Symmetry of STRUCTURE

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Abstracts

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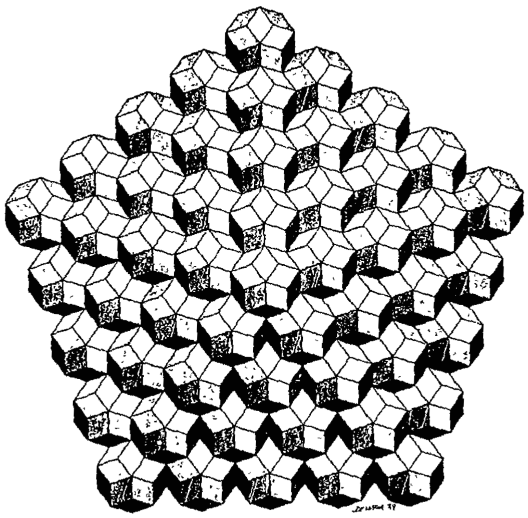

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THE USE OF SYMMETRY PRINCIPLES IN THE COMPUTER MICROTOMOGRAPHY AND IN THE DIAGNOSTICS AND DEVELOPMENT OF NEW METHODS OF INFORMATION PROCESSING AND OBTAINING

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The use of the Computer Microtomography (CM) Methods for diagnostic purposes demanded a wide use of the general symmetry principles under the development and designing either of the hardware or of the software of a Microtomography unit The high needed resolution level (1-10 micrometers) of the CM unit and also the application of standard sources of scanning radiation and the detector complexes, as a rule, lead to considerable scanning times, during which in a number of cases it is difficult to provide the demanded functioning stability of the sources electronic and detecting complexes. The consideration of peculiarities of their long-time work, as a rule, is provided by the software means, what from its side leads to the increase of complexity of the CM software and to the rise of the corresponding demands for the resources and productivity of the software, and correspondingly of their prices. On the other hand, the scanning process time comes sometimes in the contradiction to the demands on the complex productivity under the use of the Microtomograph in the product quality control systems.

The application of Symmetry Principles under the processing of the shadow function measurement results allows to realize new regimes of scanning: the stretch regime and the regime of more detailed analysis of the found zone of suspicion. On the other hand, such an approach allowed to develop a new method of shadow function analysis, based upon the calculation of spacial moments.

In the CM among a variety of different problems there are two main calculation problems which take a particular place. The first of them is to reconstruct the shadow function of the spacial variables $f(x-y)$ in accordance with the results of an indirect experiment, representing itself projections of the function $f(x+y)$ along a set of straight lines. The second problem is connected with the analysis and classification of the function $f(x+y)$ on the basis of the same projection data

These problems are being solved nowdays independently one from another. However we can consider them in a complex The basis for it can serve the fundamental theorem of Hu, according to which the display function $f(x-y)$ is in a one-to-one connection (through the characteristic function) with an infinite number of its moments $\{m_{\alpha\beta}\}$. The moment of the $(\alpha+\beta)$ -order is defined in the following way

$$m_{\alpha\beta} = \int_{-\infty}^{+\infty} x y\beta - (x,y)\,dx\,dy \qquad (1)$$

Consequently the set of moments $\{m_{\alpha\beta}\}$ of the function $f(x-y)$ can serve as the initial data for the reconstruction algorythm of this function. On the other hand, this set of moments can be used as a system of secondary signs for the analysis of the function $f(x,y)$ with the application of the image recognition methods. It is worth to mark that the sign-moments have recieved wide practical application.

To the advantages of this method we can correspond primarily the fact that the ansamble of moments of this function of different orders can be easily increased, and the probability of the right discovery of deviations from symmetry rises along with the growth of body (power) of moments, used in the recognition processes. Under the use in a microtomograph, for example, of the Röntgen radiation there exists a possibility to calculate directly from the measurement results the spacial moments from the function, desribing the object under study, not recieving before this directly the tomogramm itself. This fact can be rather useful in the cases when the problem is reduced only to the control of symmetry of the object. Excluding from the control processes of the tomogramm reconstruction stage we can succeed to reduce the scanning time of the object considerably. The realized approach allowed in general to reduce the processing time of the controlled good from 45 to 80% depending on the type of the good and the level of the demanded resolution and also to transfer to the complex usage of possibilities of the CM. In this case the detailed analysis of the suspicion zone allows to obtain the complete information on the character of defects and their geometrical characteristics which allows together with the functioning of the purposeful software which describes the life cycle in the predictable and non-predictable situations, and also the evolutions of the defect parameters during this time, to forcast sufficiently exactly the working resources of the good to reconstruct the technology of its production and repairing. It is important to note that under the analysis of concrete technical parameters of the goods, including the analysis of shadow functions, the demand for symmetry is to be sensibly interpreted, because the results of every measurement are defined by a large number of factors, which means that mathematically the problem is reduced to the correct reduction of a function, depending on a large number of variables, to a function, which describes sufficiently exactly the regulations, but depending on a smaller number of variables.

The main problem of approximation in its classical formulation is stated in the following way On a certain point set M in the space of arbitrary number of changes two functions $f(P)$ and $F(P, A_1, \dots, A_n)$ of the points PAM are given. The second of them depend also from a set of parameters $A_1, \dots, A_n$. These parameters are to be found in such a way that the deviation in M of the function $F(P, A_1, \dots, A_n)$ from the function $f(P)$ under the deviation F from f. It is worth to be mentioned that modern methods of the applied mathematics and the computer techniques allow to solve practically any problem of the approximation in the classical formulation if the class of approximation functions is given - F and the type of deviation. And how shall we do in the case when the class of approximation functions is not given ?

If we speak about the function $F(P)$, depending on a single variable, then the choice of the class of approximation functions (the empirical formulae) F with the help of geometrical interpretation (the graphical analysis) doesn't meet any difficulties For this case there exists a number of guidelines. The question of the choice of the suitable class of approximating functions, if there are not sufficiently reliable theoretical considerations, is sharply complicated, when we study the functions of three (sometimes two) or more variables.

We have elaborated a method which allowed to reduce the analysis of functions of many variables to the analysis of functions of a smaller number of variables and simultaneously to construct the approximation function. This method of approximation of a a function of many variables by a superposition of summs of functions of a smaller number of variables gives in the parallelepiped $\mathcal{D} = \{a_i \leq x_i \leq b_i, \ i = \overline{1,n}\}$ the approximation in the form $\varphi = (x_1, \dots, x_n) \approx \mathcal{L}\{\sum_{i=1}^{m} f_i(x_{i_1}, \dots, x_{i_g})\}$, where $\varphi(x_1, \dots, x_n)$ - the approximating function, depending on independent variables, $\mathcal{L}$ - a certain beforehand unknown function such that the inverse function for it is a polynom of the k-th order, i.e. if $y = \mathcal{L}(x)$ than $\mathcal{L}^{-1}(y) = \sum_{i=1}^{k} \alpha_i y^i$; $f(x_{i_1}, \dots, x_{i_g})$ is a certain beforehand unknown function from a definite combination of variables, the number of which is smaller than n.

The realization of this method on a computer allows to find functions $\mathcal{L}^{-1}$ (and consequently $\mathcal{L}$) and $f(x_{i_1}, \dots, x_{i_g})$, which give a minimum of the expression relatively to the middle square deviation.

The joint use of the described approaches allows to provide high indexes in the practice of realization of tomography systems.

The questions of diagnostics of electronic equipment components (chips, multi-layered printed surfaces etc.) become even more actual in connection with a wider use of the electronic equipment and computer techniques in different branches Within the frameworks of the realization of a number of scanning devicea for quality control, including the methods of CM, described above, we have widely used the symmetry principles, based on a comparative analysis of the parameters of ethalon systems and diagnostized systems. Under the realization of such an approach the creation of ethalon systems are possible in two ways, one of which consists in the obtaining and fixation of the processed signals of detectors, recieved as a result of scanning of non-defectable good, and the second in the obtaining and use of specially developed methods of numerical modelling of scanning processes, for example, the Monte-Carlo methods for the three-dimensional models of scanning cells taking into account the pecularities of work and spectral characteristics of the scanning radiation source of the detecting complex and the pecularities of evolution of the scanning beam characteristics under its passage through the object.

In particular, for the problems of beam tomography and primarily the Röntgen tomography a complex mathematical model has been developed for a scanning complex, consisting of a radiation source and the collimating system, the scanning object and a detector, which also has a collimating system

We have considered a stationary equation of the transfer of photons in non-uniform medium in the form

$$\vec{\Omega}\cdot\nabla\Phi(\vec{r},E,\vec{\Omega})+\Sigma(\vec{r},E)\cdot\Phi(\vec{r},E,\vec{\Omega})=$$
$$=\int K(\vec{r},E'\to\vec{\Omega}'\to\Omega)\cdot\Phi(\vec{r},E',\vec{\Omega}')\,dE'd\vec{\Omega}'+Q(\vec{r},E)$$

where $\Sigma(\vec{r},E)$ - the complete macroscopic section of interaction of a photon with the medium; $K(\vec{r},E'\to\vec{\Omega}'\to\Omega)$ - the differential section of photon scattering, $Q(\vec{r},E)$ - a function, describing the photon source; $\Phi(\vec{r},E,\Omega)$ - a differential flux in a point of phase space $(\vec{r}\,E\,\vec{\Omega})$ which was solved by the Monte-Carlo methods in the multi-group transport approximation for a three-dimentional model.

As our experience showed, the second approach allowed to use widely different additional possibilities of classification and discovering of defects in the microelectronic items Obviously, the use of such principles is possible in the other branches of diagnostics.

Thus the wide use of symmetry principles in the CM and the diagnostics leaded to the nesessity of the development and creation of complexes of new methods of obtaining and processing of information.

Symmetrical Structures in Building Construction (Extended Abstract)

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1. Background

First we look back at the history of prefabricated building systems. Prefabricated elements in buildings are almost as old as the history of house building itself. many parts of the house, such as the wall and roof, have throughout the history been made of identically formed elements. Since the industrial revolution, mass production has become an important part of the building industry. The first major milestone in the development of industrial methods in the architecture was probably Paxton's Crystal Palace erected in 1851 [Kie84]. The Crystal Palace as well as all subsequent prefabricated buildings possess attributes which are different from those of traditional buildings. Construction problems have usually more consequences throughout the design of prefabricated buildings then in the design of traditional ones. Thus, the design of construction parts play an important role in the design of prefabricated buildings. Many famous architects, such as Buckminster Fuller, Jean Prouve o. Konrad Wachsmann perceived the necessity of studying construction problems. Wachsmann especially, who led the Division of Building Research at the University of Southern California in the 60's, has made many contributions to the development and analysis of construction parts of prefabricated building systems.

The design of construction parts is a complex task, involving geometric, static, kinetic, esthetic, and other problems. Geometric problems, however, are often dominant and have larger impact on the design then other related problems. Since repeating symmetrical patterns can provide economically and esthetically appealing solutions, architects became interested in crystallography, cell biology and group theory [Pie78], [Wil72]. Although symmetrical structures spread fast in the architecture and interesting constructions emerged by designers such as Buckminster Fuller, Felix Candella, Pier Luigi Nervi or Frei Otto, not much research was done on the geometry of construction parts leaving numerous problems still unsolved.

Here we don't attempt to solve any of these problems, however, we show connections between symmetry concepts, such as tilings and building constructions.

2. The construction problem

Let us consider a number of square slabs, without specifying the function of the slabs as construction parts. The slabs are connected as shown in Figure 1 by male and female elements on each edge. One can easily see that the connection of the slabs is realizable if and only if neither of the two slab patterns to be connected has a concave boundary. For example, we can connect three slabs as in Figure 1(b) but we cannot connect a forth one to them. We can solve the problem by connecting two pairs of slabs as in Figure 1(c), or connect all four slabs at once by rotating the slabs as in Figure 1(d). In both cases there are other restrictions according to the number of slabs to be connected by one movement. Another method of connecting slabs as, e.g. in Figure 1(e) has other cosequences, caused by the static properties of the connecting parts.

Summing up the consequences of these examples, we can see that the geometrical form of the slabs and the

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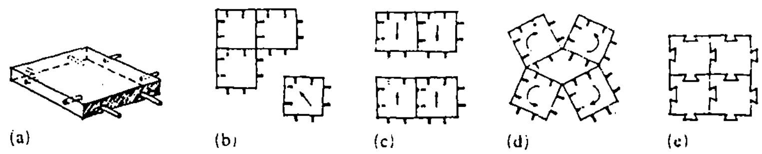

Figure 1: Connection rules for building tesselation from square slabs

connecting parts determines important properties of the entire system Such properties are:
1. the oder of assembly of the elements (which elements can be connected to which one and what kind of pattern can be achived),
2. the motion of the elements to produce a connection,
3. static characteristic of the system.
However, according to these points some geometrical properties of the slabs do not play any role, e.g. the size of the slabs or the fact that they are square. The above properties are valid for any rectangular tesselation. Furthermore, the exact form of the connection is irrelevant because all connections that allow only one motion in one direction in the plane of the tesselation possess the same properties.

Analysis of several construction systems has shown that such geometrical or topological characteristics imply a couple of fundamental rules. The knowledge of these rules can support the design process and lead to solutions that will probably not be found without them. By following the rules, buildings can be constructed from elements with very few data, in contrast to the usual documentation that contains a lot of unnecessary, often redundant information. In the realisation phase, it will be possible to use the rules for the automation of the building procedure. With a set of building elements, a list of connection rules and a description of the pattern, the assembling procedure can be done by robots, as is the case for other industrial products (machines, cars, etc.).

3. Tiling the plane

The pattern of a brick wall surface, a prefabricated curtain wall facade or the surface of a space grid are two dimensional tilings. We are quite fortunate that tilings of surfaces, especially plane surfaces were studied extensively throughout history.

It is not our purpose here to present various tiling problems, but in order to find connections to construction problems we have to know some basics about plane tilings. A tiling T is a family of entities (closed sets) $T = \{t_1, t_2, \dots\}$ called tiles that cover the plane without gaps or overlaps of non-zero area. Furthermore, we assume that the tiles are topological discs, obtained from a circular disc by continuous deformation (i.e. no tile has disjoint parts or holes in it). We also want to consider here only tiles with finite areas. A partition of the tiles into congruence classes is possible, i.e., there is a family $S = \{P_1, P_2, \dots, P_k\}$ of closed sets such that each tile $t_i \in T$ belongs to exactly one of these sets and all the tiles of a set P_j are congruent.

A tiling T is called *k-hedral* if the number of congruence classes of S is k. For k =1,2 and 3 the tilings are called monohedral, dihedral and trihedral, respectivelly. Because of the preferably small number of different elements of prefabricated building systems we are interested only in small k values. Figure 2 shows an example of a monohedral tiling that occures commonly in architecture. Given a family S of congruence classes or tile types, we assume unlimited supply from tiles of each tile type. The first basic question is whether any given set of tile types allows a tiling of the plane. This question is equivalent to ask for an algorithm (or computer program) that can decide whether such a tiling exist. It has been shown that this question can not be answered, even for just a subclass of plane tilings, suggesting that the general tiling problem is undecidable, too. We will have the same difficulty in order to answer the opposite question: what kind of tiles can tile the plane? Although no such general characterization exist, there are methods providing partial answers.

If a family of tile types allow us to tile the plane then the next question is how many different ways are possible. If we have only regular hexagonal tiles then we have only one tiling possibility. This tile family is called *monomorphic*. On the other hand, with square tiles there are uncountable many tiling possibilities, since each row

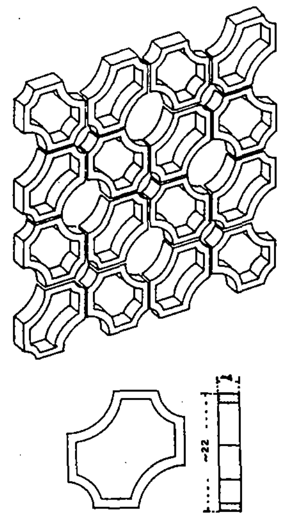

Figure 2: Monohedral tiling of a facade grid designed by L. Costa and O. Niemeyer.

(or column) of a square grid can be "slide" in infinitely many ways relative to the next row. Figure 3 shows slabs whose connections allows "sliding rows". Between these two extremes, however, there are many other possibilities. There are also many open questions about tiling possibilities to the same set of tiles [Grue81].

Another basic property of tilings is their symmetry. A monohedral tiling T is called *isohedral* if given two tiles t_i and t_j there is a symmetry transformation of the entire tiling which maps t_i onto t_j. These transformations form groups, where the most general of them is the automorphism group of the tiling. As K. Reinhard showed for the Euclidian space [Rei28] and H. Heesch for the plane [Hee35] there are monohedral tilings that are not isohedral.

Any isohedral tiling belongs to one of 81 classes that are presented in [Gru77]. Although there have been many previous attempts (such as [Kep40], [Fed00], [Haa32], [Sin38], [Wol74], [Hee63], or [Hee68]) to enumerate a certain class of plane tilings, non of them can be applied as multifarious as isohedral tilings. Since on the one hand we know the construction rules for isohedral tilings and on the other no attempt have been taken yet to apply them to architectural design, isohedral tilings provide an excellent teaching and research topic for Computer Aided Design as it was shown in [Car86].

4. Non-periodic tilings

A tiling is called *periodic* if there exist two non-parallel translations which map the tiling onto itself. Periodic tilings can be constructed from periodically repeating "patches". Isohedral tilings are certainly periodic, but not all periodic tiling are isohedral. To the discovery of non-periodic tilings led a recreational puzzle of P.A. MacMahon [Mac21]. It has square dominoes (or rather monominoes) with coloured edges and a square grid or checkerboard. The coloured dominoes, or tiles can be rotated but not reflected. We also have an unlimited supply of tiles of each type and the problem is to tile an area of the checkerboard such that a) the tiles cover the fields of the board and b) abutting edges have the same colour.

If we replace the colours by connection elements and the tiles by (rectangular) prefabricated elements then we have a familiar construction problem, schematically represented in Figure 4. If we also change the connection rules a little bit then the analogy between MacMahon's puzzle and construction problems becomes even more apparent.

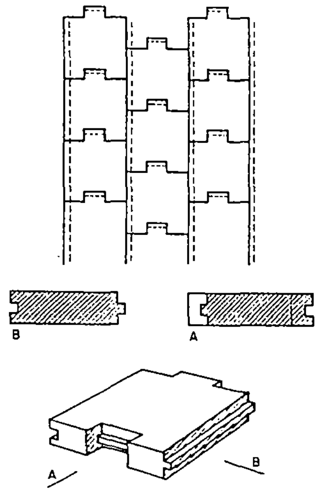

Figure 3: Prefabricated elements (slabs) that provide an arbitrary number of different tilings. The geometry of the joints determines not only the shape of the tiling but also many other features (assembling method, static, dilatation, etc.).

Let's assume that we have pairs of colours rather than just an arbitrary set of colours such that each colour has its complement, e.g. blue-red, yellow-green, black-white, etc., and abutting edges are of complementary rather than the same colour. An example of such complementary connections is shown also in Figure 4. We can easily recognize that all we did is to translate colours into different joints.

Because of the flexible and variable nature of design problems, we are interested in indefinitely enlargeable tilings, i.e., those that can tile the entire plane. However, as mentioned earlier, the tiling problem is undecidable.From the undecidability of the tiling problem follows that there are sets of tiles that tile the plane but not in a periodic fashion.

The results on non-periodic tilings have strange consequences for the design of construction parts: We can design prefabricated construction elements which can be assembled only in a non-periodic fashion. Or we may have a set of prefabricated elements that can be arranged in periodic as well as non-periodic fashion. This means that the discovery of non-periodic tilings provides us new, previously unknown design possibilities. Recently H. Lalvani has drawn attention to such a possibility for space frames (see [Lal86]). This marks, however, just the beginning of an extensiv and exciting research in both; structural and architectural design.

5. Space tilings

Although all tiling problems of the plane have a counterpart for the space, much less is known about three dimensional tilings then about two dimensional ones. Since regular and semiregular polyhedras play an important role in crystallography, polyhedral packing problems are fairly well explored. Polyhedral packings and their applications in architecture and design are represented in [Pic78] and [Wil72]. Space filling polyhedral packings (or tilings) are common in today's architecture, see [Bor68j, [Mak65] [Men75] or [Ful73]. An entirely new branch of civil engineering emerged that is concerned with the study of space structures. In particular morphological aspects of space structures as it is discussed e.g. in [Lal86] show the close relation between space structures and tilings. Let us demonstrate it by the following example.

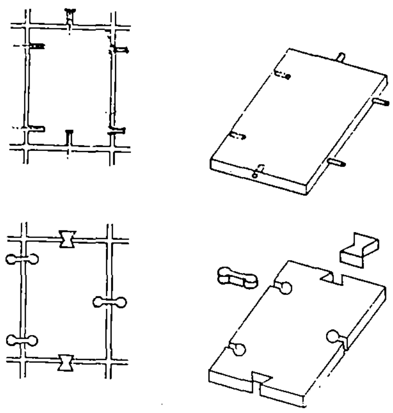

Figure 4: Tiling problem with prefabricated elements. Different connections can occur on each side of the panel or slab.

MacMahon extended the idea of the coloured dominoes for three dimensions. In the following example we also have coloured cubes as a model for a panel system. The system I have in mind might be the one of Gropius and Wachsmann. Wachsmann improved the panel joints later such that in his new system there was no need for additional connection parts (locks), the panels were held together by themselves. In both systems the joints of the panels are the same horizontally and vertically (Figure 5).

A basic question we can ask is in how many different ways can a room be built from the panels and how can rooms be added to each other in all 6 directions. Without loss of generality we can simplify the problem by looking at square panels and cube units (cells) built from the panels. Thus, the question is how many different cells are possible and how they can tile the space. This can be, however, easily transformed into a square domino problem with two colours. Each cell is built from 6 dominoes whose edges are marked with either -let us say- blue or red colour. Since the dominoes can be rotated also in the space there are 4 different dominoes shown in Figure 6. Here the connection rules of the MacMahon's puzzle are modified such that abutting edges have complementary colours. Also each domino is coloured on both side such that each side is the complement of the other (see Figure 6).

After defining the connection rules, the number of different cells can be easily calculated by Burnside's lemma (a combinatorial method for counting equivalence classes of permutations that form a group). This number is 186 and both, the generation method and a complete list of cells are represented in [Hal86]. Having the set of all cells or a subset of it we can try to fill the space by glueing cells together. Two cells can join if they share a common face (domino). It is not difficult to see that the 186 cells can fill (or tile) the space in many ways. Obviously, we are interested only in tilings with convenient properties. In [Hal86] there are tilings represented that are built from identical dominoes.

It simplifies the representation and allows us to consider cells rather then dominoes if we convert the colouring of the domino edges into the colouring of the cell edges. Each edge of a tiling contains four coloured domino edges. As Figure 7 shows there are two configurations possible: the colours define either a clockwise or a counter clockwise "rotation". The same is true for a single cell.

By looking at the set of the 186 cells we can find three subsets of cells which are built from exactly one domino type. Two of them have just one cell and the third one has 8. Each of the two sets with one cell allows only one tiling shown in Figure 8.

Using the elements of the third set the authors found 5 tilings which are, however, probably not the only ones.

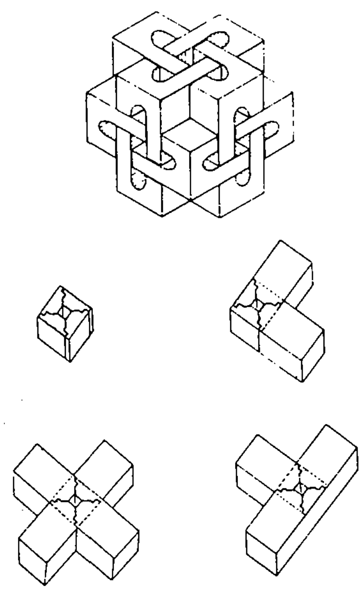

Figure 5: Connection examples of prefabricated panels from [Wac61]

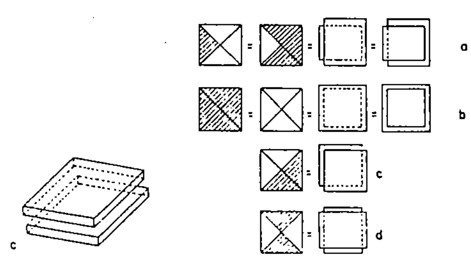

Figure 6: The four different dominoes and the correspondig panel schemata.

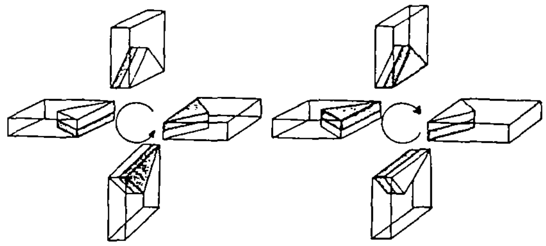

Figure 7: Colouring the edges of the tiling or cubic cells with the help of the colours of the dominoes.

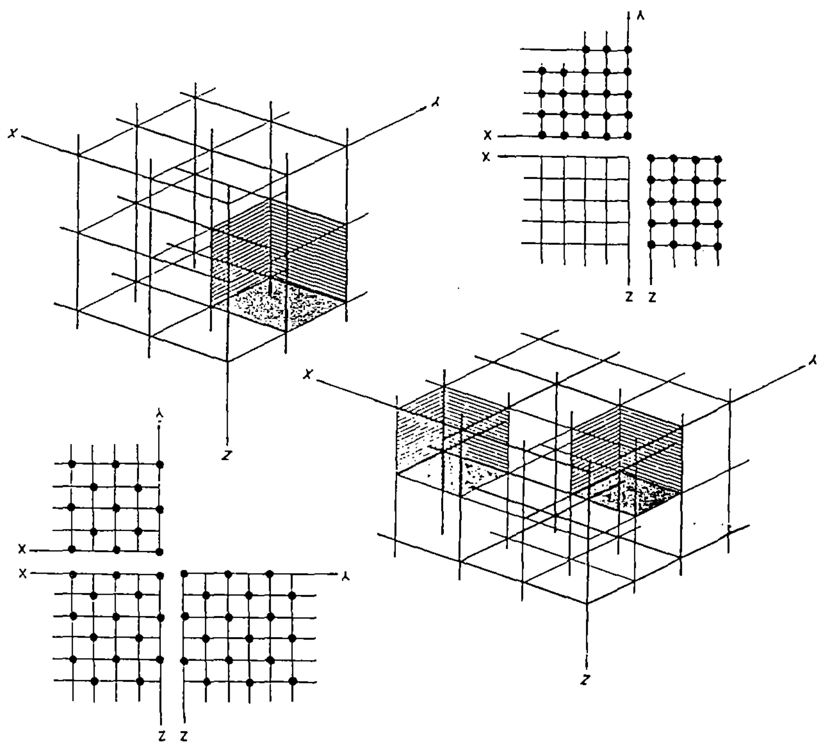

Figure 8: The only possible tilings with the domino types c and d in Figure 6. The thick lines represent clockwise, the thin ones counter clockwise rotation (or vice a versa). The "translation" of this model into e.g. Wachsmann's General Panel System is straight forward.

