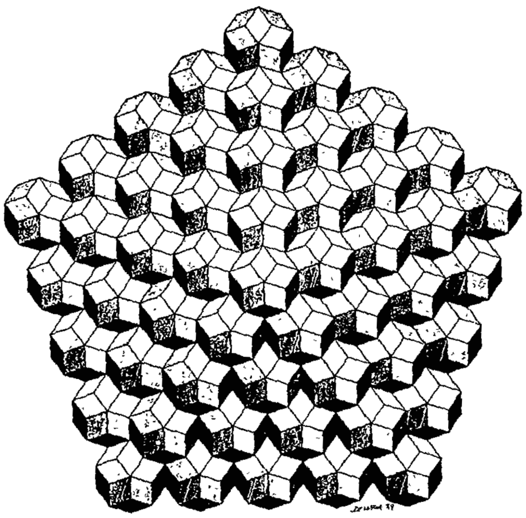
Symmetry
of
STRUCTURE

an interdisciplinary Symposium

Abstracts

II.



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VISUALIZATION
OF SYMMETRY AND OTHER MATHEMATICAL IDEAS IN TEACHING ART

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1 The >DIAPORAMA< Project

>The Bonn University Department of Mathematics and Mathematics Teaching< has realized a research project on >visualization< of mathematical aspects by using the AV-medium >DIAPORAMA<.
The theoretical foundations of the >Bonn Visualization Project< (Peters 1987) are derived from an analysis of >δείκνυμι<, the word EUCLID used for "to prove". This term means: to show, to make evident, to demonstrate, and also to teach. As the usage of this term shows, visible evidence, "demonstratio ad oculos", visualization in constructing a mathematical line of arguments seem to be the fundamental concept of Greek mathematics (Szabo 1969,p.246ff and Peters 1985,p.31-48). The learning objectives for which our research project planned and realized software range from classic topics of geometry to attempts at pupil-adequate explanations of limit, further to analyses of strategies of problem solving, and even to interdisciplinary relations between mathematics, philosophy, and art.
The >DIAPORAMA<-hardware (fig.1) is an audio-visual presentation unit. It consists of two exactly lined-up slide projectors, a dissolve unit that allows several modes of fading, and a stereo tape recorder with the possibility of recording audio signals and control informations on different tracks. The slide first shown by projector A is continually superimposed by the slide in projector B. This dissolving can be changed from a slow superimpose to a hard cut, and also to a >flip<, i.e. a sudden change from one projector to the other, e.g. to simulate reflexion.
Fundamentally the >DIAPORAMA< dissolving technique produces a "third picture", which for its part generates a pseudomotion so that this dynamic slide projection takes up the mid-position between traditional static slide presentation and moving film.

