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Abstracts

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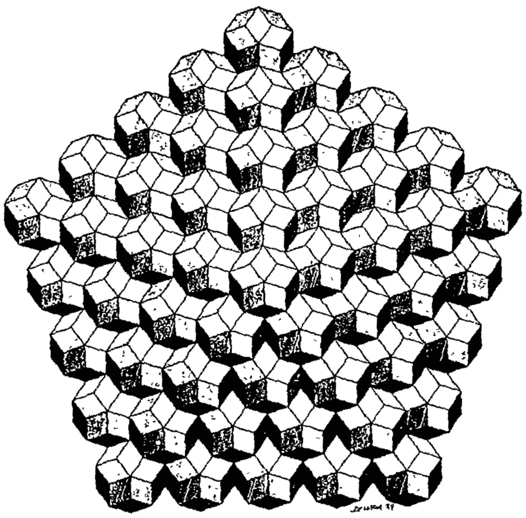

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Rational Geometry of Unidirectional Fiber-reinforced Multi-link Bamboo-like Tubes.

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This paper discusses one of the biomechanical problems, namely the problem of rational design of composite tube structures and how the geometrical symmetry of strong tube biostructures is connected with the fracture criteria.

1. Unidirectional composite tubes under axial compression show a particular mechanism of fracture with axial symmetry of the n-th order. They fail as a chinese lantern when multiple splitting and buckling of n circle segments take place . Besides, they can fail in the form of macrobuckling or in the form of local mikrobuckling under compression.

The axial compressive strength for the local microbuckling is denoted by

$$\sigma_{cr} = \sigma_c \qquad (1)$$

The critical stress for the chinese lantern mode of fracture for the thin-walled tube, acoding to Rabotnov, Polilov (1983), is given by

$$\sigma_{cr} = \sigma^* = 1.2(\gamma^4 E^5 / R_a^2\, l^2)^{1/9} \qquad (2)$$

where R_a - the average radius, l - the length of the tube, E - the longitudinal Young's modulus (along the fiber), γ - the spesific energy of splitting.

The critical stress for macrobuckling of the tube may be written by Euler's formula

$$\sigma_{cr} = \sigma_e = \pi^2 EI/Fl^2 \qquad (3)$$

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where $I = \pi\delta R_a^3$ - moment of inertia with respect to axis through a center of attraction, $F = 2\pi\delta R_a$ - cross section, δ - wall thickness.

These three failure modes are shown in Fig. 1a for unidirectional glass-fiber reinforced plastic (GFRP) with the following characteristics: E=50 GPa, σ_c=500 MPa, γ=30 KN/m, R_a=7.5 mm.

For a tube element with optimal geometry the various modes of failure must occur simultaneosly.

Then the rational tube length for simultaneos~~ly~~ occuring two fracture modes may be derived:

from (1) and (2) in the following form

$$l = 2.27(\gamma^2 E^{5/2})/(R_a \sigma_c^{9/2}) \qquad (4)$$

from (2) and (3) in the following form

$$l = 2.22(E^4 R_a^{20}/\gamma^4)^{1/16} \qquad (5)$$

from (1) and (3) in the following form

$$l = R_a \pi (E/2\sigma_c)^{1/2} \qquad (6)$$

Three graphs of these equations for the GFRP are shown in Fig. 1b.

From (4) and (5) the optimal average radius and the optimal length for the given material are:

$$R_{opt} = 1.01 E\gamma/\sigma_c^2 \qquad (7)$$

$$l_{opt} = 2.24 E^{3/2}\gamma/\sigma_c^{5/2} \qquad (8)$$

For the GFRP R_{opt}= 6.06 mm and l_{opt}= 134.4 mm.

2. Under bending condition each link of a multi-link tube structure is loaded by different stress, and therefore the optimal length to radius ratio and the optimal thickness change from link to link. The optimal thickness is found from (1). It

$$\delta = R-((R^3+4M_x/\pi\sigma_c)R)^{1/4} \qquad (9)$$

where R - the external radius, M_x - the bending moment.

The optimization problem for multi-link tubes can not be solved with the thin-walled approximation since the thickness is a parameter of optimization. Therefore it is necessary to receive the exact equation like (5), with the following expression for the moment of inertia of the circle segment:

$$I=(\alpha+\sin\alpha\cos\alpha)(R^4-r^4)/4-4\sin^2\alpha(R^3-r^3)^2/9\alpha(R^2-r^2) \qquad (10)$$

where r - internal radius, $\alpha = \pi/n$ (n - the number of segments, $n\gamma\delta l$ - the work of splitting), and to solve this equation by some numerical methods.

Our numerical research of this model shows that for $\delta/R_a \ll \alpha^2$ the optimal link length l_i decreases with the R_{ai} (average radius of the i-th link) increasing, and for $\delta/R_a \geqslant \alpha^2$ the optimal link length l_i increase with the wall thickness δ_i increasing.