Thus, this is still an unsolved problem. But there are many other unsolved problems with the coloured cells yet that can probably solved by further investigation of the related tiling problem.

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FIBONACCI NUMBERS AND THE GOLDEN SECTION : AN INTEGRATIVE,COMPUTER-ASSISTED CURRICULUM

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This lecture will describe in outline ideas associated with the Fibonacci numbers and the Golden Section forming the basis of an integrative-interdisciplinary study program encompassing their application in mathematics, science, architecture, and the arts. Part of the curriculum involves the creation of computer programs which reveal in depth the interrelationships between the Fibonacci numbers, the Golden Section , and the logarithmic spiral. So that these programs will be accessible to secondary school pupils, the programming language employed will be LOGO.

Such a curriculum seeks to achieve the decompartmentalization of knowledge inherent in many of today's programs in the sciences and the arts. An integrated approach in education is vital today in view of the fragmentaion of disciplines stemming from the exponential increase in man's knowledge over the past century (Phenix 1958: 325). This fragmentation has had dire consequences for our educational system robbing it of many of its humanistic values and degrading it to the level of preliminary vocational training in the three R's (Wynne 1970: 192). This tendency towards fragmentation is most accentuated in the divergence of the sciences from the arts. The British scientist and man of letters, Lord C.P. Snow, put the case most forcefully in his famous "Two Cultures" (Snow 1956).

This reality requires any educational philosophy to take a stand. If integration is desired, study topics and programs must reflect this. As a result topics will be selected not based solely on traditional considerations, but also on their value in bridging the gap separating aesthetics from science. Such is the case of the Fibonacci numbers and the Golden Section.

<u>Curriculum Content</u>

Fibonacci represents a towering figure in the history of mathematics. Fibonacci was important primarily as an agent for the introduction of the mathematical achievements of the ancient Eastern Cultures to the Western World emmerging from the Middle Ages in the first half of the Thirteenth Century. As such he deserves attention from the point of view of the intellectual development of Western Culture and its debt to the achievements of the Eastern one (Cajori 1961:120-126; Eves 1980: 160-168).

The Fibonacci series represents a simple, divergent, and non-proportional infinite series whose terms nevertheless converge upon a significant, finite ratio, i.e.: the Golden Mean. This finite ratio is an irrational number ($(\sqrt{5}-1)/2$). Such an irrational ratio was, as Spengler put it "alien to the Classical soul" (Spengler 1956:2324; also Lawlor 1982:38; and Eves 1980:53) and this aversion survived in Western Culture for another thousand years at least.

Fibonacci numbers (1,1,2,3,5,8,13,21,34,55,89,...) appear as important organizing principles in nature most notably in the process of phyllotaxis (Hambidge 1967: 146; Coexter 1961: 169-172) . They can be generated in a number of ways: Fibonacci's original "Rabbit Problem"(Vogel 1971: 607), a bee's traverse of a honeycomb (Lake and Newmark 1977:190), bee genealogy (Gardner 1981:162-164), the "Checkerboard Paradox" (Hunter and Madachy 1963:12-13), or Hofstadter's method of "expanding nodes" (1979: 135-137) among others.

Fibonacci numbers posses special properties which should be explored(Lake&Newmark 1977:188-192; Gardner 1981:159-62; Wells 1986: 61-7;Hunter and Madachy 1963:19-22; Alfred,B.U. 1963:57-63).

The cognitive content at this stage draws on arithmetic, combinatorial, and deductive skills. The above-mentioned generating options differ widely in terms of inherent difficulty thus permitting a concentric approach whereby one arranges them by ever-increasing degree of difficulty. With each additional example the students' anticipation of the imminent appearence of the "magic" numbers increases.

Once a recursive method for generating the numbers is available, the students can proceed to program a computer to produce these numbers. The transition from concept to recursive formula to algorithm to computer program is an essential part of comprehending the art/science of programming. Rarely is it so forecefully but easily exemplified. Witness the almost ubiquitous appearence of the Fibonacci numbers in introductory texts in computer programming(Tremblay&Bunt 1979:447-449; Forsythe 1969:9-10).

The Golden Mean,also known as the Golden Section,has been a central pillar of aesthetic philosophy from Aristotle's time down to the present. The Golden Mean represented an aesthetic discovery of immense importance which was applied by the ancient artists and artisans in the fields of the plastic arts (sculpture, pottery,painting) and in perhaps their supreme achievement: their architecture (de Lucio-Meyer 1973:61-67; Eves 1980: 48-49).

What they learned has been repeatedly applied in these and other related fields. Islamic Art is a blend of the organic and the geometric. Central to the designs of Islamic Art are the Vedic square and the Fibonacci series (Albarn et al 1974:12-32). The Renaissance artists applied, often consciously, the principles of proportion and perspective based on, among other principles, the Golden Section (Lawlor 1982:53-63; Walker 1978:53; Kline 1962: 203-230).In his <u>Divina Proportione</u> (1509), Pacioli attributed remarkable properties to the Golden Mean. As illustrator for the book he chose none other than Leonardo da Vinci ! (Pedoe 1976: 263-265).

Even in this century of "free", "uninhibited" creation the Golden Section has been exploited by such prominent artists as Frits Mondrian and le Courbusier. The two Twentieth Century movements de Stijl and le Section d'Or Group (Honour and Fleming 1982: 598-603; Hill 1977: xii; Wittkower,R 1978: 120) drew their inspiration from the contemplation of symmetry relationships such as the Golden Section.

The Golden Mean can be derived from the Fibonacci numbers by both empirical and analytical means (Huntley 1970:46-7; Wells 1986 :36-40; Wilf 1986: 30-31). Again, a concentric approach is utilized whereby the students will be led to appreciate the way in which the laws of geometry and algebra can be used to verify in a general way the results of a finite number of trial calculations.

Here too a number of computer activities offer themselves. Using "LOGO", students can construct collections of rectangles based on various proportions, including that of the Golden Mean, testing their perception of proportion. They can even create a psychological testing model by offering their drawings to impartial subjects and measuring their relative preference for the "Divine Proportion".

Again, using "LOGO", the students can create a composition of "Golden Rectangles which when viewed in "Real Time" creates the sensation of "gnomic growth" (Huntley 1970: 169-170;Lawlor 1982: 65-66). This is in fact the formula for the construction of the logorithmic spiral (recursive applications of Golden Rectangle+Turn).

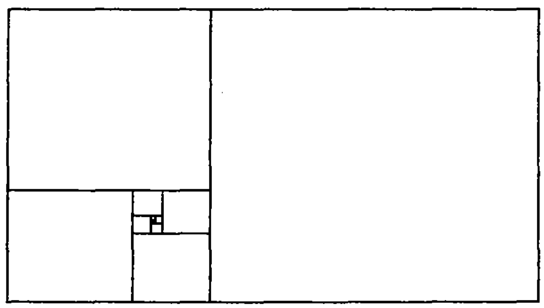
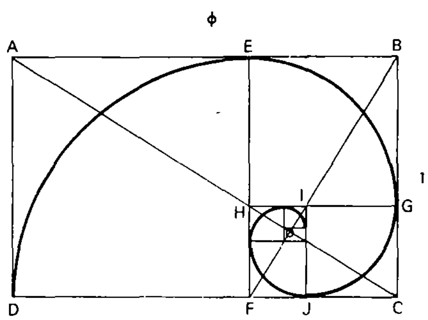
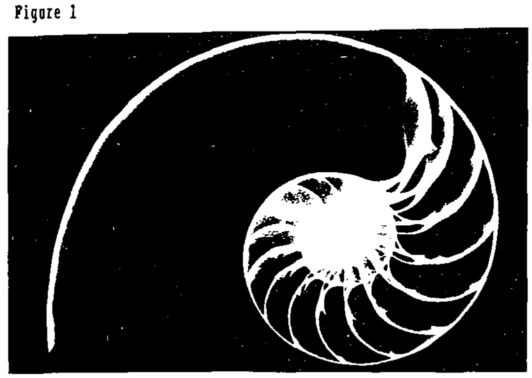


Figure 1



Logarithmic spiral

Figure 2a



Nautilus pompilius
Figure 2b

The logorithmic spiral itself is an exemplary integrating theme for studies in botany, zoology, and architecture(Lake&Newmark 1977:188-90; Stevens 1974:159-66; Bergamini 1972: 93; Huntley 1970: 164-76;Lawlor 1982:65-73).

Building a Bridge to Aesthetics

Music, and the fine arts in general, have a unique contribution to make in any educational program. The arts' contributions include : the refinement of sensory perception, the deepening of appreciation for the individual and the particular as opposed to the general and universal; the heightening of respect for the products of one's and others' creativity (Phenix 1958: 424-440).

Any program possessing a fine arts component will undoubtedly have aesthetic and emotive effect. What the specific effects will be cannot necesssarily be predicted or prescribed with the same degree of precision as its cognitive content. It would seem, however, that the program described here should include, inter alia, the following aesthetic and emotive values:

Organizing principles in artistic expression :

Some envisage art as pure expression while others as the "working-out" of potentialities within an axiomatic framework of artistic ideals. This dispute has been heavily debated down through the centuries(Wittkower 1978).

Even accepting that the artist is unaware of symmetry considerations in his creations, the inescapable conclusion lies in the completed work itself: a large body of artistic creation throughout the millenia embody the "working out" of problems of proportion and composition that can be expressed in mathematical terms even if the artist did not deign to do so (Arnheim 1962: 218; Ghyka 1977: 174; Hedian 1976).

The mutual affinity of math and music :

The integration of music and mathematics in an integrated curriculum seems to be particularly apt and effective. Beginning with Pythagoras' discovery of the laws of harmony through Helmholtz's classic experiments in acoustics it is clear that music and mathematics are inevitably intertwined (Huntley 1968: 51-6; Reik 1953:121)

It has been suggested that part of the reason for this mutual affinity lies in the shape of the auditory organ, the cochlea of the inner ear, with its spiral shape whose shape is ruled by the pattern of logarithmic growth (Lendvai 1966:190; Anfilov 1966: 112-3).

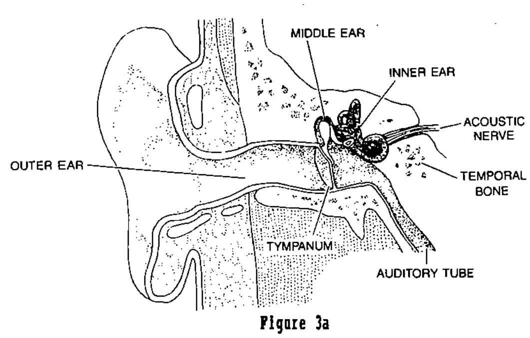


Figure 3a

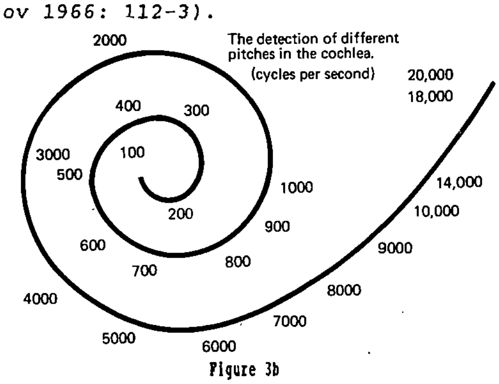


Figure 3b

The artisans who built the stringed instruments of the Renaissance (the "luthiers") made liberal use of the Golden Section (Coates 1985). In general, instrument design is based on three considersations : acoustics, ergonomics, and aesthetics, which harmonizes and humanizes the previous two. The use of geometry here was different from that architecture or painting. Proportions were not planned nor perceived directly. They resulted from considerations which superficially had nothing to do with proportion and geometry. The fact that the products of these designs embody basic proportions such as the Golden Mean implies that it is inherent to them and provides the necessary principle for their expression. The artisan employing precepts such as the Golden Section could enjoy the certainty and security that its application provides. The resultant symmetry provided him with a sense of "rightness" and was communicable leading to a universality of form. The luthier thereby embraced the two Platonic arts of music and mathematics by applying geometric considrations to his design.

From these two examples alone it should be clear that polyaesthetic education need not be separated from the study of mathematics and science. Building bridges between the fields does not merely serve to increase the amount of knowledge transmitted, but to enrich one's appreciation for the universality of the knowledge thus acquired.

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MORPHOGENESIS OF PLANTS AS A DYNAMICAL SYSTEM

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Functional differentiation of cells allows to consider an organism as a family $F = \{L_\alpha, \alpha \in A\}$ of non-intersecting subsets L_α which form this organism. Such a structure is called fibre bundle F with the fibre L_α. The set A marks functional specifics of fibres L_α. We deal with one fibre L_{α_0}, responsible for reproduction of apical leaves on a plant shoot. We will consider this fibre an independent cell organism interacting with other fibres equally with the environment.

The paper presents arguments which allow to consider the functional organization of fibre L_{α_0} as a hypercycle [1] stipulating the integrated action and the co-ordinated evolution of self-replicating units (cells) of this fibre. Hypercycle is a ring network of co-operative cyclic catalysis reactions:

$$I_1 \xrightarrow{E_1} I_2 \xrightarrow{E_2} \dots \longrightarrow I_{n-1} \xrightarrow{E_{n-1}} I_1$$

The intermediates I_k of external co-operative hypercycle are internal autocatalytical cycles themselves. Hypercycle can be treated as a dynamical system, i.e. one-dimensional group of transformations G^1 in a phase space of variables q^i (q^i being population of cells of i-th genotype). This dynamical system is not ergodic, and its phase space can be decomposed into invariant tori $S^1 \times S^1$. The movement on the torus is marked by angular coordinates (ψ, φ) which have the sense of chemical "movement" along external and internal autocatalytical cycles with frequencies Ω and ω respectively. Movements along the neighbouring internal cycles have a phase shift. At a certain phase of the k-th internal cycle, the apical leaf is born as a product of the hypercycle.

The paper analyses possible reflections of manyfold events in the phase space of the organism on the three-dimensional Euclidean macroscopic space of an external observer. The parameters of hypercycle which bring the phillotaxis phenomenon in the external space, have been studied.

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NOTATION AND NOMENCLATURE

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The notation and nomenclature for two-dimensional symmetry groups developed in the context of crystallographic research is not necessarily most suitable for use in an art-historical or design context. There are even differences between crystallographers and solid-state scientists in the manner in which they deal with symmetrical patterns.

Fundamental to crystallographic notation is the concept of a lattice, a collection of all points in a pattern related to each other by translational symmetry. This emphasis on translational symmetry is the result of the translational symmetry of the X-ray beam used by crystallographers to determine the location of the crystal elements: diffraction of the beam by the crystal is a direct result of this translational symmetry of the crystal. Solid state scientists, however, are more concerned with the symmetries of the fields, electrical, magnetic or quantum-mechanical, around each crystal element than with the absolute orientation of these fields. Accordingly, Fischer et al.[1] developed the notion of the lattice complex, a collection of all points related by any symmetry operation. Directly coupled to the concept of lattice is that of unit cell, whereas the lattice complex corresponds to the concept of fundamental region.

Rudolf Arnheim[2] recently criticized crystallographic nomenclature for not distinguishing between what he calls rotational and centric symmetry. In his reply to Arnheim's critique, Loeb[3] surmised that Arnheim refers to the distinction between centers of rotational symmetry located on lines of mirror symmetry on the one hand, and on the other hand those not on mirror lines. The notation in the International Tables (IT) for X-ray Crystallography is contrasted with that of Loeb and Le Corbeiller (L) for three patterns illustrated in Figures 1a, 1b and 1c, demonstrating that the L-notation does indeed make this distinction .

Figure 1: Patterns described respectively as 33'3'', 33'3'', 33~3 (L-notation) or p3, p3m1 and p31m (IT notation)

Rotational symmetry is fundamental in the L-notation; rotational symmetry may exist even in the absence of reflection symmetry, but the coexistence of reflection lines invariably implies rotation (in special cases translation) symmetry. Centers of rotational symmetry (L-nomenclature calls them rotocenters) form lattice complexes called roto-complexes whose symmetry values are determined by a single diophantine equation:

$$k^{-1}+l^{-1}+m^{-1}=1,$$

$\underline{k}$, $\underline{l}$ and $\underline{m}$ being the symmetry values of the respective roto-complexes. Patterns are classified according to the five solutions of this equation: 1∞∞, 22∞, 236, 244, 333; ∞-fold rotational symmetry amounts to translational symmetry. In the L-notation a k-fold roto-complex in which all centers lie on mirrors is denoted by an underline: $\underline{k}$. Distinct rotocomplexes having the same symmetry value are distinguished by a prime: k and k', and enantiomorphically paired rotocomplexes are denoted by a ~: k, k~. Significantly, the IT notation makes no distinction between rotocenters having the same symmetry value but belonging to different roto-complexes; in Design such centers will generally accomodate different motifs, so that the distinction is indeed fundamentally important.

In Design the relative positions of rotocenters and reflection lines make a great deal of difference. In the absence of reflection lines patterns tend to be very, even overly dynamic (Figures 1a and 2a), and they will exist in two mutually enantiomorphic manifestations. Conversely, when all rotocenters lie on mirror lines, the patterns tend to be static; the best balance is found when some of the roto-complexes lie on mirrors and others are enantiomorphically paired. (Compare, in Figure 1, the patterns 33'3", having no reflection symmetry, $\underline{33'3}$", having all rotocenters on mirrors, and 33~$\underline{3}$', having 3 and 3~ enantiomorphically paired.) It is easily shown that such balance is not possible in the 236 system.

Further examples contrast the notation 244’ with p4, both of which represent the pattern shown in Figure 2a, $\underline{244}$’ with p4m (Figure 2b), and $\underline{2}$44~ with p4g (Figure 2c). The notation p4 does not tell us that there are *three* distinct rotocomplexes, one having symmetry value $\underline{2}$, and *two* separate and distinct ones having symmetry value $\underline{4}$. The notations p4m and p4g have created the mistaken impression that the former corresponds to patterns having only *mirror* lines, the second only *glide* lines, when, in point of fact, both p4m-patterns and p4g-patterns contain mirror lines as well as glide lines. The notations $\underline{244}$’ and $\underline{2}$44~, on the other hand, show that in the former case all rotocenters lie on mirror lines, whereas in the latter only the two-fold rotocenters lie on mirror lines, while the fourfold sets are mutually enantiomorphic.

Figure 2: Patterns described respectively as 244’, $\underline{244’}$, $\underline{2}$44~ (L-notation) or p4, pm and pg (IT-notation)

The five groups having two-fold rotational symmetry only, are denoted in the IT respectively as p2, pmm, cmm, pgg and pmg; only the first of these notations indicates the rotational symmetry. By contrast, the respective L-notations 22’2"2’", $\underline{22’2"2’"}$, 22~$\underline{2’2"}$, 22~2’2’~(g/g’) and 22~2’2’~(m/g) show all four sets of rotocenters and their interrelations, and specify the reflection lines to distinguish the two cases which have the same sets of rotocenters.

Above, we noted the L-notation for the patterns of Figure 1. The IT-notation for the first of these (33'3" in the L-notation) is p3; the fact that there are three distinct sets of three-fold rotocenters is not shown. This is unfortunate, because visually it is not always easy to distinguish between p3 and p6 patterns, and the L-notations 33'3" and 236 point up the differences, the presence or absence of 2-fold rotocenters marking the difference. The IT-notation for the remaining two of the 33'3" groups is p3m1 and p31m, but there has been some confusion as to which is which, as there does not appear to be a logical distinction between the two sets of symbols.

In the mid 'seventies one of us (W.K.C.)[4] examined many systems of notation in her search for a suitable language and notation to study and classify Islamic geometrical patterns. She found the ones most pertinent to the arts to be Hermann Weyl's Symmetry, H.S.M.Coxeter's Introduction to Geometry, and A.V.Shubnikov' and V.A.Koptsik's Symmetry in Science and Art.[5] These books expanded on the discussion of symmetry, the second using IT notation, the third including immensely detailed and exhaustive enumeration far beyond the needs of art historians, not being designed to meet the specific needs of artists and designers. After some years of study she found that A.L.Loeb's Color and Symmetry [6], even though initially published as a monograph in Crystallography, presents the language most appropriate for art-historical studies. In contrast to the other symmetry notations the L-notation indicates the symmetry values of all rotocenters, distiguishes between distinct and mutually enantiomorphic rotocenters, and indicates whether rotocenters do or do not lie on mirror lines.

The L-notation was originally designed as part of an explicit program for developing a more sophisticated or linguistically more highly developed language of structure, aiming at precision in the communication of relevant details. The L-notation has been taught quickly and effectively to art historians and designers.[7] At a recent symposium previous students, now Design professionals, who were trained with the L-notation, demonstrated that with use this notation easily becomes vernacular.

In conclusion, then, we would have to say that the IT-notation, based primarily on lattices and translation symmetry, which are fundamental in X-ray diffraction, do not necessarily best serve the purposes of art historians and designers, for whom the L-notation has the advantage of explicit indication of all rotocenters and their interrelationships.

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Figure 1a

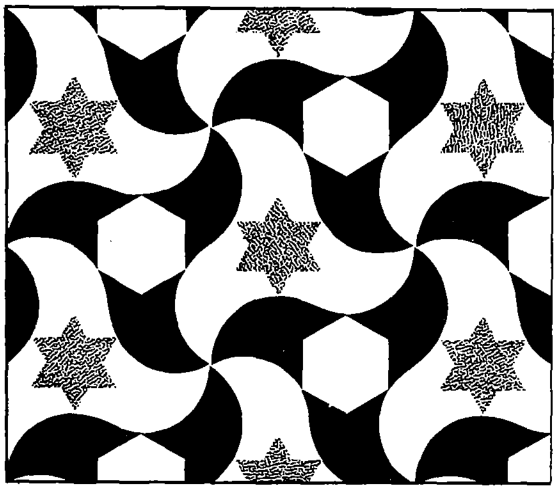


Figure 1b

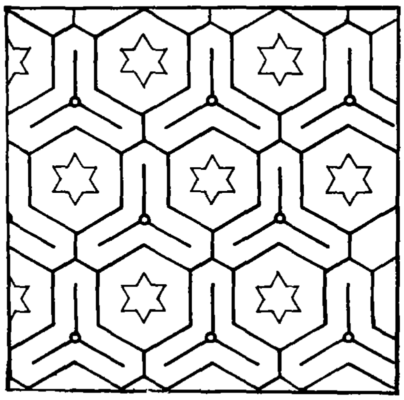


Figure 1c

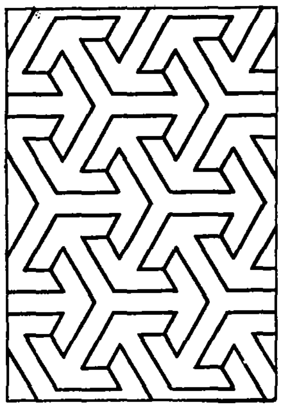


Figure 2a

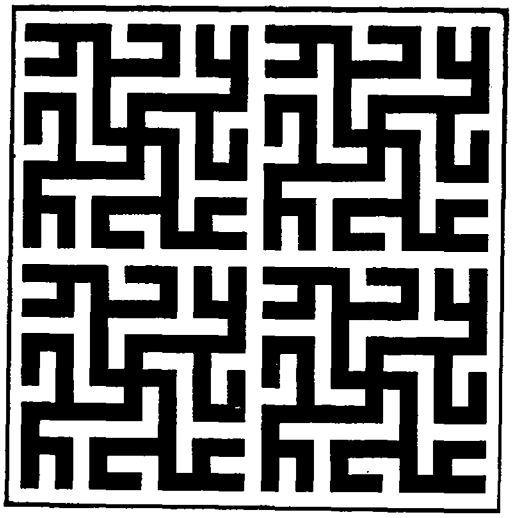


Figure 2b

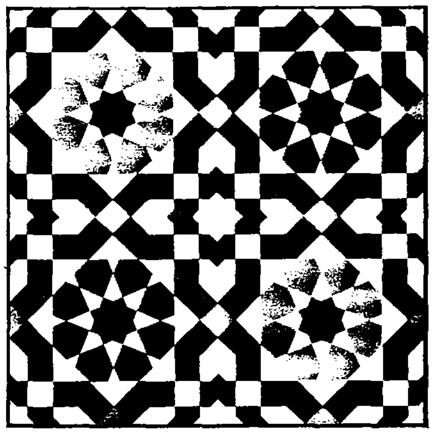
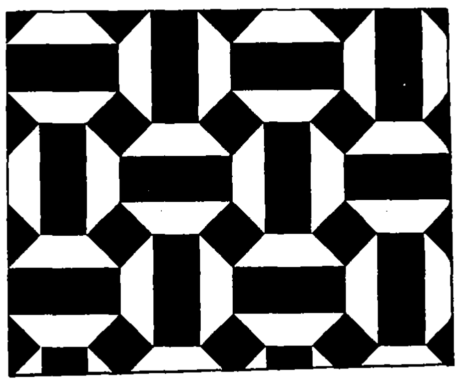


Figure 2c



COSMIC SYMMETRIES WITH MICROCOSMIC SYMMETRY BREAKINGS

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Summary: We argue that symmetries of the Universe may be due to those of microphysics.

