# Symmetry of STRUCTURE

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Abstracts

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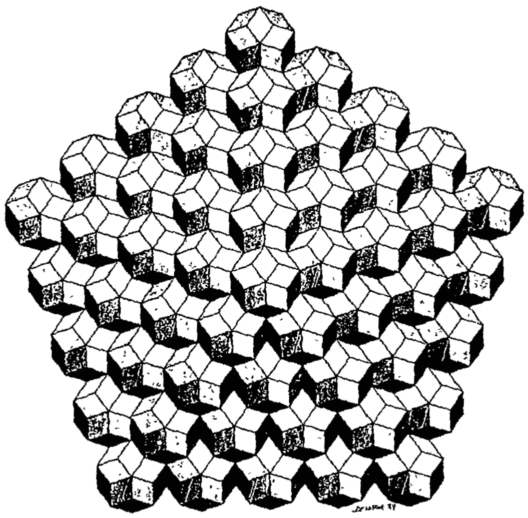

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# Symmetrical Structures in Building Construction (Extended Abstract)

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## 1. Background

First we look back at the history of prefabricated building systems. Prefabricated elements in buildings are almost as old as the history of house building itself. many parts of the house, such as the wall and roof, have throughout the history been made of identically formed elements. Since the industrial revolution, mass production has become an important part of the building industry. The first major milestone in the development of industrial methods in the architecture was probably Paxton's Crystal Palace erected in 1851 [Kie84]. The Crystal Palace as well as all subsequent prefabricated buildings possess attributes which are different from those of traditional buildings. Construction problems have usually more consequences throughout the design of prefabricated buildings then in the design of traditional ones. Thus, the design of construction parts play an important role in the design of prefabricated buildings. Many famous architects, such as Buckminster Fuller, Jean Prouve o. Konrad Wachsmann perceived the necessity of studying construction problems. Wachsmann especially, who led the Division of Building Research at the University of Southern California in the 60's, has made many contributions to the development and analysis of construction parts of prefabricated building systems.

The design of construction parts is a complex task, involving geometric, static, kinetic, esthetic, and other problems. Geometric problems, however, are often dominant and have larger impact on the design then other related problems. Since repeating symmetrical patterns can provide economically and esthetically appealing solutions, architects became interested in crystallography, cell biology and group theory [Pie78], [Wil72]. Although symmetrical structures spread fast in the architecture and interesting constructions emerged by designers such as Buckminster Fuller, Felix Candella, Pier Luigi Nervi or Frei Otto, not much research was done on the geometry of construction parts leaving numerous problems still unsolved.

Here we don't attempt to solve any of these problems, however, we show connections between symmetry concepts, such as tilings and building constructions.

## 2. The construction problem

Let us consider a number of square slabs, without specifying the function of the slabs as construction parts. The slabs are connected as shown in Figure 1 by male and female elements on each edge. One can easily see that the connection of the slabs is realizable if and only if neither of the two slab patterns to be connected has a concave boundary. For example, we can connect three slabs as in Figure 1(b) but we cannot connect a forth one to them. We can solve the problem by connecting two pairs of slabs as in Figure 1(c), or connect all four slabs at once by rotating the slabs as in Figure 1(d). In both cases there are other restrictions according to the number of slabs to be connected by one movement. Another method of connecting slabs as, e.g. in Figure 1(e) has other cosequences, caused by the static properties of the connecting parts.

Summing up the consequences of these examples, we can see that the geometrical form of the slabs and the

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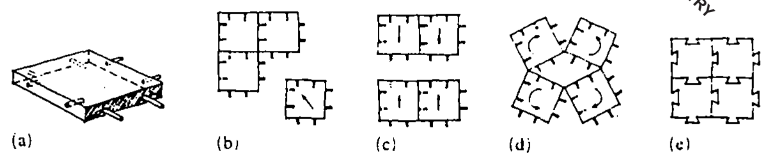


Figure 1: Connection rules for building tesselation from square slabs

connecting parts determines important properties of the entire system Such properties are:
1. the oder of assembly of the elements (which elements can be connected to which one and what kind of pattern can be achived),
2. the motion of the elements to produce a connection,
3. static characteristic of the system.
However, according to these points some geometrical properties of the slabs do not play any role, e.g. the size of the slabs or the fact that they are square. The above properties are valid for any rectangular tesselation. Furthermore, the exact form of the connection is irrelevant because all connections that allow only one motion in one direction in the plane of the tesselation possess the same properties.

