

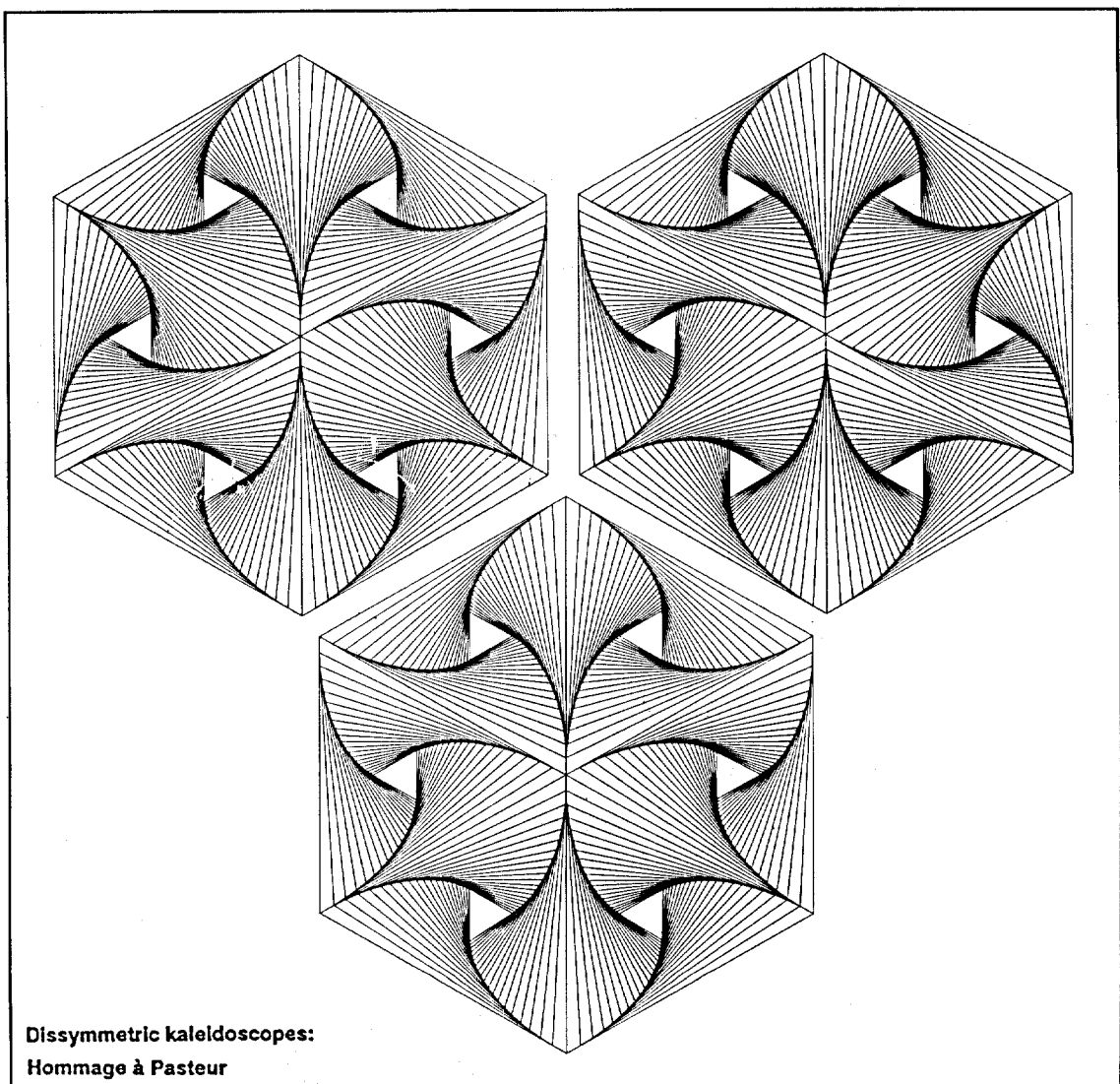
Symmetry: Culture and Science

SPECIAL ISSUE
Symmetry in a Kaleidoscope 1

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Dissymmetric kaleidoscopes:
Hommage à Pasteur

SYMMETROSPECTIVE: A HISTORIC VIEW

Introductory essay to the Special Issue

THE KALEIDOSCOPE AND SYMMETRY (OR, A SYMMETROSCOPE)

