# Symmetry: Culture and Science

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Symmetry in a Kaleidoscope 1

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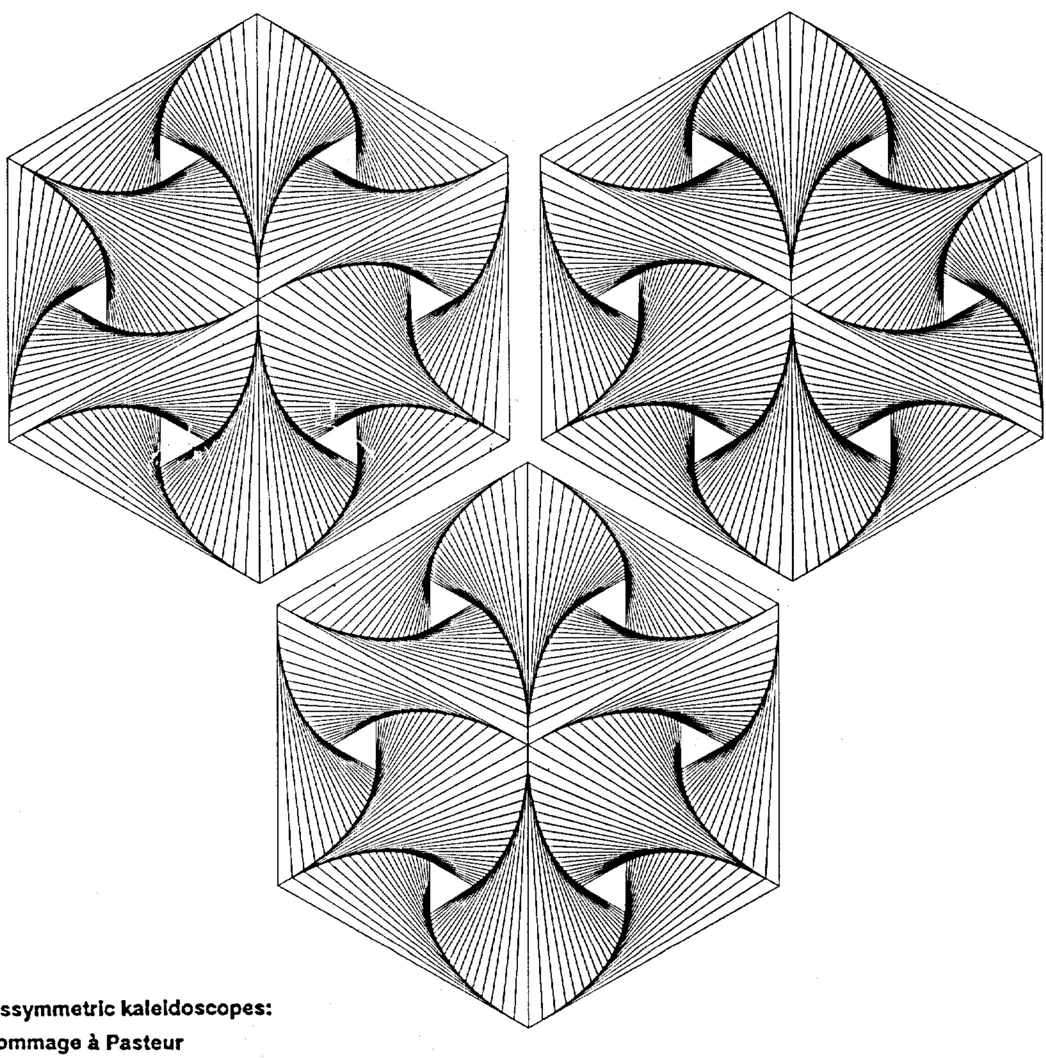

Dissymmetric kaleidoscopes:
Hommage à Pasteur

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# MANIFESTO ON (DIS)SYMMETRY WITH SOME PRELIMINARY SYMMETRIES

## INTRO-SYMMETRO-DUCTION

*Let there be symmetry! The first chapter gives a brief summary of the development of the concept of symmetry, emphasizing the possibility of applying it in various fields of culture and science. The second one presents a "discrete" historic survey without following the developments continuously, but only focusing on the cyclic peaks of the interdisciplinary studies of symmetry. Some far-reaching artistic works and disciplinary discoveries, which coincide with the considered peaks, are also discussed. The most important results may figure in both chapters, giving together a deeper insight. The general statements are often illustrated by concrete examples referring very briefly – mostly in parentheses – to basic ideas or the activity of scholars or artists (the initials of names are given only in those cases where two or more persons can be confused). Those readers who are not interested in such details may omit these parenthetic remarks. Later we will publish a more detailed historic survey on the interdisciplinary aspects of the topic, and the permanent section "Symmetro-graphy" will include a related bibliography. Currently no list of references is given: the mentioned monographs can be easily located in general catalogs, and the few papers that we cite are discussed extensively in the literature of the corresponding disciplines. Although our survey reflects the fact that the focus of interest in symmetry has moved in the 19-20th centuries from the arts to the direction of "hard sciences", the symmetry-related thinking in the modern arts and humanities is also emphasized. As a matter of fact, the real manifesto, in the concluding third chapter, is an appeal to the representatives of both sides of culture.*

## 1. WHY SYMMETRY?

**(Symmetry and broken symmetry from science to art, from cottage cheese to scent bottle)**

The Greek and Latin *symmetria*, the Sanskrit *sammita*, or the Hebrew *ketzev* and *toam*, the Chinese *dùey-cheng*, the Japanese *taishô* and *kinsei*, and the modern *symmetry*, together with *asymmetry*, featured in very many artistic and scientific discussions throughout the ages. The classical Greek meaning of symmetry as common measure and proportion was supplemented or even overshadowed by a modified one from the 18th century as mirror symmetry and later as any geometric operation which transforms a given figure or system into itself. In other words, the geometric symmetry leaves the whole figure or system unchanged (invariant) although it may permute its parts (cf., mirror, rotational, translational, and combined crystallographic symmetries). Group theory provided a "mathematical language" for the treatment of symmetries. In the early 20th century crystallographers and physicists continued to generalize the concept, taking into consideration the invariance of non-geometric properties or the conservation of various quantities assigned to a given system (cf., colored symmetries in crystallography, conservation laws in physics). We may also speak about the invariance of laws of nature, or the invariance of equations describing the corresponding laws, under given transformations or their group. The basic idea of generalization of symmetry can be easily understood when one walks in the thick of a more or less homogeneous and regular forest on a flat territory, such as a pine forest or a coconut plantation. Upon leaving the footpaths one may feel lost, as running from one tree to another does not present any difference of view: the surrounding tree patterns look alike. Our shifts from tree to tree can be considered as symmetry transformations of the forest if we consider a big enough territory (cf., discrete crystallographic symmetries). More generally we may think not only about the invariance of the view (geometric arrangement), but also of the results of experiments or the forms of equations. We may allow not only jumps between discrete positions, but also continuous transformations. For example, a passenger of a jet flying over clouds usually has an invariant view under the continuous motion of the jet; dropping a spoon in the same way, or repeating any simple physical experiment at different times, gives invariant results (cf., the homogeneity and isotropy of space-time). As a further step we may consider a larger set of objects or events if we focus not only on global symmetries, which are applicable uniformly elsewhere, but also accept local ones: the so-called gauge (or calibration) symmetry refers to the invariance of laws under a change of length scale locally at every point of space-time. This very abstract symmetry can be easily imagined by the analogy of looking up places in a world atlas. Leafing through the pages one has to consider maps made by unified principles, but very different scales: the whole earth, a continent, a country, a city. Gauge symmetry deals with those properties which remain invariant during the search, in spite of the change of scale (cf., the theory of electromagnetism). It should also be emphasized that there is a relevant practical importance of symmetry in modern physics and related fields in general. In the case of less complicated deterministic systems it is often possible to describe the events directly by the dynamic equations of classical physics, without dealing with the basic symmetries or conservation laws. However, considering very complicated or probabilistic systems – such as quantum mechanics, where we cannot determine the exact position of an electron in a given time – we usually have no other choice than to rely on carefully formulated conservation laws and predict from them which are the possible phenomena and which are the excluded ones. It is somewhat similar to the work of a bookkeeper who is not aware of the details of a business, but after balancing the data in two columns, he or she will still able to suggest which transactions can be selected and which should be excluded (see, in particle physics, the conservation of quantum numbers and the selection rules).

The concept of symmetry and asymmetry – in addition to simple geometrical or aesthetical associations – became useful in classifying various objects, processes, and phenomena, not only in crystallography and physics (pioneered by Haüy, Bravais, and P. Curie, respectively), but also in other fields of science and technology. Moreover, the symmetry considerations may help to determine all the possible elements or structures of given properties, systematizing the known ones and predicting the missing ones for further experimental search (cf., Mendeleev's periodic table of the chemical elements; van't Hoff's list of isomers with an asymmetric carbon atom, E. Fisher's analogous list for the sugar group, A. Werner's coordination theory in inorganic stereochemistry, and later Barton and Hassel's conformational analysis; Fedorov and Schoenflies's crystallographic symmetry groups, and later the colored generalization of them for magnetic structures; and, more recently, Gell-Mann and Ne'eman's SU(3) symmetry in a class of elementary particles). Many of the modern applications of symmetry are based on the late 19th century and early 20th century mathematical developments of group theory by which we may consider the appropriate symmetry transformations of a system

(Jordan, Klein, Lie, Poincaré, and later Cartan; cf., also Cayley and Sylvester's preceding invariant theory). In crystallography the discrete groups play an important role (Sohncke's 65 motion groups, Fedorov and Schoenflies's 230 space groups), while in the theory of relativity, and later in quantum mechanics and in the theory of atomic spectra, the continuous Lie-groups lead to major breakthroughs (cf., Lorentz, Poincaré, Einstein, and Bethe, Weyl, Wigner, respectively). The group concept is general enough to be useful not only in the everyday practice of crystallography, theoretical physics, and structural chemistry, but also in such distant fields as hydrodynamics (G. Birkhoff), theory of perception (Cassirer), social structures (Lévi-Strauss), and even in the arts. True, the application of group theory also has some limits: a leading physicist jokingly spoke about "Gruppenpest" (Pauli).