Analysis of several construction systems has shown that such geometrical or topological characteristics imply a couple of fundamental rules. The knowledge of these rules can support the design process and lead to solutions that will probably not be found without them. By following the rules, buildings can be constructed from elements with very few data, in contrast to the usual documentation that contains a lot of unnecessary, often redundant information. In the realisation phase, it will be possible to use the rules for the automation of the building procedure. With a set of building elements, a list of connection rules and a description of the pattern, the assembling procedure can be done by robots, as is the case for other industrial products (machines, cars, etc.).

## 3. Tiling the plane

The pattern of a brick wall surface, a prefabricated curtain wall facade or the surface of a space grid are two dimensional tilings. We are quite fortunate that tilings of surfaces, especially plane surfaces were studied extensively throughout history.

It is not our purpose here to present various tiling problems, but in order to find connections to construction problems we have to know some basics about plane tilings. A tiling $T$ is a family of entities (closed sets) $T = \{t_1, t_2, \ldots\}$ called tiles that cover the plane without gaps or overlaps of non-zero area. Furthermore, we assume that the tiles are topological discs, obtained from a circular disc by continuous deformation (i.e. no tile has disjoint parts or holes in it). We also want to consider here only tiles with finite areas. A partition of the tiles into congruence classes is possible, i.e., there is a family $S = \{P_1, P_2, \ldots, P_k\}$ of closed sets such that each tile $t_i \in T$ belongs to exactly one of these sets and all the tiles of a set $P_j$ are congruent.

A tiling $T$ is called *k-hedral* if the number of congruence classes of $S$ is $k$. For $k$ ,=1,2 and 3 the tilings are called monohedral, dihedral and trihedral, respectivelly. Because of the preferably small number of different elements of prefabricated building systems we are interested only in small $k$ values. Figure 2 shows an example of a monohedral tiling that occures commonly in architecture. Given a family S of congruence classes or tile types, we assume unlimited supply from tiles of each tile type. The first basic question is whether any given set of tile types allows a tiling of the plane. This question is equivalent to ask for an algorithm (or computer program) that can decide whether such a tiling exist. It has been shown that this question can not be answered, even for just a subclass of plane tilings, suggesting that the general tiling problem is undecidable, too. We will have the same difficulty in order to answer the opposite question: what kind of tiles can tile the plane? Although no such general characterization exist, there are methods providing partial answers.

If a family of tile types allow us to tile the plane then the next question is how many different ways are possible. If we have only regular hexagonal tiles then we have only one tiling possibility. This tile family is called *monomorphic.* On the other hand, with square tiles there are uncountable many tiling possibilities, since each row

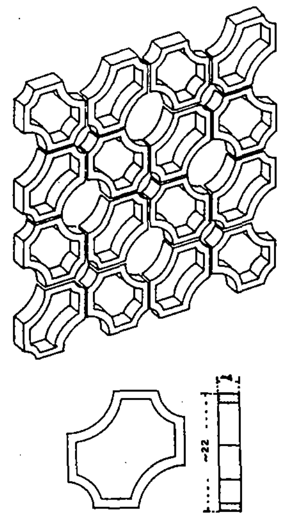

Figure 2: Monohedral tiling of a facade grid designed by L. Costa and O. Niemeyer.

(or column) of a square grid can be "slide" in infinitely many ways relative to the next row. Figure 3 shows slabs whose connections allows "sliding rows". Between these two extremes, however, there are many other possibilities. There are also many open questions about tiling possibilities to the same set of tiles [Grue81].

Another basic property of tilings is their symmetry. A monohedral tiling $T$ is called *isohedral* if given two tiles $t_i$ and $t_j$ there is a symmetry transformation of the entire tiling which maps $t_i$ onto $t_j$. These transformations form groups, where the most general of them is the automorphism group of the tiling. As K. Reinhard showed for the Euclidian space [Rei28] and H. Heesch for the plane [Hee35] there are monohedral tilings that are not isohedral.

Any isohedral tiling belongs to one of 81 classes that are presented in [Gru77]. Although there have been many previous attempts (such as [Kep40], [Fed00], [Haa32], [Sin38], [Wol74], [Hee63], or [Hee68]) to enumerate a certain class of plane tilings, non of them can be applied as multifarious as isohedral tilings. Since on the one hand we know the construction rules for isohedral tilings and on the other no attempt have been taken yet to apply them to architectural design, isohedral tilings provide an excellent teaching and research topic for Computer Aided Design as it was shown in [Car86].

