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Abstracts

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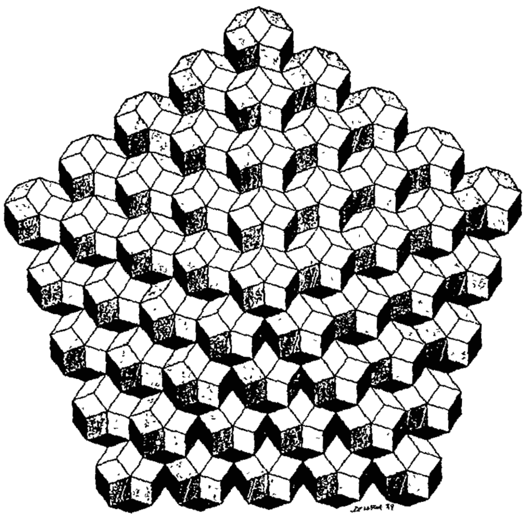

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KINEMATIC SYMMETRY AND "HYBRID" INVARIANTS IN A QUASIMONOCHROMATIC LIGHT BEAM

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1. Introduction

The category of symmetry has long been interpretted and used differently and efficiently by the human thought /1, 2/. Not yet formed as a philosophical conception, symmetry served as one of the indications of harmony, order and beauty. Just because this category was among the first to receive the mathematical status: geometric and arithmetic image.

In full measure the conception of symmetry has been embodied in the group theory. Without limiting the number and picture this theory adequately takes account of and discloses with weightly arguments the essentials of the symmetry conception both in the static and dynamic aspects. Its effectiveness resides in matching up the initial symmetries with the laws of conservation,i.e. with invariants of a transformation group.Though the main invariants were obtained long before the advent of the group theory, their real meaning and fundamentality were understood only in the making of the group-theoretical:no matter how we transform the surroundings by a group of transformations there exists a set of values that do not vary as result of these transformations.

At present, based on a more wide and deep understanding of the mathematical aspects of symmetry, it has become possible to construct a new class of the so-called "hybrid" invariants /3/.

2. Approach

In this construction the first prerequisite is a somewhat different kinematic interpretation of the conception of symmetry. As applied to the discussion of a quasi-monochromatic light beam propagating through a linear optically transparent medium in a remote field, a set of possible states of the system forms a two-dimensional complex space C(2) (spinor space from complex enveloping components of the electric vector). By fixing a certain initial state of the beam (system) one finds all other states by making corresponding linear transformations. This type of symmetry serves to "enumerate and classify" all states of the system, and is "kinematic" in this sense /4/.

Let us now consider how the quasi-monochromatic radiation passes through a linear medium. Since a light beam at an arbitrary instance of time is identified by the spinor $\zeta(t)$ (or cospinor $\tilde{\zeta}^{*}(t)$) its state on leaving the medium is determined by the finite fundamental representation $D(1/2,0)$ of the group SL(2, C) or is expressed in terms of the matrix of transformation as $\zeta'^{\mu} = U^{\mu}_{\nu}\zeta^{\nu}$.

A product of spinors and cospinors time-spaced by a value τ constructs two-point time formations in a ray. This representation is a mixed spin-tensor construction S with a valence (1.1), which transforms according to

$$S'^{\mu\dot{\nu}} = U^{\mu}_{\varkappa}\tilde{U}^{\dot{\nu}}_{\dot{\lambda}}S^{\varkappa\dot{\lambda}} \qquad (1)$$

Analogously, a set of products of n spinors and m cospinors, taken at N = n + m arbitrary instances of time, is transformed according to the expression

$$S'^{\{\nu_i\}\{\dot{\mu}_j\}} = \prod_{\substack{i=1,n\\ j=1,m}} U^{\nu_i}_{\varkappa_i}(\tilde{U}^{-1})^{\dot{\mu}_j}_{\dot{\lambda}_j}S^{\{\varkappa_i\}\{\dot{\lambda}_j\}} \qquad (2)$$

Here, {...} is the sign of the set, ν_i, $\varkappa_i$, $\dot{\mu}_j$, $\dot{\lambda}_j$ are the indexes of the i-spinor and j-cospinor. In the conventional symbolic notation the expression (2) takes the form

$$(S')^{N} = D^{n}(1/2,0)\otimes D^{m}(0,1/2)S^{N}$$

3. Invariants

The expression (2) demands averaging because of the presence of random element in the sequence of spinors. The matrix of transformation may be taken outside the averaging sing <...> in the class of stationary transformations. Thus, the representation (2) specifies essentially the transformation of correlation functions $K(\tau_2, \tau_3, \ldots, \tau_N)$ /3/.