The crucial importance of the geometric symmetries, or invariances, of the space-time continuum was demonstrated by the general theory of relativity (Einstein). Soon after this achievement the symmetries of nature and the conservation laws were connected in mathematical physics (Noether). Later the symmetry principles (invariances) and the corresponding conservation laws became the basis of the studies of quantum mechanics and particle physics (Wigner), as well as, more recently, of the theory of chemical reactions (Woodward and Hoffmann). Maybe it is worthwhile to see some further details of mainstream conceptual development of symmetry in modern physics and chemistry, although it is not easy, and the reader, in case of any "asymmetric" feelings, is encouraged to jump to the next paragraph. After the success of the theory of relativity, where very fast moving objects were described, interest turned to very small ones (Bohr and others). These studies, partly inspired by mathematical equations of classical mechanics (Hamilton, Jacobi, Poisson), led to the formulation of quantum mechanics. It had in the early period two different, but equivalent interpretations (Heisenberg's matrix mechanics, Schrödinger's wave mechanics). These, however, did not satisfy the invariances of theory of relativity: they had different mathematical forms if the observer moved from one position to another. In other words, the equations could not account for those cases where the particles moved very fast. This problem was solved by a new mathematical formulation of quantum mechanics being "relativistically invariant" (Dirac). The new equations led not only to a better understanding of the properties of the electron, but also to the prediction of a dual particle called the positron (anti-electron). It has the same mass as the electron, but is positively charged. The connection between the electron and its antiparticle can be considered as a non-geometric internal symmetry, namely a "reflection" (conjugation) that interchanges the positive and negative charges. Relativistic quantum mechanics – beyond its "mathematical beauty" – inspired both theoretical and experimental research. Later it became clear, however, that it is necessary to develop a more advanced theory to eliminate certain difficulties in the case of electromagnetic interactions (this quantum electrodynamics was developed by Tomonaga in Japan and Feynman, Schwinger, and Dyson in the United States). In the meantime, ever larger numbers of particles and their antiparticles – "building elements" at the subatomic level – were discovered, shaping the new field of particle physics. The interaction of particles, using the earlier metaphor, is a very complicated "business" with many surprising events, where typical predictions are based on careful "bookkeeping", that is, on the consideration of symmetries, or invariances, of nature and the corresponding conservation laws (they are interrelated, according to the Noether-theorem cited earlier). Firstly, the most obvious symmetries of nature were considered: the "reflections" of the charge, space, and time (C, P, T conservation) and their combinations, which are humorously referred to as "physicist's mirrors". Further laws state the conservation of concrete quantities called quantum numbers (baryon number, lepton number, various quark numbers, isotopic spin, etc.). It turned out, however, that some of the new conservation laws and the corresponding symmetries are not exact. Experiments demonstrating symmetry violations gave new tasks to researchers and the Nobel Prize Committees (Lee and Yang's symmetry violation of the conservation of P, parity; later Cronin and Fitch's symmetry violation of the combined CP). The symmetry groups helped to classify some types of elementary particles, for example, the hadrons (we already referred to Gell-Mann and Ne'eman's independently introduced SU(3) symmetry), but the comprehensive classification of all the particles is not yet available. The discoveries also led to a better understanding of the basic forces of nature. Namely, four types of interactions were identified according to the relative strengths, as well as the type of carrying particles: strong, electromagnetic, weak, and gravitational interactions (this picture emerged following the findings of Fermi, Yukawa, later Feynman and others; more recently there are conjectures about a fifth interaction, but that theory is not widely accepted). Gravity and electromagnetism are well-known, while the strong and the weak interactions can be observed, for example, during nuclear processes or radioactive decay, respectively. Local gauge symmetry had already been used intuitively in the formulation of a single theory for electricity and magnetism (Maxwell), while the more recent unification of the electromagnetic and weak interactions are based specifically on it (Glashow, Salam, Weinberg). This means that these two interactions are not entirely different, but they represent the two sides of a more general electro-weak unified field. The possibility to continue this unification to include the remaining two interactions, that is, the strong interaction (Grand Unified Theory) and, finally, gravity, is still a central problem of theoretical physics. One of the suggested theories called supergravity considers a supersymmetry, where each ordinary particle has a hypothetical superpartner with the same properties except the so-called spin (the intrinsic angular momentum). The findings in particle physics also helped to interpret astrophysical processes, especially the earlier history of the Universe (Hawking, Weinberg, Wheeler, Wilczek, and others). Thus the smallest and biggest objects of modern science are interlinked. In chemistry one can observe the importance of both geometric symmetries (crystallographic symmetries, the already mentioned symmetry principles in stereochemistry from van't Hoff to Barton and Hassel, the Cahn-Ingold-Prelog classification system of stereoisomers, etc.) and the symmetries in quantum chemistry based on discoveries in physics (the molecular orbital theory, the ligand field theory, various results in spectroscopy; see the works of Herzberg, Hückel, Pauling, and others). Here we refer only to two theories of great practical importance: molecular geometry for describing structures by electron pair repulsion (Gillespie) and the conservation of orbital symmetry for predicting the course of chemical reactions (Woodward and Hoffmann, see also Fukui).

Some central figures of quantum mechanics and particle physics contributed not only to very many parts of the subject, but also displayed a considerable literary skill (see Heisenberg's book of essays *Der Teil und das Ganze*, in English translation *Physics and Beyond*, for a very broad audience, or Wigner's collection *Symmetries and Reflections* for readers with a scientific background), intuitive and graphical approaches (see the Feynman diagrams representing the interactions of particles, as well as very many didactic ideas in his textbooks). There is even humor in the field: Gell-Mann named a new quantum number "strangeness"; adopted the Buddhist phrase "eightfold way" to refer to the number of components in his classification model of the hadrons; and later introduced three fundamental particles called "quarks", borrowing the name from the quote "Three quarks for Muster Mark" in James Joyce's *Finnegans Wake*. The mysterious expression quark has no meaning in English; in German it is a type of cottage cheese. The symmetry properties of "cottage cheese" became a subject of further investigations. Moreover, as in the food industry, as well as the theory of colored symmetry in crystallography, this "cheese" was even flavored and colored. The three types of quarks are described by "flavors", and each, according to the theory of quantum chromodynamics, has three varieties identical in all measurable qualities, but different in an additional property called color (Greenberg). Although the scholarly literature speaks about red, green, and blue quarks, which convention was adopted in some colored illustrations, these ones have only symbolic meaning, not a visual one. A further play with colors took place when the antiquarks were marked with the following anticolors: cyan, magenta, and yellow. This truly colorful theory, where even the gauge symmetry must be generalized by colors, could solve a problem with the original colorless quark model (namely, the new version does not contradict the Pauli exclusion principle). We should not overstate, however, the humorous aspects of the subject. Many of the generalized symmetries in physics and chemistry, especially those which are difficult to visualize or intuitively imagine, lost their meaning not only for the broader audience, but also for many physicists working in other fields. There are many forums on symmetry in particle physics for specialists, while the lay person may find some popular books written by a few "great masters" (Feynman, Hawking, Heisenberg, Ne'eman, Salam, Weinberg, Wigner). Note that the technical details of these highly abstract and complex theories do not belong to the sphere of interest of ISIS-Symmetry, but the general philosophical backgrounds and the interdisciplinary connections may have some importance.

In biology the consideration of symmetries led, as in physics, from the simple geometric classifications to the direction of the idea of conservation. Interestingly, physicists also became involved in the basic questions of biology (Schrödinger, Szilard). The geometrical-morphological classifications, and then descriptive models and mathematical approaches to biological symmetries (Haeckel, D'Arcy Thompson, respectively) were followed by the experimental study of the development of symmetric bodies at the "micro level" of embryology (Driesch, Spemann), and later by mathematical models of morphogenesis (Turing, Thom). The idea of generalizing the crystallographic symmetry principles to biological structures also appeared (Bernal, Vernadskii). Specific fields, such as the patterns of plant growth and the structures of viruses attracted special attention. The idea of homeostasis (Cannon) and the entropic-thermodynamic approach to biological systems (Prigogine) is related to the idea of conservation in a broad sense, while the predictions based on the genetic code (Crick and Watson) can be compared to the selection rules in particle physics. The aspects of cyclicity, equilibrium, and self-organization are discussed in various systems, including such fields as the theory of chemical reactions (Belousov, Zhabotinskii), chemical origins of life (Eigen), biological time-organization (Goodwin, Winfree), chronobiology (Halberg), population dynamics (Volterra, Lotka), automata studies (von Neumann, Wiener), economy (Arrow, Hicks, Leontief, Kondratev, Tinbergen), and even in recreational mathematics (Conway's life game). In structural engineering some spatial symmetry principles, based on both theoretical geometry and experimental dynamics, helped to build geodesic domes of maximum span with a minimum of materials (Buckminster Fuller). Some connections between stability and symmetry of space structures were also established. In this way the analysis of the usually complicated conditions of stability, a typical problem in engineering and architecture, can occasionally be replaced by a simpler consideration of some symmetries. The higher dimensional symmetries gained importance not only in pure science, but also in computer architecture (Caltech's hypercube).

Apart from the perfect symmetries, the missing elements of symmetry and the departures from the exact laws were also considered (cf., Pasteur's and P. Curie's dissymmetry of molecules and of physical phenomena, respectively; the Jahn-Teller distortion of crystal structures or molecules in quantum chemistry; Crick and

