biological cyclomerisms are observed in relatively autonomous subsystems of the body such as horns, fins, etc. Besides, the concept of autonomy is generally closely related to the genetic inheritance of shapes characteristic of the biological species, independently of the environment. The relative autonomy of the subsystems in an organism is a prerequisite for their efficient functioning. All this confirms that our approach is correct in interpreting the biological cyclomerism and its ensembles in terms of autonomous growing automata (3). Other authors used growing automata, assuming timed control of the automaton state from outside while the morphogenetic significance and potential of autonomous automata were neglected. One should not overlook the well-known noise stability of iterative algorithms which may make them especially desirable in living organisms.

Such modeling can be implanted, for instance, in a cellular automaton which is usually a uniform array of numerous identical cells, each cell having several possible states and interacting only with a few neighboring cells. The idea of such an automaton is nearly as old as that of electronic computers. The research in this field was pioneered in the early 1950's by John von Neumann. The "life game" devised in 1970 by J. Conway and capable of simulating some aspects of biological development is the most widely known cellular automaton. Numerous problems in cellular automata are conversed by so-called "information mechanics". What is important is that von Neumann's automata include cells which are placed in the cells of the Cartesian network and this "Euclidean" disposition of cells, introduced from outside, is taken up in later papers. Breaking with this tradition, we used the data on the biological significance of non-Euclidean cyclomerisms to justify the use of cellular automata also based on networks with cyclic groups of non-Euclidean automorphisms in biological modeling. In these "non-Euclidean" cellular automata, the timed change of state of the autonomous automaton is treated as an attachment of a new motive unit of a non-Euclidean cyclomerism.

Our research has revealed new promising lines for the development of the theory of growing automata to be applied to biological morphogenesis (see the author's book, Petukhov, 1988, pp. 32-34). The idea that numerous organism subsystems function in the norm and in pathology as autonomous automata and that their ensembles may prove important in medicine and biology and may suggest certain therapeutic methods of treatment, professional selection and training procedures, as well as ways to optimize working conditions for operators of complex machines, etc.

Research in biosymmetries enhances the comprehension of the unity of living nature in the same vein that science discovers ever new general biological laws and mechanisms such as genetic codes and bioenergy mechanisms. This article sheds light on and analyzes new, non-Euclidean, symmetric and algorithmic properties of general biological phenomena in biological morphogenesis, the knowledge of which is indispensable for a broad range of theoretical and applied areas, including anthropo- and zoomorphological robotics, ergonomics, biomechanics, biotechnology (above all, "morphoengineering", or directed morphogenesis control), etc. These results emphasize the morphological significance of the internal environment of the organism and suggest new approaches to understanding the relationship of morphogenesis and the mechanisms of genetic encoding and biochemical cycles. Our are in favor of not only the argument that biology is a fertile field for introduction of various symmetrical approaches, methods, and tools of group-theoretic analysis, but that development of theoretical biology and biomechanics at this stage is largely dependent on vigorous utilization of group-theoretic methods with non-Euclidean geometries.

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