Symmetry of STRUCTURE

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Abstracts

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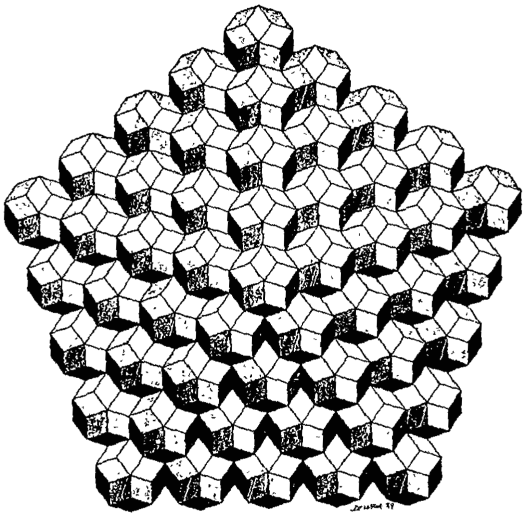

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ABSTRACT Fermat's Search for Symmetry of Triangular Numbers
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In Fermat's philosophy of mathematics, his search for symmetry as a fundamental intellectual principle is nowhere else as clear as it is in the problem complex surrounding the Last Theorem. We show that he conceived his attempted proof as a symmetry between triangular numbers and their powers in case they also constitute triangular numbers. This strategy of proof poses the question: Was Fermat justified in his claim that he had invented "a truly remarkable proof" to his Last Theorem? The key into FLT is the fact (Prop. 1) that if there is a non-zero integer solution $(x,y,z,n>2)$, (x,y,z) are sides of a triangle.

I Fermat's justification is studied by outlining first an historical scenario of the antecedents of FLT as a working hypothesis. It consists of three propositions and lemmas (Props. 1—3 and Lemmas 1—3, with sketches of the proofs). These are elementary statements well within Fermat's reach and yet give a geometrical illustration of FLT. Their novelty is Prop. 1 first suggested in (13:153—154). These antecedents are called Fermat's heuristics. They were presented at The 8th Int. Congress of Logic, Methodology and Philosophy of Science in August, 1987 in Moscow, USSR (in Section 1: Foundations of Mathematical Reasoning).

II The power of the outlined antecedents is measured by means of conclusions drawn from them (Prop. 4 and Lemma 4), comparing the conclusions with modern results. In drawing these conclusions, only methods known from Fermat's own or his predecessors' works are employed. The comparisons indicate, however, that Fermat anticipated (granting his heuristics consisted of Props. 1—3 and Lemmas 1—3) much later results. In particular, Prop. 4 is more general than Terjanian's result in 1977 at C.R.Acad.Sci. Paris 285, and Lemma 1 gives a better bound than M.Perisastri in 1969 at Amer. Math.Monthly 76. Lemma 4, in turn, offers a more promising way to an estimate of the exponent ($n = p$ an odd prime) than Grünert's lower bound for an eventual solution to Fermat's equation in 1856 at Archiv Math. Phys. 27. These are the first mathematical results.

III Further conclusions and comparisons are made possible by Lemmas 5-6. They transform the problem and set the question of Fermat's justification into a new light. Lemma 7 gathers together some results depending on Prop. 4. But that is only a watershed. A definitive answer is possible only if the final Prop. 5, FLT with odd exponents in one version, can be proved by Fermat's methods. Aiming at the proof, Porisms 1-3 and Lemma 8 are given. Enter Proof Reconstruction.

IV In the philosophical part, the implications of the foregoing heuristic, historical and mathematical considerations are outlined. They constitute, in our opinion, Fermat's true legacy with an impact on modern philosophy of mathematics and philosophical cosmology. In fact, this philosophical legacy runs parallel to Hamilton's research program which he gave up in favour of the quaternions (184 . Although Fermat's FLT and his Principle of the Least Time in optics are parts of his legacy, they are but the tip of the iceberg.

FERMAT'S HEURISTICS

It is 350 years since Fermat scribbled his "Last Theorem" (FLT) in the margin of his copy of Diophantus [3]. Despite recent advances, esp. Gerd Faltings' result (1983) and Yoichi Miyaoka's near-proof (1988), neither the mathematical nor the logical efforts nor yet computer calculations have been sufficient to solve the problem [cf. 15:2-3]. In the beginning of our century, Hilbert believed that the solution will be found [8]. In mid-1930's ,however, unsolvable problems were gathering and the Theory of Algorisms was developed by Church and Turing. After the works of Post, Markov and others (c. 1947-1952), a negative solution was suggested to Hilbert's Problem X by Davis, Davenport, Putnam and Robinson (1953-1960). In 1970 it was found by Ju.V.Matijasevič and G.V.Čudnovskij [13 :136-7]. This negative solution to the decision problem of a general Diophantine equation, although it does not rule out the possibility that the particular Diophantine equation FLT could either be positively solved or proven impossible to solve, reduced much of the hope [13:153].

Today, especially in Analytical Philosophy, FLT is often quoted as an example of Gödel's "true but undecidable statements" [15 :216-8]. This is intellectual laziness. Gödel's result is of existential character and must not be used as a problem-killer. It is not worthwhile to claim conceptual command of a particular problem that one cannot solve. There is no rational reason for believing that just FLT is undecidable.

Other attempts having failed (so far), we suggest an additional study of Fermat's antecedents [13: 153-4]. For it is fairly sure that he did not invent anything like the abstractions of modern Number Theory, and definitly did not anticipate the latest results of Theoretical Physics (which Miyaoka made use of). Ours is, therefore, a Requiem to Fermat's predecessors, in particular to the Pythagoreans and Euclid, Diophantus and Pappus.

There are two separate problems: (i) to prove FLT using concepts and methods available to Fermat, and (ii) to prove FLT by whatever means. The present day is inclined to the latter approach. The former one is more demanding, probably more elegant, and certainly closer to rules of fair play.

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