Watson's right-handed double helix of the DNA; Lee and Yang's, later Cronin and Fitch's, symmetry violation in particle physics; Salam and Weinberg's spontaneous symmetry breaking in their unified field theory; etc.). Some of these works inspired further studies in connection with biochemical evolution and the astrophysical history of the universe. In the case of any developing system – such as in biology or cosmology – the "asymmetry" in time is a crucial component. While any motion in a homogeneous space can be repeated back and forth, time has an arrow, or in other words, it is irreversible. An analogous problem was investigated in connection with the propagation of heat and the kinetic theory of gases in physics from the mid 19th century; it resulted in the introduction of the concept of entropy as the quantitative measure of disorder in a system (Clausius, Boltzmann). Indeed, lukewarm tea will not get hot again, and the molecules that evaporated from an open scent bottle will not return to it by chance. In both cases the original order vanished, which is fortunate for manufacturers of home appliances or perfume. The second law of thermodynamics postulates the increase of entropy (disorder) in a closed physical system. Somehow it underlines the importance of symmetries, where, in a local part of a bigger system, this rule is defied, producing crystals, living organisms, and other forms of order (cf., also by Onsager's and Prigogine's non-equilibrium thermodynamics). The idea of entropy also led to the study of chemical equilibria (Gibbs) and the formulation of statistical mechanics (Maxwell, Gibbs, Boltzmann, Ehrenfest), where the positions of a large number of molecules are described statistically. The probabilistic approach also became relevant for the shaping of quantum mechanics (Planck, Heisenberg, Schrödinger; cf., also the wave-particle duality, Heisenberg's uncertainty principle, and Bohr's principle of complementarity). Later, various aspects of order-disorder were investigated in the thermodynamics of phase transitions (Landau) and of critical phenomena (K. Wilson). Returning to the idea of entropy, an analogous concept was introduced in information theory as the measure of uncertainty: how many yes or no questions (binary digits or, briefly, bits) we need in order to guess an object selected from a given set, as in the case of the well-known game of twelve questions. The mathematical definition of entropy is based on an obvious and always useful strategy that the number of possibilities should be reduced by half in the case of each question (thinking about 2, 4, 8, 16, etc. objects we need 1, 2, 3, 4, etc. questions). The thermodynamic and information theoretic entropies, that is, the measure of disorder or uncertainty in a given system, respectively, are analogous not only by their mathematical definition, but they are more deeply connected in the case of some complex problems (cf., Szilard's interpretation of the problem of Maxwell's demon; here the demon uses information at the door between two chambers to sort the gas molecules and defy the second law of thermodynamics). These questions lead to the "thermodynamics" of data processing and to realizing the physical limits of computing (Bennett and Landauer). Very recently some new models and approaches gained popularity in mathematical-physical sciences to deal with order-disorder, or specifically with symmetry breaking in various natural and some social systems (cf., Haken's synergetics, Mandelbrot's fractal theory, Prigogine's non-equilibrium thermodynamics, Thom's catastrophe theory; note that here "synergetics" refers to cooperative efforts to study various disorder-order transitions in physics, chemistry, and other fields, and it is not identical with the "synergism" of the pharmacologists or the "synergy" of Buckminster Fuller). The recent research on chaos theory (with its preliminaries going back to Poincaré) also demonstrated the limits of the predictions in complex systems: often the minimal modification of the initial conditions may lead to a very different scenario and, ultimately, different patterns. The classification of related configurations has gained not only mathematical importance, but it may generate also aesthetical impressions.

The artists, rather than looking for all the possible structures, or their classification, are interested in finding some special ones. There are, however, examples when the method of composition requires working with the full set of symmetries as in the case of Bach's fugues (where all the four symmetric images, a basic theme, its rectus, inversus, and recto-inversus, are used alike); Schoenberg's dodecaphonic serial music (based on the total 48 versions of a twelve-note series given in cyclic order, where all the four symmetric images and all the twelve starting possibilities are considered); and the experimental poetry of the Russian futurists and the French "pataphysicists" (see, for example, Khlebnikov's variations of sounds, or Queneau's combinatorial methodology). The almost mathematical commitment of Bach made possible in the 1920s the geometrical-symmetrical reconstruction of the mixed-up manuscripts of his *Kunst der Fuge* (Graeser). The topic of musical symmetries also became exciting for scientists (the Bolyais, Mach, Speiser) and writers (Thomas Mann in his *Doctor Faustus* describes a method of composition which is very similar to Schoenberg's, the composer even challenged it; while the Dutch author, Mulisch, in his short novel *Symmetrie* refers specifically to Mach). The repetition of some basic motifs and the combinatorial manipulation of a given set of tones and rhythms are typical in oriental music, see, for example, the Arabic *maqam* system and the Indian *ragas* and *talas*. In the case of ornamental art, especially in those places where the figurative representation was disliked or forbidden, the intuitive "ethnomathematical" knowledge of generations could lead to finding the full set of repetition types (symmetry groups), although, most likely, it is not the central interest of artists. Interestingly, all the seven frieze groups are figured in a single art work in Fiji; all the 17 wallpaper groups are included in the decorations of the Moorish palace Alhambra (Gómez, Montesinos; cf., Müller, and Grünbaum et al.); all the 46 black-and-white wall-paper groups are described in the framework of textile design (Woods). The patterns of the Alhambra also inspired Escher's periodic drawings, which are frequently used in the teaching of geometry and crystallography (Coxeter, Ernst, MacGillavry, Schattschneider, and others; see also the movies by Emmer). A further step was to realize that a small modification of a periodic structure may enrich not only the physical properties of an object (see various "dissymmetry principles" from the ancient architecture to the semiconductor design), but also the aesthetic impressions (see Escher's metamorphic drawings, Vasarely's op-art, Reich's repetitive music). In the fine arts and in architecture the real challenge is to find a balance between the two sides of an individual object, one based on more complex arrangements than just the simple bilateral symmetry. In the case of larger scale of compositions, for example in town planning, the order (or orderliness) of the elements may have both aesthetic and practical importance. On a similar basis, the symmetry and asymmetry principles gained an importance in industrial design. New forms of high-tech arts often use mathematically defined transformations in space or time, although the observers may enjoy them without being aware of these "secrets" of the artist-engineer.

The situation in the humanities is similar to that in the arts: the classification schemes and exact methods based on symmetry do not definitely play a relevant role, although there are concrete examples of their application in linguistics, anthropology, history of arts, and other fields. Truchet and Douat's recently rediscovered combinatorial methodology for pattern generation in the early 18th century almost foreshadowed the era of computer aided design. There is a long tradition going back to the mid-19th century of classifying the patterns of decorative art by quasi-mathematical methods, including *The Grammar of Ornament* by O. Jones and many similar works. Haddon's classification of Papua friezes in the late 19th century includes a type for which he could not find any examples; it is a step towards the later geometric-crystallographic approach which emphasizes all the theoretical possibilities. Turning to other fields of the humanities, negative examples can also be observed where the "fashion" of applying exact methods became too artificial and more or less irrelevant. Usually the simple "mathematical package" of common sense ideas does not help in the better understanding of arts, but leads to confusion. The few positive examples often connected with symmetry-related considerations (see, e.g., Saussure's structuralist approach to linguistics, and later Lévi-Strauss' structural anthropology, including the analysis of symmetrical and asymmetrical kinship systems and the use of group theory; a similar interest of the Linguistic Circle of Prague, and Jakobson's formalist approach to language and literature, emphasizing the binary oppositions and the asymmetries of semiotic systems; the semiotic schools in Bloomington, Moscow and Tartu, Paris and Strasbourg; G.D. Birkhoff's aesthetic measure, and later Bense, Gunzenhäuser, and Moles' information theoretic aesthetics). Chomsky's transformational-generative grammar became useful not only in linguistics in a narrow sense, but also in computer science, and it provided a model for further formalizations in other fields. The computer is a useful tool for surveying the possible variations of given elements in numerous problems of design, architecture, urban planning, and even choreography. Here we are also facing the question of the "language" and "grammar" of the arts. In the case of ornamental art the symmetry groups can be considered to be the "deep structure" of the language of composition, which may help in comparative studies. In other fields the "language" may be much more complicated (see the recent studies in the generative theory of music, shape grammar, and hyperstructures).

Interestingly, the concept of symmetry in psychology had a very similar evolution to that in crystallography and physics: after a rather experimental period, the Gestalt movement formulated symmetry rules in the early 20th century; later the schools of Jung and of Piaget gave a broader understanding of the concept. Namely, Jung considered archetypes in the history of culture, while Piaget investigated the idea of conservation of quantity, length, and so on, during the development of children through a succession of intellectual equilibria. More recently the left and right symmetry of the cerebral hemispheres was challenged (Sperry), analogously to the works in particle physics (Lee and Yang). In both of these cases the theoretical findings were supported by experimental data. The distinctions between left and right lead to further studies (in psychology and brain research by Corballis, Gazzaniga, and others, see also the research on schizophrenia and dyslexia; in physics by Landau, Cronin, Fitch, and others). Finally, both topics inspired one to look at the origins of the violation of left and right symmetry from the evolutionary point of view in humans and the universe, linking these questions to further fields such as archaeology or astrophysics. Incidentally, the preference for the right hand goes back as early as 1.4-1.9 million years ago, as the experimental-archaeological research demonstrates (Toth). The distinction between left and right, as well as other dual symbolic classifications, often found by anthropologists, although they are not definitely based on the biological fact of handedness (R. Needham). It may be possible that the physical, chemical, and biological asymmetries are linked in more sophisticated evolutionary models, but very many questions still remain unanswered.

