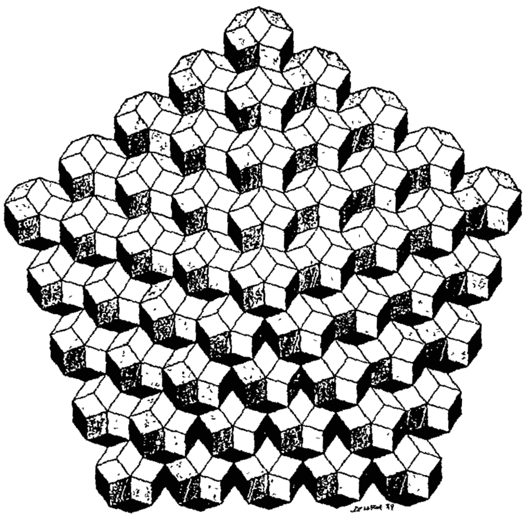
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

II.



Edited by Gy. Darvas and D. Nagy

Buda pest

August 13-19, 1989

hungary

FIBONACCI NUMBERS AND THE GOLDEN SECTION : AN INTEGRATIVE,COMPUTER-ASSISTED CURRICULUM

LIVNEH, Yosef

School of Education,Haifa University, Israel

This lecture will describe in outline ideas associated with the Fibonacci numbers and the Golden Section forming the basis of an integrative-interdisciplinary study program encompassing their application in mathematics, science, architecture, and the arts. Part of the curriculum involves the creation of computer programs which reveal in depth the interrelationships between the Fibonacci numbers, the Golden Section , and the logarithmic spiral. So that these programs will be accessible to secondary school pupils, the programming language employed will be LOGO.

Such a curriculum seeks to achieve the decompartmentalization of knowledge inherent in many of today's programs in the sciences and the arts. An integrated approach in education is vital today in view of the fragmentaion of disciplines stemming from the exponential increase in man's knowledge over the past century (Phenix 1958: 325). This fragmentation has had dire consequences for our educational system robbing it of many of its humanistic values and degrading it to the level of preliminary vocational training in the three R's (Wynne 1970: 192). This tendency towards fragmentation is most accentuated in the divergence of the sciences from the arts. The British scientist and man of letters, Lord C.P. Snow, put the case most forcefully in his famous "Two Cultures" (Snow 1956).

This reality requires any educational philosophy to take a stand. If integration is desired, study topics and programs must reflect this. As a result topics will be selected not based solely on traditional considerations, but also on their value in bridging the gap separating aesthetics from science. Such is the case of the Fibonacci numbers and the Golden Section.

Curriculum Content

Fibonacci represents a towering figure in the history of mathematics. Fibonacci was important primarily as an agent for the introduction of the mathematical achievements of the ancient Eastern Cultures to the Western World emmerging from the Middle Ages in the first half of the Thirteenth Century. As such he deserves attention from the point of view of the intellectual development of Western Culture and its debt to the achievements of the Eastern one (Cajori 1961:120-126; Eves 1980: 160-168).

The Fibonacci series represents a simple, divergent, and non-proportional infinite series whose terms nevertheless converge upon a significant, finite ratio, i.e.: the Golden Mean. This finite ratio is an irrational number ($(\sqrt{5}-1)/2$). Such an irrational ratio was, as Spengler put it "alien to the Classical soul" (Spengler 1956:2324; also Lawlor 1982:38; and Eves 1980:53) and this aversion survived in Western Culture for another thousand years at least.

Fibonacci numbers (1,1,2,3,5,8,13,21,34,55,89,...) appear as important organizing principles in nature most notably in the process of phyllotaxis (Hambidge 1967: 146; Coexter 1961: 169-172) . They can be generated in a number of ways: Fibonacci's original "Rabbit Problem"(Vogel 1971: 607), a bee's traverse of a honeycomb (Lake and Newmark 1977:190), bee genealogy (Gardner 1981:162-164), the "Checkerboard Paradox" (Hunter and Madachy 1963:12-13), or Hofstadter's method of "expanding nodes" (1979: 135-137) among others.

Fibonacci numbers posses special properties which should be explored(Lake&Newmark 1977:188-192; Gardner 1981:159-62; Wells 1986: 61-7;Hunter and Madachy 1963:19-22; Alfred,B.U. 1963:57-63).

The cognitive content at this stage draws on arithmetic, combinatorial, and deductive skills. The above-mentioned generating options differ widely in terms of inherent difficulty thus permitting a concentric approach whereby one arranges them by ever-increasing degree of difficulty. With each additional example the students' anticipation of the imminent appearence of the "magic" numbers increases.