PART 1, FROM ART TO SCIENCE (19TH CENTURY)

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In the 5th c. B.C. the Greeks opened a new chapter in geometry and introduced the term *symmetria*, while in the mid-1810s David Brewster – later Sir David – invented, during his physical studies, a "new optical instrument called the kaleidoscope" (U.K. Patent No. 4136 in 1817). It is not surprising that he coined the word from Greek expressions: *kalos* (beautiful), *eidos* (form), and *skopien* (to see). The instrument is really simple, but it links both scientific ideas and aesthetic impressions. This last statement is also true in the case of the concept of symmetry, and the main goal of the new International Society, ISIS-Symmetry, is to emphasize precisely such interdisciplinary connections.

Another common factor in the cases of both symmetry and the kaleidoscope is that the related ideas had existed long before the birth of the terms themselves. The sense of symmetry goes back to the prehistoric period, while the modern kaleidoscope also had some preliminary versions. Brewster wrote, after publishing a short paper in French (1818), a whole monograph in English on the kaleidoscope where he discusses some earlier instruments as well (Brewster, 1819). One may think that it is not a good idea for a patent-holder to do so, but Brewster always made clear the differences between his version and the earlier ones. For example, he discusses Athanasius Kircher's instruments of 1646, but immediately makes the criticism (pp. 147-148) that there the allowed angles between the two plane mirrors are $360^\circ/n$ (where $n = 3, 4, 5, \dots$), instead of restricting them to $180^\circ/n$ (where $n = 2, 3, 4, \dots$). Of course, all of the latter angles are also included in Kircher's list as $360^\circ/2n$, but Brewster excluded the other half of the list where the denominators are odd numbers.

Most commercial kaleidoscopes consist of a tube enclosing two plane mirrors inclined at an angle of 60° along a common edge ($360^\circ/6 = 180^\circ/3 = 60^\circ$). An object in this sector – in a shallow glass-box at one end of the tube – will be reflected by these two mirrors, as well as by the virtual mirrors of mirrors, and so on, presenting ultimately a symmetric pattern to the person viewing it from the other end. The eyehole should be very close to the common edge of the two mirrors. The six sectors seen in this kaleidoscope are geometrically the same (isometric), but not equally bright: the best sector is that one in which the original object is positioned, while the intensity of light drops off during the successive reflections. If the object between the two mirrors is a reduced image of a left hand, then we see three left hands and three right ones, forming three symmetric pairs. The whole system has a threefold rotational symmetry, as well as three mirror lines (three is one half of the number of sectors). Interestingly, the frequent characterization of this rosette-like pattern by "hexagonal symmetry" is incorrect; we need a symmetric object in a symmetric position to provide true sixfold rotational symmetry with six mirror lines. This observation may help us to understand what is wrong in the cases excluded by Brewster, where the number of sectors is odd. In these cases – using an arbitrary object without special symmetries – the mirror images do not fit together to form a symmetric pattern (one half of an odd number is not an integer). It is easy to understand the ambiguity and the related blurring of the pattern in a simple drawing: if we consider the mirror images of the original object successively in both directions, they will finally "clash" at the opposite side of the circle. The resulting ambiguity can only be eliminated by placing a symmetric object in a symmetric starting position; in that case the images that clashed earlier superimpose on one another.

Those who are familiar with the theory of symmetry groups – useful for the mathematical approach to periodic structures and invariances in many fields, including crystallography and ornamental arts – may reformulate these observations in terms of simple theorems about cyclic (rotation) groups and dihedral (reflection) groups. Some of Brewster's remarks intuitively anticipated the modern approach to thinking in terms of group theory; moreover, we may find in his monograph some preliminary ideas on colored symmetries, when he refers to the combinations of forms and colors (Brewster, 1819, chaps. 11 and 17, respectively). His related knowledge was probably based not only on the often-emphasized optical studies, specifically on his investigations of the polarization by multiple reflections, but also on his interest in the morphology of crystals. He even connected these two fields, contributing to the beginnings of optical crystallography.

