Symmetry of STRUCTURE

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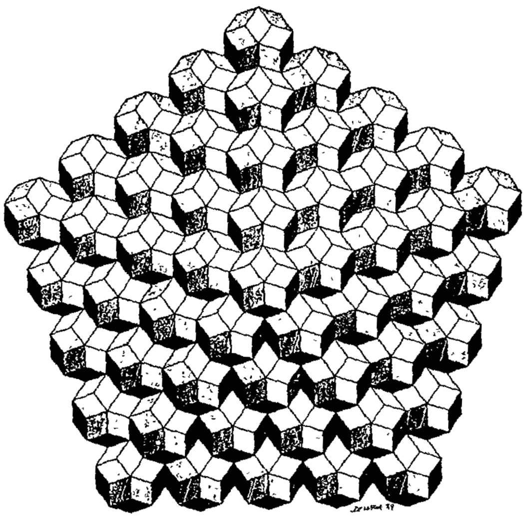
Abstracts

II.



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Highest symmetries and iterative algorithms in self organization of living matter. Cyclomer biology and cyclomer arts.

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Many problems of biological objects and processes are connected with geometrically ordered multiblock structures like the lumbar spine, fins of fishes, flowers, mollusc's shells, cardiac rhythms, unperiodical but scale rhythms of crustacea's moulting, etc.

Within the existing variety of geometrically legitimate biological structures, we concentrate on supramolecular biosystem structures whose components are integrated into an entirety in compliance with certain rules or algorithms which are the same along various lines and on various levels of biological evolution. These structures, which may be referred to as algorithmical, are of special interest for theoretical biology and physiology, also as for related sciences such as biomechanics, biotechnology, bionics, informational mechanics, etc. What is important is that, in addition to regularly shaped biological objects, there are some in which the conjugation of components is less regular, if existing at all. The report will consider algorithmical biostructures which are chains or manifolds decomposable into commensurable and regularly positioned elements (or motive units S_k). Figure 1 shows such manifolds.

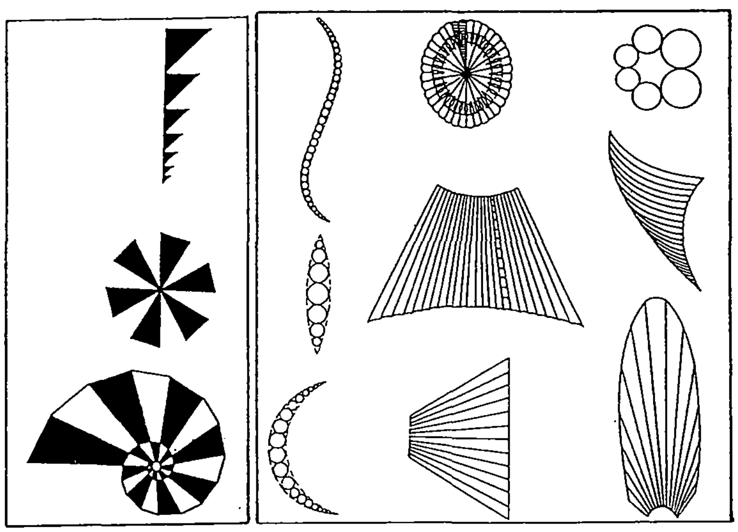

Figure 1. Geometric examples of similarity cyclomerisms (left, according to A.V. Shubnikov [1960]) and of Möbius, affine and projective cyclomerisms (right).

The general rule of representing decomposable manifolds is that the preceeding motive unit is transferred into the succeeding one by a certain fixed transformation g; in other words, the neighbouring motive units S_k are mutually conjugated by an iterative algorithm:

$$S_{k+1} = g^* S_k \qquad (1)$$

Consequently, by reapplying the generating g transformation m times to a motive unit S_k, a component S_{k+m} is obtained; mathematically speaking, in the set S_k, a cyclic (semi-) group of transformations, G, is active which contains elements g^0, g^1, $g^2, \ldots, g^m, \ldots$ (a finite number of motive units in a biological object is neglected where necessary). In other words, this decomposition of the manifold, thus organized, includes a cyclic group of automorphisms and their motive units are aligned along the orbit of the appropriate cyclic group. For brevity, such configurations will be referred to as cyclomerisms, a term known in biology, no matter whether g is Euclidean or not in an iterative algorithm (1).