Once a recursive method for generating the numbers is available, the students can proceed to program a computer to produce these numbers. The transition from concept to recursive formula to algorithm to computer program is an essential part of comprehending the art/science of programming. Rarely is it so forecefully but easily exemplified. Witness the almost ubiquitous appearence of the Fibonacci numbers in introductory texts in computer programming(Tremblay&Bunt 1979:447-449; Forsythe 1969:9-10).

The Golden Mean,also known as the Golden Section,has been a central pillar of aesthetic philosophy from Aristotle's time down to the present. The Golden Mean represented an aesthetic discovery of immense importance which was applied by the ancient artists and artisans in the fields of the plastic arts (sculpture, pottery,painting) and in perhaps their supreme achievement: their architecture (de Lucio-Meyer 1973:61-67; Eves 1980: 48-49).

What they learned has been repeatedly applied in these and other related fields. Islamic Art is a blend of the organic and the geometric. Central to the designs of Islamic Art are the Vedic square and the Fibonacci series (Albarn et al 1974:12-32). The Renaissance artists applied, often consciously, the principles of proportion and perspective based on, among other principles, the Golden Section (Lawlor 1982:53-63; Walker 1978:53; Kline 1962: 203-230).In his <u>Divina Proportione</u> (1509), Pacioli attributed remarkable properties to the Golden Mean. As illustrator for the book he chose none other than Leonardo da Vinci ! (Pedoe 1976: 263-265).

Even in this century of "free", "uninhibited" creation the Golden Section has been exploited by such prominent artists as Frits Mondrian and le Courbusier. The two Twentieth Century movements de Stijl and le Section d'Or Group (Honour and Fleming 1982: 598-603; Hill 1977: xii; Wittkower,R 1978: 120) drew their inspiration from the contemplation of symmetry relationships such as the Golden Section.

The Golden Mean can be derived from the Fibonacci numbers by both empirical and analytical means (Huntley 1970:46-7; Wells 1986 :36-40; Wilf 1986: 30-31). Again, a concentric approach is utilized whereby the students will be led to appreciate the way in which the laws of geometry and algebra can be used to verify in a general way the results of a finite number of trial calculations.

Here too a number of computer activities offer themselves. Using "LOGO", students can construct collections of rectangles based on various proportions, including that of the Golden Mean, testing their perception of proportion. They can even create a psychological testing model by offering their drawings to impartial subjects and measuring their relative preference for the "Divine Proportion".

Again, using "LOGO", the students can create a composition of "Golden Rectangles which when viewed in "Real Time" creates the sensation of "gnomic growth" (Huntley 1970: 169-170;Lawlor 1982: 65-66). This is in fact the formula for the construction of the logorithmic spiral (recursive applications of Golden Rectangle+Turn).

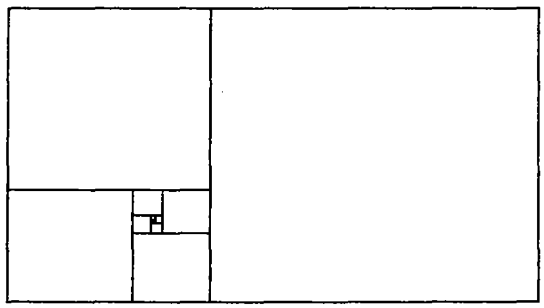
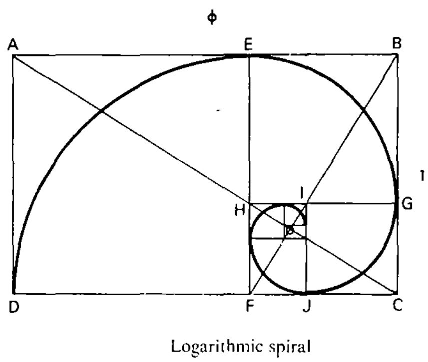
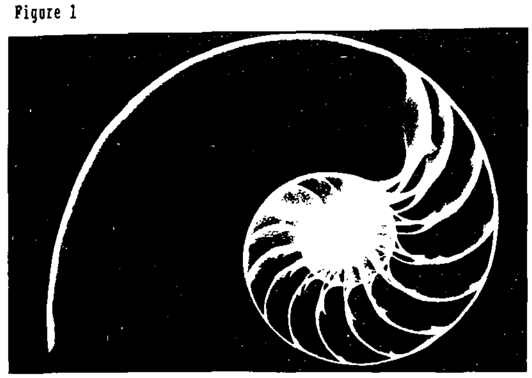


Figure 1



Logarithmic spiral

Figure 2a



Nautilus pompilius

Figure 2b

The logorithmic spiral itself is an exemplary integrating theme for studies in botany, zoology, and architecture(Lake&Newmark 1977:188-90; Stevens 1974:159-66; Bergamini 1972: 93; Huntley 1970: 164-76;Lawlor 1982:65-73).