The kaleidoscope made a real boom on the market; in a similar way to the success of the Rubik cube in the early 1980s, which is also associated with *kalos* and *eidos*, but the manipulation of the cube requires more than just *skopien* (to see). Brewster (1819, p. 7) proudly remarked in his monograph on the kaleidoscope that "two hundred thousand instruments have been sold in London and Paris during three months". Ironically, he did not receive much money for his patent: the kaleidoscope, like many other toys, was quickly pirated. Brewster achieved, however, a reputation among the general public for the creation of the kaleidoscope. Not only was the instrument itself successful, but also its name, coined by Brewster.

Lord Byron was among the first people in 1818-19 who used the expression "kaleidoscope" in a metaphorical sense (*Don Juan*, 2, 93):

... this rainbow look'd like hope –
Quite a celestial Kaleidoscope.

The word also appeared in the titles of collected essays or poems and of some periodicals. The earliest example known to us has an additional linguistic "mirroring": the Greek-rooted English word was translated into Latin: *Kaleidoscopiana Wiltoniensia, or, A Literary, Political, and Moral View of the County of Wilts, during the Contested Election for its Representation, in June 1818...* (London, 1818). The expression made its way across the Atlantic very quickly. It appeared in the title of a newspaper, *The Kaleidoscope*, which came out in Boston, Massachusetts (first issue November 28, 1818).

It is interesting to note that there are many ancient Greek texts on aesthetics where the concept of *kalos* (beauty) is interpreted by *symmetria*. Among others, both Plato (*Philebus*, 64e) and Aristotle (*Metaphysica*, 1078a) discussed this connection. Interestingly, the emphasis on the connection between "kaleidoscope" and "symmetry" did not come immediately after the invention of the instrument. The reason is very simple: in that time the term symmetry, as in the Greek tradition, was associated rather with proportion and beauty, while such meanings as mirror or bilateral symmetry and rotational symmetry spread only a little later.

Characteristically, Brewster (1819) contributed also to that process. In his monograph on the kaleidoscope he uses the term with the new meaning as well. The adoption of the expression in this context is really very fortunate: the patterns in a kaleidoscope are symmetric in both the traditional sense (proportion, or beauty in general) and the modern geometric-crystallographic one (mirror and rotational symmetries). Probably Brewster was aware of Andrien-Marie Legendre's *Éléments de géométrie* (originally published in 1794), where the French mathematician pioneered the modern approach to the understanding of symmetry. This famous textbook was published many times and used throughout the decades. In fact, Brewster edited in 1822 the English translation of this book. In his later works, Brewster also popularized the expression "symmetry" in the context of physics: he spoke, for example, about "symmetrical refraction" (*A Treatise on Optics*, London, 1831, p. 338) and about the "axis of symmetry" of a horseshoe magnet (*A Treatise on Magnetism*, Edinburgh, 1837, p. 39). Interestingly, the *Oxford English Dictionary* (1961, vol. 10) gives credit to a crystallographic monograph, written in 1823 by Brooke, for the first use of the term in modern geometric sense, while Brewster is only second in line thanks to the cited paper on magnetism. Obviously the linguists missed the earlier monograph by Brewster (1819).

"Symmetry" became a colloquial expression very quickly in various geometrical contexts, including the domain of kaleidoscope. Thomas H. Huxley in his *Physiography* (London, 1877) wrote not only about the "hexagonal symmetry" of snow crystals (p. 56), but also about the "symmetrical shapes" seen in a kaleidoscope (p. 62). James Blyth (1880) in his article "Kaleidoscope" for the *Encyclopaedia Britannica* explained the universal fascination with the instrument by referring to the "endless variety and perfect symmetry" of the presented forms. Since the late 19th century almost every discussion of the kaleidoscope has referred to symmetry. Moreover, the symmetry-kaleidoscope relationship became "symmetric": the system of Subject Headings of the Library of Congress, which has been adopted by many contemporary bibliographies and computer information services, suggests, in the case of "symmetry", the cross-reference "kaleidoscope" and vice versa.