The concept of symmetry – although the focus of interest moved to the direction of crystallography, mathematics, physics, chemistry – never lost its double meaning in science and art. There are even specific analogies between distant fields which may help both sides, see, for example, the crystallographic and ornamental symmetries (Washburn and Crowe, Zaslow and Dittert), geometric and urban patterns (March and Steadman), theory of polyhedra and geodesic domes (Coxeter's and Buckminster Fuller's mutual inspiration), geometric and musical compositions (Lendvai, Graeser, Werker), symmetry principles in modern physics and oriental philosophy (see Lee and Yang's remarks on Taoism, or more recently the popular-scientific books by Capra, Zukav, and similar authors). In other cases the symmetries or asymmetries of biological structures may inspire technical developments; see the application of biosymmetries in mechanical engineering (Petukhov), the idea of the asymmetry of the brain in advanced architecture of computers (ISIS-Symmetry workshop in 1989), or, more generally, recent research in bionics, biomechanics, and neurocomputing. The "betweenness", thinking about art and science or other distant fields, was very impressively demonstrated by the Alice stories of Lewis Carroll, alias Dodgson, which is a bravura from the point of view of both literary text and mathematical ideas, with very many obvious and hidden symmetries in space and time. The interest in mathematical concepts, including symmetry-related questions, is an interesting aspect of the poems and essays of some writers in the first half of the 20th century (Barbu, Belyi, Khlebnikov, Valéry, and others). Even in our age there are a few artists-scientists or scientists-artists who contributed significantly to both sides. "Mathematical writers", such as Gardner in English, Husimi in Japanese, and Yaglom in Russian, popularized many aspects of symmetry. A broad range of artistic-scientific connections are also discussed in Hofstadter's *Gödel, Escher, Bach*, a Pulitzer-prize winning monograph. Of course we do not suggest by these analogies to mathematize the arts or make mathematics more artistic in general, but such types of connections may have some advantages in specific problems. For instance, they may emphasize the unity of culture in education, provide new tools and methods for artists, or raise new questions in science (as the Rubik cube did). Generations of great scholars from Einstein to Planck, from Schrödinger to Hoffmann, demonstrated the importance of knowledge in the humanities and its indirect impact on scientific research. Waddington, a leading theoretical biologist, devoted an entire monograph to summarizing the analogies between biological and artistic structures. These types of connections could also help the "humanization" of science in higher education during periods when the interest declines (see, e.g., Coxeter's geometric textbook with many literary quotes, analysis of Escher's drawings, etc.). There are also a few special cases where artistic contributions could directly inspire advanced studies in science, for example, Heisenberg introduced symmetry groups into particle physics following his reading of Plato's dialogue about the five regular polyhedra; a mistake in the enumeration of the colored symmetries was discovered by Escher's periodic drawings (MacGillavry, A. Loeb); the incompleteness of the list of convex pentagons that tile by congruent copies was found by amateurs (cf., Schattschneider's survey). The artistic problem of the symmetries and proportions of the human body led from the ideal canon of the Greeks (Polyclitus or Polykleitos) to a set of alternative ones (Leonardo, Dürer), and finally to ergonomic principles in design and architecture (Le Corbusier's modulor, see also the Japanese *tatami*). Computer-aided manufacturing makes it possible to go beyond the standardized sizes of clothes, furniture, and other objects, taking into consideration individual ergonomic data in mass production. There is a tradition of generalized symmetries in Japanese culture, based not on simple mirror reflection, but on a more complex balance (cf., ikebana, garden art, calligraphy, origami, martial arts), which could help indirectly in microchip design. The need for maximum efficiency on the very limited planting area and the strictly invariant cycles of rice culture have shaped a workforce with a commitment to optimum planning and precision which is necessary in high-tech industry. (Note that the symmetric pictograph of the rice field, a square divided into four equal squares, is one of the most frequently used characters in family names.) Interdisciplinary cooperation and team-work also have traditional roots in the Japanese social life. Creating harmony between man-made objects and the surrounding environment, which often leads to the adaptation of symmetries of organic nature, is a central feature of traditional Japanese design and architecture; it helps to live more comfortably in the crowded cities. The interests of the Society for Science on Form – the Japanese expression *katachi* is more concrete than the English expression "form" – and the scope of some art-science exhibitions (Sakane, as well as Miura, Miyazaki, Yoshimoto, and others) cover many symmetry-related topics. In the West the isolation of art and science is stronger, and the connections between them are only rarely emphasized in our age. The notable positive examples are some constructivist and neo-constructivist groups, the Center for Advanced Visual Studies at the MIT (founded by Kepes), and the Institut de Recherche et Coordination Acoustique/Musique (IRCAM) in Paris (directed by Boulez). All of these movements or institutions turn regularly to questions associated with symmetry. There are examples of artists contributing to scientific periodicals (Escher, Hill, Queneau). The journals *Structure* (1958-1964), *Leonardo* (1968-) and *Topologie Structurale/Structural Topology* (1979-), founded by Baljeu, Malina, and a group of mathematicians and architects in Canada, respectively, as well as the informal meetings of the *Philomorph Association* (Cambridge, Massachusetts) often discuss various aspects of symmetry. Without overstating the possible analogies it can be claimed that the interdisciplinary approach to the study and application of symmetry – even in a broad artistic-scientific context – is very useful.

## 2. FROM DISCRETE TO CONTINUOUS SYMMETRIES
**(Cycles of symmetry and play with numbers)**

A 35-year cyclicity can be observed in the interdisciplinary study of symmetry: 1917±3, 1952±3, 1987±3. All of these intervals can be characterized by a huge concentration of publications and events, or in other words, there was some "symmetry" (synchronicity) in the interests of artists, scientists, and engineers. If we also consider connections where only two close disciplines are involved, we may see these cycles even earlier. The period of 1847±3 was dominated by the application of geometric symmetries in crystallography and in chemistry by Bravais, Möbius, and Pasteur, respectively; the precise mathematical formulation of the conservation law of energy by Helmholtz (1847), where the main idea was also pioneered by Mayer and Joule; and the foundations of modern thermodynamics (Clausius, 1850), which led a bit later to the concept of entropy. The period of 1882±3 can be characterized by the beginning of the use of crystallographic symmetries in physics by P. Curie and Minnigerode, and the further synthesis of group theoretical and geometric ideas in the papers by Dyck, Poincaré, and the monograph by Klein (1884). Poincaré also contributed to the theory of dynamical systems, in which he presented the basic ideas of the modern chaos theory. The beginning and the end of this period are marked by two crucial geometric-crystallographic monographs authored by Sohncke (1879) and Fedorov (1885), paving the way for the independent enumeration of the 230 space groups by Fedorov himself and the mathematician Schoenflies. We may also refer to the scientific aesthetics of Henry (1885) which inspired the further studies of the connections between art and science, including the questions of symmetry. The cycles in the 20th century are connected with broadly interdisciplinary works on symmetry, although the same periods were also fruitful for specialized developments in concrete fields of art and science. In the analysis of each of the three peaks, the starting paragraph will refer to the wide-ranging interdisciplinary works, while the later ones will describe artistic and scientific achievements.

### (1) 1917±3

The central feature of this period is the first comprehensive, about 350-page, interdisciplinary monograph on symmetry by the Dutch chemist Jaeger (1917) and the monumental, more than 800-page, mathematical-biological survey on growth and form by the British scholar D'Arcy Thompson (1917). Both of them had a strong interest in the history of the subject; Thompson is also known as the translator of Aristotle's *Historia Animalium*. We may also mention the monograph of the somewhat controversial German biologist, philosopher, and artist Haeckel (1917) which focuses on "Symmetrismus" as a substance in the morphology of the organic and inorganic world; this work also pioneered some results about liquid crystals. The second edition of the popular scientific booklet on symmetry by the Russian crystallographer Vul'f (or Wulff, 1919) was also published in this period (the earlier first edition of 1908 was known only in a smaller circle). Further monographs on form and function, or order in nature, were released in these years (J. Loeb, 1916; E. Russell, 1916; L.J. Henderson, 1917). The famous German chemist Ostwald (1919) – who was a controversial figure in natural philosophy, promoting "energetism" against the theory of atoms – introduced a "symmetry principle" for colors, not in the later metaphorical sense of crystallography, but purely in connection with the visual aspects. Soon after completing his monographs on the harmony of colors, he turned to the problem of the harmony of forms, describing how to generate symmetric patterns. In the beginning of this period we have the monographs by Cook (1914), on spiral formation in nature and art, and by Bragdon (1915), on mathematical creation of symmetric ornaments by projecting 4-dimensional figures. At the end of the period the book by Church (1920) on phyllotaxis appeared. (The phyllotaxis, or leaf arrangement, is an interesting topic in botany which is connected with the Fibonacci numbers and the golden section.) The journal of Hambidge (only one volume appeared, in 1919-20) and its monographic version (1920) promoted his geometrical composition method called dynamic symmetry. It is based on a system of proportions, thus the term refers to the ancient artistic meaning of symmetry.

In the same period certain artistic movements were formed with a special interest in reunifying the artistic disciplines and connecting them with the aesthetical and practical questions of the modern machine age: *De Stijl* in the Netherlands (1917), the *Constructivists* in Russia (about 1917), and the *Bauhaus* in Germany (1919). All of them attracted international attention and had a strong interest in geometrical principles of design, including the questions of symmetry. It is represented not only indirectly by the abstract compositions of Mondrian, Malevich, and Rodchenko, the "constructions" of Gabo, Tatlin, or the "total architecture" of Gropius and his colleagues, but also by theoretical works (van Doesburg, Chernishev, Moholy-Nagy, and others). The first volume of the journal of the Dutch movement included a whole paper "Symmetrie en kultuur" by the architect Wils, the manifestoes and teaching programs of the movements in Russia and Germany also focused on the problem of symmetry and asymmetry (cf., such subjects as *Rhythm of Mass*, *Transition from Nature to Abstraction*, or *Formlehre*, respectively). Of course there were relevant developments also outside of these centers. Le Corbusier independently worked out his module-system called *Dom-ino* (1914-15) which later significantly contributed to mass-production of houses. The movement *purism* and their journal *L'Espirit Nouveau* in France – criticizing the decorative degeneracy of some cubist groups – emphasized the structural equilibrium of composition and the functional aesthetics of the machine age (Le Corbusier and Ozenfant, 1918). In a manifesto they even claimed that serious art, similar to science, is the expression of invariants. There was a relevant change in the understanding of symmetry by architects: while they often rejected symmetry in the sense of rigid bilateral structures (Gropius, the founder of the Bauhaus, called it "dead symmetry"), they gave new meanings to the concept by using more sophisticated equilibria in individual buildings, biological shapes in the "organic architecture", and carefully selected periodic arrangements in town planning. It is interesting to compare this evolution of the concept to the analogous symmetry principles in mechanics, biology, and crystallography. Although all of these movements came to an end in the early 1930s, both the Constructivists and the Bauhaus were forced to split due to the political circumstances, but the main ideas survived in the later activity of the staff and students at various schools of design and architecture around the world. Moholy-Nagy even tried to build a New Bauhaus in the United States, but it was closed after financial difficulties.

The idea of shaping a general science of signs appeared earlier in both logical and linguistic contexts (Peirce, Saussure). The posthumously published work by Saussure (1916) on general linguistics had an impact on the later structuralist and semiotic movements not only in his main field, linguistics, but also in a broader sphere. The Gestalt psychologists made important contributions in this period as well (cf., Köhler, 1917). Their search for rules of grouping figural components within the whole and their introduction the rule of symmetry had some analogy with the interest of artists and architects (the secondary name of the Bauhaus at Dessau also referred to "Gestalt", i.e., form, configuration, pattern). Other symmetry-related developments in psychology, although in an indirect sense, include Freud's studies on the distinction between conscious and unconscious, the contrast between life and death instincts, the questions of duality and doubles, as well as Jung's studies on psychological types. In the same years Schoenberg, with his students Berg and Webern, turned to atonal music; in 1917 he founded an educational association in Vienna. Schoenberg started his first experiments with the dodecaphonic method in 1920. Debussy's earlier music also eclipsed tonality by introducing many new chords, but he favored the impressionist-symbolist traditions – analogous to the movements in painting and poetry – rather than a mathematical methodology. His last works, however, use almost baroque structures (*Twelve Etudes*, 1915; *Three Sonatas*, 1915-1917). Bartók always emphasized the tonal tradition, collecting, together with Kodály, folksongs and classifying the variants of a melody. It later led to his variation technique and preference for symmetrical forms. Wittgenstein started to work on his *Tractatus Logico-Philosophicus* about 1914 and finished it as a prisoner of war in 1918 (although it was not published until 1921). His view that philosophy is basically the analysis of language led to many progressive discussions and contributed to the formation of the *Vienna Circle* (Carnap, Schlick, and others). This philosophical school of logical positivism, also influenced by Mach and B. Russell, had an impact on later interdisciplinary approaches (unified science, semiotics, etc.). In the same period the anthropologists showed increasing interest in the life of native peoples of distant places. Although these studies emphasized the social context, they also led to systematic studies of symmetry as a formal element of decorative art (Boas). The new wave of art historians (cf., Panofsky, 1915) rediscovered and studied the mathematically oriented theories of such eminent masters of the Renaissance as Dürer and Leonardo. There were even "Isisological" events in this period. The first specialized science-historic journal, *Isis* (not ISIS!), was founded by Sarton (1913-14 in Belgium, restarted in 1919, and later moved to the United States), which contributed significantly to the shaping of this new scholarly field. The broader scope of this journal covers the cultural influences of science as well, thus some symmetry-related articles are also available there. At the very end of the period, during Christmas 1920, R. Steiner delivered a series of lectures entitled *Die Suche nach der neuen Isis* (Search for the New Isis). His anthroposophical movement, uniting artistic, scientific, and religious ideas from both East and West, influenced a large number of intellectuals. The anthroposophists also studied symmetry-related questions, from Goethe's morphology to the eurhythmic expression of music. Last, but not least, the impressive culture-historic monograph by Spengler was finished in 1917 (and published in 1918). In it he suggested a "cultural morphology" and emphasized the notion of cyclic development.

