Symmetry: Culture and Science

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Symmetry in a Kaleidoscope 1

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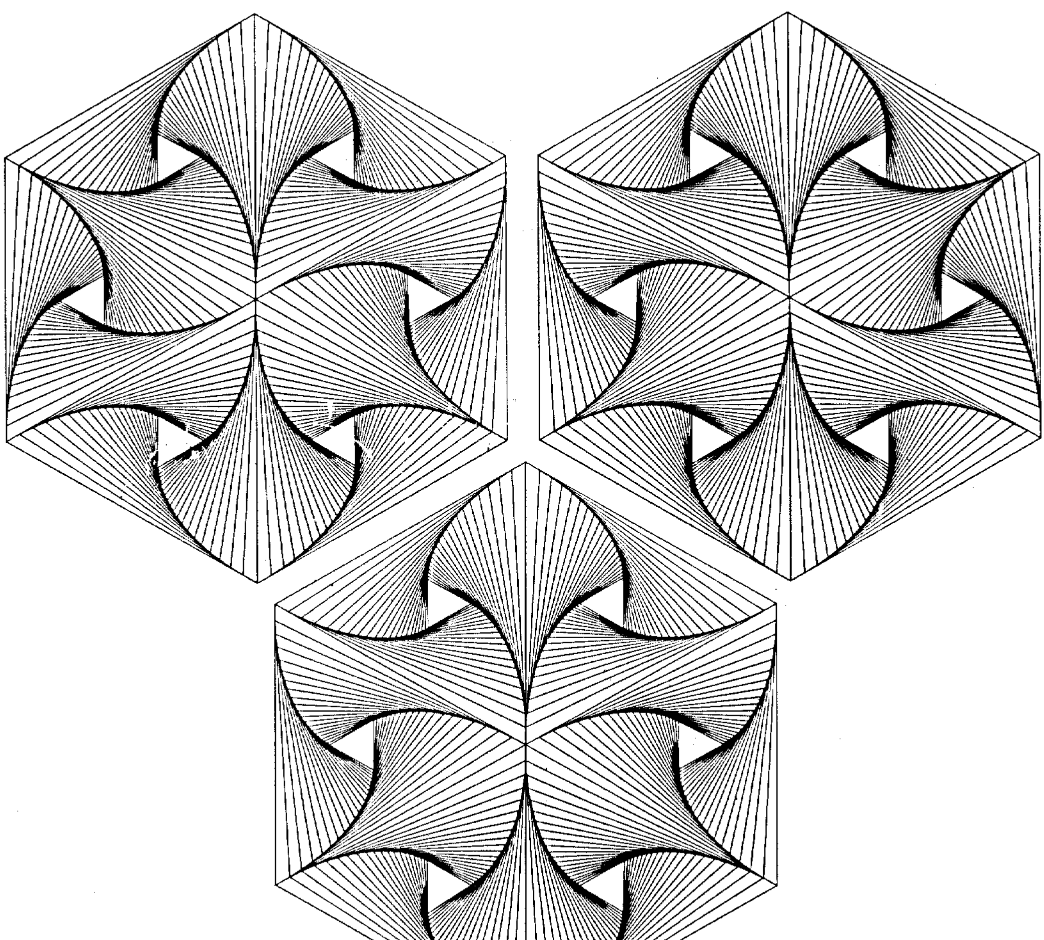

Dissymmetric kaleidoscopes:
Hommage à Pasteur

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SYMMETRY: SCIENCE & CULTURE

SURFACE STRUCTURE AND DEEP STRUCTURE OF THE SYMMETRY PHENOMENON

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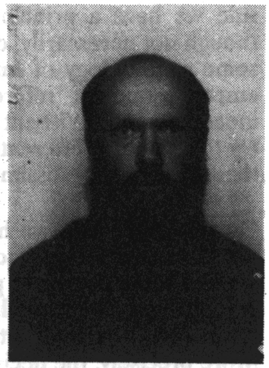

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Fields of Interest: Local and global aspects of discrete structures -- in particular the study of topological, algebraic, combinatorial and algorithmic criteria for global consistency of local geometric data in the theory of tiling, in matroid theory and in distance geometry; equivariant combinatorics and representation theory; combinatorial optimization and discrete dynamics; mathematical aspects of sequence analysis in molecular biology; mathematical aspects of classification theory.