Elementary particles and interactions are highly symmetric because of the small number of data characterizing them. However, as soon as building up macroscopic structures, any kind of asymmetry becomes possible via nonsymmetric combinations. So, symmetry, a *necessity* on microscopic scale, tends to survive only as a *possibility* on macroscopic ones. Although natural laws are (almost completely) symmetric in space and time, they do not lead to symmetric phenomena without symmetric initial conditions.

On macroscopic scale, indeed, various asymmetries appear, however, still some symmetry is present as well, which may be

1) remainder of a possible primordial symmetry via symmetric laws of motion;
2) product of an artificial intervention; or
3) outcome of an evolution, when symmetry is a condition of extremal energy (equilibrium), optimal functioning, &c.

On the largest scale Possibility 2) is ruled out because of highly superhuman sizes involved, and 3) too, because of the autocracy of gravity precluding equilibrium or organized functioning there. Still, with growing size symmetry increasingly reappears. Thus chaotic disorder in our medium scale is environed by asymptotic regions of ordered symmetric behaviours both above and below.

To explain the returning symmetry either a "finger of God" is needed to form initial conditions sufficiently symmetric, or His laws may somehow play the rôle of His finger to take care of symmetry (the latter being certainly the more elegant way). Indeed, it is the (not quite absurd) purpose of contemporary cosmology to grasp the whole Universe, together with its space, time, matter and forms, as a direct consequence of the laws of Nature.

The high symmetry of the present Universe means that, despite the fact that it contains the maximum number of constituents, its state can be characterized by means of a minimum number of data, just like a *microobject*. Such an initial state is highly unnatural unless the Universe itself had been a primitive microobject (Lukács & Paál, 1988), an idea quite conform with those of such great cosmologists as Lêmaitre (1958) and Hawking (1984). So the observed symmetry is really intelligible only in a vastly and uniformly expanding Universe. This new-fashioned *argument for the expansion* is quite independent of the observed redshift of the galaxies (and its usual Doppler interpretation), and also of any theory of gravity, which latter ones, however, fortunately point to the same direction.

The most familiar mechanism for driving expansion is the pressure difference, sweeping matter outwards into the *already existing* empty space, but this mechanism is incompatible with the observed *symmetric* endproduct. So there remains only the completely different alternative when the *space itself is expanding* (being "created") between the points *at rest*. Starting with microscopic type initial conditions one may hope to understand the present state of the Universe as the present standing of a competition between forces distroying and restoring symmetry, whose main steps we try to list now according to modern theories.

Even in the contemporary state of art, without knowing the details, existing theories uniquely single out *where* the Universe (a gravitation-dominated system) would become a microobject. This happens when the minimal energy (coming from the uncertainty principle of microphysics) of a quantum particle of localization L lies in the same order of magnitude as the energy correction from its own self-gravity, i.e.

$$E \approx (\hbar c/L) \sim (\hbar c/Lc^2)^2(G/L) \qquad (1)$$

(where G is the Cavendish-constant of gravity), so

$$L \sim (\hbar G/c^3)^{1/2} \equiv L_{Pl} \sim 10^{-33} \text{ cm} \qquad (2)$$

which is the so called Planck length. At this localization the quantum uncertainty energy and typical fluctuation time are

$$E \sim E_{Pl} \equiv (\hbar c^5/G)^{1/2} \sim 10^{16} \text{ erg} \qquad (3)$$

$$t \sim t_{Pl} \equiv L_{Pl}/c \sim 10^{-43} \text{ s}. \qquad (4)$$

The corresponding mass is $\sim 10^{-5}$ g, and the temperature $\sim 10^{19}$ GeV $\sim 10^{32}$ K. This implies that an object with age t_{Pl}, size L_{Pl} and energy E_{Pl} is within one quantum uncertainty from its complete absence, consequently its existence or nonexistence cannot be clearly distinguished. Only essentially different values would require extra explanation. Before the advent of a future quantum gravity theory the above data should be considered the most natural initial conditions, not requiring derivation from any set of *previous* data.

In most quantum field theories the completely particleless states do not necessarily possess zero energy density, even if in these states there is complete homogeneity, isotropy, stationarity, &c. All these uniform backgrounds are called *vacua*. So a vacuum may differ from Nothing, but still it represents the local (not absolute) minimum of energy and complete absence of any structure. Of them one possesses the maximal microscopic or internal symmetry in the sense that all of the expectation values of the fields vanish, but generally this is *not* the one with zero energy. The most natural initial condition is *maximal symmetry*, and then the nonvanishing energy of this state will be very important in the following history.

If then the Universe was in expansion, then the specific energy of its particles (temperature) was diminishing from T_{Pl}. When the radiation density is already negligible compared to this nontrivial vacuum energy density, the further expansion has no more diminishing effect on the density. Henceforth (for a while) the particle content is negligible, the rate of space creation is prescribed by elementary constants, and there is a simultaneous energy creation as well, to keep the density $(E/V)=\epsilon$ constant. Since $E=Mc^2$, we may speak of creation of mass, and matter as well. Conservation law is not expected to hold for energy, being conditional upon *time symmetry*, not present in an expanding Universe. According to the equations of General Relativity (Hawking & Ellis, 1973), any expanding system with a constant energy density ϵ_0 must have *negative* pressure P via

$$0 = dE+PdV = d(\epsilon_0 V)+PdV = (\epsilon_0+P)dV \qquad (5)$$

so $P=-\epsilon_0$, indeed. (Eq. (5) is well known from thermodynamics too, expressing the adiabaticity of expansion.) Since in the relativity theory the source of gravitational acceleration is not simply $M=E/c^2$ but $M_{eff}=(E+3PV)/c^2$, with negative pressure a negative gravitational effect appears accelerating the expansion. The distance R between two points of the substratum at rest changes according to the rule of classical form

$$\tfrac{1}{2}\dot{R}^2 = \tfrac{1}{2}v^2 = GM/R = (4\pi/3)(\epsilon_0/c^2)R^2 \qquad (6)$$

but here M is already time-dependent according to (5). Hence

$$\dot{R} \sim R \;\; \to R \sim \exp(t/t_0). \qquad (7)$$

This exponential expansion is called *inflation* (Guth, 1981). As it is intuitively clear, inflation - like that of a baloon - increases regularity and symmetry. During this very rapid expansion, therefore, there are *simultaneous creations* of

space (volume)
matter (energy or mass) and
symmetry (uniformity).

Then we have managed to find a *natural way from a symmetric microuniverse to a macrouniverse preserving or even increasing its symmetries*. However, we shall have to pay for it immediately with the decrease of microscopic symmetries. The state in a high energy *vacuum* is not stable because there are other vacua below. Indeed, today the cosmic expansion is decelerating, antigravity does not act. Therefore, the fields must have gone to another vacuum level, with lower energy *and* symmetry (maybe in subsequent steps). During this transition energy was released in the form of particles, so becoming structured. This change is analogous with the solidification of water, so it is called phase transition, and contains a *symmetry breaking*.

According to present theories as e.g. Grand Unification, there were 3 kinds of deterioration of symmetry of the actual state almost simultaneously: (Barrow, 1983)

1) Appearance of nonzero expectation values of quantum

fields. Then inhomogeneities also might appear, because of incoherent domains of "nucleation" of the new phase.

2) Generation of nonzero rest masses for *some* particles. With these, symmetries for interchanging between different particles ceased.

3) With this proper time appeared, so the possibility of spontaneous decay, permitting the start of developing *asymmetry* in the *matter-antimatter* ratio.

Now we have arrived at the hot, radiation-dominated Universe. Here P>0, therefore with the space creation energy and mass are being destroyed. Antigravity has ceased, so there is no more smoothening of irregularities. Comparing the two epochs:

	Vacuum-dominated	Radiation-dominated
Expansion:	$\dot{R} \sim R$	$\dot{R} \sim R^{-1}$
Irregularity:	$\partial\epsilon/\epsilon \sim R^{-2}$	$\partial\epsilon/\epsilon \sim R^{2}$

which are just symmetric formulae in R instead of time. Our present world is from 60 orders of magnitude from L_{Pl}, and can be reached by spending ~30-30 orders of magnitude of expansion in both epochs.

In the cooling matter below T~3000 K neutral atoms were formed. Since then light has been unable to prevent gravitational contraction of *local* density excesses. Thus there is a spontaneous breaking of (spatial) symmetry on medium scale: homogeneity & isotropy decreases to spherical symmetry centered at random places. Then formation of galaxies, stars and planets begins, giving a possibility to life.

Therefore the history of Universe can be narrated as that of the symmetries. The present state still contains a substantial number of symmetries, but the symmetry groups are not maximal. (E.g. spatial homogeneity and isotropy but not full space-time symmetry on largest cosmic scales; spherical symmetry but not homogeneity & isotropy on macroscopic scales; SU(3)xSU(2)xU(1) symmetry but not SU(5) on microscopic scales). And even the remaining symmetries may be *weakly* violated as right-left (parity) symmetry in *weak* interaction.

Without symmetries the Universe would be too disordered to permit to grow anything highly organized, and too complicated to be understood. On the other hand with complete symmetry no observer could have separated itself from the rest of the Universe. So *partial symmetry seems to be a necessary condition for the existence of a Universe habitable by intelligent beings.*

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ON THE STABILITY OF OSCILLATIONS WITH HELICAL AXIS OF SYMMETRY

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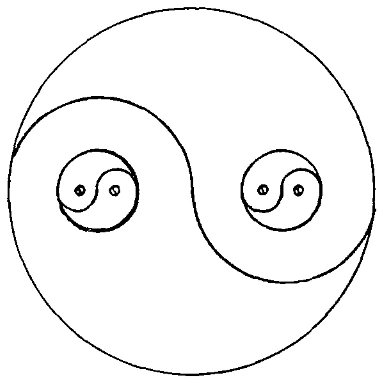
Post-Eisteinian physics seems to be excited with non-linear processes and their nature. Therefore, motions and structures with a helical axis of symmetry appear to be central to its interests. It frequently occurs so that having addressed a new problem area, the scholars surprisingly find out that mankind has already dwelt with it at a previous stage of its spiral development, though at another level. Thus, structures and motions with a helical axis of symmetry appear in remarkable symbolism of the ancient Chinese yin-yang monade.



It can be seen that the monade is visualized as a symbol with a helical axis of symmetry emblemizing two vital principles, yin and yang (they can be infinetely interpreted as the male and the female principles; night and day; light and dark; good and evil, etc.), yin being seeded with yang, and yang likewise seeded with yin. Sometimes, the embryos of yin and yang are replaced by two minor momades repeating their greater counterparts. Moreover, the monade emblemized the eternal life process ensured by the continuous interaction and rotation of yin and yang. Broadly speaking, ancient knowledge displays its being specially interested in structures and motions with a helical axis of symmetry. Thus, the concept of "hot" and "cold" processes and medicines was a fundamental acquisition of Oriental medicine. Roughly, "hot" and "cold" correspond to hyperfunctions and hypofunctions in terms of modern medicine. "Hot" drugs and food increase and "cold" ones decrease vital energies of an organ. Very interesting to us is experimental evidence obtained in 1978 by E.Yu.Kushnirenko and I.B.Pogozhev, that "hot" and "cold" drugs differ mainly in their stereochemical parametres. To be more precise, the difference is in that which winding, right or left, prevails in the molecules. Thus, either cognitively or empirically, ancient physicians appear to have been capable of capturing the link between the direction of biological molecule winding having a helical axis of symmetry and its impact on the human organism. Generally, the direction of biological spiralmolecule winding and the stability of the molecules immediately relate to the problem of life as such and its origin. Interestingly enough, V.V.Alpatov and G.I.Voskresenskaya established in 1962 that ageing lowers the optical activity of the human blood protoplasm telling of a restructuring in the molecules with a helical axis of symmetry.

The stability of structures and motions with a helical axis of symmetry has more than once been approached by various disciplines. At a macro level, astrophysicists are attracted by spiral galaxies, and crystallographers look into crystals with a helical axis of symmetry. At a molecular level, in biophysics, special attention attaches to the spiral molecules of DNA, proteins, glucosides, etc. Finally, a micro level physics is impossible without studies into, and due account of, the spin of elementary particles.

In the Newtonian age, physics described the structures with a helical axis of symmetry as the result of the combination of cyclic and linear structures. This approach makes it possible to describe spiral structures as such, but not the mechanism of their formation and the stability of their moviment. The Einsteinian age in physics brought about the understanding of the need to study into non-linear phenomena and to translate descriptions into the language of non-linear physics. Accordingly, the approach to describing helically symmetrical structures and motions has changed. To divide such motion into a cyclic and a linear component (or into two cyclic components with slightly mismetching periods in case of a spiral wound over a circumference or a torus) was earlier understood as lack of interaction there between or its being negligibly small interpreted as further enhancement introducing no changes into the overall picture. Non-linear physics points to the essentiality of cyclic to translational motion interaction and the need for it to be carefully sutdied.

In this connection much has been done to show the stability and structures and motions with a helical axis of symmetry, and the conditions in which stability is achieved. We have investigated a case of combined cyclic and translational motions whose interaction rather substantially influences the physics of the process despite its very small value. The cyclic motion was represented by a high frequency periodical process, and oscillations of a much lower frequency were taken as a translational motion. The working model was a low-frequency HF pumped pendulum. Interaction between low-frequency oscillations of the pendulum and the high-frequency field took place in a narrow zone around the pendulum's zero position. Given certain initial phase requirements and an adequate system's Q level, it acquires stationary oscillations with a frequency close to the free-running one. Depending on the initial conditions, the above pendulum oscillates with varying stationary amplitudes (to be more precise, stationary oscillation zones). The occurrence of the pendulum's stationary oscillation zones is the product of minor interaction between the components forming the system of a HF field and a LF oscillating pendulum. Their most characteristic feature is their stationary pattern associated with the stability of helically symmetrically structures, the cyclic motion being associated with a HF field, whereas translational motion, with LF oscillations of the pendulum.

Because the essential feature of natural oscillations is their stability, the above suggested physical model can lay claims on reaviling and explaining interrelations between helically symmetrical motions in nature. For example, let us consider the motion of the Earth-Moon system around the Sun. Both the Earth and the Moon move along a spiral orbit. The motion is stable over a period of billions of years. If we have a synchronized pattern for cyclic and orbital motions, the period of rotation of the Earth-Moon system around the Sun is to be connected with the cyclic period of the Earth-Moon system. The latter is known to become currently longer, thus suggesting an increasingly longer solar year or a slower rotation of the Sun on its axis, because the components in the oscillating Earth-Moon system interacts by way of the Sun's magnetic field and the solar wind. One more object allowing for a similar approach can be spiral biological molecules, which are known to be continuously oscillating Such oscillations can reveal synchronization of longitudinal and torsional vibrations. The oscillations of the unwinding spiral are interpreted by us as a HF oscillation component, and the lengthening of the structure of a molecule as a LF component. Because biological molecules are stable in both time and a wide range of external conditions, to explain this we can resort to the foregoing mechanism of stationary vibrations.

Moreover, in biology there is a whole group of oscillatory processes which fails a satisfactory description within quasilinear physics. Helically symmetrical are not the motions involved, but rather their phase images. The motions proper have a biased mirror plane of symmetry. The rest, we believe, can be considered as interaction of two vibrations--a HF oscillatory process and a LF one, which for the sake of simplicity we call translational motion. It is not impossible that the interaction of high-frequency vibrations of the wings of insects with their translational motion explains for the difference between their calculated and real flight parameters.

One more object to study via this approach is the peculiar pattern of dolphin swimming. As is known, when the dolphin swims its body in the water performs wave-like movements, but what is more remarkable, these movements are accompanied by wave-like constructions running along the body. Possibly, low-frequency vibrations of the body ensuring its translational motion and high-frequency vibrations of its skin occur in a synchronized pattern to reduce friction in the water and offer a speed yield. It is not so much the interaction mechanism of such multi-frequency motions, but the stability of such systems, if any, can be of interest for the simulation of historical and sociological processes. Thus, it is possible to analyse the motion of human thought and to show that "dissidence" is necessary for its successful development. It is rather apparent that the greatest contribution to science's headway (which is here associated with a low-frequency process) is mainly provided by young people whose views often go astray, that is deviate from the accepted point of view (these oscillations are associated here with high-frequency oscillations), and when such high-frequency oscillatian of a person's views interacts with the translational low-frequency motion of scientific progress, the result is the most natural and global contribution to the process of scientific development.

Now back to the ancient Chinese monade in which the yin and yang seeds are replaced by smaller monades. In addition to thoughts about the recurrent and unending knowledge, it now hopefully becomes suggestive of many other things. Thus, if we take smaller rotating monade as a symbol of a HF oscillation helically symmetrical process, and a greater monade as a symbol of a low-frequency oscillation process, also helically symmetrical, the pattern as a whole would become the symbol of interaction and stability of such a system as an entity. One more thing, which only naturally comes to ming; depending on a level oscillations which are believed to be low-frequency for any given scale, can be a high-frequency field for yet lower vibrations in a system of a lesser scale, etc. A view on the structure of world natural processes, based thereon, would give an uncontinuous successio of periodical oscillations would one upon another and forming a series of interactions.

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FRACTALS, RENORMALIZATION GROUP AND CHAOS IN DISTRIBUTED SYSTEMS

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Usually one connects symmetry with regular processes, temporal order, any type of structures. But researches of systems where the phenomenon of dynamical chaos (complicated non-periodic motion) can be observed have shown that concepts of symmetry are of significant importance for such systems. It was found that usually strange attractors, describing dynamical chaos, are fractals, i.e. they have structure that reproduces itself on smaller scales.

These objects proves to be invariant under certain renormalization group. Apparently, for the first time this fact has been rigorously proved in the theory of one-dimensional mappings for the Feigenbaum attractor. Symmetry enables the investigation of internal structure of the set in phase space.

In this report the fractal dimensions (characteristics used for description of fractal attractors) and numerical algorythms for their evaluation are discussed. We consider a number of examples that demonstrates the applicability of these approaches for the study of wide class of chaotic regimes in distributed systems. Among them are turbulent regimes in hydrodynamic systems, diffusion induced and spatio-temporal chaos that are characteristic of several oscillating chemical reactions.

STRUCTURAL EVOLUTION OF THE GOBELIN 'COSMOS' BETWEEN 1968 AND 1988

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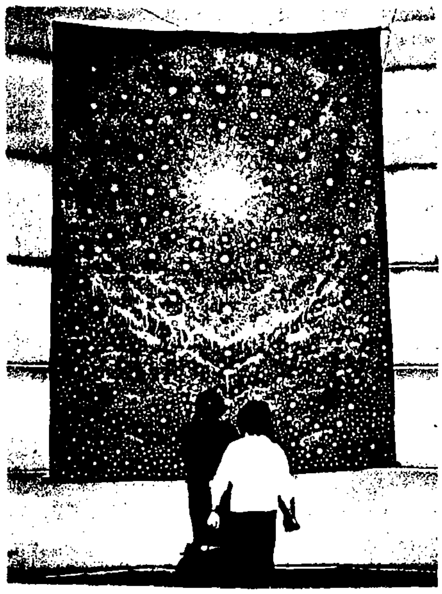
Drawing of the cartoon for the gobelin 'Cosmos' was initiated as usual for all fine artistic creations: with instinctive impulsions and intuition. After a drawing period of about one and a half years, without any geometric preconceptions, I had the drawing structurally redrawn almost completely. In fact, this was the very time when the drawing was starting to get a structure. After the development of vertical and horizontal axes, the structural order of expanding squares has been evolved, followed by the inner octagon and the diagonal series of expanding squares. Thereafter the systems of curved lines appeared, i.e. the systems of parabolas and hyperbolas, and at last, those of the circles of ever expanding radii. (Figure 1).



A further step was the appearance of sinusoidal curves, representing the directrices of this system. Thus, the concept of dynamic events has been evolved with three principal constituents:

/1/ **radial movement** ejected from the centre
/2/ **centrifugal movement** around the centre, and
/3/ **spiral movement** synthesized from the above two movements.

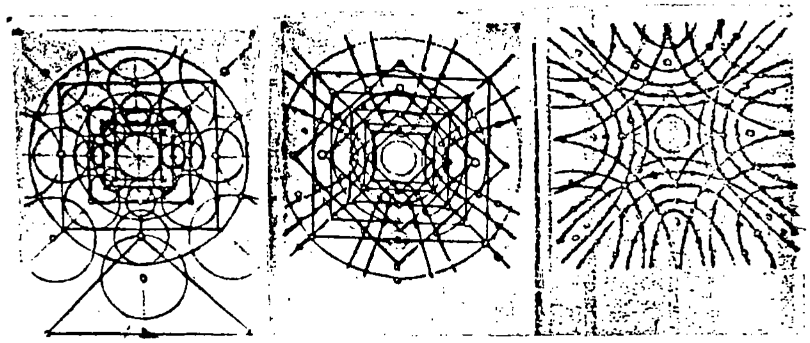


Figure 1

The transformation of square systems to wave lines produced a formation of decisive importance, namely, the vertical-horizontal and diagonal sine wave systems, respectively, reminding of trajectories of potential spaces, common in thermodynamics. Vertical and horizontal wave lines bear equipotential points, while on the diagonal ones there are points of maximal potential differences. (Figure 2).

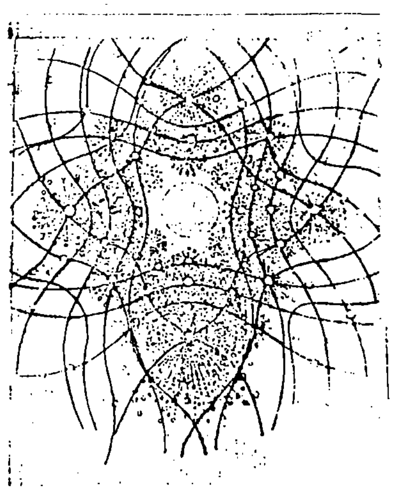
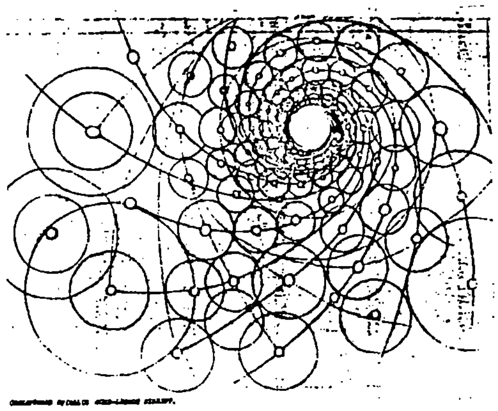


Figure 2

This important structural figure proved to be the key step for further progress. Supposing, this structural formula to be valid for the system as a whole, there should be a corresponding **network of trajectories** around each of the major galaxy centres.
After a structure elucidating work of several years, this has led, finally, to the evolvement of a **biphasic,** concentric and ever **expanding system of sine waves.** This has been the decisive step in the evolution of structural conception, permitting the interpretation of the entire structure to be expressed as an **oscillating system** around the Central Sun. At last, recognition and plotting of the **spiral construction** was an important step for the total composition and for each of the galactic stellar systems. The system of counter-moving, dextro- and levo-rotating spiral arms became a general structural formula for both of the whole system and for each of the minor stellar systems. (Figure 3).



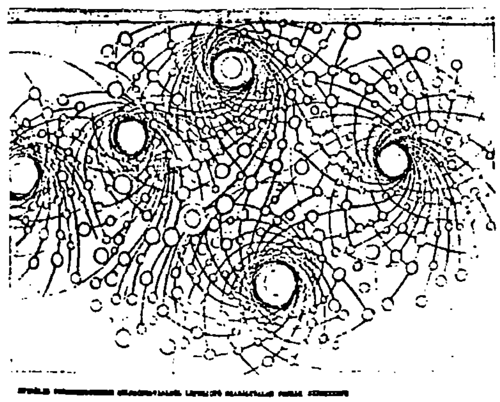


Figure 3 Figure 4

The galaxies are positioned on opposite spiral arms of the total system, at intervals increasing according to the Fibonacci sequence of numbers, as points of a logarithmic spiral. (This is in agreement with the principle of **phyllotaxy**, being a structural rule common in nature.)

Ultimately, the general dynamic formulation has been developed. Combination of explosive movement out of the centre of galaxy core, and of the circular movement, results in spiral movement. With junction of spiral arms from two neighbouring galaxies at the inflection point, sine waves are produced. Spiral movement from the centre of one system continuing in the corresponding spiral arm of the other system, so to say, 'coils up' on the core of the other force centre, then is 'ejected' by a newer explosion in the form of an opposite spiral movement toward the direction of the original, a second or an n-numbered galactic system, respectively. (Figure 4) This way, all the systems are interconnected in a 'frog' path of an endless loop.

Thus, a system of **transmission belts** originates, crossing each other in the space, wriggling under and over each other, assigning energy paths for every force centre. However, not only 'belts' but also 'plates', 'lobes', multiple curved surfaces evolve, interpenetrating, crossing each other and coiling upon the centres.

The background, so to say, basis and mounting frame of this complicate simultaneous, 'organic' type dynamic construction is a structural formula of relatively simple geometry: the edges of fundamenteal configurations, that is those of triangles, quadr - angles, octagons and other basic ones, representing the crystal structure of the Universe, as well as, the system of rotating

quadrangles, octagons, etc., representing its dynamic structure, are constituing, in a curious way, a 'planar' Universe based on Euclidean geometry. The Universe Lines are radii crossing the absolute centre, representing the Radii of the Universe.
Thus, the entire system is axial symmetric, and the axes of reflection pass the Origin vertically, horizontally and diagonally, resprectively. (Figure 5).

