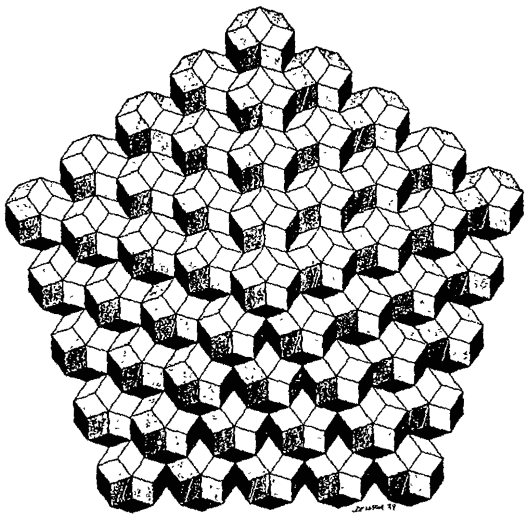
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

II.



Edited by Gy. Darvas and D. Nagy

Buda
pest

August 13-19, 1989

hungary

Extensions of the symmetry notion on geometric objects with one or more colours

Tiberiu ROMAN, BUCURESTI

The objects I'll speak about have plane images and the mathematical treatment is minimum. The succesive extensions are presented in a logical order, more than in a chronological one. For more details, there are references (and the references given there). Projection and models will be presented.

1. THE CLASSICAL SYMMETRY (ISOMETRY)

1.1. An isometry is a mapping of the euclidean space E^n onto itself, which preserves all distances. In the plane (n=2), the isometries are: a) the identity; d) reflection in a line; e) glide reflection (in a line). A symmetry of a set S (of points from E^n) is an isometry which maps S onto itself. The set of symmetries of S forms (under composition) his symmetry group G(S).

1.2. A(plane) motif M_o is a bounded and connected set of points(from E^2). A discrete pattern $\mathcal{P}$ is a (non empty) family M_i ($\subset E^2$) of pairwise disjoint, congruent copies of M_o, so that the symmetry group G($\mathcal{P}$) acts transitively[1)] on the M_i-s. There are three cathegories of $\mathcal{P}$: a) finite patterns (or rosette); b) strip patterns (frieze, band or border ornaments) with one translation vector as symmetry; c) periodic patterns (wallpaper or plane ornaments) with 2 (in different directions) translation vectors as symmetries.

1.3. If the motif M_o is asymetric (i.e. no symmetry of G($\mathcal{P}$) is its own symmetry), the pattern is called primitive and there are: a) two families of finite patterns - with c_n (the cyclic group) and d_n (the dihedral group); b) 7 types of strip patterns; c) 17 types of periodic patterns. If the motif M_o is symmetric (i.e. there is at least one symmetry of G($\mathcal{P}$) that is a symmetry of the motif also), the pattern is called non-primitive and there are: a) a familly of finite patterns; b) 8 types of strip patterns; c) 34 types of periodic patterns. For the history of these groups of symmetry, their extensions and illustrations see Grünbaum & Shephard (1987), pp.55-56, 218-256.

1.4. A chain $\mathcal{S}$ (or rod pattern) is a rotational,infinite, cylindrical surface C with a marked family $\overline{M}_i$ ($i \in I$) of pairwise disjoint congruent copies of a motif $\overline{M}_o$ (i.e. a bounded and connected set of cylinder points) , so that the symmetry group G($\mathcal{S}$)

1) For each pair M_k, M_h of the family M_i, there is a symmetry of G($\mathcal{P}$) that transforms M_k onto M_h.

./.

acts transitively on the $\overline{M}_1$ - s. Their symmetry groups were studied first in 1929. T.Roman (1969, 1985) classified these in 17 primitive and 19 non-primitive classes, using their plane development.

1.5. A major extension was performed by adding a "marking" to each motif copy. The first : colouring it in black or white so that the symmetries maintain the colour and the antisymmetries change them. Indirectly it was made in 1929-1930. Directly (but remaining fast unknown) in 1935 . The book of Shubnikov (1951) gives a new start to application of antisymmetry (and multiple antisymmetry) not only in geometry and cristallography but also in physics, chemistry, biology by scientists from U.S.S.R., and from many other countries in the 50's and 60's. See the book of Shubnikov & Koptsik (1972 in russian, 1974 in english) and the review article of Zamorzaev & Palistrant (1980, p.231).

1.6. An other acception of colouring, mathematically founded by van der Waerden & Burckhardt (1961) is the $q^{2)}$-colour symmetry for a set S as a pair (σ, π), where σ is a symmetry of G(S), and π a permutation of q indices, compatible with σ ; the compositon law of the colour - symmetry group is: $(\sigma_1, \pi_1)(\sigma_2, \pi_2) = (\sigma_1\sigma_2, \pi_1\pi_2)$. Results are given in M.Senechal (1979), T.Wieting (1982), Jarrat & Schwarzenberger (1981), T.Roman (1970, 1989). Review article: R. Schwarzenberger (1984), Grünbaum & Shephard (1987, pp.463-47o).

