Symmetry
of
STRUCTURE

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Abstracts

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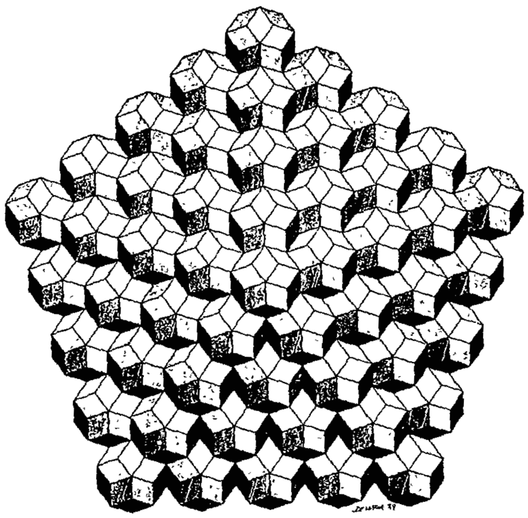

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SYMMETRY IS PROPERTY OF MATTER

I.S. Zheludev

Institute of Crystallography Acad. Sci. USSR, Moscow, USSR

In a broad sence, the word "symmetry" implies that objects and phenomena contain an element of p e r m a n e n c e, an element which is i n v a r i a n t with respect to certain transformations. This meaning is most often associated with the invariance of geometric figures or natural objects when their equal parts are transposed through rotation and reflection. In other cases, "symmetry" means the invariance of some phenomenon in relation to displacement or "reflection" in time. Sometimes both "spatial" and "temporal" invariance considered together. The symmetry science has being developed historicly differently depending on particular interest of different branches of knoledge. Later became clear that, on the final account, the symmetry is the reflection of the properties of real World, the properties of space and time.

The fundamental laws of conservation which reflect certain invariansy are connected with the gomogenety of time, gomogenety and isotropy of space. The reality of certain phenomenon in the nature, in its turn, is determined first of all by the symmetry of the matter in general, by the symmetry of space and time in particular. From this position the understanding of symmetry have to go not from the phenomenon to the Nature, but from the Nature to the phenomenon.

For the time being everyone feels the necessity for unifining different branches of the symmetry knoledge developed in different time and such unification has allready been developed. Its theoretical ground is the mathematics, the group theory and its experimental graund is based on the properties of the matter.

In our papers (Zheludev, 1983, 1987a, 1987b) the applications of symmetry in different branches of natural sciences are considered: in the electromagnetic phenomena, the physics of high energy, the solid state physics, the physical crystallography, the tensor calculation etc. We use here the knoledge of point symmetry of the geometric images representing the symmetry of space and time. They are: sphere without symmetry planes (pseudoscalar) and polar vector for time; sphere with symmetry planes (scalar) and axial vector for space. Time is anticentrosymmetric (operation of space inversion $C = \bar{1}$ is changing the "sign" of time) but space is centrosymmetric(Fig.1).

In addition to the conventional symmetry its different generalizations are considered: antisymmetry (Shubnikov, 1951), magnetic symmetry (Landau and Lifshits, 1982, p.188) and complete symmetry (Zheludev, 1960, p.346; Fig.2). The most effectiv happened to be the complete symmetry which is based for the begining on equal treatment of space and time but finaly confining to the phenomena satisfining the i n v e r s i o n o f t i m e (operation $C = \bar{1} = T$). Time has two signs but space just one (expanding Universe). The symmetry of space and time is described by group PT-4 (Fig.3).

It is shown (Zheludev, 1987c) that the symmetry of all real phenomena is confined to four rules: the rule of scale, the rule of right (left) hand, the thumb rule and the gyroscope rule. To understand this one have to consider the phenomena described by the simplest tensor relationships

$$\begin{array}{cccc} & P_i & = & a_{ij}Q_j \\ T & - & & + \quad - \end{array} \qquad (1)$$

$$\begin{array}{ccccc} & H & = & a_{ij} & S_j \\ T & + & & + & + \end{array} \qquad (2)$$

$$\begin{array}{cccc} & H & = & [NS] \\ T & + & & ++ \end{array} \qquad (3)$$

$$\begin{array}{ccccc} & H_i & = & A_{ij} & P_j \\ T & + & & - & - \end{array} \qquad (4)$$

$$\begin{array}{ccccc} & Q_i & = & A_{ij} & S_j \\ T & - & & - & + \end{array} \qquad (5)$$

$$\begin{array}{cccc} & H & = & [FP] \\ T & + & & -- \end{array} \qquad (6)$$

[a_{ij} is polar, A_{ij} is axial second rank tensors; (3) and (6) are vectors multiplications; P, Q, F are polar vectors; H, N, S are axial vectors]. All these relationships satisfy the operation of time inversion T (=C = $\bar{1}$): the right and the left sides of the relationships after this operation have the same signs what means that describing by them phenomena are real.

One can combine Eq. (1-6) in four groups representing four rules of symmetry. Some of them are well known.

The s c a l e r u l e is satisfied by phenomena described by Egns. (1) and (2). One vector is transformed into another equivalent one through a scalar either by increasing or descreasing it but preserving its direction. The simplest examples of such phenomena are electric polarization of a medium under the action of an inducing field D, magnetization of a medium in a magnetic field, the motion under the action of external forces etc.

