Symmetry of STRUCTURE

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Abstracts

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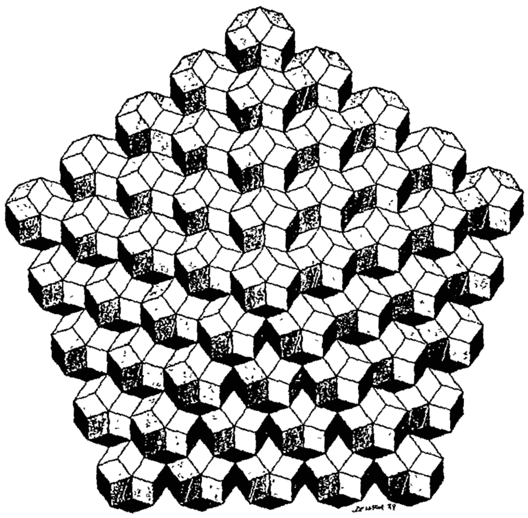

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FOLDING A PLANE -SCENES FROM NATURE, TECHNOLOGY AND ART

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SUMMARY

The purpose of this paper is to explain the solution of the geometric proposition "folding a plane into a point", and to show its appearance in natural phenomena, technology, design and art.

FOLDING A PLANE - PLATE ELASTICA -

The deformation represented typically by that of a thin sheet of paper is called in terms of geometry the isometric transformation. The meaning of "isometric transformation" is that any linear segment of the paper does not change its length through such deformation. Or it is said that the Gaussian curvature is unchanged. Since the Gaussian curvature is zero for a flat plane, this value is kept everywhere in any stage of transformation.

This basic principle is, though not rigorous, valid for thin sheet of various material, where the bending deformation is predominant over the in-plane deformation.

The proposition "folding a plane into a point" is described graphically in Fig. 1. If we consider an infinite plane, the solution must be periodic. The idea in solving this geometrical problem involves the application of the theory of elasticity which consists of the following syllogism. Let us consider it hypothetically, as if it is an elastic deformation problem of a plate. Then the problem can be solved by the finite deformation differential equation for a plate presented by von Kármán. By making the thickness of the plate unlimitedly close to zero, the first geometrical problem can be solved(1,2).

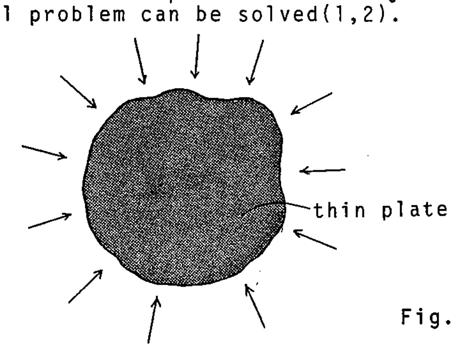

Fig. 1

Figure 2 shows arbitrarily selected 10 solutions calculated using the procedure described above. These show the contour lines of their out-of-plane deformation. When the least energy solution is sought among these solutions, it is the surface having a beautiful symmetry composed of repetition of the fundamental region which is further composed of four congruent

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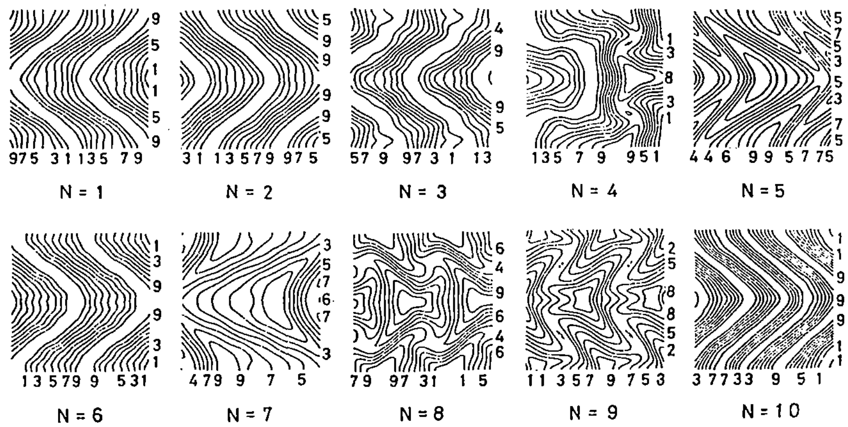

Fig. 2

parallelograms as shown in Fig. 3.

The meaning of this solution can be explained by a comparison with the corresponding one-dimensional problem, that is, folding a slender elastic column. The problem was solved by Euler and the curves thus obtained are called "elastica". In the similar manner, in case of the present two-dimensional problem the surfaces are to be called "plate elastica". We know that the column elastica has such a smooth curve which sometimes symbolizes the elastic deformation. On the contrary, the plate elastica is characterized by sharp ridge lines. This result indicates that the characteristics of deformation are essentially different between columns and plates.

