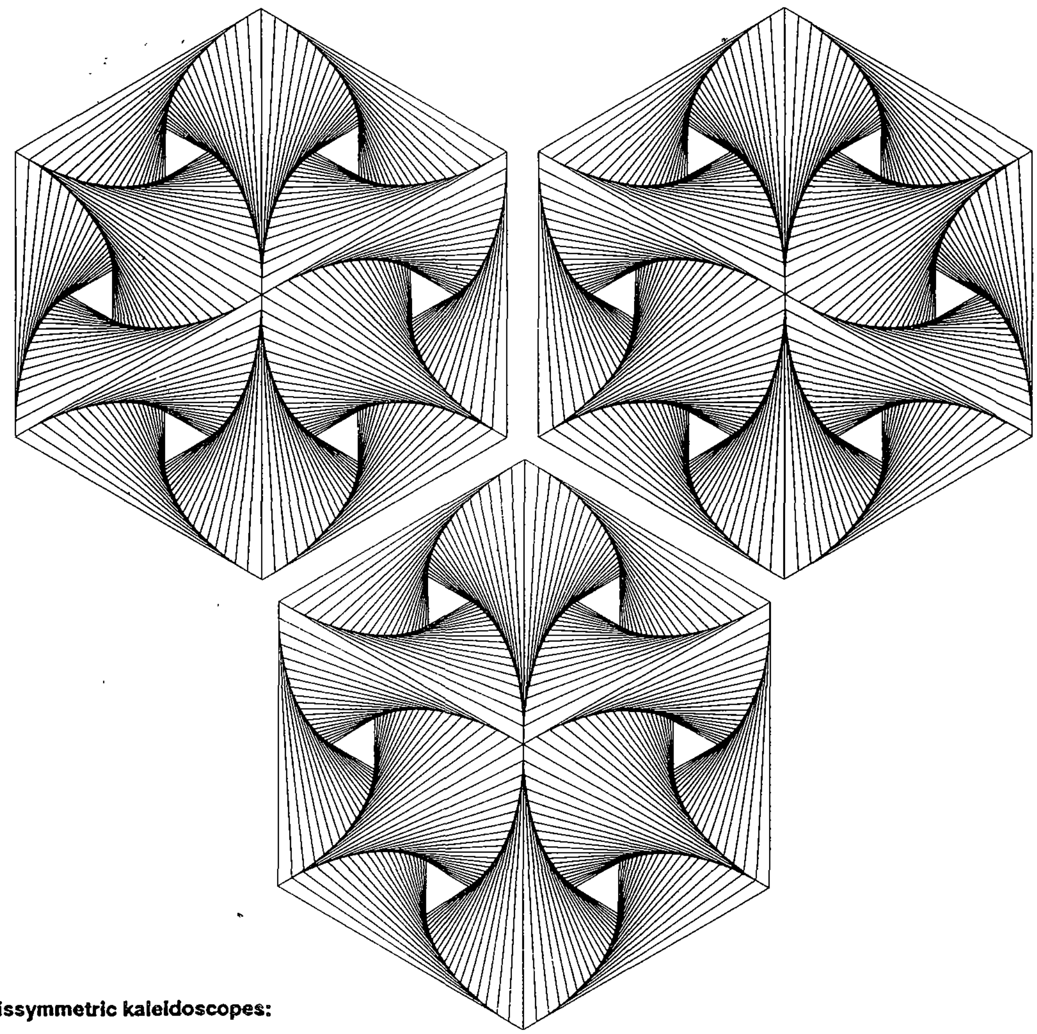
Symmetry: Culture and Science

SPECIAL ISSUE
Symmetry in a Kaleidoscope 3

The Quarterly of the
International Society for the
Interdisciplinary Study of Symmetry
(ISIS-Symmetry)

Editors:
György Darvas and Dénes Nagy

Volume 1, Number 3, 1990



Dissymmetric kaleidoscopes:
Hommage à Pasteur

Symmetry: Culture and Science
Vol. 1, No. 3, 1990, 319-328

SYMMETRY IN PHILOSOPHY AND HISTORY OF SCIENCE

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QUESTION 1

what is symmetry?