The same period belongs to the "golden age" of physics. It produced general theories and yielded special findings related to symmetry. There is no doubt that Einstein's (1916) general theory of relativity was a major achievement; it was discussed not only in mathematical-physical monographs (e.g., Weyl, 1918; Eddington, 1920; Pauli, 1921), but it made an impact in broad circles of cultural life. Philosophers, artists, and even the general public became attracted by various aspects of the new physical model, as it is reflected by very many works from artistic manifestoes to survey papers. True, this popularity also led to oversimplifications and misinterpretations. Einstein's earlier special theory of relativity (1905) postulated that all physical laws remain unchanged moving from one inertial system to another one (relativistic invariance, i.e., the homogeneity and isotropy of space-time). In the general theory he extended the idea on invariance to non-inertial, accelerating systems. It led him to a gravitational field theory where space-time is curved by the presence of matter (mass). The astrophysical consequences of the theory were discussed by Eddington, who also provided the first experimental evidence for it: he observed, during a solar eclipse in 1919, a small curvature of light rays near the sun by detecting some changes in the normal position of stars. The astronomical verification of Einstein's theory by the British scholar Eddington also had an emotional importance: after the horrors of World War I it became a symbol of international cooperation. The general theory of relativity had a further impact on the studies of the universe and the related questions of symmetry and asymmetry; for example, the far reaching theory of the expansion of the universe – the basis of the big bang theory – was formulated in the late 1920s (Hubble). The findings in mathematical physics, less known to the general public, should also be emphasized. Noether's (1918) theorem on the connection of symmetries (or invariances) and conservation laws, Weyl's (1919) paper on gauge symmetry for a unified field theory, and Kaluza's (1919-21) mathematical model giving a unified picture of the gravitational and electromagnetic fields pioneered ideas that became central in later developments of particle physics. It is quite interesting that the declarations of the importance of invariances in mathematical physics and in visual arts by Noether and Le Corbusier, respectively, happened almost at the same time. The period was also fruitful in other achievements. At the very beginning, the new atomic model of Bohr (1913) had a tremendous impact; it is based on an electron circling the proton, similar to the motion of a planet around the Sun. These studies were later continued (Bohr, Sommerfeld), until the remaining problems led in the mid 1920s to quantum mechanics, without deterministic positions of electrons, but allowing statistical distributions. A pioneer of this new approach, Heisenberg, also had a special interest in symmetry; this dates back to 1919 when he, following his cited already reading of Plato, turned to mathematical-physical models, and later to the view that "in the beginning there was symmetry" (cf., the electron-positron duality predicted by the Dirac equation in 1928). The works of Julia (1918) and Fatou (1919, 1920) are the forerunners of the modern fractal theory (cf., Julia sets). Also in this period, X-ray crystallography was shaped into a new field following a series of papers and a monograph by the two Braggs (1915). The list of all the possible symmetries in ideal crystal-structures, Fedorov and Schoenflies's 230 space groups (1890-91), became increasingly important in the classification of the observed structures. Fedorov himself led a group which provided tables for crystal-chemical analysis; this monumental volume of more than 1,000 pages was published only posthumously soon after his death (Fedorov, 1920). These developments had an impact on geometric crystallography (Niggli, 1919) and on related questions in pure mathematics, inspired also by Hilbert's famous mathematical problems (Problem 18, 1900). The results in the mathematical theory of tilings, or tessellations, were summarized in a monograph (Reinhardt, 1918, originally a dissertation). This topic was also in the focus of the old and new generations of Russian crystallographers (both Fedorov and Shubnikov contributed to it in 1916). In geology Wegener's (1915) monographic summary of the continental drift theory, based on both the jigsaw fit of continents and evidence from fossils, was a major achievement; it led later to the modern theory of plate tectonics.

**(2) 1952±3**

The novelty of the second peak is the broad international interest in the general topic of symmetry. In 1952 five interdisciplinary monographs were published in five different languages. The book *Symmetry* by Weyl (German-American mathematician) soon became a classic in both artistic and scientific circles, and is now available in more than ten languages. The monograph by K.L. Wolf (German physical chemist) and D. Kuhn (German biologist) *Gestalt und Symmetrie* was later also translated into Spanish as *Forma y simetría*. The list of further books of 1952 include a very comprehensive study on symmetry in nature and decorative art by Jaśkowski (Polish logician and mathematician), and a work on dynamic symmetry and its artistic-scientific context in classical architecture by Bairati (Italian historian of architecture), while a publishing house reprinted, with a new commentary, the previously cited book by Vul'f (Russian crystallographer). We may also mention here the 1952 edition of Speiser's (Swiss mathematician) book on the mathematical way of thinking, which discusses, among other things, musical and ornamental symmetries. This year is also a landmark in studying symmetric or asymmetric patterns in biology, and their morphogenesis, see the monographs by Portmann (Swiss zoologist), Wardlaw (British botanist), and the model by Turing (British mathematician and a pioneer of computing), as well as in 1951 an entropic approach to evolution (H.S. Blum), and a new interest in phyllotaxis (Richards). An important reference book on crystallography, the international tables of symmetry groups, was also published in 1952 (edited by Henry and Londsdale). Ghyka (Rumanian-French diplomat, engineer, aesthetician) continued his works in mathematical aesthetics, releasing three books in 1952: a practical handbook on geometrical composition, a survey on the philosophy and mystique of numbers, and finally a new edition of his essays on rhythm. There is no doubt that Ghyka was a very influential figure among artists, although he was criticized for overstating the importance of proportions and numbers in art. Max Bill (Swiss artist) discussed the main trends of form in design in an unusual trilingual book of 1952. Lurçat (French architect) presented a three-volume monograph in 1953 with the aim of developing a scientific aesthetics and a law of harmony for architectural composition. The pioneer of symmetry analysis of ornamental art, Shepard (American geologist and archaeologist), published her two related monographs "symmetrically" just before and just after this period (1948, 1956). In the very beginning of the period we have the first interdisciplinary collection of essays on symmetry as a special issue of a German journal (*Studium Generale*, 1949). Nicolle (French biochemist) published his first comprehensive book on symmetry in 1950, while a second one came out in 1955. Shubnikov (Russian crystallographer) wrote a monograph on black-and-white symmetries in 1951, which inspired many further investigations, especially in the Soviet Union. Among others, Zamorzaev (1953) and Belov with coauthors (1955) enumerated, independently, all the possible 1651 black-and-white analogues of the 230 space groups. This approach is very useful in considering the different physical properties (e.g., magnetism or spin), marked by black and white, of the geometrically equivalent units of crystal-structures. It is easy to understand the concept by an analogy to an infinitely large chess board, where we may consider both conventional color-preserving symmetries and color-changing antisymmetries. The schools of Shubnikov and Koptsik, Belov, and Zamorzaev dominated the decades that followed by discussing colored and other generalized symmetry groups, while Landau and Lifshits [Lifshitz] (1951), in their series of textbooks, independently approached the black-and-white symmetries by considering magnetic structures in solid state physics. The pioneers of these studies emphasized the artistic dimension of the colored symmetries as well, illustrating them with Escher's drawings or creating their own patterns. The study of curvilinear symmetry, where the axis of bilateral symmetry is curved, was inspired by Nalivkin (1951); it is useful in biology, geology, and even in art. This period was also important for the "rebirth" of classical geometry, a somewhat neglected field of mathematics, which inspired new interdisciplinary connections. Steinhaus (Polish mathematician) popularized in 1950 a large number of symmetry-related geometric problems in the extended new edition of his best-selling book in English which also appeared in other languages. Cundy and Rollett (British mathematicians and educators) in 1951 described many geometrical models from tilings to polyhedra, influencing generations of teachers, as well as artists and engineers. Fejes Tóth (Hungarian mathematician) in a scholarly monograph in German (1953) discussed the genetics of regular arrangement of figures by geometric optimum principles. This new field of geometry was also pioneered by the schools of Coxeter (Canada), whose very influential monograph on regular polytopes was published shortly earlier, as well as of Delaunay (or Delone, Soviet Union), and Erdős (almost continuously travelling around the world with short breaks in Hungary). Discrete and combinatorial geometry gained interest among crystallographers, morphologists and structural engineers as well. Heesch (German mathematician), who earlier pioneered the study of black-and-white symmetries, resumed his systematic work on tilings, which led later to two monographs. Bense (German mathematician and philosopher) turned to aesthetic questions, starting a series of monographs in 1954, where he introduced exact methods to study of art works (cf., information theoretic aesthetics). The interest of Zwicky (Swiss-American astronomer) in morphological questions was marked by his first papers in early 1950s; it later led to his "morphological astronomy" and the founding of the Society for Morphological Research with a broader scope in science and technology. This period brought the flowering of various holistic movements, including the cybernetics (Wiener, 1948, 1950), general systems theory (the Society for General Systems Research was founded in 1954 by Bertalanffy and others), and neo-positivism with the new version of the *International Encyclopedia of Unified Science* (resumed by Neurath, Carnap, and Morris in 1955); these approaches turned out, however, to be too general and declined after some successful contributions.