Publications: Presentations of discrete groups, acting on simply connected manifolds, in terms of parametrized systems of Coxeter matrices -- A systematic approach. *Advances in Mathematics*, 63, No.2, 1987; Repetition und Metamorphose -- zum Symmetriebegriff in der Mathematik. In: *Katalog I zur Ausstellung 'Symmetrie'*, Darmstadt, 1986; Heaven and Hell tiling. In: *M. C. Escher: Art and Science*, Amsterdam, 1986; On tiling of the plane. *Geometriae Dedicata*, 24, 1987; On the classification of local disorder in globally regular spatial patterns. In: *Temporal Order*, Berlin, 1985.

QUESTION 1

what is symmetry?

Detection of *symmetry* in everyday life is generally related to simple instances of *repetition*, mediated through translations, rotations and/or (glide) reflections. Such forms of repetition play an important role in decorative folk art as well as in many forms of professional decorative art. They also provide a wide playground for worthwhile, and not altogether trivial, mathematical analysis, investigating the possible types, simple or intricate, of the overall organization of such repetitive forms of symmetry. Consequently, the resulting interplay between decorative art and formal mathematical thinking has been fascinating to many mathematicians and even to some artists. Still, most people will agree that deeper aspects of art are rarely (if at all) related to distinct forms of repetitive symmetry. Also, for a mathematician, the analysis of regularity (including the analysis of deviation from regularity) means much more than just the analysis of highly symmetric repetitive patterns. Hence, to clarify the relevance of symmetry for mathematicians and of mathematics for symmetry, it may be misleading to overemphasize the successes of mathematics in enterprises like "wall paper analysis". Such overemphasis would do justice neither to the uses of symmetry in art, nor, even if boosted by corresponding successes in classifying three-- and higher dimensional crystal structures etc., in science, nor in mathematics.

To have a better starting point it may be worthwhile to begin by realizing that prior to any instance of observed actual symmetry or asymmetry we must have the possibility of *comparing* different things at different places and at different times by using more or less well defined and homogeneously applicable ways of measurement. Indeed, quite generally, to orient ourselves in the world in which we find ourselves living, we depend crucially on our ability, and even proclivity, to compare all kinds of things (even those which in principle are incomparable) according to all possible kinds of possible measurements: not only according to form, size and place, but also according, say, to the money we may have to pay or the time we may have to spend for them, or the particular dynamics and time scale according to which they keep changing.

By supplying and using such kinds of comparative measurements, we separate our world -- at least on the intellectual, theoretical level -- in two distinct parts. On one side we have a principally empty and *homogeneous frame of reference*, generally, though not necessarily, conceived as being of some spatio--temporal character. Using some terminology of Immanuel Kant, the presupposed global homogeneity of this empty frame of reference is an obvious "precondition for the possibility of measurement". Without such presupposed *homogeneity* it would not make sense to try to compare different things at different places and at different times or to claim their similarity or dissimilarity.

On the other side we have the actually existing, historically developing disordered world of real things and concrete situations. Even if symmetry can be observed, at least to some degree of precision, here and there, locally, in this real world, its proper domain and innate realm is the homogeneous empty frame, constructed from the "real world" by intellectually abstracting from every possible concrete "furnishing". More precisely, the decisive property of this empty frame, i.e. its homogeneity, can be expressed most adequately in terms of its internal global symmetry structure, that is, through the specification of its global *symmetry group*.

Consequently, measurements can be compared and interpreted coherently only after specifying such a global symmetry group and different specifications will certainly lead to different interpretations. This point of view was stressed in particular by Albert Einstein, whose special relativity (or better: invariance) theory depended crucially on replacing the classical Galilean group, used by Newton, by the Lorentz group. Similarly, the different versions of modern grand unified theories of elementary particles differ decisively by their choice of the universal group of (mainly hidden) symmetries, which they choose from the essentially inexhaustible stock presented by Lie group theory. Starting with such a group, they try to deduce the existing forms of matter as the symmetrically most stable, "irreducible" forms of combinations of some undecomposable elementary ingredients, while the dynamics of interaction between these forms of matter is described in terms of symmetrically admissible rearrangements of these ingredients -- actually quite in accordance with the principles of theoretical physics laid out by Plato in his famous dialogue "Timaios" more than 2300 years ago.

From this point of view, real things, observed in the real world, are only instances of *broken symmetry*, and visible local symmetries occurring in an ornament and a crystal, as discussed above, present only a very small section of the full set of global symmetries we tend to associate with such a structure. Or, to repeat our basic argument once more, prior to any such concrete form of local symmetry, which may be recognizable through exact measurement relative to a given frame of reference, there is the necessarily presupposed global symmetry of the frame itself, on which the possibility of all measurement is based.