In *philosophy and history* of science one has to refer to the whole kaleidoscope of symmetry. Historically ideas of symmetry have been very influential in the development of human thought. In technics people found out the practical advantage of balance. Pots and baskets of symmetric forms are more stable and economical than other ones. In agriculture people had to observe the periodic regularities of day and night, summer and winter, growth and death, etc. The regular movements of stars seemed to symbolize an eternal order of nature. No wonder that in mythologies, religions, and early cultures symmetric forms, like circles, were used as metaphors and symbols. On the other hand, people were fascinated by the beauty of regular patterns and ornaments. (Mainzer, 1987).

In *Greek mathematics* symmetric models were used for the first time to describe and to explain nature. In *Platonic physics* the variety of material phenomena were reduced to the regular polyhedra of Euclidean geometry. Irregularities of planets were interpreted as merely phenomenal symmetry breaking which was reduced to modified symmetric models (Hanson, 1965; Kuhn, 1966, chap. 2; Mainzer, 1980, chap. 2).

Even in the natural sciences of modern times symmetric models were often used to illustrate and visualize natural regularities, from Kepler's heliocentric model of planets to Rutherford's and Bohr's model of atoms and electronic orbitals.

Since the *19th century* symmetries are not only defined as properties of geometric models, but as properties of natural laws and theories, too. In this sense symmetry means the invariance of a theory with respect to a transformation of its coordinates by a mathematical transformation group. From this point of view a fundamental idea in *philosophy of science* can be made precise: the *unification of science.* In spite of an ever growing specialization, modern natural sciences intend to reduce their properties to some fundamental structures: physics tries to unify the different physical forces in one fundamental force; chemistry tries to explain the structure of chemical substances by the quantum mechanics of molecules; biology tries to reduce the processes of life to biochemical and biophysical laws (Mainzer, 1988a).

Mathematically the *unification* of natural science can be described by structures of *symmetry*, the specialization of science, the *variety* and emergence of new phenomena by *symmetry breaking*. In the following I want to show (1) the successes and lacks of the reductionistic program by recent developments in physics, chemistry, and biology; (2) the traditional philosophical discussion on holism, reductionism, and unification of science can be clarified by structures of symmetry and symmetry breaking.

Mathematically symmetries are defined by so-called *automorphisms* that means self-mappings of figures, spaces, etc., which leaves the structure invariant (Weyl, 1955, p. 47). In geometry the mapping of similarity is an example of an automorphism which leaves the form of a figure invariant. The relation of similarity $F \sim F'$ (figure F is similar to figure F') satisfies the conditions of an equivalence relation: (1) $F \sim F$ (reflexivity); (2) if $F \sim F'$, then $F' \sim F$ (symmetry); (3) if $F \sim F'$ and $F' \sim F''$, then $F \sim F''$ (transitivity). In general the composition of automorphisms satisfies the axioms of a mathematical group A: (1) if S and T are automorphisms, then the composition $S \cdot T$ is an automorphism; (2) the identity I which maps a figure into itself is an element of A ($I \in A$); (3) for every mapping $T \in A$ there is an inverse $T^{-1} \in A$ with $T \cdot T^{-1} = T^{-1} \cdot T = I$. Examples of discrete groups are the finite rotation groups of polygons and the Platonic groups which preserve the symmetric structure of the Platonic solids. An example of a continuous group is the rotation $R(\theta)$ with the continuous parameter θ which satisfies the axioms of a group with $R(\theta_1) \cdot R(\theta_2) = R(\theta_1 + \theta_2)$, $R(0) = I$, $R(\theta)^{-1} = R(2\pi - \theta)$, $R(\theta) \cdot R(\theta)^{-1} = R(\theta)^{-1} \cdot R(\theta) = I$.

In general, invariance is a fundamental property of mathematical structures. A structure (M, s) consists of a basis set M and a typified *structural element* s (for instance functions, functionals of M). The *structural type* $s \in \sigma(M)$ is defined as a set which is created from M by an iteration of the operation "power set (= set of all subsets) of a Cartesian product". The structural kind of (M, s) is defined by an axiom $\alpha(M, s)$ which determines the structure uniquely with respect to isomorphism, i.e. if (M, s) and (M', s') are isomorphic, then the axiom $\alpha(M, s)$ is true if and only if $\alpha(M', s')$ is true.

