Symmetry of STRUCTURE

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Abstracts

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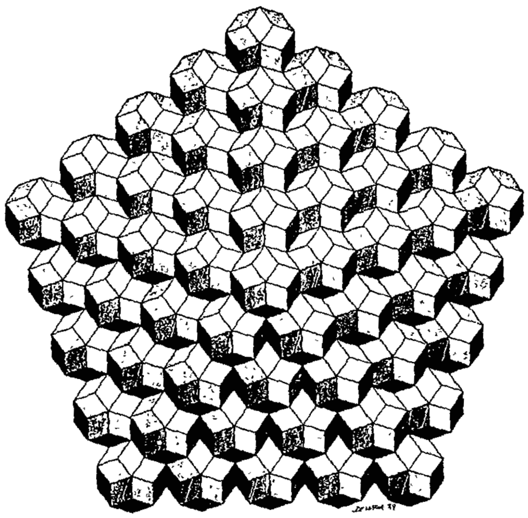

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Symmetric bracing of one-story buildings with cables and asymmetric bracing of one-story buildings with rods

(Extended abstract)

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Summary: The minimal systems of diagonal cables or rods which make a one-story building infinitesimally rigid were first studied by Bolker and Crapo (1977-79). Some of the remaining open problems were settled in the last four years (Chakravarty, Holman, McGuinness, Schwärtzler and the author). As a byproduct of these investigations we found that if the cables of such a minimal system are all parallel then the patterns of these cables are highly symmetric. This result seems to be somewhat surprising since asymmetric, rather than symmetric patterns arise in case of rods. These observations will be presented. The main tool is graph theory and network flow techniques.

Consider a 1-story building, with the vertical bars fixed to the earth via joints. If each of the four external vertical walls consists of a diagonal, the four corners of the roof become fixed. Hence questions related to the infinitesimal rigidity of a one-story building are reduced (Bolker and Crapo, 1977) to those related to the infinitesimal rigidity of a 2-dimensional square grid of size $k \times \ell$ where the corners are pinned down. Then the minimum number of necessary diagonal rods for infinitesimal rigidity was proved to be $k+\ell-2$ (Bolker and Crapo, 1977) and

Theorem A: The minimum systems correspond to asymmetric 2-component forests (Crapo, 1977).

(In what follows, every graph will be the subgraph of the complete bipartite graph $K_{k,\ell}$; the twwo subsets of the vertex set of $K_{k,\ell}$ will be denoted by A and B with respective cardinalities k and ℓ. A 2-component forest with vertex sets V_1, V_2 of the components is called asymmetric if $|V_i \cap A| \neq \frac{k}{\ell} \cdot |V_i \cap B|$.)

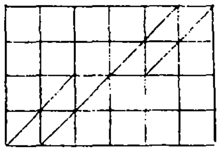
If we wish to use diagonal cables for infinitesimal rigidity, the minimum number of these cables was proved to be

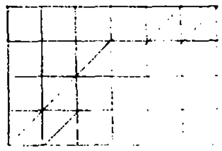
$$\begin{cases} 0 & \text{if } k=\ell=1, \\ 4 & \text{if } k=\ell=2, \\ k+\ell-1 & \text{otherwise} \end{cases}$$

(Chakravarty et al, 1986) and the minimum systems were characterized in a somewhat more technical way only recently (Recski and Schwärtzler, 1989). In a previous stage of our investigations we found the following partial result (Recski, 1988):

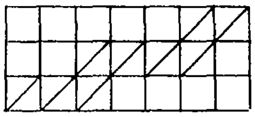
Theorem B: Suppose that all the diagonal cables are parallel. Then the system makes the grid infinitesimally rigid if and only if $|N(X)| > \frac{1}{2}|X|$ holds for every proper subset X of A, where $N(X)$ denotes the set of those vertices of B which are adjacent to at least one vertex of X.

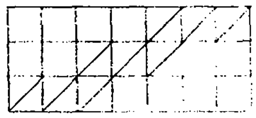
In order to illustrate Theorem A, consider the following two systems. The first one has an infinitesimal motion while the second one is infinitesimally rigid.



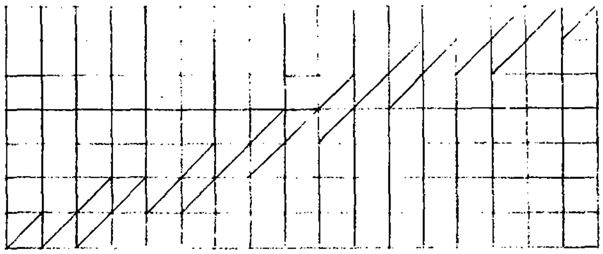


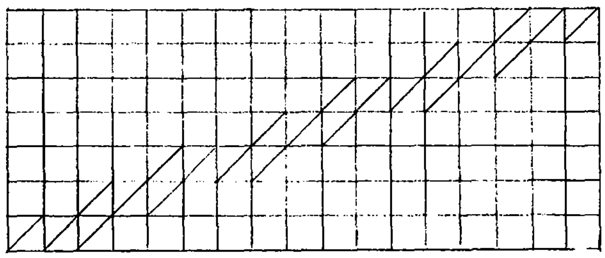
The next two systems illustrate Theorem B. Again, the first system has an infinitesimal motion while the second one is infinitesimally rigid.





The first pair of illustrations is not surprising at all; plenty of examples are known to justify the vague statement that "The less symmetric bar-and-joint frameworks are the more rigid". However, the second pair suggests another statement that "In case of tensegrity frameworks symmetry may be advantageous for minimum rigidity". Of course, symmetry alone is not enough; only one of the following two systems is infinitesimally rigid (which one?).





A possible explanation could be that cables prescribe inequalities, rather than equalities, among the shears of the rows and columns, hence each relation $a \leq b$ must be accompanied by a relation $a \geq b$ as well.