## 4. Non-periodic tilings

A tiling is called *periodic* if there exist two non-parallel translations which map the tiling onto itself. Periodic tilings can be constructed from periodically repeating "patches". Isohedral tilings are certainly periodic, but not all periodic tiling are isohedral. To the discovery of non-periodic tilings led a recreational puzzle of P.A. MacMahon [Mac21]. It has square dominoes (or rather monominoes) with coloured edges and a square grid or checkerboard. The coloured dominoes, or tiles can be rotated but not reflected. We also have an unlimited supply of tiles of each type and the problem is to tile an area of the checkerboard such that a) the tiles cover the fields of the board and b) abutting edges have the same colour.

If we replace the colours by connection elements and the tiles by (rectangular) prefabricated elements then we have a familiar construction problem, schematically represented in Figure 4. If we also change the connection rules a little bit then the analogy between MacMahon's puzzle and construction problems becomes even more apparent.

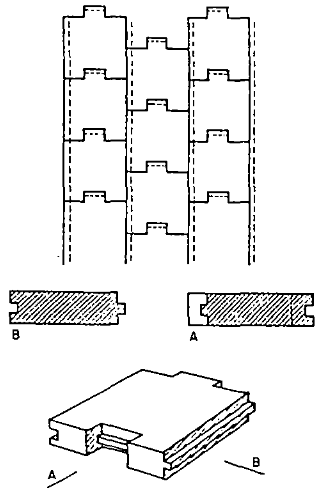


Figure 3: Prefabricated elements (slabs) that provide an arbitrary number of different tilings. The geometry of the joints determines not only the shape of the tiling but also many other features (assembling method, static, dilatation, etc.).

Let's assume that we have pairs of colours rather than just an arbitrary set of colours such that each colour has its complement, e.g. blue-red, yellow-green, black-white, etc., and abutting edges are of complementary rather than the same colour. An example of such complementary connections is shown also in Figure 4. We can easily recognize that all we did is to translate colours into different joints.

Because of the flexible and variable nature of design problems, we are interested in indefinitely enlargeable tilings, i.e., those that can tile the entire plane. However, as mentioned earlier, the tiling problem is undecidable.From the undecidability of the tiling problem follows that there are sets of tiles that tile the plane but not in a periodic fashion.

The results on non-periodic tilings have strange consequences for the design of construction parts: We can design prefabricated construction elements which can be assembled only in a non-periodic fashion. Or we may have a set of prefabricated elements that can be arranged in periodic as well as non-periodic fashion. This means that the discovery of non-periodic tilings provides us new, previously unknown design possibilities. Recently H. Lalvani has drawn attention to such a possibility for space frames (see [Lal86]). This marks, however, just the beginning of an extensiv and exciting research in both; structural and architectural design.

## 5. Space tilings

Although all tiling problems of the plane have a counterpart for the space, much less is known about three dimensional tilings then about two dimensional ones. Since regular and semiregular polyhedras play an important role in crystallography, polyhedral packing problems are fairly well explored. Polyhedral packings and their applications in architecture and design are represented in [Pic78] and [Wil72]. Space filling polyhedral packings (or tilings) are common in today's architecture, see [Bor68j, [Mak65] [Men75] or [Ful73]. An entirely new branch of civil engineering emerged that is concerned with the study of space structures. In particular morphological aspects of space structures as it is discussed e.g. in [Lal86] show the close relation between space structures and tilings. Let us demonstrate it by the following example.

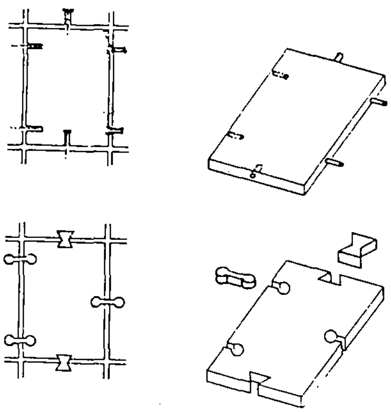

Figure 4: Tiling problem with prefabricated elements. Different connections can occur on each side of the panel or slab.

MacMahon extended the idea of the coloured dominoes for three dimensions. In the following example we also have coloured cubes as a model for a panel system. The system I have in mind might be the one of Gropius and Wachsmann. Wachsmann improved the panel joints later such that in his new system there was no need for additional connection parts (locks), the panels were held together by themselves. In both systems the joints of the panels are the same horizontally and vertically (Figure 5).