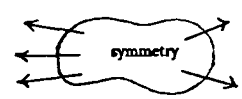
This demand means that the axiom α does not change its truth value if the structure (M, s) is replaced by an arbitrary isomorphic structure (M', s'). So the axioms of group hold for the rotations of an equilateral triangle, as well as for the real numbers. The axioms of the Newtonian theory of gravity are true for trajectories of artificial satellites, as well as for the planetary trajectories in the solar system. Isomorphisms are bijective mappings from the basis sets M onto the basis sets M' which maps the typified set s on the corresponding set s'. The types remain unchanged, because the corresponding mapping is given by $\sigma(M)$. It is obvious that the general definition of a structure is characterized by a *canonical property of invariance* (Bourbaki, 1966, chap. 4).

A mathematical example is F. Klein's *Erlangen Program* which delivers a group-theoretical characterization of geometry. Let M be a space with a special geometry and G a transformation group of the real number space $\mathbb{R}^n$. Then (M, F) is a structure with a typified set $F \in \mathrm{Pot}^2(M \times \mathbb{R}^n)$ of coordinate systems and the structural kind $\alpha_G(M, F)$. The axiom α_G means that F is a set of global coordinate systems for M on $\mathbb{R}^n$ which is complete in respect to G. It is now possible to distinguish an hierarchy of transformation groups on $\mathbb{R}^n$ and to inquire the corresponding geometrical structure. Symmetries correspond to such invariant properties of isomorphisms which map a structure on itself.

In philosophy of science symmetries can have different meanings. Heuristically symmetric models inspire scientists to find a successful problem solving. Methodologically symmetric structures are used to make theories, laws, and their invariant properties precise. An important ontological and epistemological question concerns the problem whether symmetric structures are only human inventions and projections in nature, or they correspond to structural principles of reality which determine and organize nature. A description of nature in terms of symmetry structures and symmetry breaking seems to be appropriate to grasp the diversity and complexity of reality. Some people believe in the ontological reality of symmetric structures independent of human models and ideas. But from a methodological point of view the ontological question of symmetries cannot be decided definitely.

Symmetry and symmetry breaking can at least be understood as fundamental categories of research to which the usual categories of natural science, like space, time, causality, interaction, matter, force, shape, etc., can be reduced in a logically and mathematically precise manner. This categorical framework cannot be justified as absolutely and necessarily *a priori* with a unique claim to legitimacy in the sense of Kant, but as a successful and consistent framework of research. Last but not least the principles of symmetry show that even in modern research there are some leading philosophical ideas which date back to early times of mankind and which have been fertile during the long history of human thinking.

QUESTION 2



The impact of symmetric structures which are discussed in *philosophy and history of science* is *interdisciplinary*. As an example: The different conceptions of space-time which were discussed in natural philosophy from Newton and Leibniz to Einstein can be understood as more or less complex structures of symmetry. The *mathematical group theory* offers a common structural framework in which the conceptions of Newton, Leibniz, Einstein, etc. can be distinguished as different structures of symmetry (Audretsch and Mainzer, 1988, pp. 21-51).

The structural approach to history of science shows that *Newton's* or *Leibniz'* conceptions are not simply false, but different aspects of symmetry in the physical space-time $\mathbb{R}^3 \times T$. The Newtonian structure of space-time is characterized by the so-called *elementary group* G_e which consists of the direct product of dilatations, rotations, and translations on $\mathbb{R}^3$ and the affine group of time T. While Newton believed in the existence of an absolute space and time with absolute rest and motion, Leibniz attacked these assumptions in a famous controversy with S. Clarke. His space-time is completely relative without any distinction of motions. The theological and metaphysical reasons of Leibniz may be only historically interesting today. But mathematically he described a new space-time structure. As the corresponding transformations let arbitrary continuous motions invariant, it is the *kinematical group* G_k which characterizes Leibniz' space-time. G_k is a less rich structure than G_e, i.e. $G_e \subset G_k$. It cannot explain the absolute motion of rotation. On the other hand, G_e has "too much" structure, because the absolute rest of the absolute space cannot be confirmed empirically. The adequate transformation group of classical physics is the *Galilean group* G_g which consists of the transformations of G_e and the translational velocities of the inertial systems, i.e. $G_e \subset G_g \subset G_k$.

While these symmetries of space-time have the same metric and causal structure, the situation changes in *Einstein's theory of relativity*. It is a consequence of the principle of special relativity and the velocity of light that Newton's absolute time must be abandoned. The space-time structure of special relativity is determined by the Lorentz-group. The structural approach shows that the classical space-time symmetry is not overcome by Einstein's theory of relativity, as it was suggested by some historians of science. The reason is that Einstein's theory can be embedded into the classical theory of space-time, if it is restricted to inertial systems which are moving slowly relative to the inertial system of our planetary system and the velocity of light.