KALEIDOSCOPE IN THE ARTS? (THROUGH A KALEIDOSCOPE DARKLY)

There is, however, a significant difference between the histories of "symmetry" and "kaleidoscope". The first one became a central concept connecting scientific and artistic ideas throughout the ages. Brewster himself hoped for something similar in connection with his instrument. The title of his patent document emphasized that the kaleidoscope has a "great use in all the ornamental arts" (Brewster, 1817). In his 1819 monograph he went even further, believing that the kaleidoscope would replace the work of the designer, because (p. 116) "it will create, in a single hour, what a thousand artists could not invent in the course of a year". Brewster's idea was of a new form of art created with the kaleidoscope, which would be based on the harmony of colors, shapes, and music. The general idea of such harmonies gained some support among artists, especially among composers who made experiments in the field of colored music. Even such a leading composer as Ferenc Liszt became interested in new optical devices to enrich the performance of his music; in particular, he planned to use dioramas for the *Dante Symphony*. We need not go into the details of the history of colored music, but it is important to note here that, according to various experimental studies in psychology, the synaesthetic sound-color associations are not universal, but depend on the individuals. The orchestra conducted by Liszt was in trouble – according to a famous story – when the maestro demanded that the musicians give him "more blue". In any case, there were many interesting experiments in connection with colored music, but the kaleidoscope did not become a popular tool in the field. The enthusiasts of the kaleidoscope, however, did not give up easily: Brewster himself and several later inventors created many advanced versions of the original instrument, including the *telescopic kaleidoscope* or *teleidoscope*, where distant shapes can be projected into the kaleidoscope to be multiplied by the mirrors; the *polyangular kaleidoscope*, where the angle between the mirrors can be varied; the *rolleidoscope*, where a set of object-boxes can be rotated; the *multi-mirror kaleidoscope*, where there are three or four mirrors instead of just two; and the *kaleidograph*, where the symmetrical patterns can be displayed on a screen or a glass disk. These instruments could be found, however, mostly in the children's room, sometimes in the physics teacher's collection; but not in the designer's office.

Why could not the kaleidoscope satisfy Brewster's dream for the arts? A similar question was discussed not long ago by the leading art historian Ernst H. Gombrich (1979). Let us look at Sir Ernst's conclusions in connection with Sir David's kaleidoscope. Gombrich emphasizes that he is a devotee of this instrument, moreover that he likes to share his pleasure with others. There is, however, a disappointing reaction: after a few "ohs" and "ahs" comes the anticlimax when we put the kaleidoscope aside and move on to something else. The main reason is, according to Gombrich, that the kaleidoscope with its multiple symmetries exhibits *maximal redundancy*. It leads to monotony, where there is little left for us to explore.

From the designer's point of view, it is also obvious that the kaleidoscope, apart from some general aesthetic inspiration, could not become a practical tool. First of all, the designer needs to draw his sketch on paper or some other suitable surface. To transfer a pattern from the kaleidoscope onto the new surface is not easy, although this problem can be solved by tracing the figures projected out by the kaleidograph we have mentioned earlier. There are, however, more crucial disadvantages. The kaleidoscope, although it offers infinitely many realizations of a given type of symmetry, does not go far beyond patterns which are easily available in our environment. If we need a rosette motif we may have enough inspiration – as the

name suggests – from seeing various flowers. Designers often use cut-outs or stencils of such motifs, which can be traced easier than the images projected by kaleidographic or photographic methods. Moreover, the kaleidoscopic patterns within the given type of symmetry are determined by chance, and there is no simple way of modifying them in a creative way. The situation is a little bit similar to Eddington's famous example, where monkeys are jumping on the keyboard of a typewriter. If we are lucky, or if we have infinite time on our hands (as well as well-behaved monkeys), we may produce in this way even the most beautiful poetry. Sometimes we need the pleasure of watching the kaleidoscope (or monkeys in the zoo), but it is definitely not a useful method of creating works of art.

Brewster's dream about the artistic importance of the kaleidoscope, however, survived as a myth. We may follow its elements in many books. Let us see, for example, the article "Kaleidoscope" in various editions of the *Encyclopaedia Britannica*:

- [The kaleidoscope is] of essential service in the art of the designer (1880);
- The instrument has been extensively used by designers (1911);
- ... it has real value for the pattern designer and offers an admirable illustration of the image-forming properties of combined inclined mirrors (1968);
- ... the kaleidoscope also has value for the pattern designer (1989).

The entries become shorter and shorter through the decades, but the idea of applications in design – although with less emphasis – has invariably been present. The situation is similar in the case of textbooks, where we may observe additional motifs of "scientific folklore":

- The kaleidoscope, while being a well-known toy, is also used commercially in various branches of design requiring symmetrical patterns, as in the design of pottery, wall-paper, etc. (W.H.A. Fincham, *Optics*, 7th ed., London, 1965, p. 34)
- In fact, this device has been turned to practical use in making designs for carpets and wall-papers (J.P.C. Southall, *Mirrors, Prisms and Lenses*, Reprint edition, New York, 1964, p. 48).

