Symmetry of STRUCTURE

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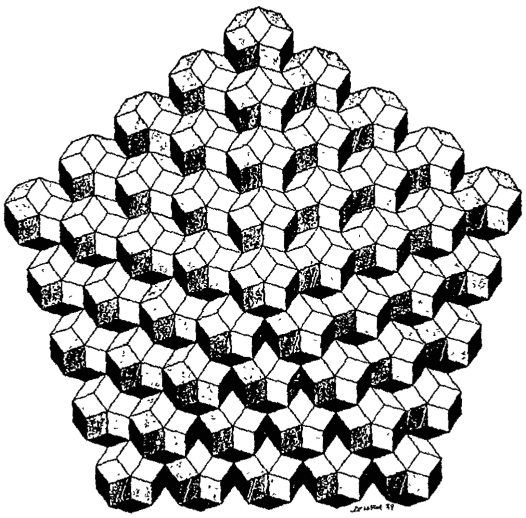
Abstracts

II.



Edited by Gy. Darvas and D. Nagy

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August 13-19, 1989

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ROTARY SHADOWS FROM THE p-DIMENSIONAL HYPERSPACE

M. Torres*, G. Pastor, I. Jiménez** and J. Fayos*****

* Instituto de Ciencia de Materiales, Sede A,

** Instituto de Electrónica de Comunicaciones,

*** Instituto de Química-Física "Rocasolano",

Consejo Superior de Investigaciones Científicas,

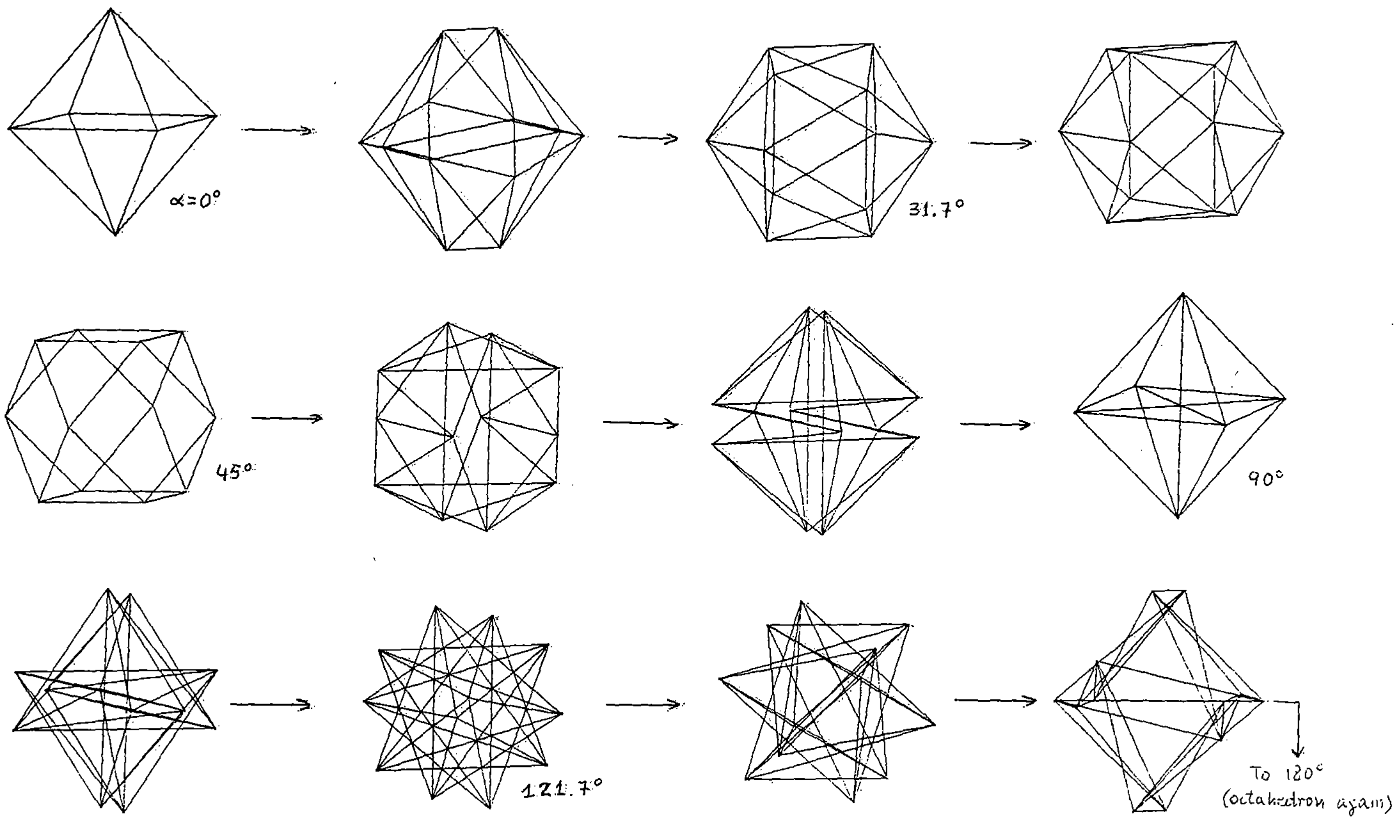
Serrano 144, 28006 Madrid, Spain.