The space-time of classical mechanics and special relativity are examples of *global symmetry*, i.e. the equations remain invariant, if all coordinates are changed by the same group transformation. Analogously, the form of a sphere remains invariant by a rotation if the coordinates of all points are changed by the same angle.

In *general relativity* the inertial systems are accelerated to each other and an observer feels an impression of force. In the geometrical language we may say that the accelerations are caused by local deviations of the global symmetry. So fields of force (gravity) must be introduced in order to compensate the deviations and to save the symmetry (form invariance) of Einstein's equation of gravity. Analogously, there are distortions on the surface of a sphere by local changes of the coordinates. The form of the sphere is preserved by the assumption of forces. We may say that in general relativity the gravitational forces are introduced by the *transition from global to local symmetry*.

An important application of the *structural approach* in *philosophy of science* is *quantum mechanics* (Emch, 1984, part 3; Mainzer, 1988a, chap. 4.2). Quantum systems (atoms, electrons, etc.) have incompatible (non-classical) observables (position, momentum, etc.) which do not commute with each other and which have not definite eigenvalues in each state. Their symmetries are defined by the invariance of the corresponding *Hamilton operators*. Examples are the rotational symmetry of atoms or the permutation symmetry of electrons in an atom which are indistinguishable in the sense of the *Pauli principle*. In simple cases the structural symmetries can be visualized at least approximately by geometric models, but not in general.

The main difference between classical and quantum systems is the following: Quantum systems which once have interacted remain in statistical correlations, even if they are separated with far distances without any dynamical interaction. This is a mathematical consequence of the so-called *superposition principle* of quantum mechanics (d'Espagnat, 1976), which is today well confirmed by the EPR (= Einstein—Podolsky—Rosen) -experiments of Aspect in 1982 (*EPR — Correlations*). In short, quantum mechanics with an unrestricted superposition principle describes a whole which is not made of isolated parts.

This unbroken wholeness of the quantum world is mathematically defined by the *logical symmetry* of the quantum world. In more technical words, it is given by the automorphism group Aut ($\overline{H}$) of the projective Hilbert space $\overline{H}$ (associated with the Hilbert space H) which corresponds to the states of a quantum system. A famous theorem of Wigner of 1931, asserts that the automorphism group Aut ($\overline{H}$) can be represented by the group of unitary operators on the state space H.

The *space-time structure* of a quantum system can be specified by a subgroup of the logical symmetries of quantum mechanics. In more technical words, the Galilean-invariance of quantum mechanics is given by a projective unitary representation of the Galilean group on the Hilbert space of state vectors.

The process of *unification* and *specialization* can be made precise by structures of symmetry and symmetry breaking. A famous example delivers *elementary particle physics* (Mainzer, 1988a, chap. 4.3). Nowadays physics distinguishes four fundamental forces: the electromagnetic, strong, weak, and gravitational forces. They can be introduced by a *transition from global to local symmetry* (as in the case of the gravitational force). Forces are interpreted as so-called gauge fields which compensate local deviations of a global symmetry.

In *electrodynamics* a magnetic field compensates a local change of an electric field (i.e. the movement of a charged body), and preserves ("saves") the invariance of electromagnetic field equations. In *quantum electrodynamics* an electromagnetic field compensates the local change of a material field (phase deviation of an electronic field) and preserves ("saves") the invariance of the corresponding field equations.

Elementary particle physics intends to unify the four physical forces in one fundamental force. Electromagnetic and weak forces could already be unified by very high energies in an accelerator ring of CERN. They can be described by the same symmetry group $U(1) \times SU(2)$. At a particular critical value of lower energy the symmetry breaks down in two partial symmetries $U(1)$ and $SU(2)$ which correspond to the electromagnetic and weak forces.

A next step of the unification program is the *big unification* of electromagnetic, weak, and strong forces, and in a last step the *superunification* of all four forces. Mathematically they are described by extensions to richer structures of symmetry (*gauge groups*). On the other hand the variety of elementary particles can be actualized by symmetry breaking.