In the arts there was a rebirth of some ideas of the constructivist movements and the Bauhaus, with the goals of furthering industrial design in the new age of mass production. The Hochschule für Gestaltung of Ulm, Germany (or the Ulm School of Design) – its German name taken from the subtitle of the Bauhaus in Dessau – was shaped in this period: the financial support was raised by German intellectuals (Schnoll, Aicher) with American contribution in 1949; the teaching started, under the leadership of the Bauhaus-educated Max Bill, in 1953; finally the new campus was officially inaugurated in late 1955. The institution became a meeting point not only for artists, but also for scientists. In 1955 the Foundation Course was reorganized by Maldonado (Argentine artist and designer), who introduced new units on symmetry, topology, Gestalt psychology, ergonomics, and visual semantics. The list of guest professors included scholars with strong interest in symmetry-related questions, among others, the previously cited chemist K.L. Wolf, who published his second major work on symmetry, together with R. Wolff, in 1956 (it is a unique two-volume set where the second one includes only figures); the philosopher-mathematician Bense, who created a program in information theory and semiotics for the School (later Moles gave similar courses); as well as the mathematician Baravalle, also known by his works on the pentagram and the golden section (1950), and on perspective (1952). Although the school closed in 1968, its influence has continued largely through its graduates, not only in Europe, but also in Northern America, Brazil, Japan and India. Some of the former faculty members contributed to the artistic-scientific topic of symmetry as well (the American architect Huff published a whole series of booklets). While the Bauhaus helped to shape a new aesthetics after World War I, the post-constructivists played an analogous role after World War II. In addition to the Ulm School of Design there were many related groups. Thus, constructivism gained a new importance in the United States (Biederman), in England (Hill, Martin), and other countries. One of the leading architects of the modern period, Le Corbusier, applied in practice his "modulor", a proportional system of the human body based on the golden section (Unités d'habitation at Marseille, 1947-52). The architectural-ergonomical composition principles of Le Corbusier focus not only on buildings and their environments, but also on the internal space designed for various activities; they shape indirectly the everyday life of the inhabitants. He also continued the theoretical work on the modulor publishing a second volume about it (1955). True, the free association with geometrical forms led to a rather sculptural architecture where the functionalism is overshadowed by visual principles (Chapel at Ronchamp, 1953-55; Punjab's new capital, Chandigarh, 1951-56). Another important figure in modern architecture, F.L. Wright, summarized his principles on organic architecture and its future in comprehensive monographs. This movement adapted various biological shapes and their symmetries.

In 1951 there were two interesting artistic events, which are connected with symmetry in a broader sense: the *First International Congress on Proportion in Arts*, organized during the Triennial Exhibition of Milan, where many leading artists and scientists came together (unfortunately we are not aware of any later congresses of this series), and the exhibition *Growth and Form* at the Institute of Contemporary Arts in London – the title refers to D'Arcy Thompson's book – which was accompanied by a collection of essays by leading art historians and biologists (edited by Whyte, 1951, see also Whyte, 1954; both books were later translated into Japanese). The topic of *Measurement of Growth and Form* was also discussed by a Royal Society Symposium in London (organized by Zuckerman, 1950). A synthesis of art history and psychology – extending the Gestalt tradition – was presented in a monograph by Arnheim (1954). It is interesting to compare this approach to the later one of Gombrich: both of them deal extensively with the question of symmetry versus asymmetry, but from different standpoints; see also the influential monograph by Gibson (1950) on visual perception including the aspect of invariance. The Dutch graphic artist Escher, whose name is often associated with symmetry, turned more explicitly to mathematical topics in this period (cf., *Double Planetoid*, 1949; *Rippled Surface*, 1950; *House of Stairs*, 1951; *Cubic Space Division*, 1952; *Gravity*, 1952; *Order and Chaos*, 1952; *Relativity*, 1953; *Spirals*, 1953; *Tetrahedral Planetoid*, 1954; *Convex and Concave*, 1955). It is in some sense a transitional period between his earlier interest in periodic drawings and the later one dominated by non-Euclidean patterns and impossible objects. The Hungarian-born American artist Kepes, who was earlier associated with the Bauhaus movement, also started to build new connections between art and science when he contributed 32 paintings, drawings, and photographs to the journal *Scientific American* (1948-1954). His exhibition, *The New Landscape in Art and Science* (1951; book version, 1956), and an article about scale, structure, and rhythm (1951) prefigured the direction which later led to the editing of the collection *Module, Proportion, Symmetry, Rhythm* and six other interdisciplinary books – containing contributions by leading artists and scientists – of the series *Vision + Value* (1965-1972). The American composer Cage organized in 1951 a group of cooperating musicians and engineers (taped music). Earlier his music had been strongly influenced by Balinese and other eastern rhythms (even the graphic symmetries of his scores are interesting), then he became involved in Zen Buddhism, resulting in 1952 in the famous silent piece (*4' 33"*). The French composer Messiaen, one of the most influential teachers in the 20th century, also studied the differences between oriental and western rhythms and summarized them in a monograph (1954). This period also led to a second climax of Schoenberg's rigidly symmetric serial technique among the new generation of composers, although many of them soon turned to other methods. Both Boulez and Stockhausen became involved in the "total serial" music applying the basic idea for both rhythmic and dynamic values (*Structures*, 1952; *Crossplay*, 1951; respectively). It was followed in the case of Boulez by more flexibility and an interest in African and Asian percussion rhythms (*The Hammer Unmastered*, 1955), while Stockhausen turned in 1952 to electronic music, searching new principles of order (*Counterpoint*, 1953; *Studies*, 1954). In the same period Xenakis criticized the serial technique and adapted different mathematical and architectural ideas for composition (stochastic music, symbolic music; cf., *Metastasis*, 1954). Boulez and Cage also used stochastic elements. Hindemith turned to Kepler's geometrically motivated "music of the spheres", adapting it to the modern style (*Harmony of the World*, 1951). The possible influence of natural symmetries in Bartók's music, especially of phyllotaxis and the golden section, as organizing principles in the time scale was investigated by a Hungarian musicologist (Lendvai, 1955).

Both the beginning and the end of the period are marked by crucial monographs by Lévi-Strauss (1949, 1955); the first one deals with the structures of kinship systems, while the second one gives his intellectual autobiography. The whole period significantly contributed to the shaping of structural anthropology. The psychologist Piaget also became interested in structuralism; his major contribution to the subject of the acquisition of scientific concepts by children was crystallized in three monographs in 1952. He also adapted the concept of algebraic group to his analysis (1954). The period was also fruitful for the study of symmetry in the context of the history of science. Indeed, some leading crystallographers understood the importance of the annotated editing of classical works, which resulted in a series of volumes in Russian with basic papers by the pioneers of symmetry (edited by Frank-Kamenetskii, Shafranovskii, Shubnikov). At first the major works of Fedorov were published (1949, 1953), then papers of other scholars – from Bravais to P. Curie – were collected in additional volumes. The more recent English translation of Fedorov's selected papers is based on the first book. The broader series *Classics of Science* has many other volumes, including a very comprehensive four-volume collection of Einstein's papers. In the German-speaking countries, Burckhardt, a leading expert of mathematical crystallography, started similar historic works. Bernal in Britain and C.S. Smith in the United States combined the interest in material sciences and symmetry with historic publications; both of them published monographs in 1951-53 contributing to both science and the humanities. Last, but not least, we should also refer to Sarton in the United States, the "father" of the history of science as an independent discipline. He organized the whole infrastructure of the history of science, from the journal *Isis* to a learned society, from publishing critical bibliographies to organizing regular symposia. Two monographs of his late period were published in 1952: a general guide to the history of science and a detailed study of ancient science with some interesting remarks on symmetry. In the same period Toynbee continued – after a long break since 1939 – his study of the cyclic development of history, identifying 21 civilizations (Volumes 7-10 were published in 1954, later he added further volumes to this series). We should also emphasize the cooperation of a leading psychologist and of a physicist, Jung and Pauli (1952), which resulted in a joint monograph on synchronicity and archetypical thinking. Jung became involved in symmetry through his interest in oriental symbols and the meaning of their structures, as well as through studies of the "Gestaltung" of the unconscious (1950). In the mentioned joint monograph. Pauli focused on Kepler's astronomical work, which is an early example of the use of mathematical models based on symmetry.

This period was the "golden age" of biological science. The most important result was the discovery of the spatial structure of DNA, where two screw-symmetric chains go around each other, similar to a spiral stair (double helix). The breakthrough was made in 1953 as a result of the cooperation of molecular biologists (Crick and Watson) with X-ray crystallographers (Franklin and Wilkins). The description of the structure of DNA – the carrier of genetic information in all living organisms, except viruses – significantly helped to visualize its replication process: to open the "zipper" of the two chains, then to synthesize the opposite half to each. The same period was also crucial in developing a broader approach to information processing, namely its understanding – beyond genetics – in mathematics, computing, engineering, biology, psychology, and linguistics, see the new fields of information theory (Shannon and Weaver, 1949), the completion of the first stored-program computers (EDSAC, 1947-49, modelled after the EDVAC, 1944-51), the idea of artificial intelligence (Turing, 1950), the shaping of a general theory of automata (von Neumann, 1952-1953), and in general the theoretical research into brain-like computers, with some overstatements (Ashby, McCulloch, von Neumann, Wiener, and others). There were important developments in structural linguistics (Harris, 1951; Jakobson, 1952), which lead to the birth of transformational-generative grammar (Chomsky's dissertation on transformational analysis and the pre-print book on the logical structure of linguistic theory were completed in 1955, while his famous monographs on syntactic structures came out in 1957). Structural linguistics inspired formal approaches not only in computing, but also in the humanities; these structuralist studies often analyze patterns in the framework of symmetry versus asymmetry. Interestingly, 1951-52 can also be considered as a "golden age" in computing, when, following the ENIAC and EDVAC projects started during Word War II, many vacuum-tube computers were completed (IAS Machine, IBM 701, MANIAC, ORDVAC, etc.), including the first machine designed for commercial use (UNIVAC). Moreover, the next two generations of computers were also foreshadowed in this period by the first multimillion-dollar contracts for the commercial manufacture of transistors, and by the very first idea of the possibility of integrated circuits (it was not realized, however, until 1959). These are clearly the first steps towards the information revolution where performance of intellectual work is done by machines on a large scale. Adding to the two basic concepts, matter and energy, a third one, information (the measure of form, according to Weizsäcker), had a great importance in both theory and practice. While the conservation laws of matter and energy help the "bookkeeping" in connection with the first two concepts, no analogous conservation law for information is available. Preserving the information relevant to us is a special task. There are, however, more specific connections between the early history of computing and the topic of symmetry than these speculative remarks on conservation. Although the connections are indirect, they are more important than is commonly believed. The analogy between living organisms and machines played a crucial role in the first period (homeostatic systems, self-reproducing machines, neural networks, etc.). The Turing test for machine intelligence compares the answers of two sides, a machine and a human being; if no difference can be detected, the machine will be considered intelligent. Binary arithmetic and the design of solid state electronics also have some connection with symmetry in a general sense. The first applications of computers attempted to provide numerical solutions to non-linear problems, including some military-related ones (ballistic curves, shock waves), but very soon other fields were also considered (meteorological prediction). In these problems, the symmetry considerations of the classical linear models are often replaced by more sophisticated initial conditions. For example, the ideal ballistic curve is a symmetric parabola, but now the emphasis is on the variations from this curve in the real practice. Interestingly, two leading pioneers of computing were strongly involved in symmetry-related questions at the same time: Turing worked on his morphogenetic model for predicting symmetric shapes in biology, while von Neumann dealt with self-reproducing cellular automata and pattern generation. It is not surprising that the recognition of symmetry, also investigated in animal psychology, became an important question in the theory of Turing machines and later in artificial intelligence. Last but not least, the developments in computing made it possible to analyze some formal aspects of art and inspired the birth of computer art. Computer graphics is very useful in providing symmetric patterns with a high level of precision.