1.7. All the above mentioned notions and results are for discrete geometric object from 1.2 and 1.4. The continuous (or semi-continuous) patterns as well as the tiling of the euclidean plane will not be tackled, but see Grünbaum & Shephard (1987).

2. THE HOMOGRAPHIC SYMMETRY

2.1. A homographic mapping of the euclidean plane $\mathbb{E}^2$ (without a point O) onto $\mathbb{E}^2 \setminus \{0\}$ is obtained by one of the elementary transformations defined through the complex function $z \longmapsto \frac{az+b}{cz+d}$ (ad - bc $\neq$ 0), i.e.: identity, rotation, homothety, inversion, reflection and their combinations. An ω-symmetry of a set S (of motifs in $\mathbb{E}^2$) is a homographic mapping of S onto itself. The set of ω-symmetries of S forms (under composition) his ω-symmetry group $G_\omega(S)$ if it contains an inversion and his similarity group $G_s(S)$ if it contains a homothety and is not a $G_\omega(S)$.

2.2. The $G_s(S)$ were applied in crystallography by Shubnikov (1960). For subsequent developments see Zamorzaev & Palistrant (1980, pp.240-241).

2.3. A discrete ω- pattern is the homographic transform of

2) r-in [15] , k - in [11] and [2] , N - in [10]

a discrete pattern . The finite ω-patterns are the homographic

Fig.1. Fig.2.

transforms of discrete strip pattern segments. The discrete angular patterns are the transforms of the discrete strips patterns. The homographic ω - patterns are the homographic transforms of the discrete periodic patterns. A conic column $\mathcal{E}$ has its plane development (i.e. an angular domain, provided with motifs) the transformed $\mathcal{S}$ chain plane development. Details about this geometric objects see in Roman (1971, 1972, 1985, 1989).

3. THE SURFACE SYMMETRY

3.1. Generalized discrete patterns on surface are extensions of the three cathegories from 1.2. a) Finite discrete patterns on surface[3] are obtained by plaâing congruent motifs in congruent cells of a rotational surface segment, included between two planes P, P' orthogonal to the rotation axis. For C one obtains segments of strip patterns; the projection on P of these on C_0, S_0, E, H_1, H_2, P_e is a finite ω-pattern. b) Strip discrete patterns on surface are obtained by placing adequate motifs on a surface segment, the parallel planes P, P' yelding hyperbolic, parabolic or right line sections; c) Periodic patterns on surface are obtained by placing adequate motifs in the cells of rotational C_0, H_1, H_2, P_e (for $z > 0$) , cells resulting by sectioning with a special family of planes orthogonal to the rotation axis and a second family of axial planes; the projection on the plane $z = 0$ is a homographic ω-pattern.

3.2. Isometries of the plane map of a bounded surface and discrete patterns on the surface are illustrated by two exemples: a) for the ellipsoid the map is an open rectangle domain: $u \in [0, 2\pi)$, $v \in (0, \pi)$. The map of a segment of E is the band between the segments $v = h$, $v = h'$. A map of 2 elliptical finite discrete patterns is given in fig. 3 and a map of a periodic discrete pattern on E, for $u = \frac{\pi m}{2}$ (m = 1,2,3), $V = \frac{\pi h}{3}$ (h = 1,2,)

Fig.3. Fig.4.

is drawn in fig. 4; b) for the torus, the map is open square $u \in (0, 2\pi)$, $v \in (0, 2\pi)$, the oposed sides beeing identical. One defines the Θ-symmetries of the torus; the discrete Θ-patterns: parallel, meridian and helicoidal finite Θ -strips and the periodic Θ-patterns are studied (Roman, 1979).

---

3) The considered surfaces are: sphere with centre O: S_0, ellipsoid E, hyperboloids H_1, H_2, paraboloids P_e, P_h, rotational cylinder C, rotational cone with apex O: C_0.

./.

4. APPLICATIONS

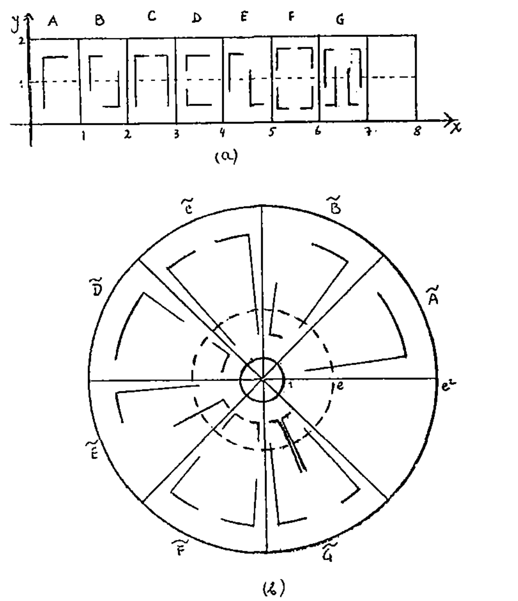
4.1. Illustrations for unicolour and bicolour discrete patterns from roumanian folk art will be projected.