The scheme of symmetry and symmetry breaking can be used to describe the *cosmic evolution*. For a short initial state (after the *Big Bang*) a fully symmetric situation of very high energy is assumed in which no particles can be distinguished, but they all can be transformed into one another. During the retardation of the cosmic evolution and cooling of its temperature, critical values were realized step by step at which symmetries break down and new particles and forces emerge: "C'est la dissymétrie, qui creé le phénomène", said Pierre Curie.

The emergence of pattern structure can be described by symmetry breaking not only in elementary particle physics, but even in *chemistry* and *biology*. In systems far from thermal equilibrium, patterns can arise suddenly if the input of energy increases to particular values and establishes a permanent metabolism with their environment (Glansdorff and Prigogine, 1971). Chemical examples are the *dissipative structures* which suddenly arise in homogeneous mixtures (Zhabotinskii reaction). A famous physical example is the *laser light* which suddenly breaks the distribution of emitted photons in an active material if the pump energy arises a particular value. Especially living organisms, which are in metabolism with their environment, are systems far from thermal equilibrium. The *morphogenesis* of these systems can be described by the same methods of symmetry breaking (Fischer and Mainzer, 1989).

In the theory of *evolution* the growth of organic forms and populations is interpreted as functional development, i.e. as an optimal adaptation to the conditions of environment. Mathematically all these examples can be understood as dynamical systems the growth of which is determined by *non-linear equations*. At a first glance the non-linearity of these macroscopic systems seems to be an insurmountable difference to the linearity of microscopic quantum systems (*superposition principle*). A philosopher of science may ask whether the theory of complex dynamical systems can be reduced to the principles of quantum mechanics.

At least one can get non-linear evolution equations out of quantum mechanics by approximate decorrelation assumptions (factorization of expectation values, neglect of higher-order correlations, etc.). In this sense the spontaneous symmetry breaking of non-linear systems can be understood at least in principle for models of lasers in quantum optics. But in detail the variety and complexity of macroscopical systems is very difficult to be explained in the framework of quantum mechanics from a microscopic point of view.

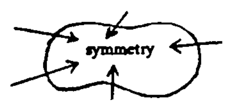
So new mathematical paradigms are introduced to describe holistic macroscopical systems with their fractal and sometimes chaotical structure (*geometry of fractals, theory of catastrophe*) (Thom, 1983). They can be characterized by forms of symmetry and symmetry breaking, too. From the viewpoint of a philosopher of science both aspects are necessary: the holistic and macroscopical view and the reductionistic and microscopic view do not exclude each other, but they are complementary.

A description of nature in terms of hierarchical symmetry structures and symmetry breaking seems to be appropriate to grasp the diversity and complexity of elementary particles, atoms, molecules, and even biological systems. Lower and higher levels in *hierarchical systems* are characterized by different time scales, a higher level having a much larger reaction time than all lower levels. A hierarchically higher level is characterized by the emergence of new qualities with symmetry breaking. But at the same time a higher-level theory has a more restricted domain of validity and is less accurate than a more fundamental level.

The great advantage of a *hierarchical view* is the common framework in which a philosopher of science can understand physical, chemical, and biological aspects of nature. Symmetry and symmetry breaking are the fundamental categories of this framework. It offers new phenomena, new problems, and new problem solutions. It shows new connections between disciplines which were regarded as separated and isolated fields of research. So it supports interdisciplinary work and gives new insight in a common structure and theory of natural science (Mainzer, 1988a, chap. 4.4; Mainzer, 1988b, p. 170).

THEORY	OBJECTS	SYMMETRIES
quantum field theories	elementary particles forces	logical symmetries of quantum systems: e.g. Aut ($\mathcal{H}$); kinematical space-time symmetries: e.g. Galileo-, Lorentz-group; dynamical symmetries: e.g. $SU(2) \times U(1)$-, $SU(3)$-forces
quantum chemistry, chemistry	atoms, molecules etc.	structural-, orbital-, crystal-symmetries
biochemistry	macromolecules	homochirality
thermodynamics	open systems with metabolism	dissipative structures
biology	organisms	functional symmetries
ecology	populations	ecological balance

QUESTION 3



The influence of *cultural background* which determined the meaning of symmetry in philosophy and history of science is mirrored in art, architecture, and religion. In the *Greek culture*, the *Middle Ages*, and *Renaissance* art, science and religion are founded by the same cosmic laws. They are an unbroken unity and wholeness. In the "quadrivium" of the Pythagoreans which consists of geometry, arithmetics, music, and astronomy, it is a fundamental aim to demonstrate the proportion and common measure (συμμετρία) of world and culture. The Pythagorean-Platonic tradition is continued in the Renaissance with a typical statement : "ars sine scientia nihil est".