Where are the pots, carpets, or wallpaper patterns created with the help of the kaleidoscope? Even what looks like a kaleidoscopic pattern is usually designed by graphical tools, or, more recently, by computer software. There is, however, a field where the kaleidoscope really survived, and even made considerable progress: in education.

KALEIDOSCOPE IN TEACHING OF GEOMETRIC-CRYSTALLOGRAPHIC SYMMETRIES (THROUGH A KALEIDOSCOPE SYMMETRICALLY)

In the beginning the kaleidoscope was studied from the point of view of geometrical optics. Brewster, as we discussed earlier, listed all the possible types of two-mirror (dihedral) kaleidoscopes, thus indicating the available types of rosette patterns. Theoretically there are infinitely many possibilities: the angle between the two mirrors should be $180^\circ/n$, where $n = 2, 3, 4, \dots$ (all the integers greater than one). In practice, however, the quality of the mirrors severely limits this list. In the case of

larger n 's some parts vanish because of the reduction of the intensity of light. Clear patterns are very hard to obtain, even in the case of relatively good mirrors, if n is greater than 10. The arrangement with two parallel mirrors facing each other, which can be interpreted as the case of an infinitely large n (i.e., 0° angle), presents images lined up perpendicularly to the mirrors in both directions (frieze pattern). It is easy to see, using, for example, a doll in front of one of the mirrors, that the images are alternating back and forth, facing each other in pairs at the virtual mirrors. Ultimately, half of the dolls are looking in one direction, the other half in the opposite direction. Of course, as in the earlier cases, only a part of the infinitely many images are visible.

The case of more mirrors in the tube which form a cylinder with an internal reflecting surface, where the cross-section is a closed polygon, was also considered by Brewster and other scholars in the early period (here we use the word "cylinder" in a very general sense; it is not necessarily based on a circle). We may also imagine the arrangement of mirrors in these kaleidoscopes as a right prism, based on a polygon, which is open from both ends. Interestingly, there are only four types of such cylindrical kaleidoscopes which always present periodic patterns without ambiguity. In the case of three mirrors, their perpendicular cross-section should form a triangle with one of the following shapes: equilateral triangle ($60^\circ, 60^\circ, 60^\circ$), half of an equilateral triangle ($90^\circ, 60^\circ, 30^\circ$), or right-angled isosceles triangle ($90^\circ, 45^\circ, 45^\circ$). In the case of four mirrors, the only allowed possibilities are the rectangles, including the square ($90^\circ, 90^\circ, 90^\circ, 90^\circ$). We cannot make a cylindrical kaleidoscope with more mirrors. These restrictions are a consequence of the obvious requirement that at each edge of the system the adjoining mirror-faces should form a dihedral kaleidoscope by an allowed angle of $180^\circ/n$. Each of the possible four types of cylindrical kaleidoscopes presents a part of a theoretically infinite periodic pattern (wallpaper-like arrangement) on a plane – or in a shallow layer – which is perpendicular to the axis of the kaleidoscope. The fact that these are the only possibilities was well known in the 19th century (Brewster, 1819, chap. 11); even the article on the kaleidoscope in the *Encyclopaedia Britannica* provided a geometric proof (Blyth, 1880). In subsequent editions, however, that part slowly disappeared: first the proof, and then the fact itself, vanished.

The repeated mirror reflections seen in the kaleidoscopes offered a natural model to represent combined mirror symmetries. In the late 19th century the latter topic, the theory of symmetries, became ever more important in geometric crystallography. It is interesting that the obvious questions in geometrical optics and geometrical crystallography have some similarities: to list all the possible types of mirror systems in kaleidoscopes, and of periodic arrangements in ideal crystals, respectively. The two problems are even more similar if we consider just the symmetries seen in kaleidoscopes and in ideal crystal structures. We discussed the solution of the first problem in the case of the classical kaleidoscope. No doubt the crystallographic question is much more complicated, because we also need other symmetry operations than mere reflections. Fortunately, these efforts were supported by such leading mathematicians as August Möbius, Camille Jordan, and Felix Klein. After many partial results the complete solution was presented independently by the Russian crystallographer Evgraf Stepanovich Fedorov and the German mathematician Arthur Schoenflies in 1890-91. Both of them described all the possible 230 types of periodic patterns in 3-dimensional space; these are the so-called spatial symmetry groups or space groups. Each of the ideal crystal structures can be characterized by one of these groups. Fedorov also discussed the analogous 2-dimensional problem and listed all the possible 17 plane groups (wallpaper groups). The intuitive meaning of this theorem is very simple: if we focus only on the "deep structure" of 2-dimensional periodic ornamentation, without considering the actual

