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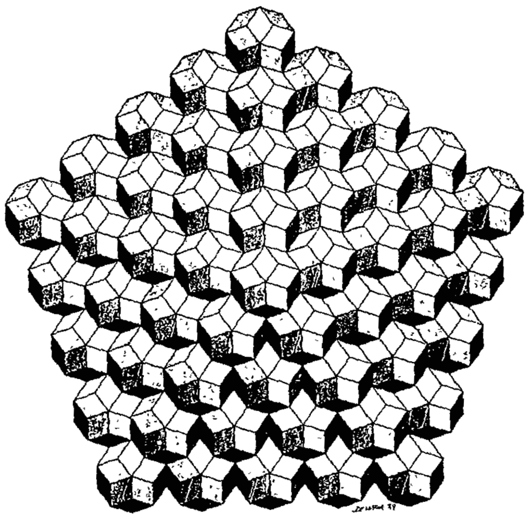
Abstracts

II.



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THE NUMBER OF DIVISIONS OF A FLAT SURFACE INTO EQUAL-SIZED ELEMENTS. CONTINUAL INTEGRAL

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The problem of counting the number of divisions of a two-dimensional surface into preassigned equal-sized elements is considered. If it is impossible to cover the surface with the given elements, this number goes to zero. In the present report the continual integral for the number of divisions is constructed and then used to formulate the criterion for the equality to zero of the sought-for number of divisions.

The lattice of nodes is constructed on the two-dimensional surface. The number of nodes is sufficiently large such that each element has the same even number of nodes, and if these elements consist of nonconnected regions, the number of nodesin each region should also be even. The requirement for evenness follows from the method of construction of the continual integral for the number of divisions. Each region or element is represented as a set of nonoverlapping dimers which cover two nearest nodes, and a separate weight is assigned to each dimer.

In the paper /I/ the continual integral is constructed for the number of possible dimer coverings of the surface with allowance for the assigned weights:

$$Z=\int\{dV\}\exp\sum_{ij=1}^{M,N}\left(-a_{1,i,j}V_{i+1,j}V_{ij}(-1)^{j}-a_{2}V_{i+1,j+i}\cdot V_{ij}(-1)^{j+1}+a_{3ij}V_{ij+1}V_{ij}\right.$$

where ij – are coordinates of the node at the lattice, V_{ij} are the Grassmann variables, is the weight allocated to the $d_{3,i,j}$-the dimer with the ij-coordinates. Using this integral as a generating function, one can construct the continual integral for the given elements. Really, each element was represented as a set of nonoverlapping dimers. The corresponding pairs of Grassmann variables commutate with each other which permits each element of division be brought into correspondence with the product of Grassmann variables in the continual integral for the number of possible divisions:

$$Z = \int \{dV\} \exp \sum_{i,j=1}^{M,N} \sum_{f} W_f \prod_{s=1}^{2m} V_{i+\Delta s,f,j+\delta s,f}$$

where $2m$ is the number of lattice nodes per element, W_f - is the weight allocated to the element of a given type.

From the equality to zero of the continual integral over the Grassmann variables it follows that there exists a transformation

$$V_{ij} = \sum_{a,b} \omega_{ijab} U_{a,b}$$

that leads the integrand containing the Grassmann variables V_{ij}, to the expression with a smaller number of variables $U_{a,b}$; the integral over these latter integrals will be equal to zero by definition /2/. The above consideration enables us to formulate the criterion for the equality to zero of the number of divisions:

If there exists a set of numbers ω_{ijab} such that

$$\sum_{\{p\}} \sum_{ij=1}^{M,N} \sum_{f} \omega_f \prod_{s=1}^{2m} \omega_{i+\Delta s,f,j+\delta s,f;\{a,b\}} = 0$$

the division of the surface into given elements is not possible.

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2. F.A.Berezin. Methods of secondary quantization. M.:Nauka, 1965, 380