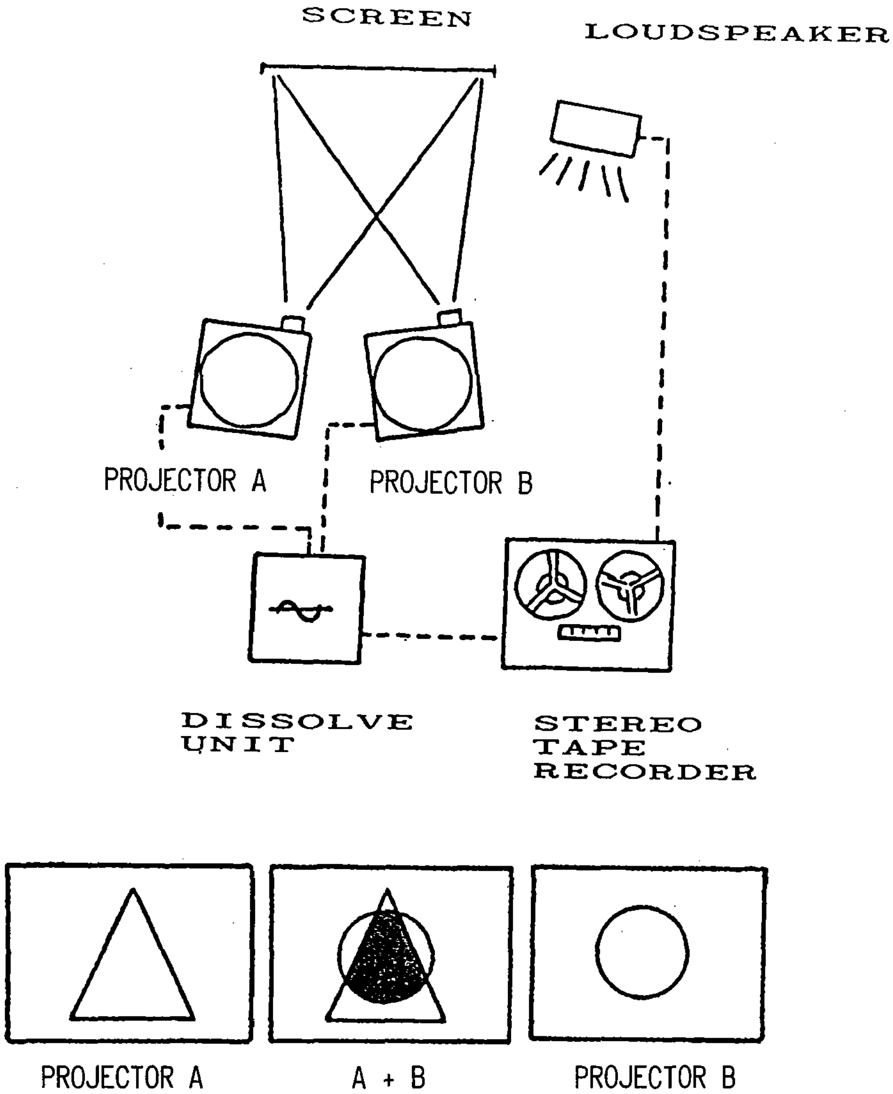

SCREEN
LOUDSPEAKER
PROJECTOR A
PROJECTOR B
DISSOLVE UNIT
STEREO TAPE RECORDER
PROJECTOR A
A + B
PROJECTOR B

FIG.1

2 Symmetry and Infinite

With the intention to discover art as a system of rules, norms, and laws, the >Bonn DIAPORAMA Project< has produced some visualization series about mathematics and art,mainly (but not only using graphs by the Dutch artist M. C. ESCHER (1889 - 1972).
The main issue of these efforts was not a new interpretation with regard to the artistic contents of ESCHER's "intellectually construc ted" works, but it was moreover the didactic reconstruction of his playing with mathematics, with proportion, reflection, perspective, and optical illusion, with zenith and nadir, with symmetry and contrast, with plane and space in simultaneous and impossible worlds.
The graph (fig.2) showing the division of a plan by birds and fishes a preliminary study about "Air and Water" (1938), enables pupils to work out the first principles of periodic drawing. The repeating pattern used by ESCHER in this case is defined by its relationship to a very simple group of the 17 plane symmetry groups, resulting from two lineary independent translations. Generating unit is a parallelogram within a parallelogram lattice. This lattice unit itself doesn't show symmetry.