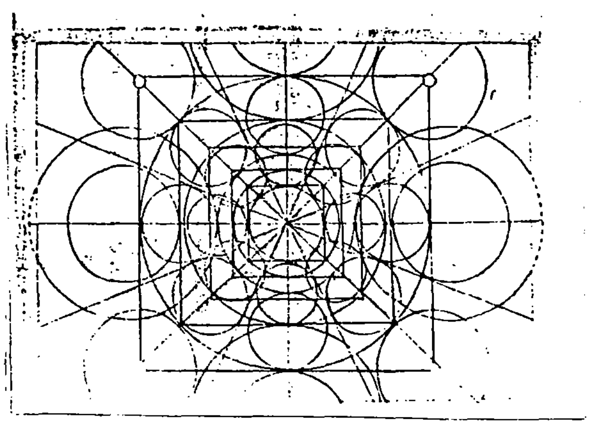


Figure 5

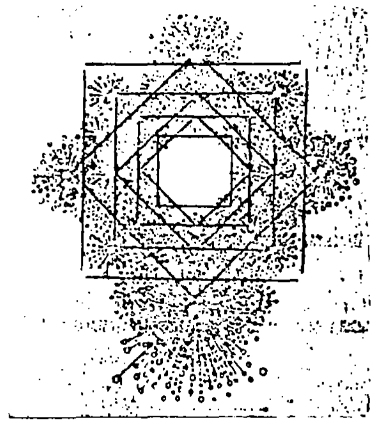


Figure 6

Nevertheless, the principle of regularity, arrangement and symmetry does not prevail for all the structure of gobelin 'Cosmos'. The principal symmetry-violating element is that there is a preferred direction, namely, the vertical axis, assigning the direction of gravitation. The other dissymmetric moment is that the most significant star, positioned under the Sun, is at a lower position than the corresponding other three stars. The 'amorphous' stellar points, condensed quasi as 'flood of light', not only tend downwards but seem to be collected, then ejected by the star of distinction. In this way, this star is representing a separating and, at the same time, a connecting role between the upper and lower spheres, and so, can be considered as a 'point' where the envelope pregnant with stellar material is ruptured, burst open, and the 'cosmic crop' is thrown out like from a poppyhead or other plant capsules. (Figure 6)
This dispersion, though, represents only an intermittent period, the centripetal force prevails again, so to say, arranges the arrays of dispersed material, bending then crumpling it under itself, and producing the spherical symmetry again but, at this time, as a sphrere of larger radius.
Thereby, by this conception, born in the field of artistic creativeness and based on visual evidences, a model of Universe is outlined with the intrinsic nature of periodic expansion (then, possibly, of retraction) in the space and time.

Miklós Maróth

TWO WAYS OF THINKING IN GREEK AND ARABIC PHILOSOPHY

There was only one Greek school of philosophy which survived the decline of antiquity: that of the Neoplatonist philosophers. Their activity was continued first by the members of the Baghdad school of philosophers and later by the schoolmen of the European Middle Ages. It was the Neoplatonic school tradition which preserved the works of Plato and Aristotle — in Neoplatonic interpretation.

The Neoplatonic theory of demonstration — first documented in the Elementatio Theologica of Proclus — has been based on the famous Tabula Porphyriana which runs as follows:

substantia

materialis immaterialis

corpus

animatum inanimatum

vivens

sensitivum insensitivum

animal

rationale irrationale

homo

This Tabula contains a hierarchic order of definitions and demonstrative syllogisms: body is a material substance, etc; and

every material thing is a substance

<u>every body is a material thing</u>

every body is a substance

The hierarchic order of sciences established by Ibn Sina was founded on this theory of demonstration. His system — the essentials of which are comprised in the table below — consists of several degrees of higher and inferior disciplines.

metaphysics

physics mathematics

medicine geography geometry arithmetics

psychology history etc etc astronomy optics harmony music

The higher sciences (metaphysics, physics, mathematics) are the theoretical ones, the inferior disciplines are the practical ones.

The structure of the human soul is parallel to this system of sciences. The vegetative part of the soul is responsible for maintaining the life, the animal soul collects the impressions through sense perception. They are genaralized by the practical intellect which is the lower part of the rational soul. We attain the first principles of the practical sciences by way of this generalization or by deduction from a hierarchically superordinate scienc. In this unified system of sciences a deduced thesis in a higher science becomes first principle for a subordinate science. The consequence of this theory is that the human mind generalizing the sense impressions sets up a lot of hierarchically ordered Tabulae Porphyrianae which form hundreds of series and sequences leading from the simplest and lowest knowledge of the inferior sciences to the first principles of the highest sciences converging on the first axiom of the metaphysics. The human mind moves *forward or backward step by step* along the concepts contained in the Tabulae Porphyrianae

The first axiom of metaphysics with the first definitions cannot be proved logically in this system. Their knowledge is of divine origin. The psychology of Alexandrus Aphrodiseus, the second century Greek philosopher says that the spheres of the heavenly regions are moved by divine intellects, the lowest of them is the intellect of the moon sphere. The human mind is originally empty, so it is a potential intellect. The potential intellect is furnished by the intellect of the moon sphere with the first forms (that is: with the knowledge of the first causes, because the meaning of *form* is "formal cause"), so it becomes actual intellect. The intellect of the moon sphere is called, because of this activity, the *active intellect.*

Thus the human mind obtains the ultimate truth without discursive thinking or long reflection by divine inspiration, that is to say by intuition.

Ibn Sina and the Arab philosophers taught that every normal scientist can get in contact with the Active Intellect, nevertheless a lot of them never attain the happiness of being inspired by divine wisdom. On the other hand, there are many believers who improving their moral habits can remain in continuous contact with the divine world and get an intuitive knowledge — without learning — of the most important things, that is of God and metaphysics.

The Greek and Arab philosophers recognized the symmetry and asymmetry of intuitive and discursive thinking: they can replace each other in certain fields, but they cannot replace one another in the domain of the first principles.

They became aware of the problem of discursive thinking and intuitition and describing them — in terms of Aristotle's philosophy — they gave an answer to the question HOW, but they could not give an answer to the question WHY. It is the modern brain research which attempts to find an explanation to it.

At the same time it is not allowed in logic to infer backward. An "if..., then ..." conditional is not reversible. Consequently, deductive reasoning can only move from the first general principles to the more individual and practical deduced knowledge. Reasoning by deduction is an asymmetric mental operation.

The syllogistic reasoning along a Tabula Porphyriana is possible moving upward and downward equally. This is the famous DEDUCTIO A PRIORIBUS ET A POSTERIORIBUS of the scholastic logic. The theory of demonstration based on the Tabula Porphyriana permitted a symmetric reasoning in the theoretical sciences which was prohibited by Plato and Aristotle (and contemporary logic as well).

HARMONY OF THE UNIVERSE

Mikhail MARUTAYEV, composer, Honoured
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The theory formulates the principle of identity of opposites, which defines harmony as a general regularity, as well as establishes three numerica-laws of harmony and their experimental substantiation in music and natural sciences.

§ 1. Definitions. Harmony is a law of integrating parts into the whole. Categories of harmony are: stability, invariance, equilibrium, conservation. They determine the integrity. Assume the following statements (axioms): 1) motion is specific and diverse; it determines qualitative difference between things; 2) all these things contain a general feature; this general feature is what recurs in phenomena, what is identical in them. The first affirms non-identity (difference) in phenomena and the second - identity. The basic properties of opposites are as follows: identity is abstract, uniform and irrelative; non-identity is concrete, diverse and relative. The connection between identity and non-identity means the identity of opposites:

$$\text{A is non-A}, \qquad (1)$$

where A is identity, non-A is multitude: non-A is B, is C, is D, but each non-A is A. Formula (1) connects the most important categories of dialectics: this connection reflects non other than the relation of general and individual (whole and parts), i.e. harmony.

A	non-A
Conent	Form
Essence	Reality
Quality	Quantity
Identity	Difference
Stability	Instability
Invariance	Variance
Conservation	Transformation
Equilibrium (rest)	Motion
Abstract	Concrete
General	Individual
Uniform	Diverse
Absolute	Relative
Whole	Parts

Let us take several examples. Consider any notion, for instance, "a tree". It contains no differences of specific trees. This is identity, i.e. general. Let A in formula (1) correspond to the notion "tree". Then non-A are specific trees - birch, oak, etc. Let A correspond to the notion "rest", then non-A are specific motions (rectilinear, curvilinear ...). The statement "each non-A is A" implies: every motion is rest, every individual is general, etc. (note that general statement: motion is rest or individual is general is false - this is identification of opposites). An example: "every motion is rest" is consistent with the principle of relativity of mechanics, due to which "... it is impossible to determine experimentally, whether the motion of a given coordinate system is accelerated or uniform and rectilinear, while the observed effects result from gravitational field ..."*. In terms of rest-motion the identity of opposites may be formulated

* Einstein A. Physics and Reality. Moscow, Nauka Publ., 1965, p. 72.

as follows: motion is diversity, where each particular case abstracted from this diversity is rest.

The essence of formula (1) appears to be a new type of generalization, which is called essential or qualitative. It means opposite to the conventional statement: individual case is generalization. This is due to the fact that the relation of individual and general determined by resides in their coincidence, identity, occurring when diversity is eliminated in a concrete, i.e. each individual case. This means such an individual case which is contained as common in all the cases of this kind. It is called as an important (or general) individual case. Examples: 1) rest is each (hence general) case of motion; 2) series Σ 1/n (1) is an important individual case of series $\Sigma\ 1/n^s$ (2). Series (2) is a quantitative generalization of series (1); series (1) is a qualitative generalization of series (2). This leads to numerical laws, since arithmetic is exactly such an individual case (basis) of mathematics. General definition: quality is a fundamental individual case inherent in all the cases of this kind; quantity is a multitude of cases containing (expressing) a basic determining case. Consequently, numbers (digits) can express not only quantity, but also quality, for instance, a golden number.

Analysis of space and time categories from the point of view of qualitative generalization makes it possible to assert harmony as the essence of space-time. The expression of space-time essence in space-time coordinates loses its sense. Therefore, the laws of harmony are formulated in the form of new mathematical principles based on a successive chain of qualitative generalizations.

§ 2. Law 1 - qualitative symmetry (S_q). According to Minkovsky, a uniform and rectilinear motion corresponds to a straight world line and an accelerated motion - to one of the curves. According to the general theory of relativity, an accelerated system is indistinquishable from an inertial one. This provides a means for interpreting formula (1) as a relation of a straight line and a curve and expressing this as the following equation

$$a^n = na, \qquad (2)$$

i.e. in the form of relation of additive Σ a=a+a+a...=na (straight line) and multiplicative Π $a=a\cdot a\cdot a...=a^n$ (curve) principles, where n is an integer or a fraction. The important individual case of solving equation (2) when a=n=2 leads to constructing a qualitative symmetry. In accordance with principles na and a^n two symmetries are constructed: arithmetic (S_A) a=x=x-b with center $x_A=(a+b)/2$; geometric (S_g) a/x=x/b with center $x_g=\sqrt{ab}$. The essence of S_g is in connection of inverse numbers a^{+1} and a^{-1}. The essence of S_A is connected with numbers 2^n (n - integer) and generalized by the qualitative equivalence formula

$$a \eqsim 2^n a, \qquad (3)$$

where symbol $\frown$ means qualitatively equal. Formula (2) reflects the dichotomy principle which forms the basis for many phenomena, specifically in biology: division of cells by half; in music: octave similarity (melody when transferred from one octave to another retains its quality). Nex, the relation or generalization of S_g and S_A is established. This occurs if relationship $x^{+1} \frown x^{-1}$ is satisfied for x_g. In accordance with formula (3) this means $x^{+1}/x^{-1}=2^n$, hence $x=(\sqrt{2})^n$. This case S_g is called a qualitative symmetry (S_g) with center $x_q=x_g=\sqrt{ab} = (\sqrt{2})^n$.

§ 3. Transformations of S_q. The numerical intervals between two adjacent powers of $\sqrt{2}$ are called S_q ranges and the powers of $\sqrt{2}$ - range bounds. The ranges are designated as $\overset{i}{R}$ or $\overset{j}{R}$ (i. j - range numbers). Let: $\dots \overset{-3}{\div} (\sqrt{2})^{-2} \overset{-2}{\div} (\sqrt{2})^{-1} \overset{-1}{\div} (\sqrt{2})^{0} \overset{+1}{\div} \sqrt{2} \overset{+2}{\div} \dots$. The numbers above are range numbers. Let number is in $\overset{+1}{R}$. This means $\sqrt{2}>a>(\sqrt{2})^0$. The two adjacent ranges cover the interval of an actave. One range is equal to a half-octave. Transfer of a number from one range to another according to formula (6) (see below) is transformation. The transformation of number of into b is designated as $a \underset{\frown}{\perp} b$, or $a_i \underset{\frown}{\perp} a_j$. The transformation of S_q takes the form

$$a \underset{\frown}{\perp} a^k \cdot 2^n, \tag{4}$$

where k= +1 or -1, alternating in each subsequent range; n is integer changing every other range by one. Let us designate each range bound as $\perp$ and introduce number a into $\overset{+1}{R}$. Its transformation will be as follows:

$$\begin{aligned} &\perp a \overset{+1}{\cdot} 2^0 \perp a^{-1} \overset{+2}{\cdot} 2 \perp a \overset{+3}{\cdot} 2 \perp a^{-1} \overset{+4}{\cdot} 2^2 \perp a \overset{+5}{\cdot} 2^2 \perp \dots \\ &\perp a^{-1} \overset{-1}{\cdot} 2^0 \perp a \overset{-2}{\cdot} 2^{-1} \perp a^{-1} \overset{-3}{\cdot} 2^{-1} \perp a \overset{-4}{\cdot} 2^{-2} \perp a^{-1} \overset{-5}{\cdot} 2^{-2} \perp \dots \end{aligned} \tag{5}$$

The general formular of transformations for any a_i (or law 1):

$$a_i = a_j^b \cdot 2^c, \tag{6}$$

where a_j is a preset number; $b=k_i \cdot k_j$ and may assume only two values: $b_1=+1$, $b_2=-1$; number c depending on b may also take only two values: $c_1=n_i-n_j$, $c_2=n_i+n_j$. If $b=b_1$, then $c=c_1$; if $b=b_2$, then $c=c_2$. Values k and n are determined by expression (4) or (5). The transformations of S_q form a group. For instance, let $a_{+2}=1.618$ (golden section). Find a_{-1}. From (5) $k_{+2}=-1$; $n_{+2}=+1$; $k_{-1}=-1$, $n_{-1}=0$. According to formula (6) we obtain $a_{-1}=0.809$.

§ 4. Law II - disturbance of symmetry (S_d). Law II - the essence (invariant of S_q - arises from the fact that in the general case $x_g \neq x_A$. Law II gives rise to numerical series by the formulae $C_k^{(1)} = (1+2^k)_i$ and $C_k^{(2)} = |\sqrt{2}(1+2^k)|_i$, where k is the integer, i means that the number obtained in the brackets should be transformed into the i-th range by formula (6). When i=-1 we obtain the following 10 numbers of S_d: 0.713, 0.718, 0.729, 0.750, 0.800, 0.884, 0.943, 0.970, 0.985, 0.992. These 10 numbers unevenly divide S_q into 11 parts. The uniform tempering of the S_q range (qualitative generalization of S_d numbers) is achieved by means of 10 integral powers of the number $\alpha=(\sqrt{2})^{-1/11}=0.96898...=$ $=0.969$ (R^{-1}). Number α (measure of disturbing symmetry) is a shift from one. The main center of S_q is $x_q=\sqrt{2}=\alpha^{-11}$; a shift from $\sqrt{2}$ into R^{+1} is set by number $\alpha^{-10}=$ $=(\sqrt{2})^{10/11}=2^{5/11}=1.3703509\ ...=\beta$, which is the essence of S_d. Number β in the first 6 characters coincides with the constant $\hbar c/e^2=1.3703598\cdot 10^2$. The tempering of the S_q ranges connects numbers 1.37 and 10. This connection also arises from equation (2) $a^n=na$, where at $a=10$ $n=0.137128857\ ...$, i.e. $10^{0.137}=$ $=0.137\cdot 10=1.37$.

§ 5. Law III - golden section. From equation $\Phi^n+\Phi^{n+1}=\Phi^{n+2}$ it follows: $\Phi=(\sqrt{5}+1)/2=1.618\ ...$ and $\Phi^{-1}+\Phi^{-2}=1$ ($\Phi^{-1}=0.618$, $\Phi^{-2}=0.382$). Number Φ was know, but here due to S_q it was given a wider interpretation. Law III follows from law II. Therefore, numbers Φ and 1.37 are connected. Let $a_{-2}=\Phi^{-1}$, $b_{-3}=\Phi^{-2}$. From formula (6) we find a_{+1} and b_{+2}; $x_g=\sqrt{a_{+1}\cdot b_{+2}}=1.37$. This connection points to the heuristic nature of S_q. Thus, the proposed theory integrates the three problems (disturbed symmetry, number 137 and golden section) posed by modern science and considered to be different into one.

Besides, the theory is supported by extensive experimental material: the author descovered the laws of harmony in musical series, periodic table, planetary distances, in musical works, micro- and macrocosmos, biology, genetics, etc. The theory posed new problems: enigma of number 0.417, number 3, 123, etc.

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Wolfgang Mastnak

Symmetric Groups - Music - Pedagogics
(Abstract)

Abstract structures, which are the topics of mathematics and which take part in Poppers World 3 are created or discovered - this depends on the philosophical meaning - by the human mind. They are inspired both by perception and rational reflection and sometimes moreover by an aesthetic feeling, so e.g. Platon's explanations in "Timaios" construct the theory, that triangles and minimal euklidian solids are the first components of the physical world.

Let's start now from the point, where we accept the existence of abstract algebraic, topologic etc. structures.
In an integral way of thinking analogies play an important role, and under this proposition it is ingenious to discover analogies between mathematical structures and structures of music. The central theorem of the statements the author holds is, that those analogies are not a priori but must be based on a special way of constructing them. In algeba something like this (in a very strong way) is called isomorphism.
In a very similar way isomorphisms or principles of transformations can be created, so that it is possible to say, that two objects, e.g. algebraic structures and musical events, show analogies.
Here three kinds of such isomorphisms are explained and explored as a basic of pedagogic reflections: a formal, an aesthetic-heuristic and a depth-psychological one.

1.The formal isomorphism:
Let us take an algebraic structure, especially a symmetric group. We want to create a formal isomorphism between this group and music.
1.1.First the question arises, which elements here should be adjoined to which elements there. Concerning the groups we take "permutations", the real elements of the group, concerning music we choose pitches of a note. This is totally arbitrary thus it would also have been possible to choose special rhythms, the duration of notes etc. This is the step, where we regulate the Objects, we deal with.
1.2.The next problem is the range of the notes, quasi a kind of mode. In an example already put into practice we worked with the symmetric group S4, which has 24 elements. It seemed to be natural to take either an octave divided into 24 quatertones or a chromatic scale over the range of two octaves. With regard to the possibilities of the school, the second way has been chosen, so that it was possible to make experiences with classic instruments.
1.3.The next problem is the fixing of the coordination between group-elements and the pitches. Like the former stipulations also the solution of this problem is arbitrary, perhaps heuristically influenced. The modus, which was taken in our pedagogic process was influenced in this manner: (1234) was adjoind to C (the idendity), (1243) to Cis, (1324) to D, (1342) to Dis etc. This yields a mathematical function between the set of the elements of the symmetric group S4 an the notes of the

range over two octaves.

1.4. The next question is what to do with this function. First it is intended to form music which is connected with the structure of the symmetric group S4. There two fundamental ideas were found:

1.4.1. A group has special sub-structures, especially subgroups, that is to say those subsets, which build together with the groupdefining multiplication a group too. According to the fixed function these subgroups yield special accords. These can create a set of tropes, on which improvisations may be based.

1.4.2. The multiplication of two group-elements is a well defined group element. Now we can begin with some subset of the set of group elements. This means beginning our piece of music with a special accord. Each element of our subset can be multiplicated with another element of this subset, so that the set of all possible multiplications is a new accord, which has been created by the former accord. A set of three elements normally formes another a set of three elements (a result of two elements is possible, too). This set yields a new one and so on. The piece is (not in the rhythmic way) totally determined by the beginning accord.

1.5. In our pedagogic process these reflections were fundamental. First the pupils learned that analogies do not exist without depending on special identification rules. Analogies are built by the human mind, or at least, only these analogies are - in the sense of Kant - recognizable by a human beeing.

The next step was the recognition of the possibility, to transform algebraic structures (which are beautiful in a special sense) to sound, a possibility, which is in a special sense much more determined than a piece based on a heuristically built dodekaphone trope.

The next pedagogic sequence is based on the improvisation over this sound-structures, where both possibilities 1.4.1. and 1.4.2. are practised.

2. The aesthetic-heuristic isomorphism:

Intermedial transformations are a central problem of polyaesthetic education. This chapter can be submitted to this field of questions. You can try, once having unterstood the structure of symmetric groups (as well as other abstract structures), to transform them to a piece of art, to music, to concrete poetry etc. This demands a complex act of creativity and depends moreover both on the cultural experience of the transforming individuum and the internally represented adventure of the moment.

Here the transformed results are not formally determined but must be explored in regard to their social and psychic, and sometimes somatically influenced, sources, especially those that are highly relevant for arts.

This aspect completes the process as a pedagogic one, in which the pupil exercises his abilities in creative problem solving activities (especially in both an artspecific and an abstract way, the latter sometimes influencing the creativity in respect to other problems) and cultivates his experiences with the arts. Moreover those processes concerne not only a problem of learning but may also affect more vital aspects and even lead to the borders of meditation, of philosophical reflections concerning the "beeing" as it is, of an idea of eternal "beeing" and of an eternal circular course, as e.g. the biblical Kohelet expresses (Ecclesiastes or the Preacher): "A generation goes, and a generation comes ... the sun rises and the sun goes down, and hastens to the place where it rises ... round round goes the wind, and on its circuits the wind returnes ...". Such reflections influence the realisation of the improvisation of musical drama.

3.The depth psychic isomorphism

In point 2 the transformations were mainly done during a status of vigilant conciousness. Now mainly unconcious processes should create transformations of symmetric groups to music.
First the pupils learn to know the symmetric groups, permutations and their multiplications etc. Then a light trance induction is done in class. During a light hypnotic status the pupils get the suggestion of letting permutations sound, and letting symmetric groups become audible. This is possible because of the phenomenon, that in trance as well as in dreams man can adjoin special medial characters to objects, characters, which they dont posess in reality. E.g. words can get a human form, qualities begin to talk etc. Therefore it is possible (and it was so), that a unconcious transformation is done, which depends mainly on a depth-psychic work.
Such transformations are quite different to the strongly determined ones of 1. and the heuristic-aesthetic ones of 2. Both the result and the way of creating them are different. It is to suppose, that this processes could become imporant in a psychoanalytic diagnosis and therapy. The results first are internal, which means, that - if we want to hear the creation of the unconciousness - the sound, the pupil heard during his trance, must be realised. This is as problematic as e.g. Tartinis translation of his famous violin sonata, which he heard during a dream (where it is to point out, that man when dreaming recognizes in a different way than when beeing awake) play by the devil and which became his most famous work although he maintained, that his sonate was so weak, that it would not be compared with the "originel" sonate of the devil.
Somehow this way of creating music can be compared with that way, where musicians take drugs for being stimulated and even carried off and produce music in a status of tranformed conciousness. In our example of school not drugs but natural possibilities of the human psyche are used.

(The report of our researches concerning this theme will show the pedagogic way and some results.)

ABSTRACT Fermat's Search for Symmetry of Triangular Numbers

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In Fermat's philosophy of mathematics, his search for symmetry as a fundamental intellectual principle is nowhere else as clear as it is in the problem complex surrounding the Last Theorem. We show that he conceived his attempted proof as a symmetry between triangular numbers and their powers in case they also constitute triangular numbers. This strategy of proof poses the question: Was Fermat justified in his claim that he had invented "a truly remarkable proof" to his Last Theorem? The key into FLT is the fact (Prop. 1) that if there is a non-zero integer solution $(x,y,z,n>2)$, (x,y,z) are sides of a triangle.

I Fermat's justification is studied by outlining first an historical scenario of the antecedents of FLT as a working hypothesis. It consists of three propositions and lemmas (Props. 1—3 and Lemmas 1—3, with sketches of the proofs). These are elementary statements well within Fermat's reach and yet give a geometrical illustration of FLT. Their novelty is Prop. 1 first suggested in (13:153—154). These antecedents are called Fermat's heuristics. They were presented at The 8th Int. Congress of Logic, Methodology and Philosophy of Science in August, 1987 in Moscow, USSR (in Section 1: Foundations of Mathematical Reasoning).

II The power of the outlined antecedents is measured by means of conclusions drawn from them (Prop. 4 and Lemma 4), comparing the conclusions with modern results. In drawing these conclusions, only methods known from Fermat's own or his predecessors' works are employed. The comparisons indicate, however, that Fermat anticipated (granting his heuristics consisted of Props. 1—3 and Lemmas 1—3) much later results. In particular, Prop. 4 is more general than Terjanian's result in 1977 at C.R.Acad.Sci. Paris 285, and Lemma 1 gives a better bound than M.Perisastri in 1969 at Amer. Math.Monthly 76. Lemma 4, in turn, offers a more promising way to an estimate of the exponent ($n = p$ an odd prime) than Grünert's lower bound for an eventual solution to Fermat's equation in 1856 at Archiv Math. Phys. 27. These are the first mathematical results.

III Further conclusions and comparisons are made possible by Lemmas 5-6. They transform the problem and set the question of Fermat's justification into a new light. Lemma 7 gathers together some results depending on Prop. 4. But that is only a watershed. A definitive answer is possible only if the final Prop. 5, FLT with odd exponents in one version, can be proved by Fermat's methods. Aiming at the proof, Porisms 1-3 and Lemma 8 are given. Enter Proof Reconstruction.

IV In the philosophical part, the implications of the foregoing heuristic, historical and mathematical considerations are outlined. They constitute, in our opinion, Fermat's true legacy with an impact on modern philosophy of mathematics and philosophical cosmology. In fact, this philosophical legacy runs parallel to Hamilton's research program which he gave up in favour of the quaternions (184 . Although Fermat's FLT and his Principle of the Least Time in optics are parts of his legacy, they are but the tip of the iceberg.