shape of the basic motif, we have exactly 17 types of repetition. The arts and crafts have produced through the ages a very rich collection of periodic patterns on surfaces – from woven textiles to lattice-works, from mosaics to mural designs – but all of them can be classified according to these 17 types.

It is not surprising that Möbius (1849, 1851) and later Fedorov (1883, 1885) became interested in kaleidoscopes. Both of them approached the crystallographic symmetries using geometrical methodology, while Jordan, Klein, and Schoenflies used an algebraic treatment. Incidentally, the cited Möbius is well-known because of the famous strip named after him, which became a constant topic of "mathematical folklore". Another relevant motif which could lead both Möbius and Fedorov to the kaleidoscopes was their regular work with optical instruments. Möbius, although he is best known for his mathematical achievements, had various appointments as an astronomer; at one point he became director of the Leipzig Observatory. Fedorov's involvement in the subject was connected purely with crystallography and mineralogy: he developed optical instruments for measuring the angles of crystals (Fedorov goniometer, Fedorov optical stage). For illustrating crystallographic symmetries the earlier kaleidoscopes, which present planar patterns, were not really satisfactory. The topic requires the study of 3-dimensional structures. Möbius, and later Fedorov realized, however, that by using three mirrors it is possible to shape such "corners", which are suitable for the representation of spatial symmetries without ambiguity. Such a trihedral kaleidoscope can be obtained for example by standing a dihedral device perpendicularly on a third horizontal mirror (Fig. 1). In addition to these types, there

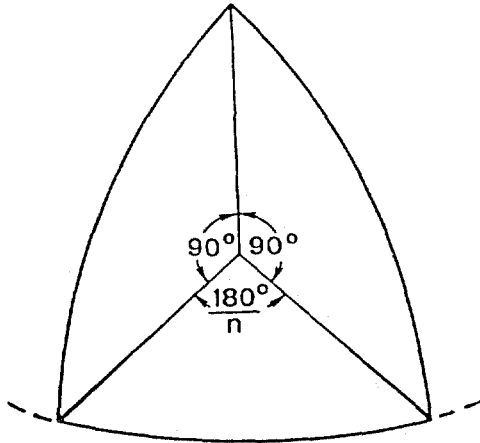


Figure 1: A class of trihedral (or Möbius-Fedorov) kaleidoscopes. This arrangement may represent infinitely many variable types ($90^\circ, 90^\circ, 180^\circ/n$; where $n = 2, 3, 4, \dots$). Indeed, if the vertical edge is hinged so that the adjacent mirrors can be moved along the horizontal one, that is, we may "open" and "close" the two vertical mirrors, the system can be easily transformed from one type to any other.

are also three further ones. We may characterize them, similarly to the illustrated types (Fig. 1), by listing the angles of the three faces at the vertex of the kaleidoscope: (1) $70^\circ, 55^\circ, 55^\circ$; (2) $55^\circ, 45^\circ, 35^\circ$; (3) $37^\circ, 32^\circ, 21^\circ$ (the numbers are rounded). Möbius analyzed this question by investigating the triangular tilings, or tessellations, generated by reflections on a sphere. Using modern mathematical interpretation, Möbius determined not only all the types of the suitable triangles (the so-called Möbius triangles), and the related trihedral kaleidoscopes, but also the types of the spatial reflection groups with a fixed point. Fedorov, who was aware of Möbius's works, dealt with similar geometric questions in connection with his own theoretical