In order to verify the applicability of the proposed criterium of optimization to natural biological structures like a bamboo trunk, we consider the bamboo trunk as multi-link tube structure under bending condition due to wind load. For a fixed radius of the lowest part of a trunk the thickness of the tube was found from (9). Then the optimal first link length was found from the exact equation like (4) for the chinese lantern mode of fracture.

We consider the linear decreasing of the trunk radius from the lower part to the top and assume the following properties of the wood: E = 30 GPa, γ = 20 KN/m, the total length l = 14 m, the first link radius R_1 = 0.1 m, σ_c=350 MPa, and the critical bending force P = 8 KN. The results of computer modelling show that the link length varies from 0.14 to 0.2 m, and monotonously

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increase from the ends of the trunk to the middle (See Fig. 2). Quantatively it is in agreement with the real changing of.bamboo link lengths.

CONCLUSION

The considered failure modes take into account the scale effect, and so the optimal dimensions of the tube structures may be found using the criterium of simultaneos mixed-mode failure initiation.

The proposed approach was developed in order to prove that some strong biological structures agree with optimal relations connected with simultaneos initiation of different fracture modes.

This method may be used in the design of tube constructions (e.g. ferms) using unidirectional composite materials (CFRP, GFRP and others).

REFERENCES

1. Ju.N.Rabotnov, A.N.Polilov, On the Chinese Lantern Mode of Composite Pipes Fracture, Mechanica Kompositnyh Materialov (in Russian),1983,3,p.548-550.

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Fig. I

Three modes of fracture under axial compression.

a) Stress versus length; b) length versus radius.

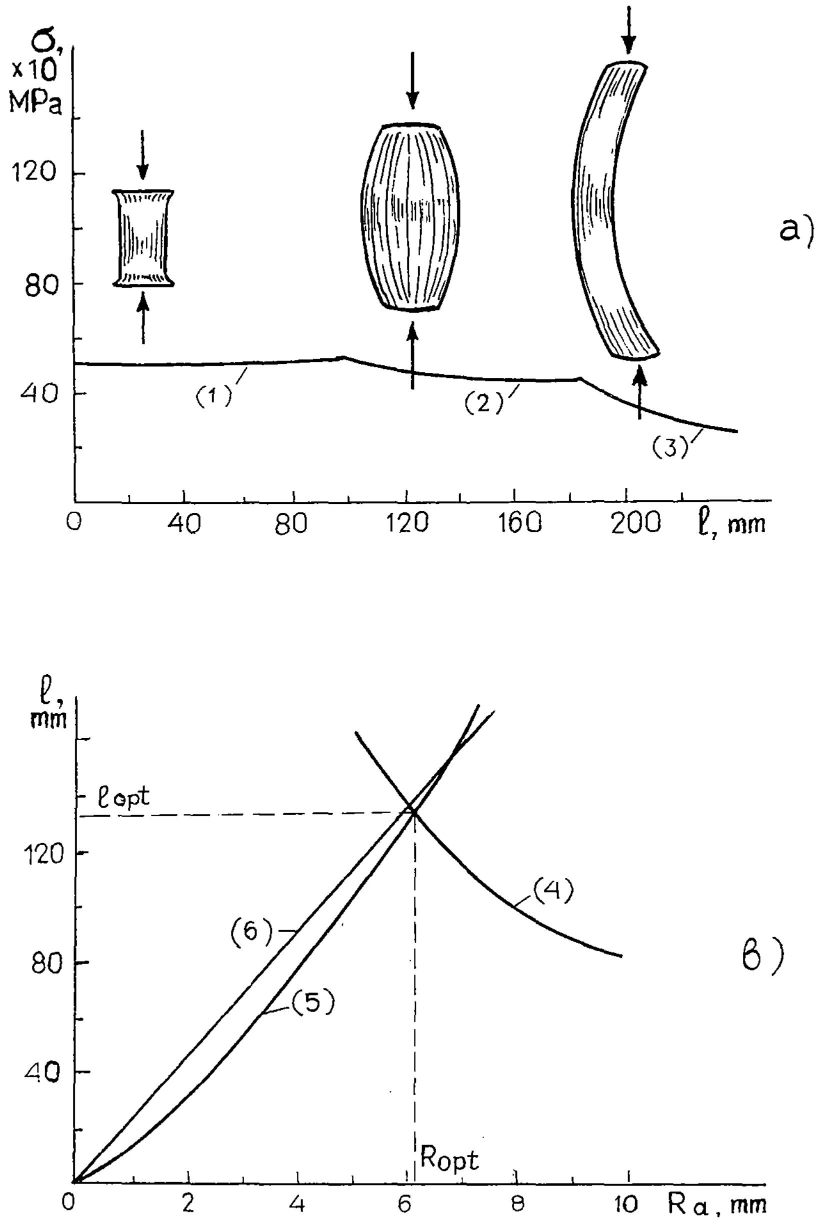

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Fig. 2

The link length versus the link number.