L. B. Alberti said in *De re aedificatoria* (1485) that beauty of an organism, building, or statue is the harmony of its parts in a determined number, proportionality, and order. The intentions of Renaissance crystallized in one person: Leonardo da Vinci. He realized the unity of an architect, painter, engineer, and philosopher. In architecture I only mention his studies of central symmetry which dominates many buildings of the Renaissance style and which enlarged the traditional meaning of symmetry. The German "Leonardo" was Albrecht Dürer who analyzed the visual laws and initiated the inquiring of perspectivity.

Beside the European tradition, we must not forget the *Chinese*, *Arabic*, and *Hindu cultures* which developed fascinating patterns of symmetry in their art and architecture. They expressed cosmic and esthetical laws independently of the European influence. In the past there were only some rare interactions between the European tradition and these cultures (for instance Leibniz and the Chinese science and religion).

Nowadays the natural philosophy of *Taoism* is rather popular. Some people who believe in "New Age" and other modern mythologies assume that Chinese and Hindu philosophy with the idea of cosmic waves and holisms form a much better cultural background for modern sciences like quantum mechanics or ecology than Democritus's atomism or the Cartesian mechanism. It is a pity that these people are not aware of some facts of the European history of philosophy. Historically it is well known that the Hellenistic *stoa* developed models of fields and waves in order to explain matter, too. In the beginning of the 19th century the German literary and philosophical movement of *romanticism* (Schelling, Hegel, Novalis, etc.) assumed universal forces and spiritual fields which heuristically influenced physicists like Ritter, Oerstedt, Faraday, and others when they were doing their first steps towards electrodynamics.

Nevertheless the unity of science, technics, and art were broken after the Renaissance. The painter W. Hogarth made fun about the "strange idea" that our thinking, looking, and hearing should be governed by the same laws of harmony. For D. Hume *beauty* is reduced to *subjective perceptions* and feelings of the observer. Mathematics and natural science developed their own conceptions of symmetry and symmetry breaking which became fundamental today (compare Questions 1 and 2).

But my thesis is that even *modern art* and *architecture* is searching for a new center and new structural laws (Mainzer, 1988a, chap. 5.4). There was an obvious analogy between modern art, modern mathematics, and modern natural science since the beginning of this century. All these different kinds of cultural activities gave up their traditional naturalistic and intuitive view of the world and became more and more *abstract*. This tendency can be observed as well in the abstract art of Picasso, Braque, and others as in the abstract formalism of modern axiomatic mathematics (Hilbert, Bourbaki, etc.) and quantum mechanics (von Neumann, Dirac, etc.). On the other hand, there was no direct interaction between "both cultures". Neither Picasso, Braque or other founders of the abstract cubism read Planck, Einstein etc. nor vice versa.

There seems to be a "preestablished harmony" (Leibniz) or "hidden harmony" (Heraclitus) between these different cultural activities. In the sense of Hegel we may say that philosophy, art, and science are different expressions of a structural evolution in which mankind develops new forms and figures of its mind with a tendency to more and abstract complexity. In cubism, for instance, painters analyze elementary geometric forms like triangles which are the invisible unities behind reality and which can be composed in the complex forms and figures of our experience. The analogy with the abstract mathematical formalism of modern physics which analyzes elementary particles and atoms is obvious.

During the *Weimar Culture* in Germany (the "Golden Twenties") the so-called *Bauhaus* tried to find a new unity of industrial and technical culture. The new esthetics should express the new conditions of civilization which are determined by industry, technics, and science. So this movement of famous architects and painters suggested new measures of proportion and beauty which are only justified by their *functionalism*. Several manifestoes of the *Bauhaus* proclaimed a functionalistic unity of technics, natural science, and modern way of life. Like Leonardo and Dürer, Oskar Schlemmer studied the canonical proportions of human bodies. But man is not only an isolated atom. He is regarded in his cosmic environment by means of the new sciences of physics, biology, anatomy, physiology, psychology, and philosophy. Body, mind, and soul, but also form, function, and economical conditions should be brought in an optimal relationship.