and experimental works in crystallography. It is hard to reconstruct whether the trihedral kaleidoscopes did or did not play any relevant role in Fedorov's pioneering work on space-filling polyhedra, where he refers very briefly to kaleidoscopes (Fedorov, 1885, see, e.g., part 3, remark 1), as well as in the process of enumeration of the crystallographic symmetry groups. There is no doubt, however, that the new kaleidoscopes were very useful in illustrating some basic ideas for students. Fedorov employed various tricks in adapting these instruments for educational purposes. For example, we may represent the local arrangement of symmetric systems of points around the vertex of a trihedral kaleidoscope (atoms in a crystal-structure around a lattice-point) by inserting in it a small ball. It is also easy to produce images of various symmetric polyhedra by pouring liquids into the kaleidoscope. Of course the whole instrument should resemble an ice-cream cone, in order to hold the liquid. Moving the kaleidoscope to another position we can easily change the "object", the triangular surface of the liquid. Fedorov recommended the application of mercury which produces exciting silver-colored figures. However, it is better, for reasons of safety, to use non-poisonous liquids. Another possibility is to use, instead of a liquid, appropriate vertex-figures made of wire. Obviously it is better to try to make such experiments with kaleidoscopes than to speak about them! Indeed we plan to organize a workshop on kaleidoscopes in the near future: see the *Call for Kaleidoscopes* in *SFS (Symmetric Forum of the Society)* of this issue.

In the second part of this paper the 20th century developments of the kaleidoscope will be discussed. Interestingly, some kaleidoscopes gained importance even in modern arts; in this way the focus will, for a while, shift from science back to art. We will also provide a "periodic table" of all the possible kaleidoscopes shaped by plane mirrors.

THE PRESENT VOLUME: A "SYMMETROSCOPE"

Let us now turn to our own "kaleidoscope". Of course it is only a metaphorical kaleidoscope, but it has many properties common with the real one. It presents different pictures at each turn, all related to symmetry. Moreover, it has, similar to the classical Brewster kaleidoscope with an angle of 60° , a threefold symmetry with three "axes", that is, questions which were addressed by all of the contributors. This "kaleidoscope" may also give insight into many disciplines and artistic fields.

On the cover of this issue we have images which are connected not only with art and mathematics, but also with an interesting event in the history of the concept of symmetry. All of the three images are derived from a well-known "kaleidoscopic" figure, which originally had three mirror lines (Fig. 2.a). If we cut this pattern into six equal triangular sections through the center, just as we cut a birthday cake, we obtain slices – basic units – which are congruent (isometric), and each of them is suitable to generate the whole hexagonal image in an ordinary dihedral kaleidoscope of 60° (Fig. 2.b). This composite image appears in many books on computer art and computer graphics. The figure is, however, older than generally believed. It was the subject of a composition exercise on the "whirling triangle" at the Basic Design Studio of the American architect William S. Huff in 1961, while in the same year an Indian mathematician, Prakash Sharma, was led to a similar structure by a note on triangular doodles (see *The Mathematical Gazette*, 43 [1959], 34-35 and 45 [1961], 26-27). An analogous illustration was used by Rutherford Boyd in an earlier article on mathematical ideas in design (*Scripta Mathematica*, 14 [1948], 128-135).

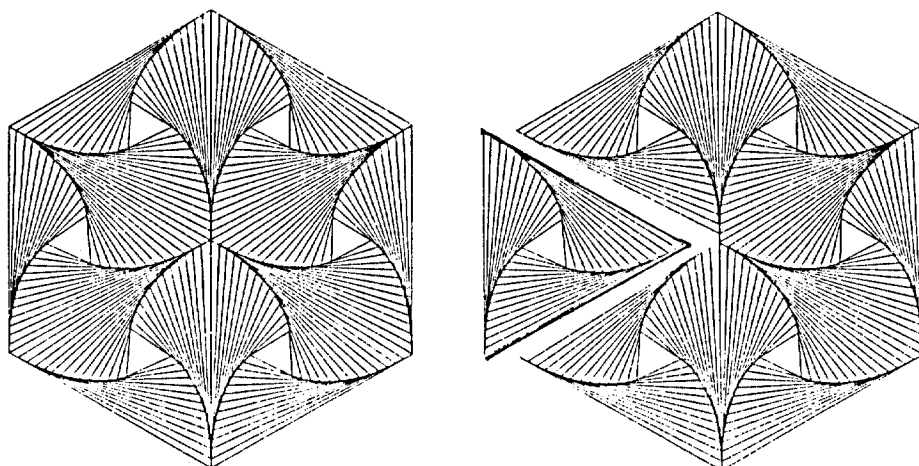


Figure 2: (a) Programmed design *Fan Fun* by David Kos from the Basic Design Studio of William S. Huff, Carnegie Institute of Technology, 1961.
 (b) The same design with a marked triangular section (basic unit).