In the general case this is a reducible representation, it splits into unreducible representations

$$\begin{array}{lll} D(n/2,\alpha)\ldots\ldots & D(\beta+1,\alpha) & D(\beta,\alpha) \\ D(n/2,\alpha+1)\ldots\ldots & D(\beta+1,\alpha+1) & D(\beta,\alpha+1) \\ \ldots\ldots\ldots\ldots\ldots\ldots & & \\ D(n/2,m/2)\ldots\ldots & D(\beta+1,m/2) & D(\beta,m/2) \end{array}$$

$$\beta=\begin{cases} 0, \text{at } n = 2K \\ 1/2, \text{at } n = 2K+1 \end{cases}$$

$$\alpha=\begin{cases} 0, \text{at } m = 2J \\ 1/2, \text{at } m = 2J+1 \end{cases}$$

for the specially symmetrized (by the indexes of formula (2)) correlation functions $K'(\tau_2, \tau_3, \ldots, \tau_N)$ which are combined into invariant ensembles similary to the Dicke model for a system of N two-level molecules and represent coordinates of the vector in the corresponding representation space.

If the light source forms the optical process of the chaotic class, then for the description of correlation constraints it issufficient to have second-order correlation functions. But in the case of a nonthermal, partially coherent, radiator there is a cause to use a systematic description of coherence of higher orders; here, the constructed invariant structures may be substantial.

In the constructed space representations the convolutions of vectors with the coordinates of the enumerated correlation functions K' are simplest invariants, or more complex convolutions of tensors composed of the time-spaced components of like vectors. Thus, the ensemble of possible invariants in the light beam propagating through a linear optically transparent medium has a developed hierarchy not only in the sense of their affiliation to one or other order of the correlation functions, but also from the view point of belonging to one or other unreducible ensemble.

We cite as an example the transformation (1) of correlation functions. Using the formulas

$$K_{11}(\tau) = S^{1\dot{1}}(\tau) = S_0(\tau) + S_1(\tau) \qquad K_{21}(\tau) = S^{2\dot{1}}(\tau) = S_2(\tau) + iS_3(\tau)$$

$$K_{22}(\tau) = S^{2\dot{2}}(\tau) = S_0(\tau) - S_1(\tau) \qquad K_{12}(\tau) = S^{1\dot{2}}(\tau) = S_2(\tau) - iS_3(\tau)$$

the components of these spin - tensors can be matched up with points ($S_\alpha(\tau)$, $\alpha = 1,2,3,4$) of the four-demensional complex space C(4). We can construct from the components of complex four-vectors $\{S_\alpha(\tau)\}$ an invariant coupling equation for the group

$$\det S^{\mu\dot{\nu}} = S_0^2(\tau) - S_1^2(\tau) - S_2^2(\tau) - S_3^2(\tau)$$

In the case of unitary transformations U this invariant decomposes into

$$I_1(\tau) = S_0(\tau) \qquad I_2(\tau) = S_1^2(\tau) + S_2^2(\tau) + S_3^2(\tau)$$

For spin-tensors $S^{\mu\dot{\nu}}$ with Hemitian matrices (that corresponds to $\tau = 0$ in the product $\mathcal{E}(\tau) \otimes \tilde{\mathcal{E}}^*(\tau)$), the representtation (2) is none other than the Lorentz transformation.

4. Remarks

In closing it may by said that the effect of the medium through which radiation propagates may be interpretted as generalized operation of symmetry in a set of constructed invariants. Here, we have essentially a certain structural symmetry: absence of anysimilarity in the small (for a concrete correlation function) and appearrance of symmetry for polynomial formations of correlation functions grouped in a definite manner.

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