By the way, I believe that similarly the occurrence of symmetry in art, architecture and in social contexts (e.g. in religious and political symbols, ceremonies etc.) is very often meant to symbolize some otherwise invisible deeper relationship or, in the social context, some hopefully powerful "higher order", transcending the individual and his claim to unrepeatable personal uniqueness.

Coming back to physics and mathematics, it is also important to observe that the underlying global symmetries of the frame of reference allow one not only to recognize actually occurring local symmetries, they enable one also to study systematically how things (or rather their state parameters) change when transformed by global symmetry transformations. In other words, they allow to study invariance as a particular case of it *covariance*. Such forms of covariance are studied in representation theory or "harmonic" analysis. Very often one can use such analysis to reduce the complexity of complicated dynamical systems by introducing "symmetry adapted state parameters". In turn, such parameters may help to recognize the invariant dynamical law according to which the system in question evolves.

Indeed, recognizing continuously changing systems (like the planetary system) as special manifestations of some (hopefully forever) unchanging dynamical law (like Newton's gravitational law) appears to me to be the most demanding and the deepest application of symmetry analysis in mathematics and physics.

To answer the question "What is symmetry" from my point of view as a mathematician, I would therefore like to stress the following four points:

- The fascination, associated with the occurrence of visible forms of concrete, local symmetry, is based very often, at least to some extent, on the feeling or conviction, that such visible symmetry symbolizes some form of invisible "higher order" or some "deeper relationships".

- In accordance with this, to understand what symmetry means in mathematics and physics, we have to start with the observation that concrete instances of symmetry can be recognized only relative to a given frame of reference and hence depend crucially on some presupposed and not directly observable symmetry structure of that frame which in turn can be expressed most adequately through specification of its global symmetry group.

- Given such a global symmetry group of the abstract frame, to study its implications for whatever fits into that frame one need not restrict one's attention to what remains invariant under some system of symmetry transformations, taken from this group. A much wider and much more important goal is to study how things *change* covariantly with respect to such symmetry transformations and to adapt their description accordingly to the observed forms of covariance by an appropriate choice of a symmetry adapted state parameter system.

- Using such descriptions the ultimate goal is to unravel what after all may be invariant in a world which on every possible level appears to be changing continuously: The basic, simple laws according to which the world (or, more modestly, its subsystems in question) evolve.

QUESTION 2

The implications of mathematical symmetry analysis as characterized in section 1 are obviously so fundamental for all sciences and so manifold that it would be hopeless to try to start listing, explaining and classifying them in a few pages. Instead, I would like to describe one particularly striking example, based on the work of two former Ph.D. students of mine, Heike Schuster and Martin Gerhardt (Dress et al., 1988; Schuster and Gerhardt, 1988). They set out to develop a mathematical model for certain heterocatalytic reactions on metal surfaces in terms of some appropriately defined "cellular automaton" (cf. Wolfram, 1986). This particular kind of modelling forces any user to try to find very simple and homogeneously applicable local rules, according to which the global system is supposed to change its state by local interaction. Generally, such local rules (re-) produce an amazing wealth of rarely predictable phenomena.

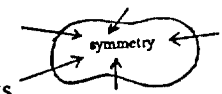
Motivated by a thorough analysis of the experimental results (Jaeger et al. 1985), H. Schuster and M. Gerhardt considered essentially the following 'ansatz': The state of each local subsystem (cell) is characterized by some integer, say, between 0 and 100. Without local interaction each cell would continuously enlarge its state parameter value until it reaches the threshold value 100 from where it drops back to 0 to start growing again. Local interaction is determined by simple "diffusion", forcing cells to accelerate or to slow down the growing process, according to whether or not the average of the state parameter value of the neighboring cells is higher or lower than the state parameter value of the cell itself (cf. Dewdney 1988 for more details).

Starting from a randomly chosen field of initial state parameter values these simple rules produced highly organized and intricate patterns of circular and spiral wave fronts (cf. Figure 1), modelling not only the chemical reaction, we initially considered, but also Belousov--Zhabotinskii type reaction patterns (cf. S.C. Müller et al. 1987).

Figure 1 Wave fronts, generated by the hodge--podge machine, designed by Heike Schuster and Martin Gerhardt. These pictures were made at the Max--Planck--Institute für Ernährungsphysiologie, Dortmund, with support of the Volkswagen Foundation which is gratefully acknowledged.