There is an interesting mathematical approach to the basic unit of this composition by the "three-bug problem" (see the marked triangle in Fig. 2.b). The bugs are put at the corners of an equilateral triangle, and at the same time each one starts to crawl clockwise towards its neighbor. It is easy to see that each bug will approach the center of the triangle along a path of a logarithmic spiral, and at any instant they determine the corners of an equilateral triangle. Drawing these triangles periodically, according to a given time-interval, we derive what is really a set of whirling and decreasing equilateral triangles, although it is the three spirals that strike the eye. Note that here we have an angle-preserving process: the angles of the decreasing triangles are invariant. (Interestingly, some insects fly toward a light bulb along a logarithmic spiral, because their orientation is based on maintaining the visual perception of light at a constant angle; after a collision they often shift to a circular motion around the bulb, which corresponds to keeping an angle of 90° with the radii.) Fitting together six copies of the basic unit, three clockwise and three counterclockwise versions, in an appropriate way – or using just one in a dihedral kaleidoscope of 60° – we obtain the required figure. Martin Gardner, discussing the three-bug problem, also presented the composite hexagonal image, adopting it from Rutherford Boyd's earlier article ("Mathematical Games" in *Scientific American*, 213 [1965], no. 1, 100-104). Gardner's article was probably the main source of many later adaptations, taking the topic from classical drawings to computer graphics.

On our present cover, however, the original symmetry is "decreased". We have a mirror symmetric version with just one vertical mirror line, as well as two centrosymmetric versions which are mirror images. In 1848 Louis Pasteur came across similar symmetries. He was working on a problem in molecular chemistry: how the congruent molecules of the same chemical composition may have different physical properties, specifically left and right optical activity (rotating the plane of the polarized light to the left or to the right). He realized that the answer to the problem lay in the 3-dimensional structure of the molecules. Although the left- and right-handed molecules are geometrically equivalent by a mirror reflection, in real physical space no rigid motion can produce this transformation (it would require a 4-dimensional space, as Möbius remarked). However, if a molecule is mirror symmetric (has a plane of symmetry), the left-handed and right-handed mirror-

molecules are not distinguishable (they are superimposable), consequently, there is no optical activity. The same inactivity can be observed in the case of racemic compounds, where the distribution of the two kinds of molecules is about equal. Biochemical processes, however, favour mostly one kind of molecules, which indicates the possible importance of the topic when studying the origins of life. Returning to the original geometrical question: exactly those figures have two versions which miss some elements of symmetry. Specifically, in the 2-dimensional case it is necessary to exclude the mirror symmetry, while in the 3-dimensional case, the mirror, the central, and the so-called mirror-rotation symmetry. Pasteur suggested referring to those molecules which have two versions as "dissymmetric" ones, in the sense of lack of some, not necessarily all, symmetries. Note that the concept of dissymmetry is not identical with asymmetry where all the possible elements of symmetry are missing. Indeed, the asymmetry is the extreme case of dissymmetry.

Using this term in connection with the three non-kaleidoscopic images on the cover, we can observe that there is a symmetric one with a vertical mirror line, while the two others give a dissymmetric pair (but they still possess central symmetry, or twofold rotation). Moreover, adopting the terminology of textile industry as S- and Z-twists of yarns (left- and right-hand screws), the three images may represent the letters I, S, Z. It is even possible to write ISIS with these shapes. Huff's "dissymmetric kaleidoscopes" also reflect our goals: we should not overstate the symmetries in art and science, because often the dissymmetry is more important.

Of course we are aware that our "kaleidoscope" cannot compete with the success of those of Brewster, Möbius, Fedorov – and of such contemporary scholars or inventors as Coxeter, Koptsik, and Schwabe – and we can even be criticized, as was Kircher, for adopting too wide an angle of view. Our goal is, however, to present a variety of pictures on symmetry before putting the kaleidoscope aside. We hope that in our case the multiple symmetries do not exhibit maximal redundancy. Let us turn this "symmetroscope"...

(A related bibliography will be published simultaneously with Part 2 of the article, appearing in the section *Symmetro-graphy*, which will include all the cited works.)