The r i g h t- (l e f t-) h a n d r u l e described by Eqn. (6) is well known. This rule governes such phenomena as the Hall and Magnus effects, propagation of electromagnetic waves, etc.

The mutual orientation of vectors in the Magnus effect obeys the following rule - a real rotation corresponds to the rotation of the "force" vector (not changing its direction upon operation of time reversal R) towards the flux vector in a way to make the angle between them smaller. In order to determine the direction of the motion of a conductor with a current in a magnetic field one may use the right-hand rule. In this case, the extended fingers of the right hand indicate the direction of electron motion (which is opposits to the direction of the current). The sign of the North pole of a magnet (perpendicular to the palm) is determined in such a way that if we look from the side of the pole from which magnetic force lines outcome, the electrons forming electric current move in the clockwise direction (the direction of the current is anti-clockwise). In this case the conductor with a current moves in the direction indicated by the thumb of the right hand. Note that in both cases, irrespectively of the phenomenon nature, the mutual orientation of all three vectors in the above considered phenomena is described in the complete symmetry by group $\underline{m}m\underline{2}$ and in the conventional symmetry by group m.

The t h u m b r u l e determines the direction of the linear motion of a rotating screw [eqns.(4) and (5)]. A right-hand screw proceeds into the body if it is rotated in the clockwise direction, whereas the left-hand screw, being rotated in the same direction, in unscrewed from the body. The right-hand screws thus correspond to the mutual orientation of polar and axial vectors depicted in Fig.3.

It is worth noting that sometimes the thumb rule is used to determine the direction of the magnetic field of a direct current. First of all, in this case the directions of the current and the magnetic field are mutually perpendicular (and not parallel as is determined by the thumb rule). Sometimes the well known concept on magnetic field lines outcoming from the North pole and closing at the South pole is used. The direction of these lines is taken to be the direction of the magnetic field often denoted by a polar vector. In the actual fact, the magnetic pole is axial and its "direction" (the pole) is uniquely determined by the current direction in magnetic Ampere turns if we look at the North pole, we see that electrons in such turns (i.e., in solenoids) move in the clockwise direction, if we look at the South pole, electrons move in the anticlockwise direction.

The g y r o s c o p e r u l e is applied to the phenomena described by relationship (3). As is known, if to tilt a rotating gyroscope (apply a force to it), it starts rotating about the third axis which is normal to the rotation axis and the axis of rotation provided by the applied force. For a gyroscope fixed at the center of gravity, the application of such a force gives rise to a moment and its precession which may also be interpreted as a rotation of the gyroscope about the third axis. The phenomenon as a whole is described by three mutually perpendicular axial vectors two of which change its sign upon operation R and the third one (the moment of a force) does not change its sign. The gyroscope rule also describes such phenomena as electron paramagnetic and nuclear magnetic resonance.

In complete symmetry the configuration of three mutually perpendicular axial vectors is described by the centrosymmetric group $\bar{3}\underline{m}$ (in the conventional symmetry it is described by group $\bar{3} = C_{3i} = S_6$).

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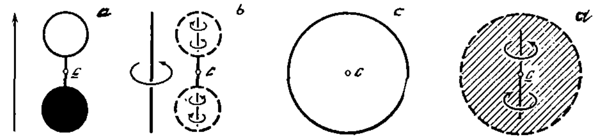

Fig. 1

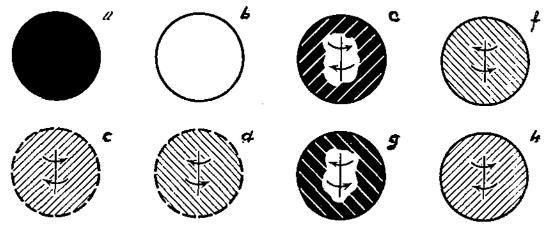

Fig. 2

Bases				Irreducible representation	Operations and characters			
Polar vector	Axial vector	Scalar	Pseudo-scalar		1 $(TP)^2$	$\bar{1}$ T	$\underline{1}$ P	$\underline{1}$ TP
—	—	—	—	A	1	1	1	1
$\pm A_{xy}$			A	B	1	−1	1	−1
	$\pm a_{xy}$	a		C	1	1	−1	−1
$\pm A_{xy}, \pm a_{xy}$				D	1	−1	−1	1

Fig. 3

CAPTIONS TO FIGURES

Fig. 1. Geometric images of scalars and vectors and their complete symmetry (see below). a - polar vector and two scalar sphere "opposite" sign, complete symmetry $\infty/\underline{m}mm$; b - axial vector and two pseudoscalar sphere having opposite signs of enantiomorphs, complete symmetry $\infty/\underline{mmm}$; c - scalar sphere (operation $\bar{1}$ for it is symmetry operation, but operation $\underline{1}$ converts it into sphere of opposite "colour". d - pseudoscalar sphere (operation $\underline{\bar{1}}$ for it is symmetry operation but operation $\bar{1}$ converts left sphere into right one).

Fig. 2. Simplest geometric images of complete symmetry. a,b - scalar spheres (+) and (-); c,d - pseudoscalar spheres left (+) and right (-); e,f - left "black" and right "white" spheres; g,h - right "black" and left "white" spheres.

Fig. 3. The characters of irriducible representations and the basis invariants of group PT-4.