The *Bauhaus* intended to found a new "logos of modern times", a new idea of center and symmetry after the loss of the Pythagorean-Platonic tradition. This functional view of the world is the cultural background of *modern philosophy*, too. I remind of Wittgenstein's *Tractatus logico-philosophicus*, Carnap's *Logischer Aufbau der Welt* or Neurath's *Einheitswissenschaft* which expressed the *logocentrism* of this epoch.

But the unified functionalism and structuralism of modern times grows old. Its *symmetry breaking* has become obvious in the architecture. The international style of a boring functionalism without any fantasy, which dominates the business centers of our cities, has become hostile against man and nature. Since some years there is a critical movement of the so-called *postmodern architecture* which is enlarged to a general cultural critique of the *postindustrial society*, *post-structuralism*, *postmodern philosophy of science*, etc. The common idea is the *loss of center* and the critique of an universal *logos* of technical and scientific rationality which perhaps aims at a totalitarian functionalism and bureaucracy like Orwell's *Big Brother*. In Germany during 1933-1945 symmetry and unity was a symbol of a totalitarian architecture.

So the postmodern architecture of the present tries to relax the purism and functionalism of modern buildings with different elements of historical styles. But the variety of historical reminiscences and asymmetrical elements in architecture does not mean a movement back to historicism or eclecticism. It is the expression of a sceptic and ironic view of the world which does no longer believe in an omnipotent technical rationality and its claim to solve all human problems. It underlines individuality and the importance of accidental details, and doubts in universal harmony and rationality. So it prefers *symmetry breaking* as a *chance of variety, pluralism*, and *individual freedom*.

But variety and pluralism must not be a contradiction to unity. It was Leibniz who suggested that the unity of the world can only be experienced by man under special aspects. So his motto was "unity in variety". It dates back to the old philosophical idea of Heraclitus that even symmetry breaking is related to a sometimes *hidden symmetry*.

REFERENCES

Audretsch, J., Mainzer, K., eds. (1988) *Philosophie und Physik der Raum-Zeit*, [Philosophy and Physics of Space-Time, in German], Mannheim: B.I.—Wissenschaftsverlag.

Bourbaki, N. (1966) *Elements de mathématique: Théorie des ensembles*, [Elements of Mathematics: Set Theory, in French], Paris: Hermann.

d'Espagnat, B. (1976) *Conceptual Foundations of Quantum Mechanics*, London: Benjamin.

Emch, G.G. (1984) *Mathematical and Conceptual Foundations of the 20th-Century Physics*, Amsterdam: North Holland.

Fischer, E.P. and Mainzer, K., eds. (1989) *Was ist Leben? Zum 100. Geburtstag von Erwin Schrödinger*, [What is Life? Dedicated to Erwin Schrödinger on the Occasion of his 100th Birthday, in German], München: Piper.

Glansdorff, P. and Prigogine, I. (1971) *Thermodynamic Theory of Structure, Stability, and Fluctuations*, New York: Wiley.

Hanson, N.R. (1965) *Patterns of Discovery: An Inquiry into the Conceptual Foundations of Science*, Cambridge: Cambridge University Press.

Kuhn, T.S. (1966) *The Copernican Revolution: Planetary Astronomy in the Development of Western Thought*, Cambridge, Mass.: Harvard University Press.

Mainzer, K. (1980) *Geschichte der Geometrie*, [History of Geometry, in German], Mannheim: B.I.--Wissenschaftsverlag.

Mainzer, K. (1987) Philosophie und Geschichte des mathematischen Symmetriebegriffs, [Philosophy and history of the mathematical conception of symmetry, in German], *Historia Mathematica*, 14, 183-185.

Mainzer, K. (1988a) *Symmetrien der Natur: Ein Handbuch zur Natur- und Wissenschaftsphilosophie*, [Symmetries in Nature: A Handbook for Philosophy of Nature and Science, in German], Berlin: De Gruyter.

Mainzer, K. (1988b) Symmetries in nature, *Chimia*, 42, No. 5, 161-171.

Thom, R. (1983) *Paraboles et catastrophes: Entretiens sur les mathématiques, la science et la philosophie*, [Paraboles and Catastrophes: Discussions about Mathematics, Science and Philosophy, in French], Argenton-sur-Creuse: Flammarion.

Weyl, H. (1955) *Symmetrie*, [Symmetry, in German], Basel: Birkhäuser; Original English ed., *Symmetry*, Princeton, N.J.: Princeton University Press, 1952.