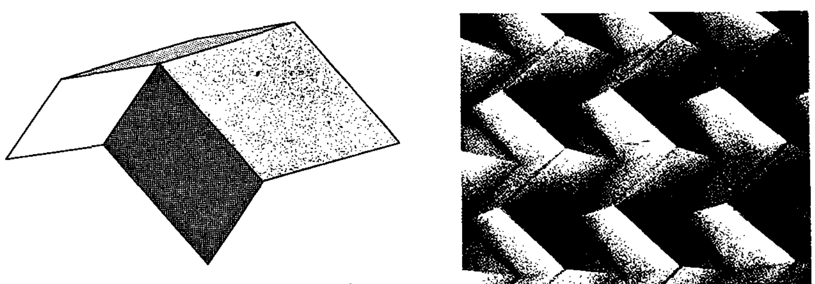

Fig. 3

DESIGN OF A FOLDED MAP

There are three problems with maps folded at right angles in the conventional manner. First, an orthogonal-folded map requires an unduly complicated series of movements to fold and unfold it. Secondly, once unfolded there is a strong possibility that the fold may be "unstable" and turn inside out. Finally, right-angled folds place a lot of stress on the paper inducing, almost without exception, tears which begin where two folds intersect.

The key to an alternate system of map folding lies in the "plate elastica". It is to use a variant on concertina folding to produce a slightly ridged surface composed of a series of congruent parallelograms, that is, the shape of "plate elastica"(3). The most important point of difference from an orthogonal folded sheet is that the folds are interdependent. Thus a movement along one fold line produces movement along the other. In other words, the user can open the map by just one pull at a corner(Figure 4 from Ref. 4).

The new method also solves in part the other problems. Interdependence of folds means that it is very difficult to reverse them and the amount of stress place on the map sheet is also reduced because only one thickness of paper comes beneath the second fold, avoiding the need to fold several sheets.

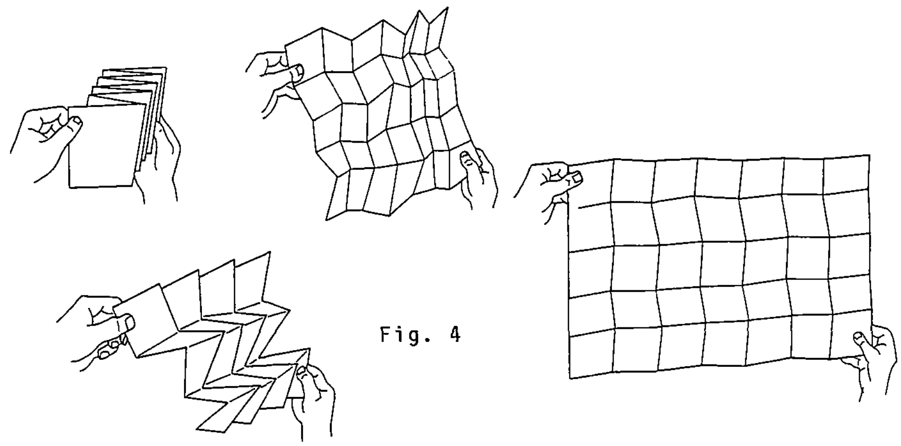

Fig. 4

APPEARANCE IN NATURE AND ART

The exact symmetry of the geometric solution will never appear in the natural phenomenon because of the fuzziness inherent in the nature, though the essence of the symmetry is always kept in it. It means that the deformed surface of thin plates are much like the irregular patterns appeared in Fig. 2. Such examples are observed in our everyday life as a crashed paper, a body shell of a crashed automobile etc. Furthermore, such irregularity is used for the means of expression by some artists. Figure 5 is a part of the bronze relief at the entrance hall of our Institute by a sculptor Hisao Yamagata, and we could see both the regular and the irregular pattern merging with each other.

Fig. 5

APPLICATION TO SPACE TECHNOLOGY

Large planer membranes are mandatory for many of space missions in near future. Solar arrays, solar-power satellites, solar sails, space radars are typical examples. Therefore, the technology necessary for the construction and packaging of these large membranes on the ground and their deployment in space must be established. The technology described in the section of the folded map was originally invented in order to cope with such problem of packaging of large membrane space structures. For the purpose of testing its feasibility, the project are under way to launch a two-dimensional deployable array on board the space platform scheduled for 1992(5). It is a large thin-membrane solar cell array which will be simultaneously deployed in two orthogonal directions(Fig. 6).

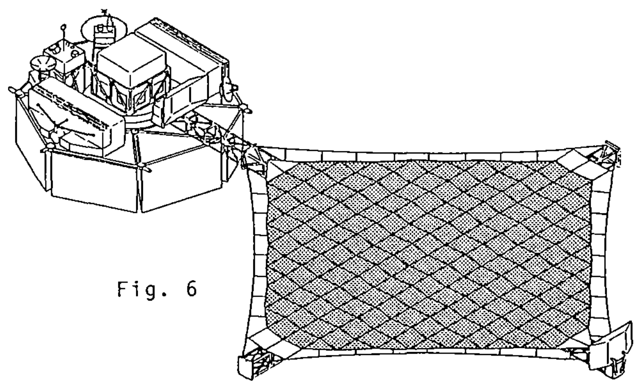

Fig. 6

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