There were basic results in physics, too. In the late 1940s the shaping of quantum electrodynamics gained a special attention, which was stimulated also by the experiments made with microwave techniques during World War II. The mathematical equivalence of the theoretical works of three scholars working independently (Tomonaga, Schwinger, and Feynman) was demonstrated in 1949 (Dyson). The Jahn-Teller distortion, playing an important role in quantum chemistry, was predicted in the 1930s, but it was not observed experimentally until 1952. Actually, some degenerate orbital states of electrons in molecules or crystals are connected with special symmetries of the whole system, therefore the degeneracy can be removed by a small distortion of this symmetry. Very many important discoveries were made in the early 1950s in particle physics. The first modern particle accelerator, the Brookhaven Cosmotron, began operating in 1952, giving experimental support to the theoretical investigations. In the same time Wigner, who pioneered the application of continuous symmetry groups to quantum mechanics in the 1930s, focused on the generalization of symmetry and conservation laws for the new circumstances. The conservation of various properties of particles, described by quantum numbers, were considered by him and other physicists. An interesting step was the introduction of "strangeness" of particles, which is conserved in some interactions (strong and electromagnetic ones), but not in the weak interaction (Gell-Mann, Nishijima, 1953). The fundamental symmetry law on a "universal mirror" with combined components was formulated in 1953-1955 (CPT theorem, i.e., combined conservation of charge conjugation, parity, and time reversal, by Schwinger, Lüders, Pauli). Other symmetry-related results include further developments in connection with the gauge symmetry (Salam and Ward, 1950-51; Yang and Mills, 1954), and the first observation of an antiproton (1955). Shortly afterwards the most surprising symmetry violation (non-conservation of P, parity) was announced – following the theory of Lee and Yang and the experiments of Wu (1956) – in the case of a concrete process during weak interaction (beta decay of cobalt). It means that nature, similar to our everyday practice, does distinguish between left and right (moreover the combined CPT-conservation cannot be split into conservation laws of the individual components, considering just P-conservation). Although there were some preliminary remarks about what could happen if the corresponding symmetry is violated (Feynman, Wigner), but almost nobody believed that it was a real possibility. The success of physicists from the East, including China, Japan, India, Pakistan, is notable. Although many of them moved to western countries, taking the advantage of the scientific infrastructure there, the cultural-philosophical influence of the oriental thinking cannot be excluded.

**(3) 1987±3**

In the case of this peak, the central year has no special importance, but rather the whole period is highlighted by a series of events. Namely, from 1984 we witness many art-science related symposia, including *Shaping Space*, Northampton, Massachusetts, 1984 (organized by Senechal and Fleck); *M.C. Escher: Art and Science*, Rome, 1985 (Emmer); *Science on Form*, Tsukuba, 1985 (Ishizaka et al.); *Symmetrie*, Darmstadt, 1986 (Wille et al.); *Symmetry in a Cultural Context 1 and 2*, Tempe, Arizona, 1987 and 1988 (Nagy); *Symmetry - Asymmetry*, Bombay, 1988 (Ramanna et al.); *Symmetry of Structure*, Budapest, 1989 (Darvas and Nagy); as well as other interdisciplinary events inside the scope of science, for example, *Spontaneous Symmetry Breakdown*, Karpacz, Poland, 1984 (Michel, Mozrzymas, Pękalski); *Symmetries in Science, 2, 3, 4*, Carbondale, Illinois, 1986 and Lochau, Austria, 1988 and 1989 (Gruber et al.); *Crystal Symmetries*, Moscow, 1987 (Vainshtein); *Symmetries and Nonlinear Phenomena*, Paipa, Colombia, 1988 (Levi and Winternitz). We may mention also a science-historic symposium shortly before the considered period: *Symmetries in Physics (1600-1980)*, Sant Feliu de Guíxols, Catalonia, 1983 (Doncel); its proceedings came out in 1987. In the Soviet Union the *Fedorov Sessions* in Leningrad (mid-May of each year) provide a regular forum for symmetries in material sciences and related questions (organized by Shafranovskii, Frank-Kamenetskii et al.). Some symposia were accompanied by symmetry exhibitions in distinguished galleries: Mathildenhöhe, Darmstadt, 1986 (organized by Krimmel), Hungarian National Gallery, Budapest, 1989 (Beke). In other cases mathematicians became guest curators of Escher exhibitions (Emmer in Rome, 1985; Schattschneider in Betlehem, Pennsylvania, 1987). We have already mentioned the popularity of art-science symposia in Japan (Sakane). This worldwide high "density" of symmetry-related interdisciplinary events is new, while in crystallography and in particle physics there are regular forums. (We know of only two earlier symposia where both the artistic and scientific aspects of symmetry were featured: Northampton, Massachusetts, 1973; Vienna, 1978; as well as three other ones where the scope was very broad: Rome, 1969; Venice, 1970; Leningrad, 1971. Interestingly, these events are condensed around half-way between 1952 and 1987, that is, 1969-1970). In the period of 1987±3 about thirty interdisciplinary monographs and five collections of essays were published specifically on symmetry in English, French, German, and Russian (the latter were edited by Hargittai twice, Noël, Stork, Tyukhtin and Urmantsev, respectively). This rich set of books includes such unique volumes as W. Hahn's monograph on symmetry as a developmental principle in nature and art (more than 800 illustrations), Mainzer's handbook on symmetries of nature (more than 750 pages), Washburn and Crowe's monograph on the symmetries of patterns in various cultures (about 550 illustrations), and the two collections of essays entitled *Symmetry: Unifying Human Understanding*, edited by Hargittai (each more than 1,000 pages). Another symmetry-related milestone is Grünbaum and Shephard's monograph on tilings and patterns, which combines a detailed critical survey of the past with new research of the field (more than 700 pages). The geographic diversity of the interest can be impressively demonstrated by an educational pamphlet on symmetry published in Kiribati, formerly Gilbert Islands, Oceania (edited by Barry, 1989). At the end of the period we have the birth of ISIS-Symmetry (this final name was suggested by the German mathematician Dress) and the idea of interdisciplinary journals on symmetry. As an interesting synchronicity it crystallized independently at three places in 1989-1990: *Symmetry: Culture and Science* (Budapest: ISIS-Symmetry); *Symmetry: An Interdisciplinary and International Journal* (New York: VCH Publishers, a subsidiary of VCH [Verlag Chemie], Weinheim; it was terminated after the first issue in 1990, but some of the planned special issues will still be published as individual books edited by Hargittai); *Tetrahedron: Asymmetry, The International Journal for Rapid Publication* *on all Aspects of Asymmetry in Organic, Inorganic, Organometallic, Physical and Bio-organic Chemistry* (Oxford: Pergamon Press). We may mention another periodical entitled *Chirality* (New York: Liss; this term suggested by Lord Kelvin refers to the left-right handedness of molecules and other objects, and it is equivalent to Pasteur's dissymmetry). There were interesting developments not only in the Western world, but also in the East. The Society for Science on Form was founded in Japan in 1984. Their journal *Science on Form*, more recently renamed as *Forma*, is published semiannually from 1985. Although the central year of the period, 1987, has no special emphasis, but it has some importance to us: the informal network of the first Arizona workshop on symmetry set off the events which led to the founding of ISIS-Symmetry. The *Symmetry of Structure (First Interdisciplinary Symmetry Symposium and Exhibition)* in 1989 – probably the first international artistic-scientific conference on the topic in English with a call for papers – attracted about 200 participants from 26 countries and all the five continents, while most of the cited symposia only had invited speakers. There is no doubt, however, that the *Projekt Symmetrie*, organized jointly by the City of Darmstadt and the Technische Hochschule Darmstadt in 1986, had a broader scale of programs; it can hardly ever be surpassed. The planned triennial ISIS-Symmetry Symposia, initiated by the 1989 event, focus on the relationships between art and science, while the symposia *Symmetries in Science* (organized by B. Gruber et al.) emphasize the problems in mathematical-physical sciences with occasional papers on related chemical or biological questions. These two forums may supplement (not substitute for!) each other, as well as other disciplinary meetings.