Classical biomorphology (see classical works of W. D'Arcy Thompson [1917], H. Weyl [1952], A.V. Shubnikov [1960], et al) paid great attention to only those biological cyclomerisms which have generating transformation "g" from the similarity transformation group (the last consists of rotational, translational, mirror and scale transformations only).

Similarity transformations in biomorphology are also known with reference to the scale of three-dimensional growth which is fairly frequently observed in animals and vegetation over extensive periods of individual development and is accompanied by mutually coordinated growth behaviour of small zones distributed in the volume of the body, a behaviour which is geometrically described as a scale tranformation. With the transformation of as few as three points of the growing configuration known, the transformation of the continuum of its points may be assessed.

Do the similarity symmetries and the scale of the volume growth exhaust all geometrically legitimate kinds of mutual conjugation of parts in a structure and ontogentic transformations in living bodies? Or do they act in biology as very particular cases of the kinds which are built around non-Euclidean groups of transformations containing similarity sub-groups? The writer's research has provided a positive answer to this latter question. It is well known that there are two basic ways to extend the similarity transformation group, either to the Möbius transformation group or to the projective transformation group. Both these ways have a biological value according to our research.

The report consists of many examples of parallel existence of Euclidean and non-Euclidean cyclomerisms in biological structures: segmented horns of animals, antennae of insects, vertebrae of animals, shells, etc. One of these examples considers a regular non-linear reduction of average diameter d_k of conductive airways in adult human lung as a function of the order of generation k of dichotomous branching (according to E. Weibel and D. Gomez [1962]); the author's research reveals that this reduction for all the 28 generations of branching is well described by the iterative algorithm (1) with the following generating transformation g of the local-similarity kind: $d_{k+1} = (0{,}748\ d_k + 0{,}01):(-\ 0{,}013d_k + 1)$. Consequently, this important property of conductive airways may be interpreted (like some other biocyclomerisms) from the viewpoint of general biological value of local-similarity transformations.

Euclidean and non-Euclidean cyclomerisms have also had a direct bearing on the kinematics of a broad range of biological movements which can, on numerous occasions, be interpreted as a process in which cyclomerisms replace one another (so called "cyclomeric polymorphism"). The author's research also revealed the existence of non-Euclidean kinds of three dimensional growth which had not been known before, notably Möbius and affine; the three-dimensional growth of living bodies can be interpreted in terms of cyclomeric polymorphism.

Researchers in various countries have for a long time been studying time biorhythms and this field of natural sciences can boast of its own traditions, terminology, and challenging findings. Still, it has concentrated attention on periodic rhythms of physiological processes such as breathing and walking, repeating processes occurring simultaneously with periodic diurnal and seasonal changes, etc. The reader of the literature on biorhythms may think that no biorhythms other than perodic are significant or possible. In point of fact, however, the range of biologically significant rhythmns is broader and the periodic one is but a particular, albeit important, subclass. Nontrivial Euclidean and non-Euclidean iterative algorithms are obviously at work in certain rhythmic processes by Arenicola marina, Bonasa umbellus, some kinds of cardiac arrhythmias (Wenckebach periods) and other disturbances of normal periodic bioprocesses, etc. (For more details see S. Petukhov [1988, p. 34-36]). This is the way to develop a special cyclomeral theory of biorhythm disturbances under a wide class of internal or external influences; in this way, the author achieved first theoretical results.

The writer's findings are in favour of not only the argument that biology is a fertile field for introduction of various symmetrical approaches, methods and tools of group-theoretical analysis, but that development of theoretical biology, physiology and biomechanics at this stage is largely dependent on vigorous utilization of group-theoretic methods with non-Euclidean geometries. These findings permit the development of "cyclomer biology" as a science about general biological (including physiological) value of symmetrical-algorithmical principles of structurization and interaction. Cyclomer arts, combining music, mathematics, art and dancing, are also being developed by the author on the basis of application of the general cyclomerism principle with biologically valuable generating transformations.

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