FERMAT'S HEURISTICS

It is 350 years since Fermat scribbled his "Last Theorem" (FLT) in the margin of his copy of Diophantus [3]. Despite recent advances, esp. Gerd Faltings' result (1983) and Yoichi Miyaoka's near-proof (1988), neither the mathematical nor the logical efforts nor yet computer calculations have been sufficient to solve the problem [cf. 15:2-3]. In the beginning of our century, Hilbert believed that the solution will be found [8]. In mid-1930's, however, unsolvable problems were gathering and the Theory of Algorisms was developed by Church and Turing. After the works of Post, Markov and others (c. 1947-1952), a negative solution was suggested to Hilbert's Problem X by Davis, Davenport, Putnam and Robinson (1953-1960). In 1970 it was found by Ju.V.Matijasevič and G.V.Čudnovskij [13:136-7]. This negative solution to the decision problem of a general Diophantine equation, although it does not rule out the possibility that the particular Diophantine equation FLT could either be positively solved or proven impossible to solve, reduced much of the hope [13:153].

Today, especially in Analytical Philosophy, FLT is often quoted as an example of Gödel's "true but undecidable statements" [15:216-8]. This is intellectual laziness. Gödel's result is of existential character and must not be used as a problem-killer. It is not worthwhile to claim conceptual command of a particular problem that one cannot solve. There is no rational reason for believing that just FLT is undecidable.

Other attempts having failed (so far), we suggest an additional study of Fermat's antecedents [13: 153-4]. For it is fairly sure that he did not invent anything like the abstractions of modern Number Theory, and definitly did not anticipate the latest results of Theoretical Physics (which Miyaoka made use of). Ours is, therefore, a Requiem to Fermat's predecessors, in particular to the Pythagoreans and Euclid, Diophantus and Pappus.

There are two separate problems: (i) to prove FLT using concepts and methods available to Fermat, and (ii) to prove FLT by whatever means. The present day is inclined to the latter approach. The former one is more demanding, probably more elegant, and certainly closer to rules of fair play.

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PERFECTION AND GENEALOGY OF STRUCTURES, IN PARTICULAR, OF CRYSTAL STRUCTURES

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Whatever its nature, each structure has properties of three types: I - the properties which vary continuously and monotonically as external conditions are varied continuously and monotonically, the structure remaining unchanged; 2 - the properties which are stable in the range of external conditions permitting the existance of a given structure, but which can vary monotonically and continuously in the region of existance of a structure differing from a given structure; and 3 - the multiplicity, the properties which are fixed for any structures of a set and can in none of these structures vary continuously.

The parametric description of structures enables one to include the number F of mutually independent free parameters, the number H of differing hard parameters and also to choose a list of multiplicities M_i which form the identificator of a structure. The numbers F and H characterize any of the mutually identical elements of a structure and in combination with multiplicities M_i - perfection of a structure which means the relation between its free (F) and hard (H) origins. Changes in external conditions lead to changes in the values of free parameters up to the stabilization of some of them, accompanied by the change of the F/H ratio and, if any of M_l is changed, to the transition, i.e. the change of the symmetry and of the degree of perfection of a structure.

For crystal structures whose elements have a given internal symmetry (Laue class) and a given internal structure, it is possible to construct a family tree including structures with the Fedorov groups of different levels. The lowest level belongs to the Laue class of structure elements (the symmetry level is determined by the numbers F and M). The results of the parametrization are displayed in tables such as table I compiled for Laue classes (a) and Bravais lattices (b). These tables are more informative as compared with the scheme of group coordination or with the scheme of the second-kind transitions between Laue classes (Indenbom, 1960). The information needed to solve the problem about the kind of transitions is contained in the parametric descriptions of structures (Mazo, 1984): the second-kind transitions, in particular, can occur within the family tree with the participation of only one free parameter (order parameter) and it is fixed by an obligatory change of the multiplicity of identical structure elements.

There is some evidence for the adequacy of the proposed parametrization of crystal structures. The number F correlates with the number of independent components of the second-rank tensor, S (Sirotin, Shaskol'skaya, 1975). On the basis of the information about symmetry and structure of the molecule, the phase diagrams of ices and cholesteric liquid crystals were constructed. The diagrams include phases with the symmetry determined with certainty from the diffraction data, and also all the phases whose regions of existance are determined by thermodynamic methods (Lyakhov, Mazo, 1988). The numbers H correlate with the degree of anharmonisity of structures - for the F and I type lattices the Q-factors at comparable frequencies differ; the acoustic Q-factor of alloys and compounds with the lattice cF(H-4) is lower than for alloys and compounds with the lattice cI (H-3); the Q-factor at microwave freqeuncies for ferrospinels (cF) is lower than for ferrogarnets (cI) and orthoferrites (oI). It is noteworthy that simple elements do not crystallize in the lattice cP(H=I), but are most freqeuntly encountered in the cF(H=4), cI(H=3) and hP (H=2) variants.

Fig.1 (a)

F	H=0	H=1	H=2
4		$O_h(48)-T(12)$	
5	$D_{3d}(12)-C_3(3)$	$D_{4h}(16)-C_4(4)$	$D_{6h}(24)-C_6(6)$
6		$D_{2h}(8)-C_{2v}(4)$	
7		$C_{2h}(4)-C_2(2)$	
8			
9	$C_i(2)-C_1(1)$		

Fig.1 (b)

F	0	1	2	3	4
1		cP		cI	cF
2	rR	tP	hP	tI	
3		oP	oC	oI	oF
4		mP	mC		
5					
6	aP				

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COMPUTER-ANIMATED POLYHEDRAL TRANSFORMATIONS

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The five regular and the thirteen semi-regular polyhedra have been known for centuries. The regular structures are known after Plato, and the semi-regular ones after Archimedes. Composed of regular faces only, these provide models of symmetry, and have inspired both artists and scientists. The drawings of Leonardo da Vinci and the graphics of Escher are among the well-known works in the visual arts. In science, the polyhedral shapes of crystals like fluorite and galena, the lattices of silicates and borates, and the protein shells of spherical viruses are well-known examples of the use of poylhedra in nature.

A central aspect in the study of structure is transformation of one structure to another. Transformations introduce the dynamic element in our classification of structures and helps us see one structure as a "state" of another. This provides the fundamental motivation for the study of transformational structures, whereby structures can change to others by adding certain elements, or changing some others. In natural structure, the study of such transforming structures becomes extremely important. Common examples of polyhedral transformations in nature are when an icosahedral viral shell undergoes a lattice transformation to release the DNA, or when the different faces of a crystal grow at different rates to produce a "transformed" polyhedron. In architecture, the use of transformational structures are useful for designing adaptive structures that respond to the changing environment.

The dynamic aspects of space structures are best realised with the use of computers. Computer-animation provides a natural medium for studying and visualising the temporal aspects of transforming structures, and provides the basis of the present collaborative work. This animation shows continually transforming polyhedra within the three polyhedral families, namely, the tetrahedral. octahedral and the icoshedral symmetries, and extends to the infinite class of prismatic symmetries. For the purposes of this presentation, the transforming polyhedra are restricted to those with mirror-symmetry; this excludes the enantiomorphs which will be presented later. The animation is an extension of the earlier computer-animation *Sketckes of Polyhedra Transformations* , a 12 min. color video, by the authors with P.Hanrahan, and first premiered at the conference 'Shaping Space', Smith College, Mass. (1984).

The early sequences of the animation show the composition of the color-coded fundamental region of polyhedra, the transformations of this region to others, and the conversion of the fundamental region into the entire polyhedron by series of reflections and rotations. This is followed by sequences of the continuous pulsation of one polyhedron to another through the three families. The polyhedra are color-coded analogously within each family. The red corresponds to 3-fold, 4-fold and the 5-fold faces respectively in the tetrahedral, octahedral and the icosahedral families; the blue corresponds to the 3-fold faces, and the green is used for the 2-fold faces. The color-coding uses the RGB system for this medium, and corresponds to the red-yellow-blue in the pigment system.

The next set of sequences show the build-up of the cubic "reference space" for each family of transforming polyhedra. The polyhedra can transform continually to one another within this reference cube where very distinct location represents a distinct polyhedron. For the purposes of illustration, a lattice of 5 x 5 x 5 polyhedra are shown to display their transformation *in space* . A few rotations, a tumble and a fly-through bring a closer look at this space for each family. Transformations, *in time* , within this space are shown next by a single polyhedron moving through this space. As the polyhedron moves, it changes it shape. The correlation between the changing polyhedron and the changing reference space is shown in the last sequence.

The animation is produced at the Computer Graphics Laboratory, New York Institute of Technology, using NYIT's modelling and rendering software. Special real-time animation software was written by Patrick Hanrahan[1]. The computational geometric model is based on Robert McDermott's algebraic solution for the transformations[2]. The polyhedral transformation concepts were first described in Haresh Lalvani's doctoral dissertation[3]. The animation used Vax 11/780s, was previewed on Evans and Sutherland's Multi-Picture System, and was rendered on the Ikonas. The animation is 9 minutes long at the time of this writing.

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PROPOSED PAPER

JAY HAMBIDGE AND THE THEORY OF DYNAMIC SYMMETRY

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ABSTRACT

The American art and design theorist, Jay Hambidge (1867-1924) was one of the first designers in the 20th century to realize the utility of discoveries in the natural science by Cook (1907) and D'arcy Thompson (1917) for use in art and design analysis. His work has been largely forgotten but when viewed within the present day discoveries and concerns for symmetry in many fields, scientific and artistic; he can be viewed as a true leader; one who was in upon the foundations of the symmetry movement of our century. Jay Hambidge made use of principles of Euclidian geometry and the proportions of the golden section of analyze examples of classical art. (Hambidge, 1919). His first book, The Greek Vase demonstrated the principles behind classical art and design. A second, more complete work, The Greek Temple (Hambidge, 1924) documented the research which Hambidge did in Athens on his measurement at the ancient sites. His wok demonstrated the use in matters of art and design of the same symmetry principles as Cook had outlined in his book Curves of Life (Cook, 1907). The basis of Hambidge's design concepts as well as geometric analysis was based on the work of Penrose. (Penrose, 1902) Hambidge first outlined them in at lecture in London in 1902. The twentieth century art and design field had made an extensive use of the ideas of symmetry, the principles of the golden section, and the theory of dynamic symmetry.

Many contemporary artists continue the tradition and work. The following demonstrates that the golden section and the principles of dynamic symmetry have become as central to their more recent work as it was when Hambidge first wrote and lectured on this ideas.

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HARMONY IN ITS UNITY WITH SYMMETRY AND RHYTHM.

The purpose of this paper is to discuss the correlation existing between the concepts of harmony, symmetry, rhythm. The topicality of the problem under study follows from the fact that despite the frequent use of the terms "harmony", "symmetry" and "rhythm" in current publications, one cannot find there any more or less detailed treatment of the meaning attached to the above-mentioned terms. Secondly, even some outstanding contemporary scholars find the implications of such an age-old term as "harmony" either "mysterious" /Einstein, 1, 123/, or "not lending themselves to any systematic analysis" /Bohr, II, III/. Considering that the work undertaken to give a philosophical interpretation of the problem of harmony marks but the beginning of an extensive and, hopefully, collective research /III, IY/, it seems appropriate to remind the reader that it was Aristotle who, following the Pythagoreans and Heraclites, treated harmony as unity in diversity. According to A.F.Losev, in ancient Greek philosophy "harmony" implied the organized character of the Universe and Cosmos as apposed to chaos. This is in conformity with the reasoning of Aristotle: "all that is harmoniously arranged arises from the unarranged and (vice versa) the unarranged results from the harmoniously arranged /Y, 14/. True, some contemporary authors (I.Prigogin, I.Stengers) contrast chaos not with harmony but with "order" - a concept which, to all practical intents and purposes, is hard to distinguish from chaos /YI/.

In this connection it would seem more expedient to consider the correlation between symmetry and rhythm. Such an approach has its prehistory presented in the works by Leibnitz, Bayle and Shaftesbury. "Harmony", Bayle wrote "connects the future with the past as well as the present with the non-presnent. The former type of connection unites times while the latter unites places /YII, 50/, or, as Shaftesbury put it "all that is beautiful is harmonious and symmetrical, and all that is harmonious and symmetrical is true..." /YIII, 183/.

No more enviable is the position occupied in philosophy by the concepts "symmetry" and "thythm" which are contiguous to that of harmony. Thus, in some cases symmetry is taken to be synonymous to harmony /IX, 15/, in others the concept of symmetry is referred to as a "vague" one /X, 4/. The situation with the concept of rhythm is quite similar. According to some estimations, at present there are up to 50 different general definitions of rhythm /XI, 6/. Evidently, this does not only testify to the difficulties inherent in the study of symmetry and rhythm but it also displays the activity of the researchers who are making efforts to overcome these difficulties.

Still, despitr the serious divergencies in the comprehension of harmony, symmetry and rhythm, the majority of researchers agree that these concepts have as their basis a certain objective content presented by the relationships which are being formed in nature, in society and in consciousness. Starting from the eleats'idea of the contradictory character of motion according to which change and conservation are mutually comlementary notions, one can assume that symmetry is connected with momentum conservation while rhythm - with that of change i.e. with the transition from a more or less settled state to one which is still in the making.

To proceed further with our discussion of the problem concerning the relationships existing between harmony, symmetry and rhythm, let us make use of the most general and widespread treatments of symmetry and rhythm. "Symmetry is a concept which reflects the order, proportionality and commensuration prevailing between the component parts of a whole, a certain balanced state, a relative stability existing in the objective reality /X, 30/. One of the interpretations of the meaning attached to the concept of rhythm regards the latter as an "alternation in time of certain units, arrangement and succession of spatial forms, recognition of space-time continuum" /XI, 4/.

Since in this case the elucidation of the implications of concepts "symmetry" and "rhythm" has mainly propaedeutic purposes, for we believe that they may enable one to approach a really new state of matters which, in our opinion, can be attained only under conditions of a harmonious developmet, elaboration of a more comprehensive concept of harmony would contribute to a better knowledge of the structure.

Being a specific form of motion and development, harmony, if considered as a particular state of material and spiritual structures, cannot be attained momentarily but only as a result of a certain process whose indispensable aspects are symmetry and rhythem.

Taking note of all the above-said, it would be appropriate to introduce one more specification. One may wonder why, for instance, some authors maintain that harmony is the same thing as symmetry. The fact is that in these cases one is wont to consider structure as something invariable. A structuralistic approach based on such like arguments is characteristically completely disengaged from the bearer (material or spiritual) of the structure. To a certain extent this may be considered admissible in scholarly studies, yet even in these circumstances it should not be overlooked that one is actually concerned with some real systemspossessing theirown relevant structures. Yet, since the structure itself figures in such treatments as a sign of stability, as the expression of the conservative aspect of motion, it is symmetry that is playing the role of the dominant sign of the state, which gives rise to its identification with harmony.

A similar situation is to be seen in the interpretation of rhythm. In monotonic music, a phenomenon devoid, as it were, of any spatial localisation, rhythm as a "temporary and accentuated aspect of.. harmony" /XI, 230/ becomes an analogue of harmony, serving as its representative.

As for harmony proper, it is the integrity and the unity of numerous components, therefore it inevitably gives birth to something new as an indispensable link in the harmonious global development. For example, the Universe expanding after the "Big Bang" is at each moment renewing the relations between its structural components creating thus prerequisites for the emergence of new structures. So it becomes clear that the structural symmetry of our solar system is by no means an absolute one but only relative. It is in the substitution of new structures for the old ones that the rhythm complementing this particular system manifests itself, splitting the process of cosmological transformations into a chain of temporal sequences. This is confirmed by the fact that the space of time during which the material substrate is being accumulated in the form of a "black hole" serves as a precondition for a new "Big Bang", thus representing the objective expression of the rhythm of the Universe. The regularity of the rhythm displayed by our world in virtue of the open character of the Universe is also relative. Yet even in music the breach in the regular succession of a symmetries i.e. a "rhythmical asymmetry" and an irregular "rhythmical pattern" as movement from the major to the minor and vice versa /XII, 47-48/ are prerequisite to the creation of qualitatively new aesthetic ima images.

Consequently, harmony is a state of some complex system in which contradictions are being solved not by a succession of victories and failures, but by a rhythmical regularity with which the potentialities pertaining to the whole are growing in the interests of preserving the subsystems connected by symmetry. With reference to the existence of relatively closed system, one can say that the way from its birth to its disappearance (transition to a new quality) represents a harmonious form of development.

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SYMMETRY AND ASYMMETRY AND SELF-ORGANIZATION IN THE WORLD PICTURE

In the present-day scientific world picture the problems of identity between being and thought are bound up with the analysis of applicability limite of adequate means for theoretical reproduction of the development processes.

Materials for such an analysis are obtained through scientists associations, which display their interest in one or another method of synthesising scientific knowledge valid at a definite stage for universal usage. This might be said referring to fairly general methods such as symmetry and asymmetry or order and disorder.

In the present-day world picture is evident that symmetry is order. It is also evident that it does not exist in perfect hypostasis, only as an ideal construction. Hence, the natural assertion that the asymmetry exist as an absence of symmetry, as a deviation from symmetry. This deviation can be at random, with probabilities near to nought and this case it is nearer to disorder, but it can be characterized through repetition, through a sui-genezis reproduction in different structures and then it appears as complex expression of order, of a more level order.

Thus, there is a deviation from symetry as disorder and a deviation from symetry as order, as a more complex order.

Obviously, as we have a go over from a theory of hard determinism and absolute rationalism to a determinism and rationalism, in the same way the relation between symmetry and asymmetry became more complex, by discovering a more complex symmetry behind deviations from symmetry. This is, for example, the case of the conservation of combined parity.

The problem of symmetry and asymmetry can be related with the unity of space-time structures in which we can also unity between symmetry and asymmetry.

The symmetry as remarkable expression of structures constance conservation, of qualitative stability, or as a constance of repetition, the symmetry - as an introduction of qualitative differentiation through a lack of stability, and then through reordering and reconstruction.

There are many examples for it. It is crystallography, differences in the optic properties appear, the living systems having the capacity of an asymmetric diversification, that receive the most subtle forms in the case of the human brain and etc.

From a general philosophic point of view, it is possible to assert, that the asymmetry gives higher chances to qualitative diversification and to complexification of systems than the symmetry. That the asymmetry is not only a factor of diversification at random, but also a factor of order diversification, as a repetition of new structures stabilization.

Therefore we come to discuss the relation between symmetry and asymmetry, pointing out some characteristics of the unity between symmetry and asymmetry, in the case of the fundamental senses of the becoming entropic and neg-entropic.

The symmetry demonstrate the idea of equilibrating the Universe forces, the asymmetry - to the idea that every equiliorum can be disturbed. Much more, the created disturbance contains not only the older traces of equilibration but also the chance of a new equilibration. The disturbance appears in preponderent progressive and in preponderent involutive ones, that can alternate or coexist.

Represent in this case, the unity between symmetry and asymmetry offers an another face of the being complementarity submitted to the correlation between order and disorder, stability and variability, unity and differentiation.

In the present-day scientific world picture the asymmetry has a more important role than the symmetry.

The researches into thermodynamics of nonequilibrium processes instigated the progress in comprehension of a new paradigm, for they afforded an opportunity to consider dissipative structures (Prigogine I.R.), phase transitions in laser generation (H.Haken), autowaves processes in active media (Beloussov B.P., Zhabotinsky A.M.), bistability structures of a trigger type and others as self-organizing systems with disorder into order conversion.

The adoption of the general principle of subordination was an important stage of the description of such patterns as self-organization which has no reference to any taken specific form of the motion of matter, but which manifests itself in every case whenever opportunity offers necessary combination of internal and external conditions. The above mentioned subordination principle being assumed as the basic, it is possible to eliminate a large number of variables in complex systems and thereby to reduce the task to the solution of small number of variables, which represent the parameters of the order.

And last aspect of our abstracts.

The temporal symmetry refers to the big cycles in repetition, to the supposed eternal circuit of matter, to the succession of progressive and regressive phases, of evolution and involution. The temporal asymmetry refers to the variety in which the two big sense of universal becoming are achieved: the sense of the disorder increase and sense of order increase.

The symmetry leads to the idea of unity, stability and closing. The asymmetry - to the idea of difference, variability, opening.

Conclusion. Both the symmetry and the asymmetry are implied in the explanation of the being capacity of self-organisation and also in the explanation of its capacity of self-disorganization that to pass from superior order to an inferior one, from order to disorder.

SYMMETRICAL AND ANTISYMMETRICAL FORMS OF BENDING VIBRATIONS OCCURING IN SPORTING GOODS

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Many of created end employed sporting goods, for instance rackets and ski, belonged to the devices having symmetry of form.

A vibrational state of sporting goods is directly connected with achivement of competition results and may exert negative or healthful influence.

Investigation of natural frequencies of bending vibrations of sporting goods, for example a tennis racket, taking into account parameters of elasticity-plasticity of sportsman's arm, shows an essential influence of them on resonance frequencies spectrum alternation and on resonance creation possibility in the "man arm - racket system" [1]. There was determined on the basis of digital simulation a regularity of alteration of natural frequencies of the system depending on mechanical parameters of an arm, its mass, stiffness in progressive and angilar moving.

Values of mentioned parameters depends on individual pecularities of the arm, manner racket holding and may change depending on different types of shocks execution.

The first natural frequency and corresponding to it type of vibrations are defined by racket vibrations as a rigid body relatively a joint-hinge, arranged in the hand. The second and following natural frequencies and corresponding types of symmetrical and antisymmetrical vibrations defines the frequency of transition of racket vibrations as a rigit body into vibrations of racket as a flexible body. Consequently depending on correlation between elastic-inertial parameters of hand and racket there may arise as symmetrical so and antisymmetrical forms of vibrations. The influence of symmetrical and antisymmetrical forms of vibrations on the quantity of "recoil" on a hand is unequal. There may be observed also in the system "man-ski" arising of symmetrical and antisymmetrical forms of vibrations.

As was determined on the basis of digital simulation, arising of that or another frequency and form of bending vibrations of ski and its amplitudes depend on elastic-inertial connection of body (feet) muscles of a man with a ski track and on correlation between their masses, on hardness of ski track of moving [2].

The bondary and initial conditions must be formulated better for a half of the system, resulting in simplification of derivation of vibration equations. There was determined on the basis of simulation, that the first natural frequency of the system and corresponding to it symmetrical and antisymmetrical forms of bending vibrations must have insiquificant vibration amplitudes, what take place in the case of week coupling of sportsmans body with sport device, in our case with ski. Increasing of rigidity of this coupling leads to corresponding growth of symmetrical vibration amplitudes, and in this case the more ratio of sportsmens mass to ski mass, the more the vibration amplitudes.

The second form of vibration arising in the system is an antisymmetrical one with two vibration nodes.

The symmetrical forms are more prefereble to have views for the convenience of its damping, since they give smaller rise to moment componente of reaction and contains the number of antisymmetrical form garmonics by unit veniger and also needs in veniger number of dempfers.

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Topology and geometry of bar constructions made from regular 20-hedron

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The centres of 20-polyhedrons edges determine the halfregular 32-polyhedron. Vertices of the 20-polyhedron and 32-polyhedron determine the 80-polyhedron, which topology is presented on fig.1a. The centres of 32-polyhedron's edges are vertices the 92-polyhedron. Placing on the walls of the 32-polyhedron pyramids, which vertices are lying on the common spherical surface, we get the 180-polyhedron.

If on the spherical surface circumscribing the vertices of the regular 20-polyhedron are projected the centres of its walls, we get the 12-polyhedron. Each edge of the 20-polyhedron fits in one edge square with one of the 12-polyhedron. The vertices of the 20-, 12-, and 32-polyhedron compose the vertices of the 240-polyhedron having 360 edges, which topology is shown on fig.1b. The vertices of the 20-polyhedron and 12-polyhedron compose the vertices of the 60-polyhedron.

From the given polyhedrons can be formed, in turn, bigger ones for instance - in the first kind the 180-polyhedron is converted into the 540-polyhedron, 240-polyhedron into 720-polyhedron, the 320-polyhedron into 960-polyhedron etc., in the second case the 80-polyhedron is converted into 320-one, 180-polyhedron into 720-one, 240-polyhedron into 960-one, etc. In the third case the 240-polyhedron is converted into 320-one, 540-polyhedron into the 720-one, the 720-polyhedron into the 960-one, etc.. These 3 kinds of convertion enable 3 quite different constructions.

Treating the edges of each polyhedron as bars and vertices as nodes we get one layer space truss. Taking the opportunity of coverting of one polyhedron into the second one, at their concentric setting, we can join together two one layer structures, forming in this way two-layers bar constructions.

Fig.1c presents the topology of the two-layers construction consisting from the bar layer fitting in the 80-polyhedron /thick lines/ the bar layer fitting in the 240-polyhedron /thin lines/ and the layer of joining bars /punctate lines/. The calculated angle coordinates of nodes for 1/20 part are presented in tab.1.