This period can also be characterized by the information- technical revolution. While around 1952 we saw a few big computers built individually for some rich institutions, between 1984 and 1990 the personal computer became an everyday tool. In the beginning the commercial PCs were rare and very expensive, but towards the end of this period we witnessed ever cheaper and more widely adopted devices with software used throughout science and culture. The proliferation of personal computers also has consequences for the topic of symmetry: very many programs were developed to help specialized fields, from symmetry analysis of crystals to modelling the asymmetries of the universe. There are programs for teaching various aspects of symmetry. Computers are commonly used to create patterns in artistic-scientific contexts; see the achievements from simple drawings to computer-aided town planning, from musical experiments to computerized textile design. It is interesting to see that the problem of tilings served as a dissertation topic in the first peak, it became on object of further classification problems in the second one, while in the last one, beyond the comprehensive monograph referred to (Grünbaum and Shephard), it simply "married" computing. See, for example, the following software: *RepTiles*, *FunTiles* (Dress, Huson, programs by Delgado, and Westphal), the *Alhambra Program* (Fleury), and the recent study of Truchet's 18th century work (C.S. Smith, programs by M. Wilson). The popularity of the dynamics of non-linear phenomena – the theories of catastrophe, chaos, fractals – and the related broken symmetries are also connected with the computers. Many public-domain fractal programs have been written for virtually every type of PC, making it possible for the general public to explore and enjoy the visual phenomena. Indeed, the aesthetic aspects of the new developments also gained some popularity: the Penrose tilings with pentagonal symmetry were adapted not only by researchers of the quasicrystals (Mackay, Ogawa, and others), but also by artists (Fukao, Robbin). The "beauty of fractals" and related phenomena (Mandelbrot, more recently Peitgen and Richter, Pickover, Vicsek, and others) also attracted the artistic community, including composers of digital music (Pimenta). The symmetry considerations helped not only the popular side of computing, but also advanced research. For example, to design better chips, to find new architectures for devices (neural networks, neurocomputing), and to make more sophisticated programs for Computer Aided Design and Manufacture (CAD/CAM). While in the first peak the anthropologists, and in the second one the composers, turned to the symmetries and rhythms of Asian and African civilizations, now the specialists in mathematical education discovered similar sources: the "ethnomathematics" (the expression is coined by D'Ambrosio of Brazil) uses examples from African drawings to Polynesian games, giving a broader cultural context to the subject. A large number of symmetry-related educational materials were published (Dale Seymour, Tarquin Publications). Recreational-mathematical puzzles and toys – many are connected with symmetry principles – gained a new importance in this period; earlier the Rubik cube dominated the market. The Oxford University Press started a series of books entitled *Recreations in Mathematics* (edited by Singmaster), while the new AHA Gallery and Shop in Zürich specializes in mathematical – or even symmetrical – toys.

The first observation of the fivefold symmetric quasicrystal in 1984 (Shechtman et al.) – a symmetry which is theoretically excluded in ideal crystal-structures – was initially a surprise for material scientists, but soon became a new field of interest. Very quickly there appeared a large number of publications on this subject, including a monograph (Henley, 1987) and twelve collections of papers and proceedings (edited by Gratias and Michel, 1986; Kuo, 1987; Steinhardt and Ostlund, 1987; Amann et al., 1988; Janot and Dubois, 1988; Jarić, 1988; Jarić, 1989; Jarić and Gratias, 1989; Fujiwara and Ogawa, 1990; Hargittai, 1990; Jarić and Lundquist, 1990; Yacamân, et al., 1990 – five of these books were published in Singapore). In the same period the International Union of Crystallography reached a new milestone in the classical part of the field by publishing the new version of the *International Tables for Crystallography: Space-Group Symmetry*. It was a common effort of an international team (led by T. Hahn). The first edition of this monumental work was published in 1983, but in our period there were many further developments (2nd revised edition, 1987; brief teaching editions, 1985, 1988). There are comprehensive monographic surveys of such related fields as axiomatic mathematical crystallography (Galiulin, 1984; Engel, 1986; see also the series of articles of Fischer and Koch), the topology of tessellations (Montesinos, 1987), the mathematical history of the golden section (Herz-Fischler, 1987), sphere packings (Conway and Sloane, 1988), lattice points (Erdős, Gruber, and Hammer, 1989). The recent research of the mathematics of plant growth and phyllotaxis – this topic figured in both earlier peaks – led to a new monograph and the first symposium in the field (Jean, 1984; 1985). The art and science of polyhedra was represented by both monographs (Miyazaki, 1986; Coffin, 1990) and the proceedings of a symposium (Senechal and Fleck, 1988). The Synergetics Institute, the Japanese Division of the Buckminster Fuller Institute (founded in 1988 by Kajikawa), designed various models of dynamic, or flexible, polyhedra.

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Without overstating the described cyclic peaks, the international synchronicity of the events is quite interesting; even Spengler's and Toynbee's monographs on cyclic development, as well as Jung and Pauli's work on synchronicity are quite fitting. Of course we would not claim that all the major achievements happened in the indicated three periods. The peaks were identified by milestones of the interdisciplinary study, that is, the additional remarks on artistic works and disciplinary findings do not represent a careful selection from the whole history, but only a coincidence with the corresponding peaks. Interestingly, in physics, biology, partly in mathematics, computing, and crystallography, as well as in design, and music we could see in those short periods a large number of crucial events, while the disciplinary results in chemistry, which were summarized in the first chapter, had much less synchronicity with the interdisciplinary peaks. The survey is, of course, incomplete even in the case of interdisciplinary works. Thus, such relevant monographs on symmetry in science and art as Vul'f's (1908), Shubnikov's (1940), Shubnikov and Koptsik's (1972, 1974) in the Russian cultural sphere, as well as J. Rosen's (1975), Lockwood and Macmillan's (1978), and some others are missing. Focusing now on the Russian achievements, we may observe interesting synchronicities. About 1905 there were various cultural activities in connection with symmetry, including discussions by scientists and artists, lecture series (which led to the publishing of Vul'f's book), and the preparation of the Russian version of Haeckel's monumental book on the "beauty of form" (originally *Kunstformen der Natur*). These studies may have provided a good artistic-scientific background to the constructivist movements. In the late 1930s architects started a series of books on proportion and symmetry where they translated and commented on foreign works (Ghyka, Hambidge, Mössel) and published their own research (Brunov). The appearance of Shubnikov's very influential book on the broadest aspects of symmetry in science and art in 1940 coincided with the publishing of Vernadskii's work on biogeochemistry, who also dealt with the problem of symmetry as a naturalist and philosopher. About 1975 many interdisciplinary books and collections of papers were published on symmetry besides Shubnikov and Koptsik's unique monograph (Vizgin, 1972; Urmantsev, 1974; Dmitriev, 1976; Dobrokhotova and Bragina, 1977; Galaktionov, 1978; Ivanov, 1978; Kompaneets, 1978; Zamorzaev et al., 1978; see also the collections edited by Porshnov, 1976; Kedrov and Ovchinnikov, 1978). In this period there was a lively activity in symmetry in geology emphasizing its connection with crystallographic, biological, and other aspects (Shafranovskii and Plotnikov, 1975; Leningrad Symposium organized by Plotnikov et al., 1976; see also the posthumously published work by Vernadskii, 1975). It seems that the Russian cultural sphere also has a 35-year cycle, but with a different starting point: 1905±3, 1940±3 and 1975±3. Here the preliminaries include in 1870±3 Gadolin's (1867) work on the crystallographic systems and Mendeleev's (1869) periodic table.

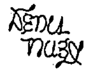
All of the peaks of interdisciplinary findings were preceded by corresponding disciplinary works, a necessary condition for a new synthesis. The late 19th century results in geometric crystallography, stereochemistry, and biomorphology helped the birth of the monographs by Vul'f, Jaeger, and Thompson; the revolutionary developments in physics in the first half of the 20th century, and the related mathematical and philosophical questions, inspired Weyl's interest which culminated in his survey of 1952. From the point of view of the very recent peak, the impact of computing and mathematical modelling should be emphasized. It is also important to remark that, although we focused here on scientific publications and artistic movements, not all relevant discoveries or inventions are manifested in such a form. It is very useful to see the main trends in a broader context, because those influences do not come directly from the person's field often remain hidden. The cultural background of specific geographic regions may also have an impact on the style of thinking of some artists and scholars. For example, the Hebrew cabbalistic figures, the Indian mandalas, the Chinese and Japanese pictographs, and the philosophy of the Yin-Yang could indirectly help some studies. The lesser importance of bilateral symmetry in the East could inspire some discoveries related to symmetry violations or broken symmetries. ISIS-Symmetry has a special interest in such "hidden symmetries". We will return to these questions, as well as to some backgrounds of the 35-year cyclicity, in the near future. Our desire is not to wait another 35 years for a huge concentration of events. We should keep the interested people connected and organized by providing – although with a lower amplitude – much shorter periods for the cycles.

## 3. MANIFESTO ON CONTINUOUS (DIS)SYMMETRIES FOR THE FUTURE: Let us have more symmetry among different fields of culture and science, among different geographic regions, and among ourselves...

Let us come together from different fields of art and science, from theory and practice, from education and research. Let us continually try to bridge our split culture. Let us keep these connections as a protection against the overspecialization and isolation of disciplines. Let us consider symmetry as a possible "bridge" or even an intuitive "language" (far from being universal or unique!) to help interdisciplinary and international communications. Let us stop forcing our papers into the "Procrustean bed" of various disciplinary proceedings or periodicals, by adding compromising new paragraphs, or cutting off some relevant parts. Let us stop the disappearance of these works into the "ocean" of publications, without reaching their real audience. Let us have a "symmetric forum" in art and science regularly!

We should keep in mind, however, that symmetry is the third most frequent symptom of compulsive obsession according to a recent psychiatric survey by Rapoport. To avoid the "symmetry disease" we should consider these interdisciplinary connections just as a supplement to our disciplinary knowledge, and nothing more. We should use these bridges to go to other fields to exchange our experiences, then to return "home". A typical danger on such bridges is that some people would make them very wide and then stop right there forever. Their contributions block healthy traffic on the bridge (submissions with zero information and crackpot papers), and often lead to the collapse of the entire bridge (as in the case of the extreme extension of meaning of such concepts as "cybernetics" and "system"). To meet people briefly on the bridge, however, is very useful, because this is the forum where we may easily find people from other places with similar interests. Another danger is the extreme narrowing of the bridge by overspecialized contributions. There can be bridges as narrow as to accommodate only one person, that is, only the contributor can (hopefully!) understand his or her own statements – this is the time during a symposium when the audience is asleep or quietly leaving for the buffet. We would not claim that one-person bridges can never lead to major contributions (no one followed even Galois' path into group theory for many years), but interdisciplinary forums are definitely not the right place to present and judge them. Overspecialized contributions can even damage the atmosphere of an interdisciplinary meeting.

The truth is, neither too general nor too specialized topics can help the discussions. Neither too soft nor too rigid approaches to symmetry can inspire other people. We need a broad, but not boundless, understanding of this concept in art, science, and technology. We should find a "dynamic symmetry" between the two extremes, similar to real bridges over a river, which are mostly neither asymmetric, nor perfectly symmetric. We should remember Pierre Curie's advice that "the dissymmetry makes the phenomenon". It is also important in the case of social symmetries: we need a fuzzy approach to the topic in an informal agora, or forum, similar to the ancient Greek or some oriental traditions. Let there be (dis)symmetry...



Seconded by