<u>Building a Bridge to Aesthetics</u>

Music, and the fine arts in general, have a unique contribution to make in any educational program. The arts' contributions include : the refinement of sensory perception, the deepening of appreciation for the individual and the particular as opposed to the general and universal; the heightening of respect for the products of one's and others' creativity (Phenix 1958: 424-440).

Any program possessing a fine arts component will undoubtedly have aesthetic and emotive effect. What the <u>specific</u> effects will be cannot necesssarily be predicted or prescribed with the same degree of precision as its cognitive content. It would seem, however, that the program described here should include, inter alia, the following aesthetic and emotive values:

<u>Organizing principles in artistic expression :</u>

Some envisage art as pure expression while others as the "working-out" of potentialities within an axiomatic framework of artistic ideals. This dispute has been heavily debated down through the centuries(Wittkower 1978).

Even accepting that the artist is unaware of symmetry considerations in his creations, the inescapable conclusion lies in the completed work itself: a large body of artistic creation throughout the millenia embody the "working out" of problems of proportion and composition that <u>can be</u> expressed in mathematical terms even if the artist did not deign to do so (Arnheim 1962: 218; Ghyka 1977: 174; Hedian 1976).

<u>The mutual affinity of math and music :</u>

The integration of music and mathematics in an integrated curriculum seems to be <u>particularly apt</u> and effective. Beginning with Pythagoras' discovery of the laws of harmony through Helmholtz's classic experiments in acoustics it is clear that music and mathematics are inevitably intertwined (Huntley 1968: 51-6; Reik 1953:121)

It has been suggested that part of the reason for this mutual affinity lies in the shape of the auditory organ, the cochlea of the inner ear, with its spiral shape whose shape is ruled by the pattern of logarithmic growth (Lendvai 1966:190; Anfilov 1966: 112-3).

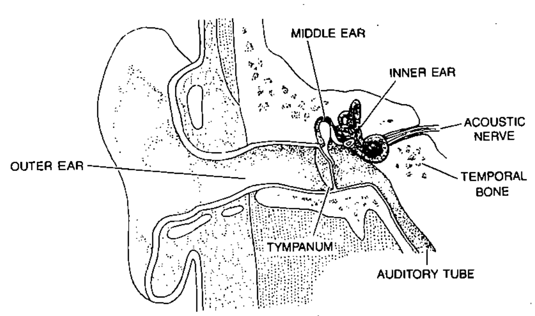


Figure 3a

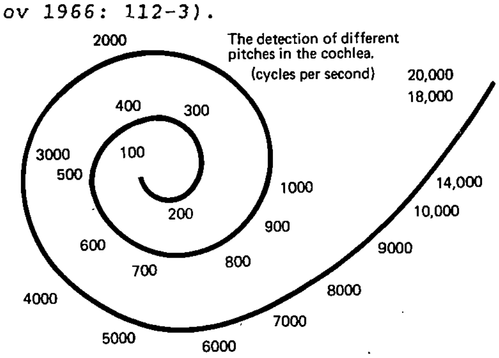


Figure 3b

The artisans who built the stringed instruments of the Renaissance (the "luthiers") made liberal use of the Golden Section (Coates 1985). In general, instrument design is based on three considersations : acoustics, ergonomics, and aesthetics, which harmonizes and humanizes the previous two. The use of geometry here was different from that architecture or painting. Proportions were not planned nor perceived directly. They resulted from considerations which superficially had nothing to do with proportion and geometry. The fact that the products of these designs embody basic proportions such as the Golden Mean implies that it is inherent to them and provides the necessary principle for their expression. The artisan employing precepts such as the Golden Section could enjoy the certainty and security that its application provides. The resultant symmetry provided him with a sense of "rightness" and was communicable leading to a universality of form. The luthier thereby embraced the two Platonic arts of music and mathematics by applying geometric considrations to his design.