TABLE 1

NODE	COORDINATES φ	COORDINATES ϑ
1	0°	0°
2	0°	16°28' 19,92"
3	36°	31°43'02,908"
4	0°	37°22'38,525"
5	23°33' 13,02"	50°39'04,9"
6	0°	58°16'57,091"
7	36°	63°26'05,815"
8	19°15'55,88"	69°28'58,31"
9	0°	79°11'15,659"
10	36°	79°54'25,755"
11	18°	90°

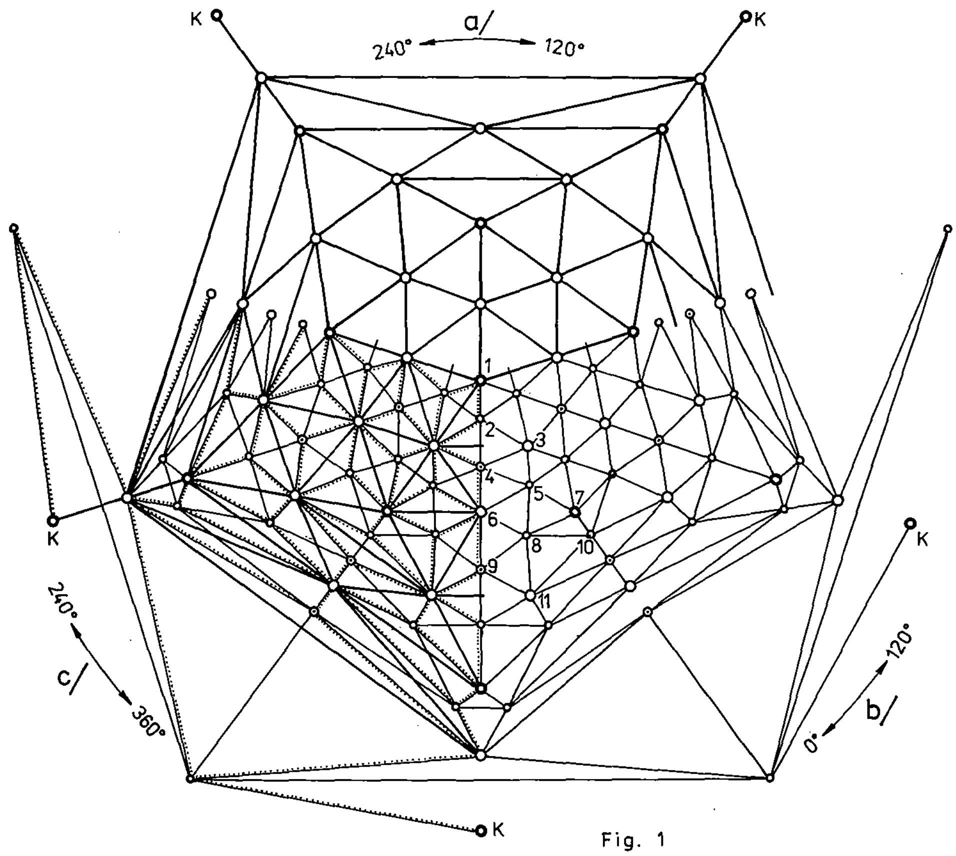

Fig. 1

Fig.2 shows the two-layer bar construction and at the same time reflects the convertion of a polyhedron into a second one. The bars of the internal layer are the edges of the 180-polyhedron. The double line is marking the mother network of this 180-polyhe-dron, which is the halfregular 32-polyhedron. The external layer bars are the edges of the 92-polyhedron which walls are: 60 isoce les triangles, 20 regular hexagons. Transposing polygons into trian gles we get 240-polyhedron. It's the convertion of the 3d kind.

Fig.3 displays the two-layers bar structure based on the conver tion of the internal 60-polyhedron into the external 240-polyhe - dron changed here into mother network of the second type /60x4/, being 92-polyhedron. The double line is marking the network of

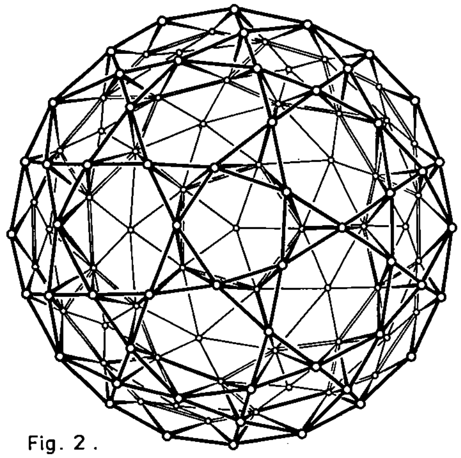

Fig. 2 .

The nodes of two-layers constructions were described by independent concentric spheres and changed the reciprocal relation of their radii calculating geometric characteristics of the constructions.

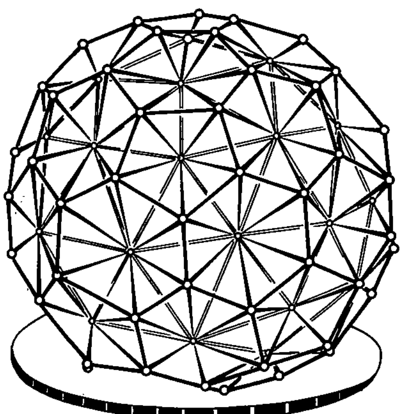

Fig. 4 .

the 60-polyhedron. Completing the polygons with triangles we get the 240-polyhedron.

Fig.4 presents the two-la--yer bar construction based on the convertion of the internal 80-polyhedron into the extrenal 240-one changed here into mother network of the first kind /80x3/ closed by 12 pentagons and 30 hexagonals.

Fig.5 presents the view of the 240-polyhedron.

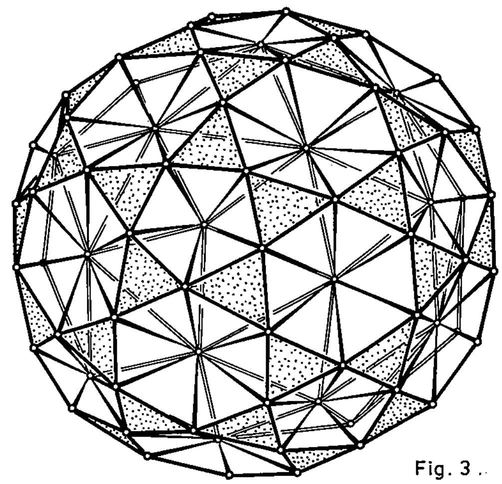

Fig. 3 .

Fig.6 illustrates an example of a change of bars length according to radii ratio R180/R60 is the radius of sphere describing construction nodes coming from the 180-polyhedron, R60 is the radius of sphere describing the nodes of construction coming from the 60-polyhedron. The lines 1 and 2 illustrate the changes of length for bars of the

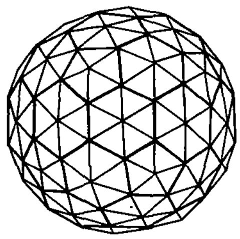

Fig. 5.

construction described by sphere with the radius R60; the lines 3 , 4 and 5 illustrate changes of length bar groups for the construction described by the sphere with the radius R180; the lines 6 , 7 present the change of groups of bar length joinig both layers.

Similarly were marked the lengthes of bars for two-layers bar construction based on 80-polyhedron and 240-one, and the results are presented on fig.7.

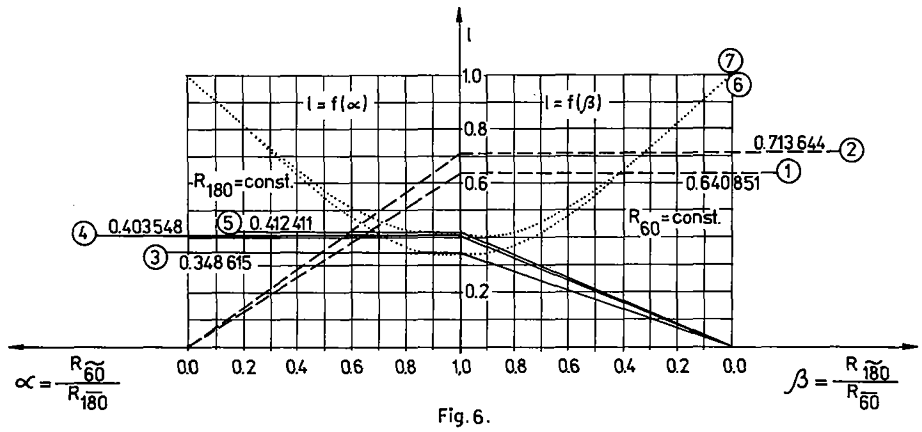

Fig.6.

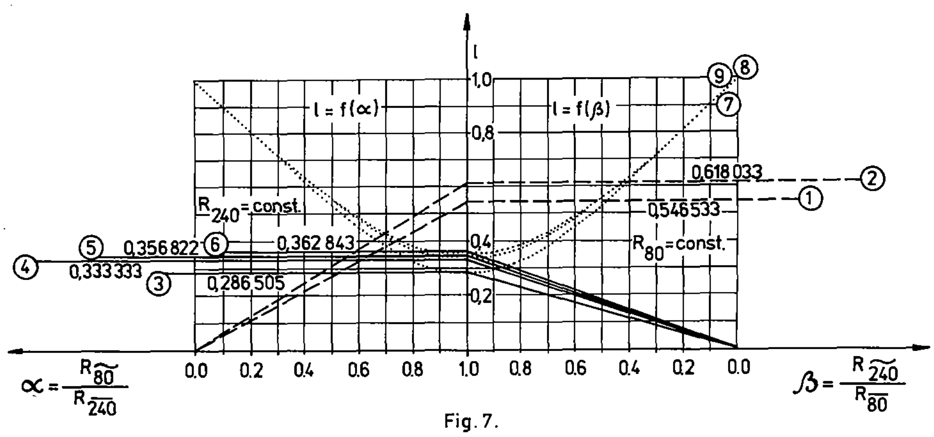

Fig.7.

FOLDING A PLANE -SCENES FROM NATURE, TECHNOLOGY AND ART

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SUMMARY

The purpose of this paper is to explain the solution of the geometric proposition "folding a plane into a point", and to show its appearance in natural phenomena, technology, design and art.

FOLDING A PLANE - PLATE ELASTICA -

The deformation represented typically by that of a thin sheet of paper is called in terms of geometry the isometric transformation. The meaning of "isometric transformation" is that any linear segment of the paper does not change its length through such deformation. Or it is said that the Gaussian curvature is unchanged. Since the Gaussian curvature is zero for a flat plane, this value is kept everywhere in any stage of transformation.

This basic principle is, though not rigorous, valid for thin sheet of various material, where the bending deformation is predominant over the in-plane deformation.

The proposition "folding a plane into a point" is described graphically in Fig. 1. If we consider an infinite plane, the solution must be periodic. The idea in solving this geometrical problem involves the application of the theory of elasticity which consists of the following syllogism. Let us consider it hypothetically, as if it is an elastic deformation problem of a plate. Then the problem can be solved by the finite deformation differential equation for a plate presented by von Kármán. By making the thickness of the plate unlimitedly close to zero, the first geometrical problem can be solved(1,2).

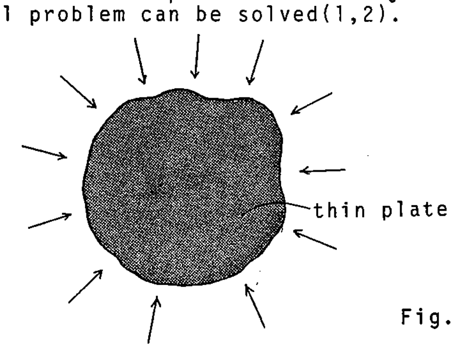

Fig. 1

Figure 2 shows arbitrarily selected 10 solutions calculated using the procedure described above. These show the contour lines of their out-of-plane deformation. When the least energy solution is sought among these solutions, it is the surface having a beautiful symmetry composed of repetition of the fundamental region which is further composed of four congruent

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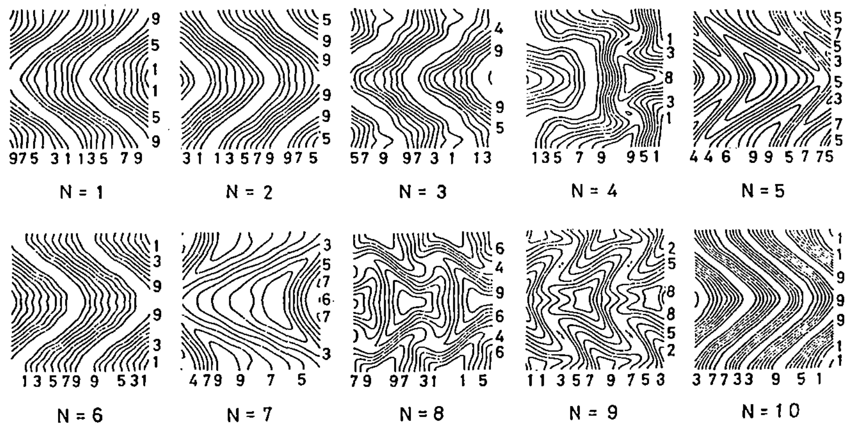

Fig. 2

parallelograms as shown in Fig. 3.

The meaning of this solution can be explained by a comparison with the corresponding one-dimensional problem, that is, folding a slender elastic column. The problem was solved by Euler and the curves thus obtained are called "elastica". In the similar manner, in case of the present two-dimensional problem the surfaces are to be called "plate elastica". We know that the column elastica has such a smooth curve which sometimes symbolizes the elastic deformation. On the contrary, the plate elastica is characterized by sharp ridge lines. This result indicates that the characteristics of deformation are essentially different between columns and plates.

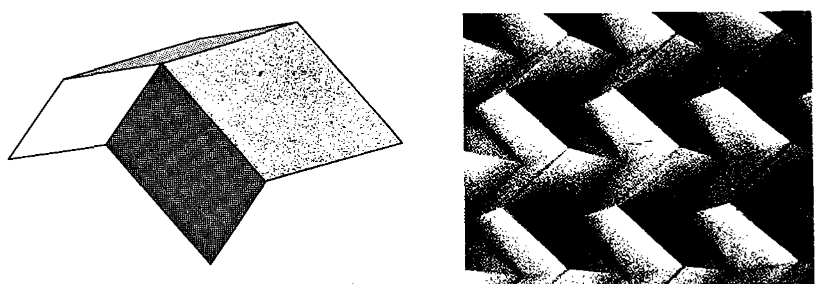

Fig. 3

DESIGN OF A FOLDED MAP

There are three problems with maps folded at right angles in the conventional manner. First, an orthogonal-folded map requires an unduly complicated series of movements to fold and unfold it. Secondly, once unfolded there is a strong possibility that the fold may be "unstable" and turn inside out. Finally, right-angled folds place a lot of stress on the paper inducing, almost without exception, tears which begin where two folds intersect.

The key to an alternate system of map folding lies in the "plate elastica". It is to use a variant on concertina folding to produce a slightly ridged surface composed of a series of congruent parallelograms, that is, the shape of "plate elastica"(3). The most important point of difference from an orthogonal folded sheet is that the folds are interdependent. Thus a movement along one fold line produces movement along the other. In other words, the user can open the map by just one pull at a corner(Figure 4 from Ref. 4).

The new method also solves in part the other problems. Interdependence of folds means that it is very difficult to reverse them and the amount of stress place on the map sheet is also reduced because only one thickness of paper comes beneath the second fold, avoiding the need to fold several sheets.

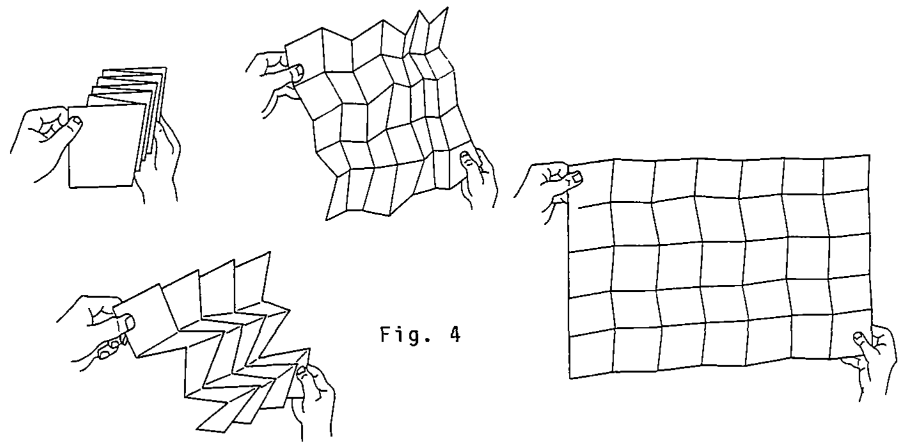

Fig. 4

APPEARANCE IN NATURE AND ART

The exact symmetry of the geometric solution will never appear in the natural phenomenon because of the fuzziness inherent in the nature, though the essence of the symmetry is always kept in it. It means that the deformed surface of thin plates are much like the irregular patterns appeared in Fig. 2. Such examples are observed in our everyday life as a crashed paper, a body shell of a crashed automobile etc. Furthermore, such irregularity is used for the means of expression by some artists. Figure 5 is a part of the bronze relief at the entrance hall of our Institute by a sculptor Hisao Yamagata, and we could see both the regular and the irregular pattern merging with each other.

Fig. 5

APPLICATION TO SPACE TECHNOLOGY

Large planer membranes are mandatory for many of space missions in near future. Solar arrays, solar-power satellites, solar sails, space radars are typical examples. Therefore, the technology necessary for the construction and packaging of these large membranes on the ground and their deployment in space must be established. The technology described in the section of the folded map was originally invented in order to cope with such problem of packaging of large membrane space structures. For the purpose of testing its feasibility, the project are under way to launch a two-dimensional deployable array on board the space platform scheduled for 1992(5). It is a large thin-membrane solar cell array which will be simultaneously deployed in two orthogonal directions(Fig. 6).

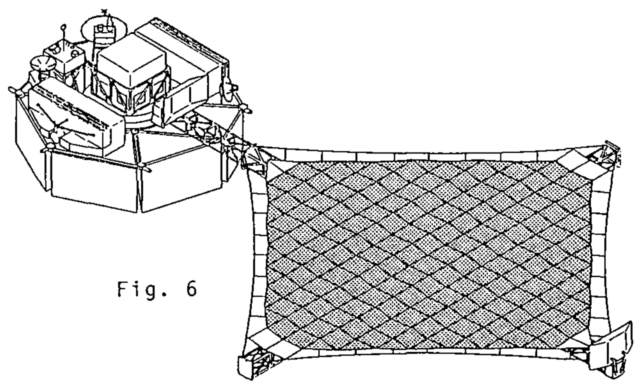

Fig. 6

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CORRELATION BETWEEN CRYSTAL SYMMETRY AND SYMMETRY OF THE FINE STRUCTURE OF ELECTRON DIFFRACTION PATTERNS

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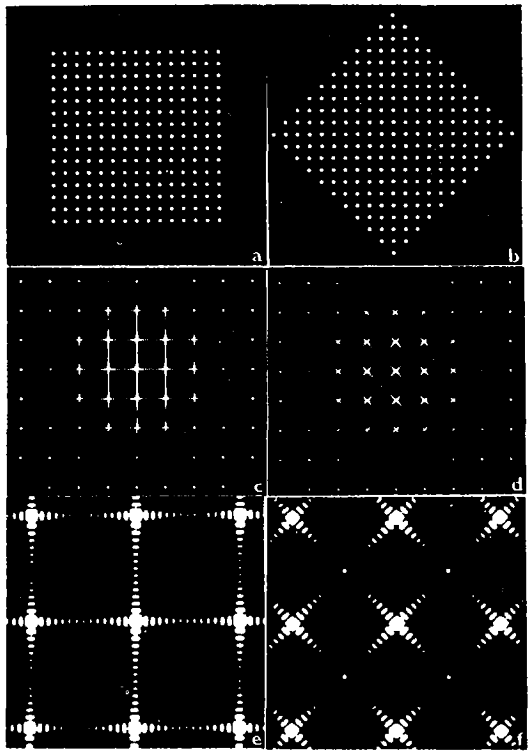
The various electron diffraction techniques are most versatile and reliable methods for the determination of the symmetry of crystalline samples. Both the point group and the space group of crystals may be determined by means of convergent-beam microdiffraction. Furthermore, the symmetry of the diperiodic crystal surfaces can be deduced from low-energy electron diffraction (LEED) as well as reflection high-energy electron diffraction (RHEED) patterns. Besides the structural symmetry obtained by analysing the geometry and the intensities of the patterns the interpretation of the shape of the reflections may reveal the symmetry of the external form of the crystals. Within the kinematical theory of diffraction, the mutual connection between the diffraction pattern and the reciprocal lattice is elucidated by the EWALD sphere construction. As the reciprocal lattice is the Fourier transform of the crystal lattice (see fig. 1), both the position and the shape of the diffraction spots are specified by the Fourier transform of the distribution of scattering objects in the crystal. The position of the diffraction spots is determined by the internal structure of the crystal, but the shape of the diffraction spots is mainly determined by the external form of the crystal as well as by the presence of crystal defects. The influence of the external form of the finite lattice on the shape of the diffraction spots is illustrated in fig. 2.

finite crystal	=	unit cell	*	infinite lattice of points	•	shape function
FT ↓		FT ↓		FT ↓		FT ↓
reciprocal crystal	=	F.T. of the unit cell	.	infinite reciprocal lattice of points	*	shape amplitude

Fig.1: Scheme of the Fourier transform of a finite crystal (• multiplication, * convolution)



(a), (b)- two-dimensional lattices of the same structure but with different external boundaries

(c)-(f)- Fraunhofer diffraction patterns from (a),(b) obtained by optical diffractometry

(c), (d)- the whole central region of the patterns

(e), (f)- a detail showing the shape of the diffraction spots

Fig. 2: The influence of the shape transform

In transmission electron diffraction of small crystals spots are frequently observed which have distinct fine structure consisting of streaks, satellites or elongations. This fine structure is determined by the intersection of the EWALD sphere with the three-dimensional intensity distribution of the diffracted beams. The contribution of the crystal shape to the shape of the diffraction spots can be theoretically described by the crystal shape amplitude $S(\vec{p})$ which in fact is the Fourier transform of the shape function $s(\vec{\xi})$ of the crystal. The shape of the diffraction spots may be related to a single shape amplitude only if the lattice is sufficently large, i.e. if it is formed by a large number of unit cells (more than 30) in any direction. This is usually the case in crystallographic applications. If this condition is not fulfilled, the shape of the diffraction spots should be related to the so called lattice amplitude, which is a periodic function expressable as a superposition of the shape amplitudes. The concept of the shape amplitude was introduced by von Laue (1936). General algebraic expressions were derived (Komrska, 1988), which enable numerical evaluation of the shape amplitude of any crystal polyhedron. The derivation is based on the repeated Abbe transform, which makes it possible to express algebraically the multiple integrals defining the shape amplitude. The shape amplitude is a special case of the Fourier transform of a real function. Hence, the algebraic expressions must possess all the properties that follows from this fact.

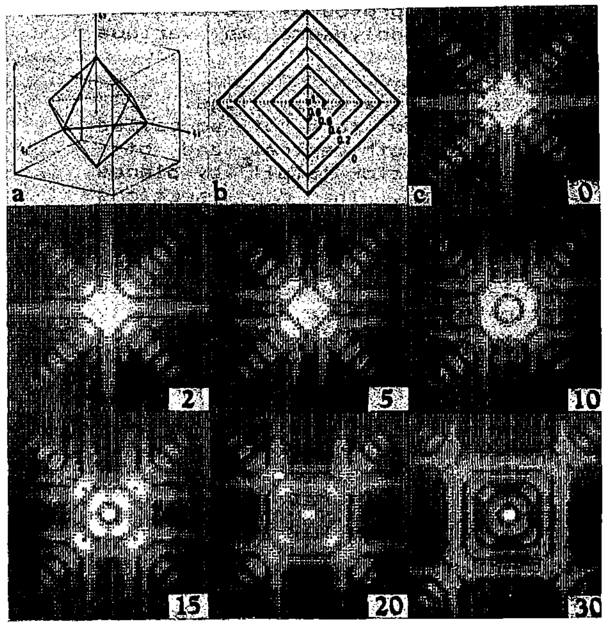


Fig.3: Crystal shape factor map for an octahedron in (001)-orientation;
(a)-orientation relationship of octahedron in circumscribed cube,
(b)-thickness isolines

(c)-cross-sections of shape factors $|S|^2$,

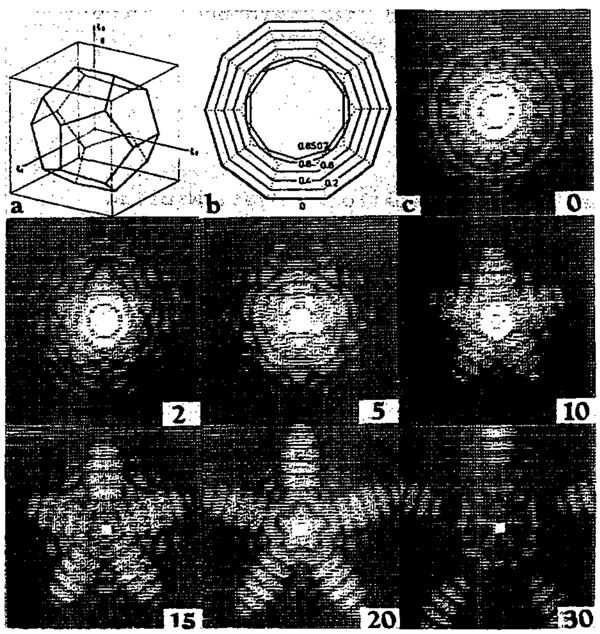


Fig.4: crystal shape factor map for a regular pentagonal dodecahedron in (0 τ 1)-orientation relationship of pentagonal dodecahedron in circumscribed cube,

(b)-thickness isolines

(c)-cross sections of shape factors $|S|^2$

The main properties are:
The shape amplitude $S(\vec{p})$ has all the symmetry elements of the shape function $s(\vec{\zeta})$ of the crystal.
The shape amplitude of a crystal with centrosymmetric shape is always a real function.
The properties of the shape amplitude were proved by calculating the shape amplitudes of the Platonian polyhedra in various orientations (Komrska, Neumann, 1986).

The computer simulation procedure is illustrated in figs. 3,4. The numerical results are presented in the form of so-called crystal shape factor maps representing both central and off-central cross-sections through the shape factor $|S(p)|^2$ by planes perpendicular to the incident electron beam. The cross-sections of the shape factor for a regular pentagonal dodecahedron in $(0\tau 1)$-orientation, *i.e.* parallel to a five-fold axis (fig. 4) might be useful for interpretation of diffraction patterns of small quasi-crystals, were a pentagonal dodecahedral shape is possible. The central cross-section of the shape factor clearly shows the tenfold symmetry corresponding to the fivefold axis of $s(\vec{\zeta})$. The off-centre cross-sections obviously exhibit the loss of the centrosymmetry which leads to the fivefold symmetrical cross-section.
The computer simulation of the crystal shape factor was applied to the interpretation of the shape and symmetry of transmission electron diffraction reflections from small polyhedral gold and palladium crystals having octahedral and tetrahedral habits (Neumann et. al., 1988). The fine structure of the experimental diffraction spots may be compared with that of simulated spots obtained from the two-dimensional intensity distribution at plane intersections with the corresponding shape amplitudes. The different off-centre cross-sections can be assigned to the corresponding components of the actual deviation parameter of a diffraction spot from the Ewald sphere for any given size of the crystal. The method can also be used to interpret the differences in the symmetry of diffraction patterns of multiply twinned particles and quasi-crystals.