In a recent paper, we found some continuous evolutions that connect the radial skeletons (center to vertices directions) of basic polyhedral forms [1]. Afterward, taking into account the works of Hadwiger and Coxeter [2] about hypercrosses shadows falling on the ordinary spaces E^2 and E^3, we have extended our above mentioned work in order to connect in a continuous way the icosahedral and cubic symmetries and orders [3,4].

Here, we use our "rotary shadow method" [4] to generate two appealing geometric evolutions. We begin finding variable vectors half-stars in a rotary subspace E^n which represent the orthogonal projections of half-crosses defined in the hyperspace E^p, p>n. So, according to the theorem of Hadwiger [2], we start from variable half-stars with p vectors which preserve the equation

$$\sum_{i=1}^{p} v_{i\mu} v_{i\gamma} = \frac{1}{n} \delta_{\mu\gamma} \sum_{i=1}^{p} v_i^2 \quad ; \mu,\gamma = 1, \ldots, n \quad , \qquad (1)$$

where $(v_{i1}, v_{i2}, \ldots, v_{in})_{i=1,\ldots,p}$ are the components of the p



α=0°
31.7°
45°
90°
121.7°
To 180°
(octahedron again)

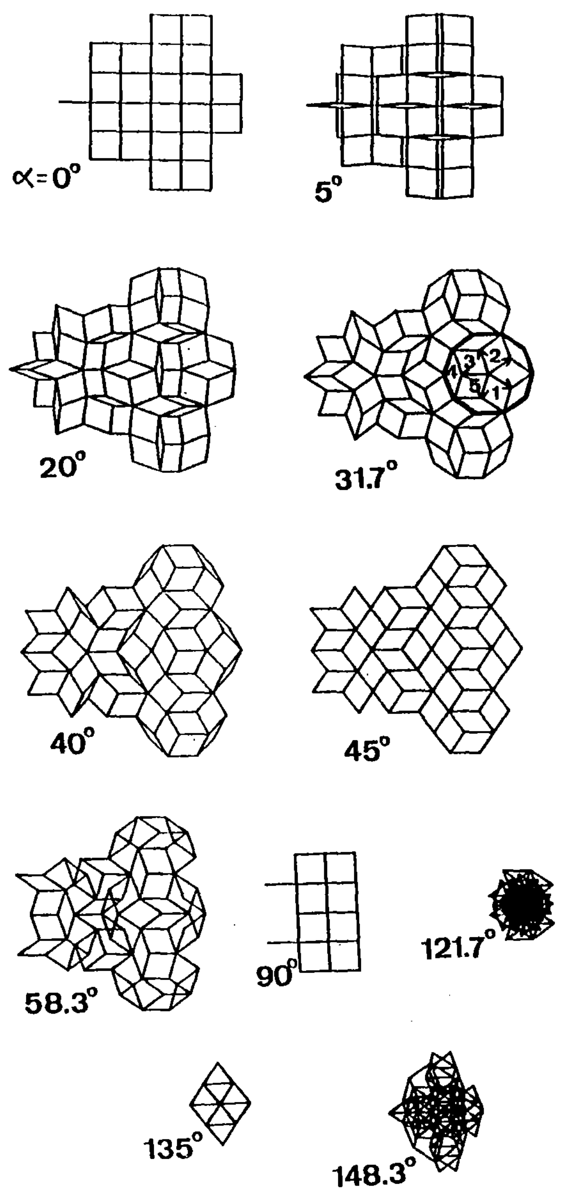


Fig. 2

vectors $\{v_i\}_{i=1,...,p}$ in the rotary subspace E^n.

The α-variable half-star $v_1=(c,-s,0)$, $v_2=(c,s,0)$, $v_3=(0,c,-s)$, $v_4=(0,c,s)$, $v_5=(-s,0,c)$, $v_6=(s,0,c)$, where $c=\cos\alpha$ and $s=\sin\alpha$, preserves eq.1 (with p=6 and n=3). By drawing an adequate set of edges which join the vertices of this α-variable star, the octahedron metamorphosis can be seen. Fig.1, where star vectors are omitted, schematically shows this cycle which connects in a continuous way some notable polyhedra (octahedron, icosahedron, cuboctahedron and small stellated dodecahedron)

The α-variable half-star $v_1^*=(c^2,-s)$, $v_2^*=(c^2,s)$, $v_3^*=(-s^2,c)$, $v_4^*=(-2sc,0)$, $v_5^*=(-s^2,-c)$ preserves eq.1 (with p=5 and n=2). When $\alpha=31.7174°$, this half-star coincides with the five pentagonal directions of a Penrose tiling [5]. When α varies, the half-star and the Penrose tiling also evolve in a continuous way. Fig.2 shows an evolutionary cycle of a Penrose tiling patch. This cycle contains a singular state of extreme folding or reversible collapse ($\alpha=121.7174°$).

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