From these two examples alone it should be clear that polyaesthetic education need not be separated from the study of mathematics and science. Building bridges between the fields does not merely serve to increase the amount of knowledge transmitted, but to enrich one's appreciation for the universality of the knowledge thus acquired.

BIBLIOGRAPY

1. Albarn,K.,Smith,J.M.,Steele,S. and Walker, D. (1974) The Language of Pattern. London:Thames and Hudson.

2. Alfred,B.U. (1963) "Exploring Fibonacci numbers". Fibonacci Quarterly,1(1): 57-63.

3. Anfilov,G. (1966) Physics and Music. Moscow: Mir.

4. Arnheim,R. (1966) "A review of proportion". In Kepes,G.(ed.). Module,Proportion, Symmetry, Rhythm. New York: George Braziller,218-230.

5. Bergmini, D. (1972) Mathematics. New York: Time-Life.

6. Cajori,F. (1961) A History of Mathematics. New York: Macmillan.

7. Coates,K. (1985) Geometry, Proportion, and the Art of Lutherie. Oxford: Clarendon Press.

8. Coexter,H.S.M. (1961) Introduction to Geometry. New York: Wiley.

9. de Lucio-Meyer,J.J. (1973) Visual Aesthetics. London: Lund Humphries.

10. Eves,H. (1980) Great Moments in Mathematics Befroe 1650). New York: The Mathematical Association of America.

11. Forsythe,A.I.,Keenan,T.A.,Organick,E.I., & Stenberg,W. (1969) Computer Science: A First Course. New York: John Wiley.

12. Gardner,M. (1981) Mathematical Circus. Middlesex: Penguin Books.

13. Ghyka,M. (1977) The Geometry of Art and Life. New York: Dover.

14. Hambidge,J. (1967) Dynamic Symmetry. New York: Dover.

15. Hedian,H. (1976) "The golden section and the artist". Fibonacci Quarterly,14(12): 406-418.

16. Hill,A.(1977)."Introduction". In Malina,F.J.,Ed. VisualArt,Mathematics,and Computers. Oxford: Pergamon Press, iv-xiii.

17. Hofstadter,Douglas R. (1979) Godel,Escher,Bach: An Eternal Golden Braid.New York: Basic Books.

18. Honour,H. and Fleming,J. (1982) A World of History of Art. London: Macmillan.

19. Hunter,J.A.H. & Madachy,J.S. (1963) Mathematical Diversions. Princeton: D. van Nostrand.

20. Huntley, H.E.(1968) The Divine Proportion. New York: Dover.

21. Kline,M. (1962) Mathematica: A Cultural Approach. Reading: Addison-Wesley.

22. Lake,F.and Newmark,J.(1977) Math as a Second Language. Reading:Addison-Wesley.

23. Lawlor,R.(1982) Sacred Geometry. London: Thames and Hudson.

24. Lendvai,E. (1966) "Duality and Synthesis in the Music of Bela Bartok". In Kepes,G.(ed.). Module, Proportion, Symmetry, Rhythm. New York: George Braziller,174-193.

25. Pedoe,D. (1976) Geometry and the Liberal Arts. Middlesex: Penguin Books.

26. Phenix,P.H.(1958) Philosophy of Education. New York: Holt,Rinehart & Winston.

27. Reik,T. (1953) The Haunting Melody. New York: Da Capo Press.

28. Snow,C.P.(1956). "The two cultures". New Statesman,52(1334): 413-414.

29. Spengler,O. (1956) "Meaning of Numbers". In Newman,J.R.(ed.). The World of Mathematics. New York: Simon and Schuster, 2315-2347.

30. Stevens, Peter S. (1974) Patterns in Nature. Boston:Little Brown.

31. Tremblay,J. and Bunt,R.B.(1979). An Introduction to Computer Science: An Algorithmic Approach. New York: McGraw-Hill.

32. Vogel,K. (1971) "Leonardo Fibonacci". In Gillispie,C.C. (ed.). Dictionary of Scientific Biography. New York:Charles Scribner's Sons,IV: 604-613.

33. Walker, D.P. (1978) Studies in Musical Science in the Late Renaissance. London: Warburg Institute.

34. Wells,D.(1986) Dictionary of Curious and Interesting Numbers. Middlesex:Penguin Books.

35. Wilf,H.S. (1986) Algorithms and Complexity. Englewood: Prentice-Hall

36. Wittkower,R. (1978) Idea and Image. London: Thames and Hudson.

37. Wynne,J.P. (1970) Philosophies of Education. Westport: Greenwood Press.