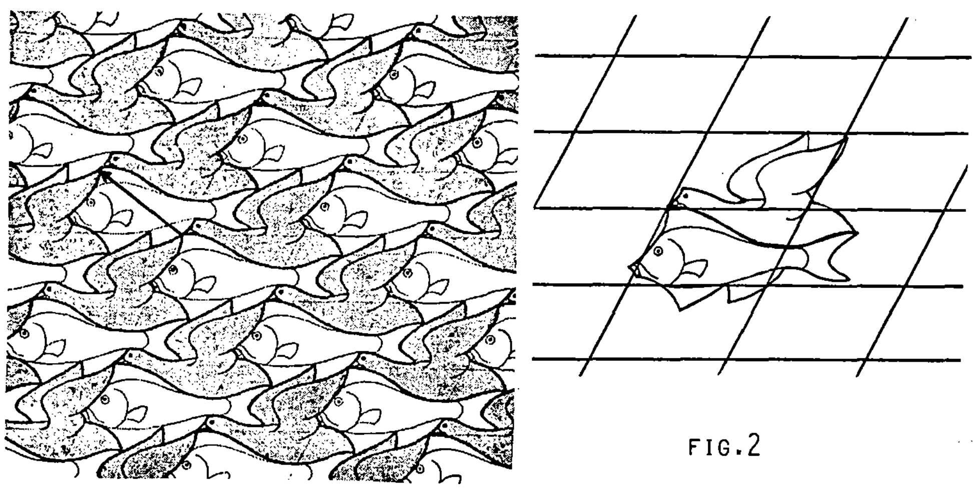


FIG.2

Planes structured by congruent rhythms are preludes to metamorphoses and circles. Conformal mapping led ESCHER to symmetries and non-euclidian elements as constructing principles e.g. in his woodcut "Circle Limit III" (1958; fig.3).

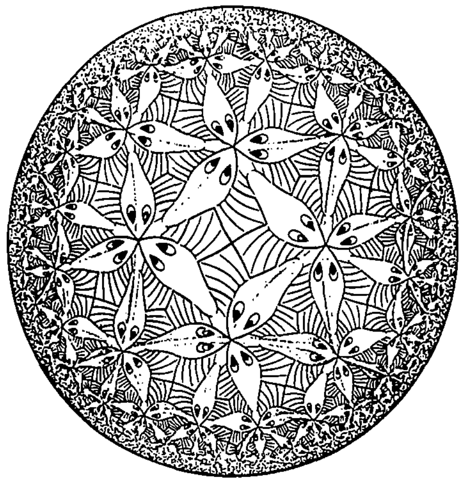


FIG.3

He found a plan for a series of attempts to a new way of "approaching the infinite" in a picture (fig.4) by COXETER,which was presente in connection with POINCARE's circle model of hyperbolic geometry. Laying bare these elements as constructive principles in classroom teaching leads to the limits of school mathematics instrumentation. Visualizations in another DIAPORAMA-series teach, how M.C. ESCHER tried to bring the totality of plane into a limited figure.

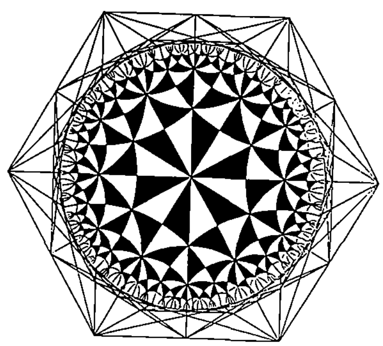
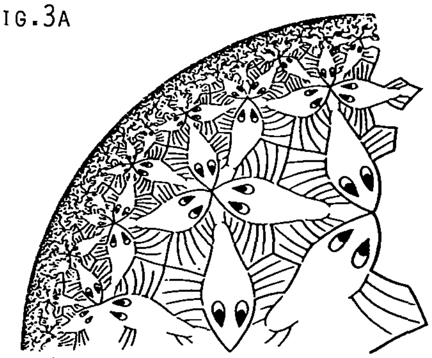


FIG.4

Pupils learn: in POICAREs model of hyperbolic geometry there is no h-point on or outside the periphery of the circle, only diameters and arcs of circles, which are orthogonal to the circular line, are h-lines.Fishes on every arc meeting head to tail rise from the circular line and fall back to it, seemingly without reaching it as a limit, increasing towards the center and decreasing when leaving. By help of more than 90 superimposes our project tried to provide an approach to a mathematical and also aesthetic interpretation of ESCHER's graph describing the following sub-aspects:

FIG.3A



1 central symmetries
2 rotations of regions
3 rotations of figures
4 meetingpoints of heads
5 meetingpoints of tails
6 arcs (h-lines) with fishes
7 "approaching the infinite"
8 symmetric structure of h-lines
9 twenty steps of the printing process (fig. 3a)

