Symmetry of STRUCTURE

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Abstracts

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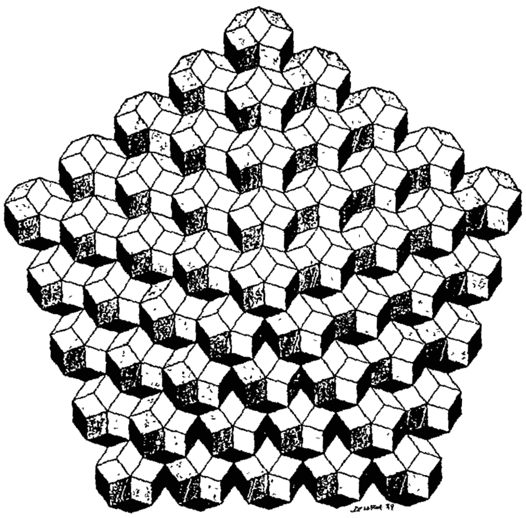

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SYMMETRY AND APPROXIMATE STUDY OF MULTIDIMENSIONAL STRUCTURES IN NONLINEAR MEDIA

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One of the problems in a study of stationary and other self-similar structures is the solution of nonlinear elliptic equations. Most of the methods being used in this field are approximate and numerical. We shall consider only the equations that have two or more spatially-homogeneous solutions (backgrounds), and the rest solutions are transitions from one background to another. Under such conditions in one-dimensional case application of method of matched asymptotics provides good results. Its generalization can be proposed for multidimensional problems if the symmetrical solutions are of interest.

We have studied the self-similar solutions of nonlinear heat equation $T_t = \alpha\Delta(T^{\sigma+1}) + T^{\beta}$, that satisfy

$$\alpha\Delta y - 0.5(\beta-\sigma-1)\cdot(\xi, \nabla y^{\alpha}) + y^{\alpha\beta} - y^{\alpha} = 0, \qquad \alpha = (\sigma+1)^{-1}, \;\; \xi = \mathbf{r}/\psi(t) \qquad (1)$$

In one-dimensional case linearization of (1) and matching the solution of linearized equation with asymptitics of $y(\xi)$ when $\xi \to \infty$ ($y \to 0$) provided good approximate solutions. One of the ways to study the multidimensional problem is to construct approximate solutions. For that, like in the Galerkin method, one can select n rays and perform matching only on them. Then approximate solution is a result of interpolation.

From the physical point of view the solutions possessing several axes of symmetry are of interest. Such symmetric approximate solutions and corresponding numerical solutions y have been found. The approximate solutions proved to describe y

qualitatively, and in some cases quantitatively for the very small number of rays. In fact, it is necessary to know the behaviour of y only along two rays, that are the halves of neighbour symmetry axes (the reflection transfers axis into another one, so the behaviour along all axes will be defined). Thus one can construct approximate solutions using only two one-dimensional functions.

To describe the behaviour of the function along the rays, one must know the solution of linearized equation with explicit dependence on the coordinates on the chosen rays. Such solutions can be efficiently received by means of separation of variables technique. In two-dimensional problem there were two coordinate systems where separation could be done (Cartesian and polar systems). The corresponding two classes of approximate and numerical solutions have been constructed. A number of results have been received for 3-D case and for the system of two elliptic equations.

As a result of this work the following hypothesis has been proposed. It is possible, that for the solutions – transitions from one background to another symmetry makes the number of essential degrres of freedom very small. Similar situation occurs in synergetics when the "order parameters" can be defined due to the presence of dissipation. The separation of variables (symmetry of differential operator) enables the approximate investigation of such systems.

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