A basic question we can ask is in how many different ways can a room be built from the panels and how can rooms be added to each other in all 6 directions. Without loss of generality we can simplify the problem by looking at square panels and cube units (cells) built from the panels. Thus, the question is how many different cells are possible and how they can tile the space. This can be, however, easily transformed into a square domino problem with two colours. Each cell is built from 6 dominoes whose edges are marked with either -let us say- blue or red colour. Since the dominoes can be rotated also in the space there are 4 different dominoes shown in Figure 6. Here the connection rules of the MacMahon's puzzle are modified such that abutting edges have complementary colours. Also each domino is coloured on both side such that each side is the complement of the other (see Figure 6).

After defining the connection rules, the number of different cells can be easily calculated by Burnside's lemma (a combinatorial method for counting equivalence classes of permutations that form a group). This number is 186 and both, the generation method and a complete list of cells are represented in [Hal86]. Having the set of all cells or a subset of it we can try to fill the space by glueing cells together. Two cells can join if they share a common face (domino). It is not difficult to see that the 186 cells can fill (or tile) the space in many ways. Obviously, we are interested only in tilings with convenient properties. In [Hal86] there are tilings represented that are built from identical dominoes.

It simplifies the representation and allows us to consider cells rather then dominoes if we convert the colouring of the domino edges into the colouring of the cell edges. Each edge of a tiling contains four coloured domino edges. As Figure 7 shows there are two configurations possible: the colours define either a clockwise or a counter clockwise "rotation". The same is true for a single cell.

By looking at the set of the 186 cells we can find three subsets of cells which are built from exactly one domino type. Two of them have just one cell and the third one has 8. Each of the two sets with one cell allows only one tiling shown in Figure 8.

Using the elements of the third set the authors found 5 tilings which are, however, probably not the only ones.

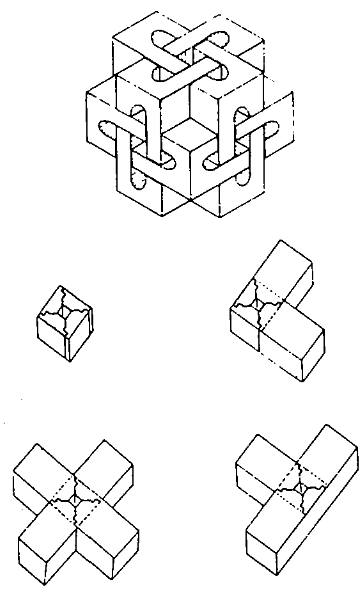

Figure 5: Connection examples of prefabricated panels from [Wac61]

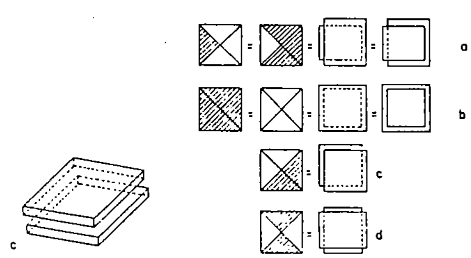


Figure 6: The four different dominoes and the correspondig panel schemata.

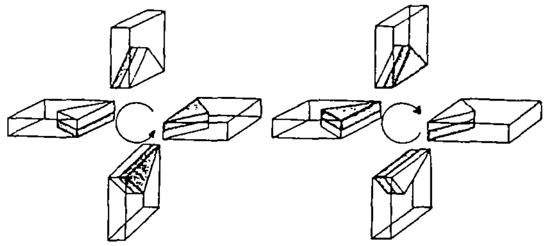

Figure 7: Colouring the edges of the tiling or cubic cells with the help of the colours of the dominoes.