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ASYMMETRY OF THE BRAIN IN SCHIZOPHRENIA

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The history of asymmetry of the Human brain reaches as far back as to the discovery of aphasia as a disease process of one brain hemisphere .
Kraeplin was one of the first who thought about neuropathological changes in endogenous-psychosis. But it took about 70 more years to establish this idea. Today they are several well controlled morphometric post-mortem studies giving evidence of pathological changes in the limbic system of schizophrenics (Bogerts 1982,1984,1985, Brown et al 1986 , Jakob and Beckmann 1986 , Kovelmann and Scheibel 1984). Nearly all studies were done on only one hemisphere , which meant that a comparison of both hemispheres was not possible so far. On a completely new set of whole brain sections from 20 schizophrenic patients and 20 age-and sex-matched controls we determined the volume of the hippocampal formation and amygdala planimetrically. The volume of the hippocampus was significantly reduced in both hemispheres in the

schizophrenic group by approximately 35 % . whereas the amygdala showed no difference compared to the controls (**Bogerts et al. personal communications**). Crow and his team filled the lateral ventricle of 19 formalin-fixed brains of schizophrenic patients and 23 controls with radio-opaque material (urographin 150) to obtain an X-ray image of the ventricle from the lateral aspect. He found that on the right side of the brains the difference between the schizophrenic and control groups in temporal horn area was negligible ; on the left side however it was in excess of 130 %. It was concluded that the structural changes in schizophrenia have an affinity for the temporal region selective to the left hemisphere (**Crow et al 1989**).

CT-and MRI-studies confirm this finding of asymmetry in the brains of schizophrenics. In the CT scans of 54 schizophrenics and 54 controls matched for age and sex , we found the left Silvian fissure most enlarged on several levels (by 79-233 %) , followed by the right Silvian fissure (42-96%) (**Bogerts et al. 1987**). In a recent study we measured the anterior horn of the lateral ventricle planimetrically on two CT-levels in 150 schizophrenic patients and 150 age-and sex-matched controls. The left anterior horn area was significantly enlarged in the schizophrenic group (**Falkai et al 1989**).

Finally **Delisi et al (1988)** found a significant volume reduction of the limbic complex - meaning the amygdala plus the hippocampal formation - in both hemispheres in the MRI's of schizophrenics.

Lack of gliosis and a lateralized volume reduction of temporal lobe structures reflects an arrest of cerebral growth in this region. Handedness and the asymmetries in the brain to which it relates are a late evolutionary development; they may be controlled by a single gene. Thus schizophrenia could result from an anomaly of this specifically human gene (Crow et al. 1989).

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DISCRETE SYMMETRIES IN COSMIC SCALES

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Summary: A lattice structure of discrete symmetries is demonstrated on the cosmic mass-size diagram.

While our world may appear confusing when studied at small details, it is simple and regular from a proper bird's-eye view. Indeed, the basic mass, length and time scales of phenomena seem to be arranged in a secret order manifested only when comparing orders of magnitude. Nowadays we are already able to recognize the *logical background* behind these regularities that can be expressed by simple *algebraic formulae* admitting in turn transparent *geometric interpretation* in terms of approximate *discrete symmetries*, as we shall see. Making use of this knowledge we may attempt to theoretically reproduce the design of Nature from elementary particles to the entire observable Universe in a simple and aesthetic way. The result is briefly explained in the present comment and summarized in the enclosed Figure where mass M vs. size R are displayed in logarithmic scales, with an origo corresponding to the proton (Lukács & Paál, 1981).

Each of the three basic physical phenomena of general competence, gravity, quantization and relativity, has one fundamental constant (Cavendish constant G, Planck constant ħ, and light velocity c, respectively). Note that c is also universal limitation for propagation of signals, so cannot be overrun by any velocity coming from other disciplines. Therefore one gets two lines

$$M = (c^2/G)R \quad \text{and} \quad M = (\hbar/c)R^{-1}$$

limiting the places of possible stable configurations on the diagram from the left. On the ascending line each mass defines a length (the Schwarzschild radius) below which irresistible gravitational collapse leads to black hole formation; on the descending line one finds to each mass a length (its Compton length), below which our naïve notions of space and time become obscure and even meaningless because of quantum uncertainty.

The objects of the real world are "well aware of the law", and apparently respect the above limitations. On the Figure the points representing astronomical objects lie close to the upper boundary, while microphysical ones are near the lower one, in the permitted region. Objects built up only from nucleons, atoms and (presumably) neutral leptons (e.g. neutrinos) are also roughly of nuclear, atomic (and "leptonic") density and therefore aligned along equidensity lines (of slope +3, since $M \sim R^3$) attached to the respective objects. So matter can form stable equilibrium configurations only with masses and sizes corresponding to these lines. The natural laws and the building blocks determine the basic features of the structures.

The above "construction of the world" is surprisingly regular: the equidensity lines happen to be just equidistant. These lines reveal the intimate connection between the micro- and macrocosmos. The neutron star (n^*) is the "sign of the neutron in the sky"; the ordinary star and the quasar are those of the atom; while the protocluster ($\circledast$), galaxy cluster now, is probably that of a neutral lepton (a kind of neutrino or other weakly interacting particle). So the astronomical macrocosmos is just the microcosmos "projected to the sky".

The central equidensity line is the most populated according to our knowledge. Here one finds the particles of cosmic dust, meteorites, biological and geological formations, moons, planets. In case of *stable* equilibrium the two extremes on this line are the atoms (purely electrically bound objects) and stars (purely gravitationally bound objects). Loosely speaking stars represent "gravitational atoms", while atoms mean "electric stars". Between these two extremities man represents *aurea mediocritas* - a "gravitationally limited electric being" - who is therefore larger than the H atom by just the same factor as smaller than the star. (Otherwise he would be broken in pieces if fallen down to the ground.) Calculating this geometric mean between star and atom directly from basic natural constants, one gets about 78 kg (!) just like our typical human mass indeed. Considering, therefore, orders of magnitude, one finds that the "measure of Nature" is "anthropocentric" both in mass and in size, but this fact has nothing to do with any kind of subjective wishful thinking.

Furthermore note on the Figure that the series *atom* (hydrogen), *bacterium* (simplest living), *man* (most evolved living), *mountain* (highest still stable) and *star* is also equidistant in a good approximation.

The above all-embracing order controls not only each individual, but also the "totality", i.e. the Universe as well. This can be made obvious by the Figure which shows that not only the above series of "ordinary" objects but also the series of extreme astrophysical objects (neutron star, minimal quasar, protocluster, observable Universe) is just equidistant. This implies that the series of equidistant equidensity lines can be extended to include the density line of the *Universe* (of $\sim 10^{-29}$ g/cm^3). The intersection point of this line with the black hole line of slope +1 correctly gives the mass and size of the entire observable Universe ($\sim 10^{80}$ proton mass and $\sim 10^{42}$ proton radius). It is worth mentioning that the point "Universe" equally well characterizes both the part of the totality observationally known at present and the part which can in principle become known via ideal observations, because signals from essentially more distant regions have not yet reached us during the entire past of the Universe beginning from its "Big Bang birth" till now.

A further beautiful expession of the all-embracing regularity is that the geometric mean between the size of the *Universe* and that of the *atom* is just about the size of the *star*. The geometric mean between the size (or mass) of the *star* and that of the *atom* is the *man* while the geometric mean between the *man* and the *atom* is the *bacterium*.

As a consequence of the well known expansion of the Universe the position of the point U is time dependent, and, according to the suggestion of the Figure, it should move just along the limiting line of slope +1 (otherwise its present precise fitting to this line would be highly improbable). This is indeed true, so that the whole past history of the Universe can also be read off the diagram. Its evolution clearly ought to have started from the intersection point of the limiting lines (Lukács & Paál, 1988), corresponding to the so called Planck length and mass, given by the formulae

$$R_{Pl} = (\hbar G/c^3)^{1/2}; \qquad M_{Pl} = (\hbar c/G)^{1/2}.$$

These Planck data represent the only physical units defined uniquely by the laws of Nature themselves without any arbitrary convention. The ratio of Planck and proton masses is

$$M_{Pl}/M_p = (\hbar c/GM_p^2)^{1/2} = 1.3 \times 10^{19},$$

a basic large dimensionless number of Nature. Its powers have important meaning in the structure and evolution of the cosmos. Integer steps by this ratio in the Figure give such series: Planck scale, presently exploding black hole scale, neutron star scale, Universe scale at the decoupling of the radiation, Universe scale at the time of final quantum evaporation of black holes of stellar mass. In the history of the Universe the corresponding time scales are: Big Bang, supersymmetry phase transition, quark confinement, decoupling from radiation, black hole evaporation.

Summing up we may say that in the structure and evolution of the cosmos one can find equidistant steps in mass, size and time on a logarithmic scale expressing orders of magnitude. Commensurable steps connect one important preferred formation or event to the other. This regularity can be compared to the discrete symmetries of crystallic lattices, although it is an essentially generalized "symmetry" connecting unequivalent (but equally important) objects or events. It is the logical interconnection what is common here, not the type of the objects.

All this is not merely a magic of numbers. Many of the found regularities are straightforward consequences of simple physical arguments, while others indeed "depend rather delicately on apparent *coincidences* among physical constants" which in turn prove to be prerequisites of our existence (Carr & Rees, 1979). A tiny distuning of initial data or strength of interaction or particle masses or asymmetries would be enough to completely destroy our comfortable Universe, which seems as a "suit tailored just to our human measure". We may therefore be surprised to find ourselves in an "anthropomorphous" cosmos. The message of modern science appears to be that both the "anthropocentric" and the "anthropomorphous" characters are properties of Nature herself, so that these attributes begin to lose their purely pejorative meaning.

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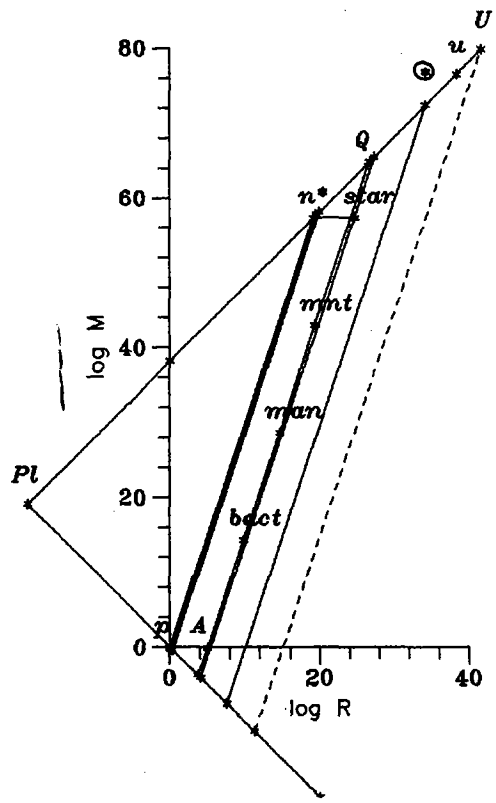

Cosmic mass-size diagram

p: proton, ν: neutrino, n^*: neutron star, Q: quasar, ⊛ : protocluster, u: Universe at decoupling, U: Universe at present, Pl: Planck data, A: atom, mnt: mountain, bact: bacterium. Heavy line: nuclear density, double line: atomic density, single line: "leptonic density", dashed line: cosmologic density.

Ursula Panhans-Bühler Gábor Császári

Copying a Work by Robert Smithson or: Another Experience With the Presence of Artworks

"A camera is a portable tomb"[1], wrote Smithson in 1969 during a trip through the Mexican peninsula Yucatan on which he placed twelve mirrors he had brought from New York to nine different sites along his way. Later he documented these mirror displacements in a poetic photo essay, but in this transposition the mirrors were gone blind, the text describing the experiences had become an obituary. About the risk involved in authentic experiences and its non-communicability knew Smithson: "There is different experience before the physical abyss than before the mapped revision."[2]

The works of Robert Smithson aimed at a notion that an acceptance of entropy would replace a current relationship to nature which is characterized by fear, revenge and the phantasmagory of control. He understood that the artist could not incorporate any longer his experiences into an aesthetic form which then could be enjoyed without any risk and at the same time without any experience by the critic and art lover.
This insight resulted in his 'Nonsites' and the later works which belonged only in a classificatory way to a tendency called 'land art'.

The artist does not want to be the hero any longer that the audience yearns to adore and at the same time the form of his art, though personal as a result of an aestetically exploring experience, is not any longer individual in a conventional way. It makes no sense to rebuild a Nonsite, but it seems to us a possibility

1) in: Nancy Holt (Ed.), The Writings of Robert Smithson, New York 1979, S.95
2) Ibid., S.84

to reconstruct one of his mirror objects thus making present as well a tricky fascination and a suspension of habitual expectations. We, who are responsible for the reconstruction in this exhibition, had done this before to translate the mute riddle of photographic reproductions and explanatory texts into a speaking one. Not through the original material and mirrors but with the same mirror constellation and similar in size and shape we try to make present the attraction and the lure.

We don't want to offend against a copywright of the original artwork with a pirated edition. But nevertheless we think that it is an advantage of many works by Robert Smithson, that they object to the fetishism as a modern cult to which other artworks respond more easily. The name of the author can't be any longer the coat of the priest which obscures the works and covers the eye of the spectator.

About the artist: Robert Smithson (1938-1973) was a crucial figure in the American art world in a period in which the diversification of styles such as Minimal Art, Land Art and Conceptual Art took place in the works of Smithson and his friends Eva Hesse, Donald Judd, Robert Morris, Richard Serra, Michael Heizer, Claes Oldenburg and others accompanied by the artists' and critics' discussions in Artforum and Arts Magazine. He was supported by the Virginia Dwan Gallery and later by the John Weber Gallery for whom the question how to sell his works was not a primary concern. Smithson's development started with an examination of the Abstract Expressionism in the fifties, then he moved from Sci -Fi parodies to Crystalline Structures, Nonsites, Mirror Objects and Mirror Displacements, Rundowns of Asphalt and Cement to Landscape Sculptures, Land Reclamation and their filmic translations in the early seventies.

The Dynamics of Structure Formation in Semiconductor Breakdown Experiments

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The universe we see is a ceaseless creation, evolution, and annihilation of structures or patterns which are endowed with a degree of stability. They take up some part of space and last for some period of time. It is a central problem of the natural sciences to explain this change of structure and, if possible, to predict it.

Nature surprises us with the fact that the tremendous amount of physical data and results can be condensed into a few simple laws and equations that summarize our knowledge. These laws are essentially qualitative and the question arises how nature generates the enormous variety and novelty of structures by starting from such simple principles. Unaware of the scope of simple equations, man has often concluded that more than mere equations is required to explain the complexities of the world. Today, we are not so sure, for already a simple random walk described by the Laplace equation provides a prototypical situation with behaviors ranging from orderly to chaotic. The formation of spatiotemporal patterns is, first of all, a geometrical problem. Thus, in our quest to understand how structures are generated by nature, the relationship between physical forces and the geometries they can shape is of fundamental importance. This relationship and the ensuing universal character of nonlinear pattern-forming processes are the central themes motivating the work outlined in the following.

The mechanisms that lead to pattern formation are best understood in continuous nonlinear systems, which to external probes appear to be governed by a few essential degrees of freedom. In such systems, structural changes are characterized by the appearance of singularities and bifurcations which occur if a balance existing between competing system-immanent effects breaks down. As a consequence, an initially quiescent system becomes unstable and, in a sequence of bifurcations, restabilizes successively in ever more complex space- and time-dependent configurations. Within particular symmetry constraints, the bifurcations occur in certain definite and classificable ways if the system is structurally stable.

So far, it appears that the subject of such complex nonlinear behavior is dominated by theoretical investigations and computer simulations, whereas experimental measurements on real-world physical systems represent the minority. Among the various objects which can be studied experimentally, solid-state turbulence in semiconductors looks highly promising (Huebener, Peinke, and Parisi, 1989). Nonlinear current transport behavior during low-temperature avalanche breakdown of extrinsic germanium comprises the self-sustained development of spatio-temporal dissipative structures in the formerly homogeneous semiconductor (Huebener et al., 1987; Peinke et al., 1988; Parisi et al., 1989a). This kind of nonequilibrium phase transition between different conducting states results from the autocatalytic nature of impurity impact ionization generating mobile charge carriers (Schöll, 1987; Schöll et al., 1987; Parisi et al., 1987). The simple and direct experimental accessibility via advanced measurement techniques favors semiconductors as a nearly ideal study object for complex nonlinear dynamics compared to other physical systems. Further representing a convenient reaction-diffusion system that exhibits distinct universal features, the present semiconductor system may acquire general significance for many synergetic systems in nature. Finally, in view of the rapidly growing application of semiconductor technologies, the understanding, control, and possible exploitation of sources of instability in these systems have considerable practical importance.

We have performed experiments with single-crystalline p-doped germanium (typical dimensions of about 0.2 x 2 x 5 mm^3), electrically driven into low-temperature avalanche breakdown via impurity impact ionization (Peinke et al., 1989a). Analogous to the corresponding phenomena in gaseous plasma discharges and atmospheric lightning, impact ionization of the shallow impurity acceptors can be achieved in the bulk of the homogeneously doped semiconductor. In the temperature regime of liquid helium (somewhat below 5 K) most of the charge carriers are frozen out at the impurities. Since the ionization energy is only about 10 meV and electron-phonon scattering is strongly reduced, avalanche breakdown already takes place at electric fields of a few V/cm and persists until all impurities are ionized (Parisi et al., 1988). The underlying nonequilibrium phase transition from a low conducting state to a high conducting state is directly reflected in strongly nonlinear regions of negative differential resistivity in the microscopic current-density versus electric-field characteristic (Schöll, 1987; Peinke et al., 1987a). Accordingly, the autocatalytic process of impurity impact ionization also leads to a strongly nonlinear curvature of the macroscopic (measured) current-voltage characteristic (sometimes with S-shaped negative differential resistance), the nonlinearity occurring just beyond the voltage corresponding to the critical electric field where the current increases by many orders of magnitude (typically, from a few nA in the pre-breakdown up to a few mA in the post-breakdown region). Under slight variation of distinct control parameters (electric field, magnetic field, and temperature in the range of some 10^{-6} V/cm, 10^{-4} G, and 10^{-3} K, respectively) the resulting electric current flow displays a wide variety of spatio-temporal nonlinear transport behavior, embracing the spontaneous symmetry-breaking emergence of both filamentary spatial and oscillatory temporal dissipative structures. One of the major issues of strong current interest is the question to what extent the break-up of spatial order during current filamentation is correlated with the onset of low-dimensional temporal chaos in the current oscillations.

The complex spatial behavior of our semiconductor system can be globally visualized by means of two-dimensional imaging of the current filament structures via low-temperature scanning electron microscopy (Huebener, 1988). As reported elsewhere (Mayer et al., 1987 a,b) in detail, nucleation and growth of filamentary current patterns in the nonlinear post-breakdown regime are often accompanied by abrupt changes between different stable filament configurations via noisy current instabilities. Moreover, the simultaneous spatial identification of oscillatory current flow dynamics in the vicinity of adjacent different conducting phases (e.g., boundary regions of the current filaments) provides a powerful tool for gaining deeper insight into the mutual interplay between spatial and temporal current structures (Mayer et al., 1988).

Self-generated current oscillations (with a relative amplitude of about 10^{-3} in the frequency range 0.1 - 100 kHz) are found to be superimposed upon the steady d.c. current (of typically a few mA) in the strongly nonlinear post-breakdown regime of the measured current-voltage characteristic. By means of slightly varying the applied electric or magnetic field, the temporal behavior of the current changes dramatically, displaying the typical universal scenarios of chaotic nonlinear systems (Peinke et al., 1985a,b, 1987b, 1989b; Rau et al., 1987). On the ladder towards higher orders of chaos we discovered a chaotic hierarchy of strange attractors (Stoop et al., 1989; Parisi et al., 1989b). The abrupt structural change between different dynamical states associated with gradually decreasing spatial correlation of different sample parts indicates a break-up of the semiconductor system from strongly coupled into more independent subsystems. In this way, new actively participating degrees of freedom are gained reflecting increasing dimensionality of the system. Turbulent dynamics may thus be ascribed to nonlinear coupling between competing localized oscillation centers intrinsic to the present multi-component semiconductor system. So far, we have demonstrated experimentally the existence of spatially separated oscillatory subsystems as well as their long-range interaction (Röhricht et al., 1986, 1987; Schöll et al., 1987; Peinke et al., 1987b; Mayer et al., 1987a,b, 1988). Moreover, the underlying nonlinear physics reveals critical phase transition behavior by varying the temperature at constant electric and magnetic field (Röhricht et al., 1988). Most importantly, we disclosed an upper bound for the onset of structure formation (first-order phase transition) and avalanche breakdown (second-order phase transition) at critical temperatures of about 5.5 K and 7.2 K, respectively.

To conclude, our experiments deal with a challenging example of a macroscopic synergetic system, consisting of spatially separated and diffusively coupled subsystems which by themselves show oscillatory behavior. Depending sensitively upon distinct control parameters, the resulting spatio-temporal current flow undergoes various symmetry-breaking phase transitions typical for nonlinear dynamical systems. So far, the crucial role of the relationship between the breakdown of spatial order and the onset of low-dimensional temporal chaos has been unfolded to some extent. There is hope that these promising advances attained in semiconductor physics via certain fundamental interdisciplinary concepts, such as symmetry and symmetry breaking, linear and nonlinear stability, frustration and constrained dynamics, may open up the possibility of a fruitful interaction with the diverse fields ranging from mathematics to biology, even the creative and performing arts, and most branches of the sciences.

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VISUALIZATION
OF SYMMETRY AND OTHER MATHEMATICAL IDEAS IN TEACHING ART

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1 The >DIAPORAMA< Project

>The Bonn University Department of Mathematics and Mathematics Teaching< has realized a research project on >visualization< of mathematical aspects by using the AV-medium >DIAPORAMA<.
The theoretical foundations of the >Bonn Visualization Project< (Peters 1987) are derived from an analysis of >δείκνυμι<, the word EUCLID used for "to prove". This term means: to show, to make evident, to demonstrate, and also to teach. As the usage of this term shows, visible evidence, "demonstratio ad oculos", visualization in constructing a mathematical line of arguments seem to be the fundamental concept of Greek mathematics (Szabo 1969,p.246ff and Peters 1985,p.31-48). The learning objectives for which our research project planned and realized software range from classic topics of geometry to attempts at pupil-adequate explanations of limit, further to analyses of strategies of problem solving, and even to interdisciplinary relations between mathematics, philosophy, and art.
The >DIAPORAMA<-hardware (fig.1) is an audio-visual presentation unit. It consists of two exactly lined-up slide projectors, a dissolve unit that allows several modes of fading, and a stereo tape recorder with the possibility of recording audio signals and control informations on different tracks. The slide first shown by projector A is continually superimposed by the slide in projector B. This dissolving can be changed from a slow superimpose to a hard cut, and also to a >flip<, i.e. a sudden change from one projector to the other, e.g. to simulate reflexion.
Fundamentally the >DIAPORAMA< dissolving technique produces a "third picture", which for its part generates a pseudomotion so that this dynamic slide projection takes up the mid-position between traditional static slide presentation and moving film.

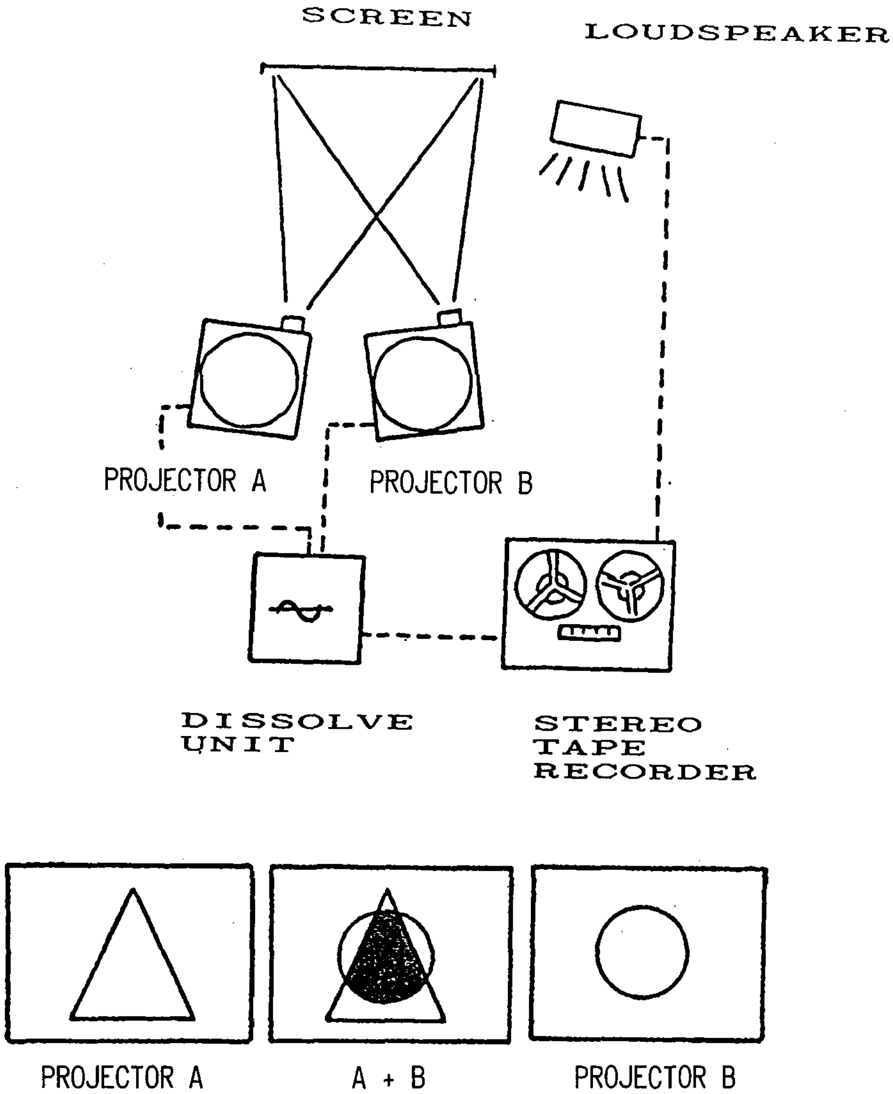

SCREEN
LOUDSPEAKER
PROJECTOR A
PROJECTOR B
DISSOLVE UNIT
STEREO TAPE RECORDER
PROJECTOR A
A + B
PROJECTOR B

FIG.1

2 Symmetry and Infinite

With the intention to discover art as a system of rules, norms, and laws, the >Bonn DIAPORAMA Project< has produced some visualization series about mathematics and art,mainly (but not only using graphs by the Dutch artist M. C. ESCHER (1889 - 1972).
The main issue of these efforts was not a new interpretation with regard to the artistic contents of ESCHER's "intellectually constructed" works, but it was moreover the didactic reconstruction of his playing with mathematics, with proportion, reflection, perspective, and optical illusion, with zenith and nadir, with symmetry and contrast, with plane and space in simultaneous and impossible worlds.
The graph (fig.2) showing the division of a plan by birds and fishes a preliminary study about "Air and Water" (1938), enables pupils to work out the first principles of periodic drawing. The repeating pattern used by ESCHER in this case is defined by its relationship to a very simple group of the 17 plane symmetry groups, resulting from two lineary independent translations. Generating unit is a parallelogram within a parallelogram lattice. This lattice unit itself doesn't show symmetry.