So far, we do not have a full understanding of these results based on pure mathematical reasoning. Still, I consider these results to be a particularly convincing evidence for the claim that the heart of mathematical symmetry analysis is not the purely geometric analysis of interesting patterns as depicted in Figure 1 (surface structure of symmetry), but the search for the invariant laws, by which such patterns will arise (deep structure of symmetry). At the same time, the results may be used to demonstrate that the visual fascination of the surface structure of symmetry is a valuable indication of some underlying deep structure of symmetry, in this case the simple system of dynamical rules designed by H. Schuster and M. Gerhardt which are applied in the same way at every stage and at every single place of the system. Incidentally, it may also be interesting to note that restricting such rules to one dimension and following their time development using a second dimension, one can generate all kinds of patterns of mollusc shells which was shown quite independently by Meinhardt and Klingler (1988).

QUESTION 3



Symmetry has always presented a great challenge to mathematics. Its study has motivated a wide range of mathematical investigations. Many of these could be used to exemplify how the particular appreciation of symmetry by mathematicians and through mathematics has contributed decisively to the development of our field. Still, I believe that there is an even more intimate relationship between mathematics and the appreciation of the charms of symmetry, which does not concern symmetry as a subject of mathematical studies, but as a guiding principle for a mathematician's particular way of searching for the truth and designing mathematical theories. The internal dynamics of mathematical reasoning seems to be governed not so much by formal logic, nor just by fantasy and free imaginative speculation, but by a certain it *sense of balance*. This sense of balance is crucial not only for distinguishing between adequate and inadequate approaches to given problems, but also to search creatively for new and promising directions of research, and to discriminate between those and idle mathematical bustle. In other words, at any possible stage the work of mathematicians is strongly guided by aesthetic criteria: uncovering fruitful areas of research, familiarizing oneself with the inherent structure of such fields, developing relevant conjectures, discovering proofs or surprising and enlightening relationships with other areas, as well as reworking all these tasks, until finally an adequate and logically, as well as aesthetically, satisfying and convincing form has been found.

To illustrate this fact one could quote from probably every autobiography, written by a mathematician. Instead, to conclude these remarks, I would like to invite the reader to pore over the picture of the rock garden of the Buddhist monastery Ryoanji Temple in Kyoto and try to understand that it is every mathematician's dream to create mathematical work whose form and content presents the same kind of perfectly worked out harmony and that for us there is indeed no difference between the artistic appeal of such works of art and the appeal of the most beautiful papers written in mathematics.

In addition, I believe that realizing the well balanced tension between the components of such a design and the strong field of force created that way, may also lend further evidence to the basic thesis I tried to expound in this note: It is not the easily discernable *static* manifestations of symmetry, which primarily deserve to be studied. It is the compelling, though much less perceptible forms of symmetry, controlling *dynamic* processes, which deserve a penetrating analysis in any scientific investigation of the symmetry phenomenon.

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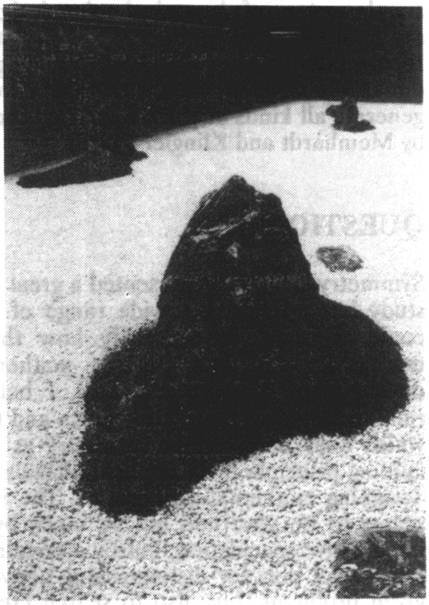

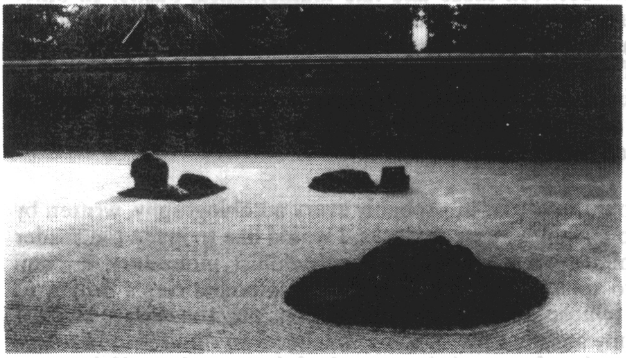

Figure 2 Pictures from the rock garden of the Buddhist monastery Ryoanji Temple in Kyoto. These pictures were provided by the "Japanisches Fremdenverkehrsbüro Frankfurt", which is gratefully acknowledged.

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