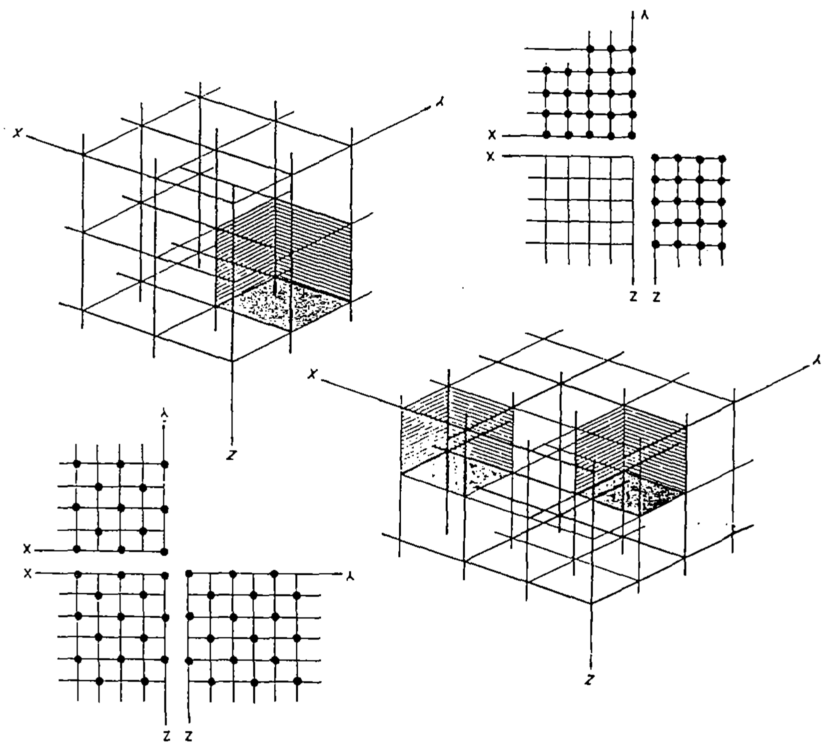

Figure 8: The only possible tilings with the domino types c and d in Figure 6. The thick lines represent clockwise, the thin ones counter clockwise rotation (or vice a versa). The "translation" of this model into e.g. Wachsmann's General Panel System is straight forward.

Thus, this is still an unsolved problem. But there are many other unsolved problems with the coloured cells yet that can probably solved by further investigation of the related tiling problem.

## References


[Bor68] Borrego, J., Space grid structures, MIT-Press, Cambridge, Mass., 1968.

[Car86] Carrera, J., Lenart, M., "Computer aided tiling design", Research Report, Carnegie-Mellon Univ. Dept. of Architecture, 1986.

[Fed00] Von Fedorow, E., Regulaere Plan- und Raumteilung, Abh. II Cl. Akad. Wiss. Muenchen, nr.20, pp.465-688, 1900.

[Ful73] Fuller, R.B., Marks, R., The Dymaxion World of Buckminster Fuller, Anchor Press, 1973.

[Grue81] Gruenbaum, B., Shephard, G.C., "Some problems on plane tilings, in 'The mathematical Gardner'", ed. D.A.Klarner, Wadsworth Int., pp.167-196, 1981.

[Gru77] Gruenbaum, B., Shephard, G.C., "The eighty-one types of isohedral tilings in the plane". Math.Proc., Cambridge Philos.Soc. 82, pp.177-196, 1977.

[Haa32] Haag, F., "Strukturformeln fuer Ebenenteilungen", Z.Kristallogr., nr.83, pp.301-307, 1932.

[Hal86] Haller, F., Lenart, M., "On the geometry of prefabricated building systems", Environment & Planning B, vol.13, pp.63-84, 1986.

[Hee35] Heesch, H., "Aufbau der Ebene aus kongruenten Bereichen", Nachrichten von der Ges.d.Wiss. zu Goettingen, Weidmannsche Buchh., Berlin, 1935.

[Hee63] Heesch, H., Kinzle, O., Flaechenschluss, Springer, 1963.

[Hee68] Heesch, H., Regulaeres Parkettierungsproblem, Westdeutscher Verlag, Koeln-Opladen, 1968.

[Kep40] Kepler, J., Harmonice mundi, 1619, Gesammelte Werke, Band VI, Beck, Muenchen, 1940.

[Kie84] Kielelstedt, F.T., 'The Crystal Palace", Sci.Am. Oct. 1984, pp.132-143.

[Lal86] Lalvani, H., "Non-periodic space structures", Space Structures, nr.2, pp.93-108, 1986/87.

[Mac21] MacMahon, P.A., New Mathematical Pastimes, Cambridge University Press, London, 1921.

[Mak65] Makowski, Z.S., Steel space structures, Michael Joseph, London, 1965.

[Men75] Mengeringhausen, M., Raumfachwerke aus Staebe und Knoten, Bauverlag, Wiesbaden-Berlin, 1975.

[Pie78] Pierce, P., "Structure in nature is a strategy for design", MIT-Press, 1978.

[Rei28] Reinhard, K., Zur Zerlegung der Euklidischen Raeume in kongruenten Polytope, Sitzungsber.d.Berliner Akad., 1928.

[Sin38] Sinogowitz, U., "Die Kreislagen und Packungen kongruenter Kreise in der Ebene", Z. Kristallogr., nr. 100, pp.461-508, 1938.

[Wan75] Wang, H., "Notes on a class of tiling problems", Fundamentae Mathematicae, LXXXII, pp.295-305, 1975.

[Wil72] Williams, R., The geometrical foundation of natural structure, Dovers Publications, N.Y. 1972.

[Wol74] Wollny, W., "Die 36 regulaeren Parketts mit dem Quadratnetz", Geometriae Dedicata, nr.3, pp.41-60, 1974.