Comprehending the constructive imagination of ESCHER's "Circle Limit III" pupils consider the woodcut as "nice" and try to draw and paint simple designs themselves (fig.5)
For many topics of mathematics instruction even ESCHER-graphics can be motivating to analyse mathematical structures or visualization of aesthetic application of mathematical or antimathematical thoughts. (Peters 1987,p.53-73)

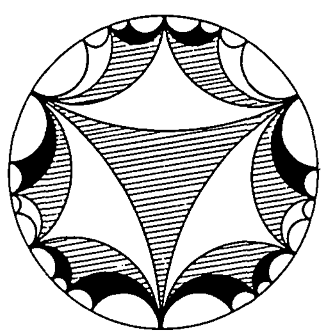


FIG.5

3 Impossible Worlds

The topic of other DIAPORAMA-series was the violation of rules in perspective mapping. It was intended to show that ervery three-dimensional object can be drawn two-dimensionally. But not every object which is presented in the drawing plane necessarily has an equivalent in the three-dimensional reality. These visualizations revealed some advantages of the dissolve technique e.g.

- the possibility to superimpose a graph by a drawing showing
- only the essential details,
- to bring into focus minor aspects,
- to add constructiv subsidiary lines,
- to mark planes,
- and to structurize complexes by painting them with new colours.

ESCHER's lithography "Belvedere" (1958) presents a pavillon with a view over a valley in the mountains. The second floor of this haunted castle seems orthogonal to the first floor. The ladder is leaning with its top against the exterior, whereas its foot is inside the pavillon.

A DIAPORAMA-series tried to visualize these violations of mathematical rules by means of descriptive geometry. Drawing the horizon into the graph and then trying to reconstruct the sagittals, we realize that there are two saggitals: one above, the other below the horizon. This is against exact perspective drawing. The top and the lower part of the pavillon show different perspectives, yet they are still connected by columns. This is only possible on the drawing plane, in the three-dimensional space, however, there is no equivalent. The reflection of the upper part of the building at the midparallel of the two sagittals results in corresponding blocks above and below the horizon,which are in perspective. The two part sagittals also form one line now. The whole "Belvedere" becomes mathematically correct,and thus it has an equivalent in three-dimensional space (fig.6).

Having understood the mathematical construction of "Belvedere" by ESCHER pupils reconstructed their "Belvedere" and found it artistically boring (fig.7-8).

