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Abstracts

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Symmetric Groups – Music – Pedagogics
(Abstract)

Abstract structures, which are the topics of mathematics and which take part in Poppers World 3 are created or discovered – this depends on the philosophical meaning – by the human mind. They are inspired both by perception and rational reflection and sometimes moreover by an aesthetic feeling, so e.g. Platon·s explanations in "Timaios" construct the theory, that triangles and minimal euklidian solids are the first components of the physical world.

Let·s start now from the point, where we accept the existence of abstract algebraic, topologic etc. structures.
In an integral way of thinking analogies play an important role, and under this proposition it is ingenious to discover analogies between mathematical structures and structures of music. The central theorem of the statements the author holds is, that those analogies are not a priori but must be based on a special way of constructing them. In algeba something like this (in a very strong way) is called isomorphism.
In a very similar way isomorphisms or principles of transformations can be created, so that it is possible to say, that two objects, e.g. algebraic structures and musical events, show analogies.
Here three kinds of such isomorphisms are explained and explored as a basic of pedagogic reflections: a formal, an aesthetic-heuristic and a depth-psychological one.

1.The formal isomorphism:
Let us take an algebraic structure, especially a symmetric group. We want to create a formal isomorphism between this group and music.
1.1.First the question arises, which elements here should be adjoined to which elements there. Concerning the groups we take "permutations", the real elements of the group, concerning music we choose pitches of a note. This is totally arbitrary thus it would also have been possible to choose special rhythms, the duration of notes etc. This is the step, where we regulate the Objects, we deal with.
1.2.The next problem is the range of the notes, quasi a kind of mode. In an example already put into practice we worked with the symmetric group S4, which has 24 elements. It seemed to be natural to take either an octave divided into 24 quatertones or a chromatic scale over the range of two octaves. With regard to the possibilities of the school, the second way has been chosen, so that it was possible to make experiences with classic instruments.
1.3.The next problem is the fixing of the coordination between group-elements and the pitches. Like the former stipulations also the solution of this problem is arbitrary, perhaps heuristically influenced. The modus, which was taken in our pedagogic process was influenced in this manner: (1234) was adjoind to C (the idendity), (1243) to Cis, (1324) to D, (1342) to Dis etc. This yields a mathematical function between the set of the elements of the symmetric group S4 an the notes of the

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range over two octaves.

1.4. The next question is what to do with this function. First it is intended to form music which is connected with the structure of the symmetric group S4. There two fundamental ideas were found:

1.4.1. A group has special sub-structures, especially subgroups, that is to say those subsets, which build together with the groupdefining multiplication a group too. According to the fixed function these subgroups yield special accords. These can create a set of tones, on which improvisations may be based.

1.4.2. The multiplication of two group-elements is a well defined group element. Now we can begin with some subset of the set of group-elements. This means beginning our piece of music with a special accord. Each element of our subset can be multiplicated with another element of this subset, so that the set of all possible multiplications is a new accord, which has been created by the former accord. A set of three elements normally formes another a set of three elements (a result of two elements is possible, too). This set yields a new one and so on. The piece is (not in the rhythmic way) totally determined by the beginning accord.

1.5. In our pedagogic process these reflections were fundamental. First the pupils learned that analogies do not exist without depending on special identification rules. Analogies are built by the human mind, or at least, only these analogies are — in the sense of Kant — recognizable by a human beeing.

The next step was the recognition of the possibility, to transform algebraic structures (which are beautiful in a special sense) to sound, a possibility, which is in a special sense much more determined than a piece based on a heuristically built dodekaphone trope.

The next pedagogic sequence is based on the improvisation over this sound-structures, where both possibilities 1.4.1. and 1.4.2. are practised.

2. The aesthetic-heuristic isomorphism:

Intermedial transformations are a central problem of polyaesthetic education. This chapter can be submitted to this field of questions. You can try, once having unterstood the structure of symmetric groups (as well as other abstract structures), to transform them to a piece of art, to music, to concrete poetry etc. This demands a complex act of creativity and depends moreover both on the cultural experience of the transforming individuum and the internally represented adventure of the moment.

Here the transformed results are not formally determined but must be explored in regard to their social and psychic, and sometimes scmatically influenced, sources, especially those that are highly relevant for arts.

This aspect completes the process as a pedagogic one, in which the pupil exercises his abilities in creative problem solving activities (especially in both an artspecific and an abstract way, the latter sometimes influencing the creativity in respect to other problems) and cultivates his experiences with the arts. Moreover those processes concerne not only a problem of learning but may also affect more vital aspects and even lead to the borders of meditation, of philosophical reflections concerning the "beeing" as it is, of an idea of eternal "beeing" and of an eternal circular course, as e.g. the biblical Kohelet expresses (Ecclesiastes or the Preacher): "A generation goes, and a

generation comes ... the sun rises and the sun goes down, and
hastens to the place where it rises ... round round goes the
wind, and on its circuits the wind returnes ...". His
reflections influence the realisation of the improvisation of
musical drama.

3.The depth psychic transphism
In point 2 the transformations were mainly done during a status
of vigilant conciousness. Now mainly unconcious processes will
create transformations of symmetric groups to music.
First the pupils learn to know the symmetric groups,
permutations and their multiplications etc. Then a light trance
induction is done in class. During a light hypnotic status the
pupils get the suggestion of letting permutations round, and
letting symmetric groups become audible. This is possible
because of the phenomenon, that in trance as well as in dreams
man can adjoin special medial characters to objects, characters,
which they don't posess in reality. E.g. words can get a human
form, qualities begin to talk etc. Therefore it is possible (and
it was so), that a unconcious transformation is done, which
depends mainly on a depth-psychic work.
Such transformations are quite different to the strongly
determined ones of 1. and the heuristic-aesthetic ones of 2.
Both the result and the way of creating them are different. It
is to suppose, that this processes could become important in a
psychoanalytic diagnosis and therapy. The results first are
internal, which means, that – if we want to hear the creation of
the unconciousness – the sound, the pupil heard during his
trance, must be realised. This is as problematic as e.g.
Tartinis translation of his famous violin sonata, which he heard
during a dream (where it is to point out, that man when dreaming
recognizes in a different way than when beeing awake) play by
the devil and which became his most famous work although he
maintained, that his sonate was so weak, that it would not be
compared with the "original" sonate of the devil.
Somehow this way of creating music can be compared with that
way, where musicians take drugs for being stimulated and even
carried off and produce music in a status of tranformed
conciousness. In our example of school not drugs but natural
possibilities of the human psyche are used.

(The report of our researches concerning this theme will show
the pedagogic way and some results.)