4.2. There will be projections for: colour Escher patterns, colour homographic patterns, colour chains - also as suggestions for their wider utilization.

4.3. Tires profiles models - as examples of Θ-patterns will be shown.

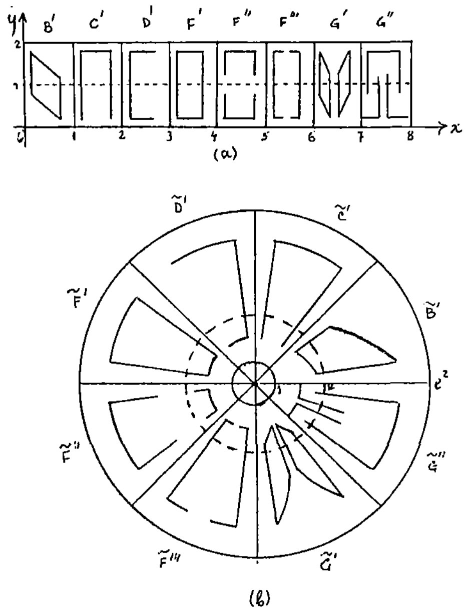
REFERENCES

1. Galiarski & Zamorzaev: Kristallografija 8, 691 (1963)
2. Grünbaum & Shephard: Tiling and Patterns; W.Freeman & Co, New York (1987)
3. Jarrat & Schwarzenberger: Utilitas Mathematica 19, 295 (1981)
4. T.Roman: Zt.f.Krist. 132, 372 (1970)
5. T.Roman: Bul.Inst.Petrol Gaze Geologie, 17, 237, (1971)
6. T.Roman: Mathematica (Cluj) 14 (37), 167 (1972)
7. T.Roman: Generalized Symmetries on the torus. Symposium on mathematical Crystallography , Riederalp (Wallis) p.15 (1979)
8. T.Roman: Crystallographic applications of new Classification extention. Ninth European Crystallographic Meeting, (ECM), Torino, (Italy), p.672-3 (1985)
9. T.Roman: Coloured Chains, without and with cristallographic restrictions, Submitted for the Twelfth ECM Moscow, August 1989
10. R.Schwarzenberger: Bull London Math . Soc., 16 , 209 (1984)
11. M.Senechal: Discrete Appl. Math. 1 , 51 (1979)
12. A.Shubnikov: Symmetry and Antisymmetry of Finite Figures , Akad.Nauk SSSR, Moscow - Leningrad, (1951)
13. A.Shubnikov: Kristallografija 5, 489 (1960)
14. Shubnikov & Koptsik: Symmetry in Science and Art , Moscow (1972)
15. van der Waerden & Burckhardt: Zt. f. Krist. 115, 231 (1961)
16. T.Wieting: The Mathematical Theory of Plane Chromatic Ornaments, M.Dekker, New York (1981)
17. Zamorzaev & Palistrant: Zt . f. Krist . 151, 231 (1980).



Asymmetric motifs for strip patterns (a) and for isometric ($\tilde{A}$, $\tilde{C}$) and homographic finite patterns (b)

Fig. 1



Symmetric motifs for strip patterns (a) and for isometric ($\tilde{C}'$) and homographic finite patterns (b)

Fig. 2

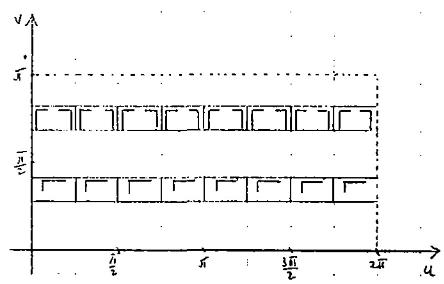


Fig. 3

Two elliptical finite patterns on the ellipsoid map.

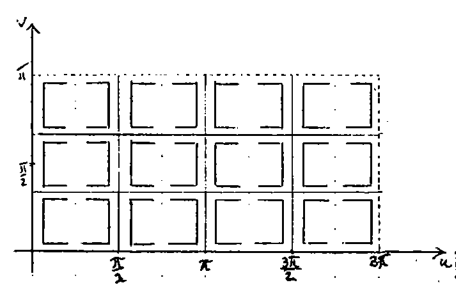


Fig. 4

Periodic discrete pattern on the ellipsoid map.