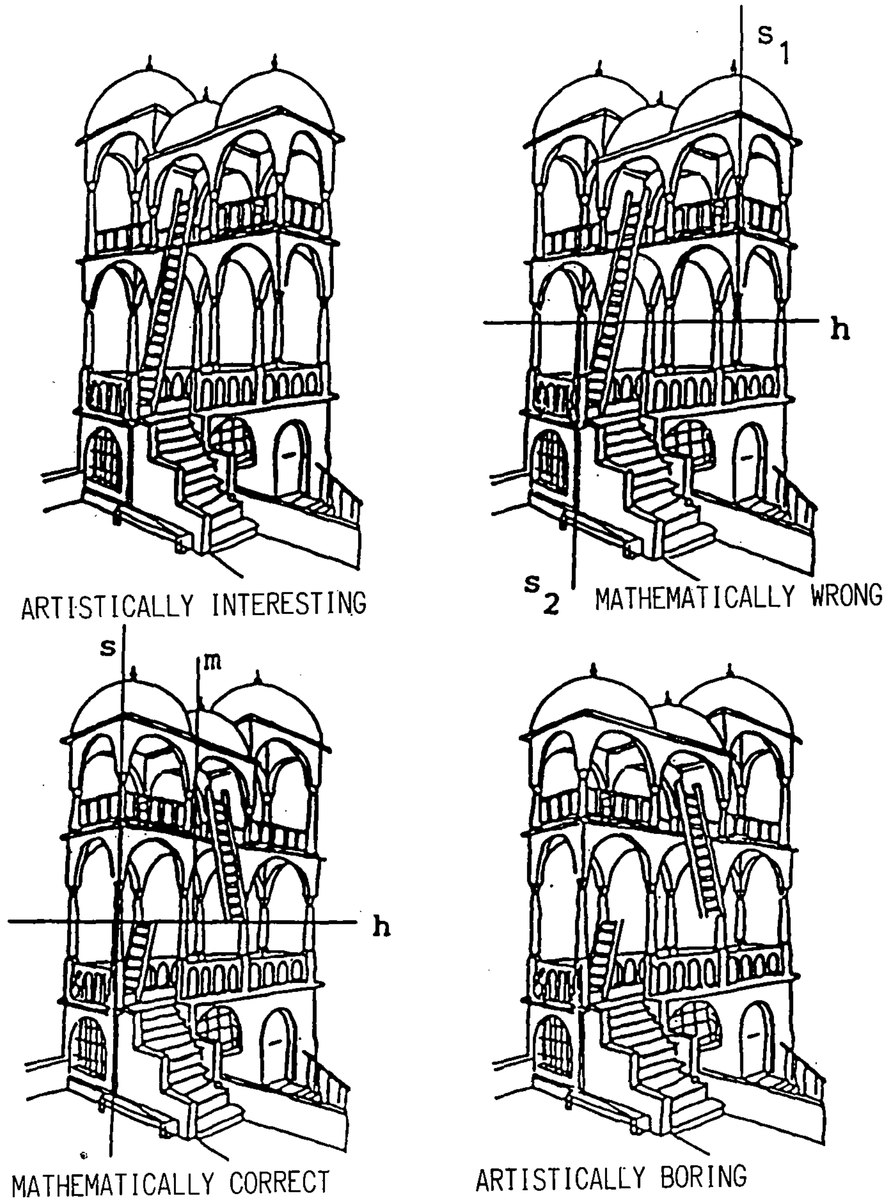

s1
h
ARTISTICALLY INTERESTING
s2
MATHEMATICALLY WRONG
s
m
h
MATHEMATICALLY CORRECT
ARTISTICALLY BORING

FIG.6

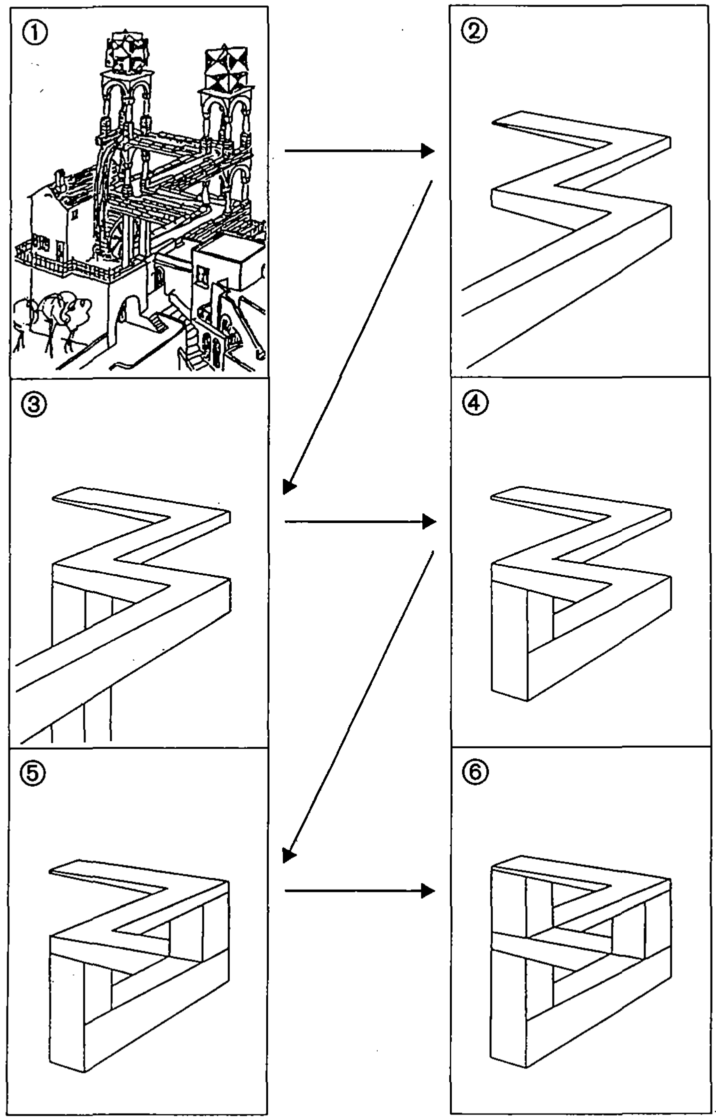


FIG.9

Similar aspects of violation of mathematical perspective may be analysed in regarding ESCHER's lithography "Waterfall" (1961) and discovering the three times hided >tribar< (fig.9).

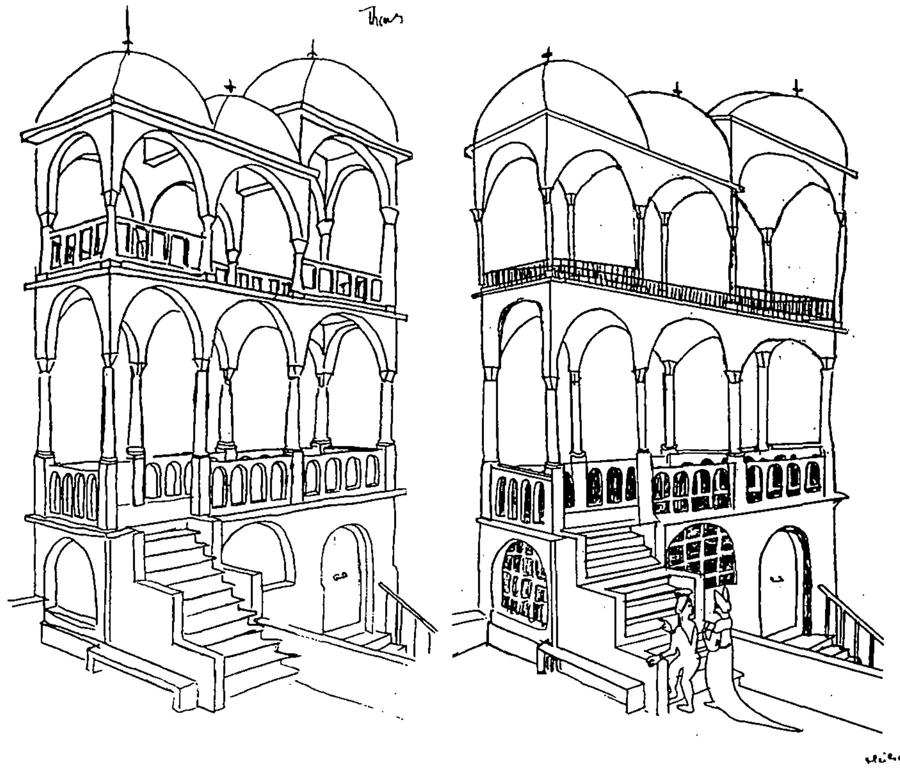


FIG.7-8

4 Visualization as Reconstruction

In contrast to verbal and also symbolic presentation visualization proves to be a more subtile method of activating mathematical structures in pupil's imaginations (Peters 1988,p.44-67). Visualization is very close to making visible mathematical theorems by interpreting "to prove" as >δείκνυμι<. The term >θεώρημα< also means: having seen, considered, examined, and understood. A theorem becomes obvious, evident, manifest, clear, because its construction was the "oject of a vision". Visualization means to demonstrate essential structures, to give insights into the mode of a constructing strategy, to look into the constitution of a theorem. Visualization is a didactic provocation for reconstructing mathematics by the pupils.

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