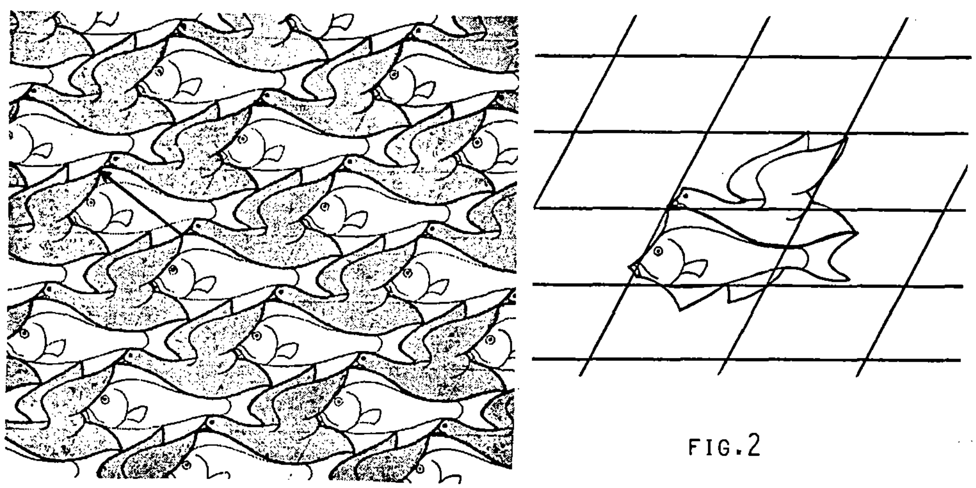


FIG.2

Planes structured by congruent rhythms are preludes to metamorphoses and circles. Conformal mapping led ESCHER to symmetries and non-euclidian elements as constructing principles e.g. in his woodcut "Circle Limit III" (1958; fig.3).

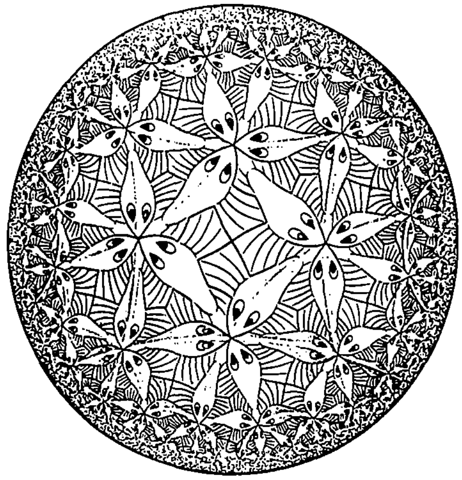


FIG.3

He found a plan for a series of attempts to a new way of "approaching the infinite" in a picture (fig.4) by COXETER,which was presente in connection with POINCARE's circle model of hyperbolic geometry. Laying bare these elements as constructive principles in classroom teaching leads to the limits of school mathematics instrumentation. Visualizations in another DIAPORAMA-series teach, how M.C. ESCHER tried to bring the totality of plane into a limited figure.

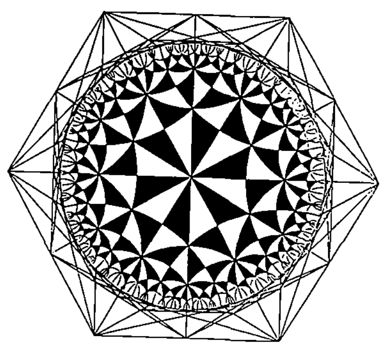
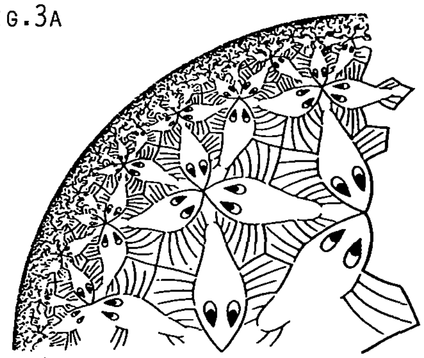


FIG.4

Pupils learn: in POICAREs model of hyperbolic geometry there is no h-point on or outside the periphery of the circle, only diameters and arcs of circles, which are orthogonal to the circular line, are h-lines.Fishes on every arc meeting head to tail rise from the circular line and fall back to it, seemingly without reaching it as a limit, increasing towards the center and decreasing when leaving. By help of more than 90 superimposes our project tried to provide an approach to a mathematical and also aesthetic interpretation of ESCHER's graph describing the following sub-aspects:

FIG.3A



1 central symmetries
2 rotations of regions
3 rotations of figures
4 meetingpoints of heads
5 meetingpoints of tails
6 arcs (h-lines) with fishes
7 "approaching the infinite"
8 symmetric structure of h-lines
9 twenty steps of the printing process (fig. 3a)

Comprehending the constructive imagination of ESCHER's "Circle Limit III" pupils consider the woodcut as "nice" and try to draw and paint simple designs themselves (fig.5)
For many topics of mathematics instruction even ESCHER-graphics can be motivating to analyse mathematical structures or visualization of aesthetic application of mathematical or antimathematical thoughts. (Peters 1987,p.53-73)

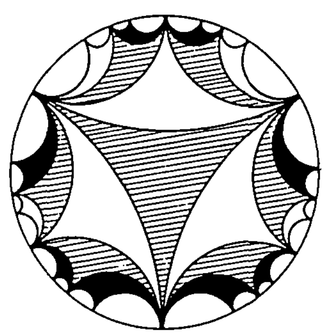


FIG.5

3 Impossible Worlds

The topic of other DIAPORAMA-series was the violation of rules in perspective mapping. It was intended to show that ervery three-dimensional object can be drawn two-dimensionally. But not every object which is presented in the drawing plane necessarily has an equivalent in the three-dimensional reality. These visualizations revealed some advantages of the dissolve technique e.g.

- the possibility to superimpose a graph by a drawing showing
- only the essential details,
- to bring into focus minor aspects,
- to add constructiv subsidiary lines,
- to mark planes,
- and to structurize complexes by painting them with new colours.

ESCHER's lithography "Belvedere" (1958) presents a pavillon with a view over a valley in the mountains. The second floor of this haunted castle seems orthogonal to the first floor. The ladder is leaning with its top against the exterior, whereas its foot is inside the pavillon.

A DIAPORAMA-series tried to visualize these violations of mathematical rules by means of descriptive geometry. Drawing the horizon into the graph and then trying to reconstruct the sagittals, we realize that there are two saggitals: one above, the other below the horizon. This is against exact perspective drawing. The top and the lower part of the pavillon show different perspectives, yet they are still connected by columns. This is only possible on the drawing plane, in the three-dimensional space, however, there is no equivalent. The reflection of the upper part of the building at the midparallel of the two sagittals results in corresponding blocks above and below the horizon,which are in perspective. The two part sagittals also form one line now. The whole "Belvedere" becomes mathematically correct,and thus it has an equivalent in three-dimensional space (fig.6).

Having understood the mathematical construction of "Belvedere" by ESCHER pupils reconstructed their "Belvedere" and found it artistically boring (fig.7-8).

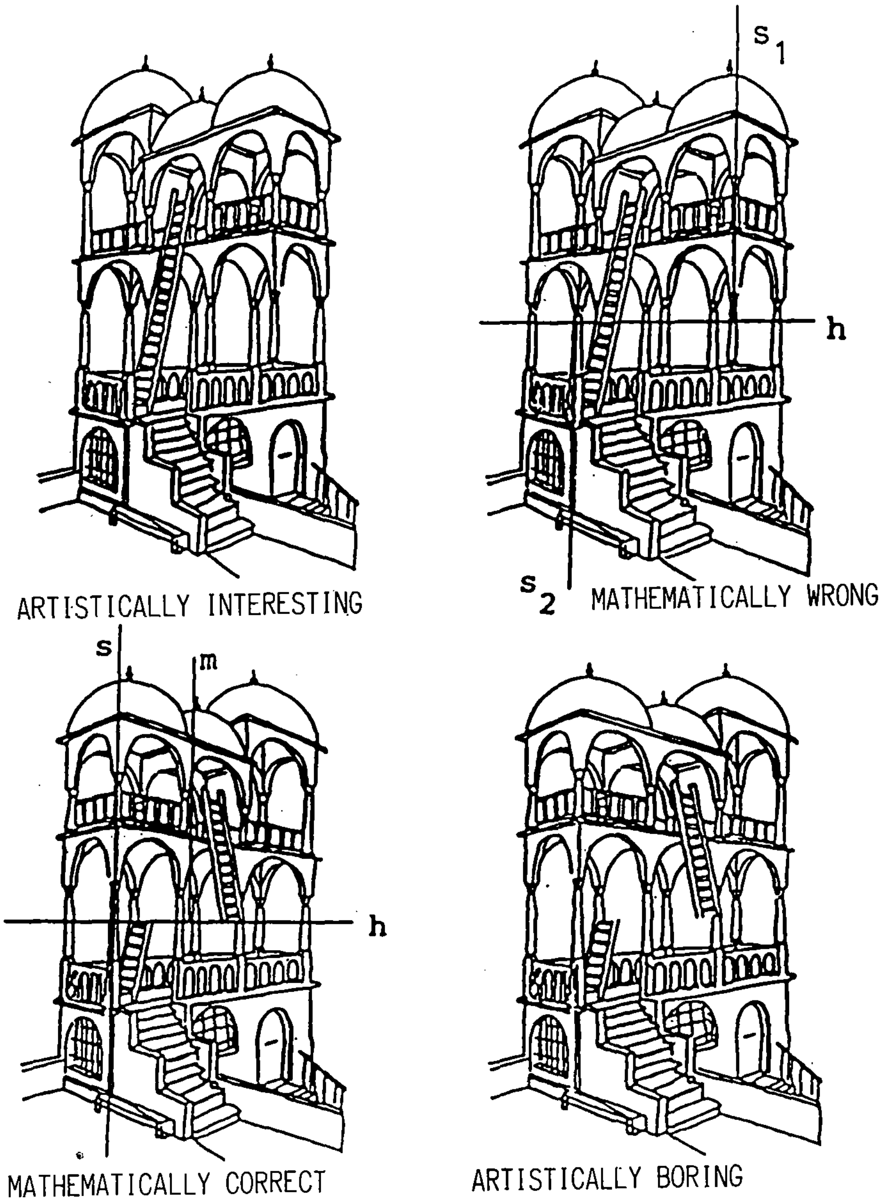

s_1
h
s_2
ARTISTICALLY INTERESTING
MATHEMATICALLY WRONG
s
m
h
MATHEMATICALLY CORRECT
ARTISTICALLY BORING

FIG.6

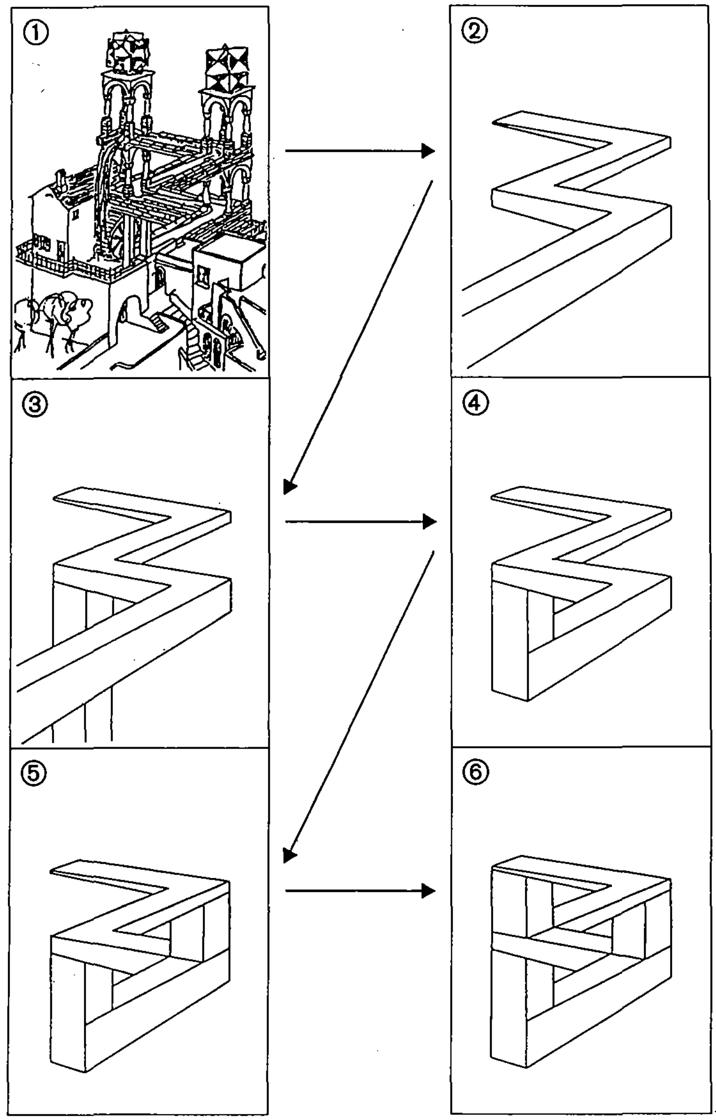


FIG.9

Similar aspects of violation of mathematical perspective may be analysed in regarding ESCHER's lithography "Waterfall" (1961) and discovering the three times hided >tribar< (fig.9).

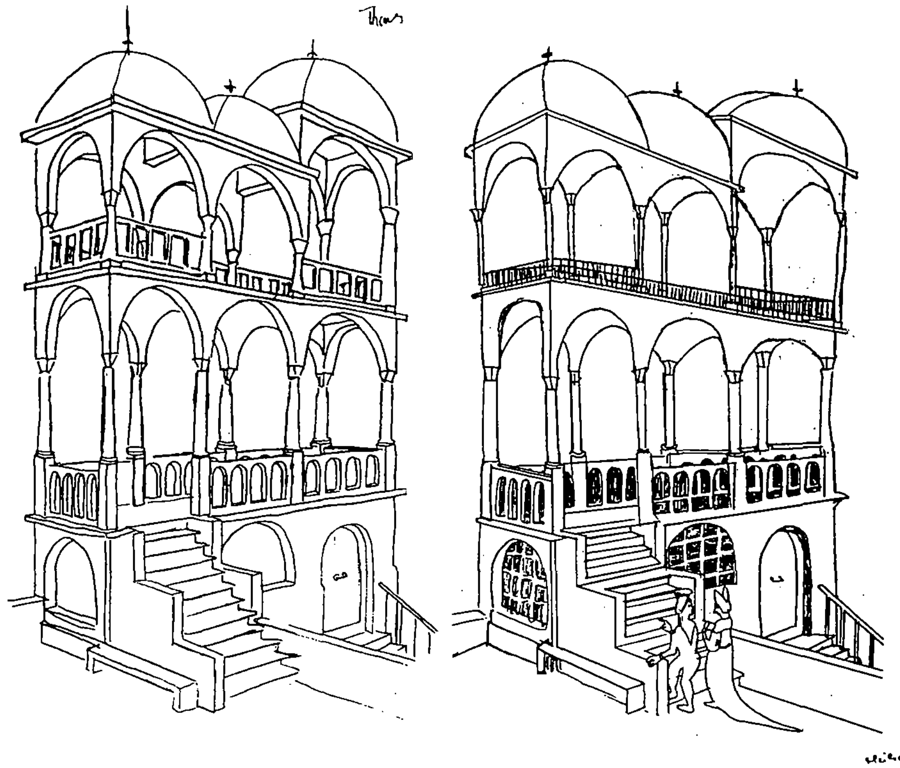


FIG.7-8

4 Visualization as Reconstruction

In contrast to verbal and also symbolic presentation visualization proves to be a more subtile method of activating mathematical structures in pupil's imaginations (Peters 1988,p.44-67). Visualization is very close to making visible mathematical theorems by interpreting "to prove" as >δείκνυμι<. The term >θεώρημα< also means: having seen, considered, examined, and understood. A theorem becomes obvious, evident, manifest, clear, because its construction was the "oject of a vision". Visualization means to demonstrate essential structures, to give insights into the mode of a constructing strategy, to look into the constitution of a theorem. Visualization is a didactic provocation for reconstructing mathematics by the pupils.

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Highest symmetries and iterative algorithms in self organization of living matter. Cyclomer biology and cyclomer arts.

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Many problems of biological objects and processes are connected with geometrically ordered multiblock structures like the lumbar spine, fins of fishes, flowers, mollusc's shells, cardiac rhythms, unperiodical but scale rhythms of crustacea's moulting, etc.

Within the existing variety of geometrically legitimate biological structures, we concentrate on supramolecular biosystem structures whose components are integrated into an entirety in compliance with certain rules or algorithms which are the same along various lines and on various levels of biological evolution. These structures, which may be referred to as algorithmical, are of special interest for theoretical biology and physiology, also as for related sciences such as biomechanics, biotechnology, bionics, informational mechanics, etc. What is important is that, in addition to regularly shaped biological objects, there are some in which the conjugation of components is less regular, if existing at all. The report will consider algorithmical biostructures which are chains or manifolds decomposable into commensurable and regularly positioned elements (or motive units S_k). Figure 1 shows such manifolds.

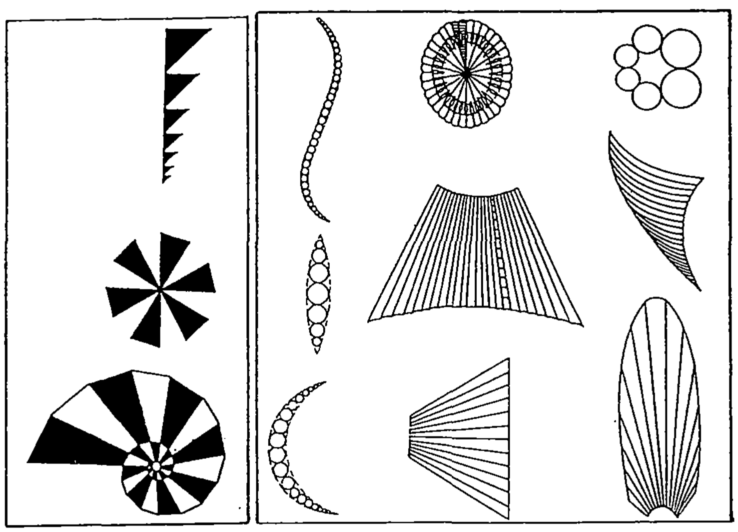

Figure 1. Geometric examples of similarity cyclomerisms (left, according to A.V. Shubnikov [1960]) and of Möbius, affine and projective cyclomerisms (right).

The general rule of representing decomposable manifolds is that the preceeding motive unit is transferred into the succeeding one by a certain fixed transformation g; in other words, the neighbouring motive units S_k are mutually conjugated by an iterative algorithm:

$$S_{k+1} = g*S_k \qquad (1)$$

Consequently, by reapplying the generating g transformation m times to a motive unit S_k, a component S_{k+m} is obtained; mathematically speaking, in the set S_k, a cyclic (semi-) group of transformations, G, is active which contains elements g^0, g^1, $g^2, \ldots, g^m, \ldots$ (a finite number of motive units in a biological object is neglected where necessary). In other words, this decomposition of the manifold, thus organized, includes a cyclic group of automorphisms and their motive units are aligned along the orbit of the appropriate cyclic group. For brevity, such configurations will be referred to as cyclomerisms, a term known in biology, no matter whether g is Euclidean or not in an iterative algorithm (1).

Classical biomorphology (see classical works of W. D'Arcy Thompson [1917], H. Weyl [1952], A.V. Shubnikov [1960], et al) paid great attention to only those biological cyclomerisms which have generating transformation "g" from the similarity transformation group (the last consists of rotational, translational, mirror and scale transformations only).

Similarity transformations in biomorphology are also known with reference to the scale of three-dimensional growth which is fairly frequently observed in animals and vegetation over extensive periods of individual development and is accompanied by mutually coordinated growth behaviour of small zones distributed in the volume of the body, a behaviour which is geometrically described as a scale tranformation. With the transformation of as few as three points of the growing configuration known, the transformation of the continuum of its points may be assessed.

Do the similarity symmetries and the scale of the volume growth exhaust all geometrically legitimate kinds of mutual conjugation of parts in a structure and ontogentic transformations in living bodies? Or do they act in biology as very particular cases of the kinds which are built around non-Euclidean groups of transformations containing similarity sub-groups? The writer's research has provided a positive answer to this latter question. It is well known that there are two basic ways to extend the similarity transformation group, either to the Möbius transformation group or to the projective transformation group. Both these ways have a biological value according to our research.

The report consists of many examples of parallel existence of Euclidean and non-Euclidean cyclomerisms in biological structures: segmented horns of animals, antennae of insects, vertebrae of animals, shells, etc. One of these examples considers a regular non-linear reduction of average diameter d_k of conductive airways in adult human lung as a function of the order of generation k of dichotomous branching (according to E. Weibel and D. Gomez [1962]); the author's research reveals that this reduction for all the 28 generations of branching is well described by the iterative algorithm (1) with the following generating transformation g of the local-similarity kind: $d_{k+1} \approx (0{,}748\ d_k + 0{,}01):(-\ 0{,}013d_k + 1)$. Consequently, this important property of conductive airways may be interpreted (like some other biocyclomerisms) from the viewpoint of general biological value of local-similarity transformations.

Euclidean and non-Euclidean cyclomerisms have also had a direct bearing on the kinematics of a broad range of biological movements which can, on numerous occasions, be interpreted as a process in which cyclomerisms replace one another (so called "cyclomeric polymorphism"). The author's research also revealed the existence of non-Euclidean kinds of three dimensional growth which had not been known before, notably Möbius and affine; the three-dimensional growth of living bodies can be interpreted in terms of cyclomeric polymorphism.

Researchers in various countries have for a long time been studying time biorhythms and this field of natural sciences can boast of its own traditions, terminology, and challenging findings. Still, it has concentrated attention on periodic rhythms of physiological processes such as breathing and walking, repeating processes occurring simultaneously with periodic diurnal and seasonal changes, etc. The reader of the literature on biorhythms may think that no biorhythms other than perodic are significant or possible. In point of fact, however, the range of biologically significant rhythmns is broader and the periodic one is but a particular, albeit important, sub-class. Nontrivial Euclidean and non-Euclidean iterative algorithms are obviously at work in certain rhythmic processes by Arenicola marina, Bonasa umbellus, some kinds of cardiac arrhythmias (Wenckebach periods) and other disturbances of normal periodic bioprocesses, etc. (For more details see S. Petukhov [1988, p. 34-36]). This is the way to develop a special cyclomeral theory of biorhythm disturbances under a wide class of internal or external influences; in this way, the author achieved first theoretical results.

The writer's findings are in favour of not only the argument that biology is a fertile field for introduction of various symmetrical approaches, methods and tools of group-theoretical analysis, but that development of theoretical biology, physiology and biomechanics at this stage is largely dependent on vigorous utilization of group-theoretic methods with non-Euclidean geometries. These findings permit the development of "cyclomer biology" as a science about general biological (including physiological) value of symmetrical-algorithmical principles of structurization and interaction. Cyclomer arts, combining music, mathematics, art and dancing, are also being developed by the author on the basis of application of the general cyclomerism principle with biologically valuable generating transformations.

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Rational Geometry of Unidirectional Fiber-reinforced Multi-link Bamboo-like Tubes.

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This paper discusses one of the biomechanical problems, namely the problem of rational design of composite tube structures and how the geometrical symmetry of strong tube biostructures is connected with the fracture criteria.

1. Unidirectional composite tubes under axial compression show a particular mechanism of fracture with axial symmetry of the n-th order. They fail as a chinese lantern when multiple splitting and buckling of n circle segments take place . Besides, they can fail in the form of macrobuckling or in the form of local mikrobuckling under compression.

The axial compressive strength for the local microbuckling is denoted by

$$\sigma_{cr} = \sigma_c \qquad (1)$$

The critical stress for the chinese lantern mode of fracture for the thin-walled tube, acoding to Rabotnov, Polilov (1983), is given by

$$\sigma_{cr} = \sigma^* = 1.2(\gamma^4 E^5 / R_a^2\ l^2)^{1/9} \qquad (2)$$

where R_a - the average radius, l - the length of the tube,E - the longitudinal Young's modulus (along the fiber), γ - the spesific energy of splitting.

The critical stress for macrobuckling of the tube may be written by Euler's formula

$$\sigma_{cr} = \sigma_e = \pi^2 EI/Fl^2 \qquad (3)$$

where $I \approx \pi\delta R_a^3$ - moment of inertia with respect to axis through a center of attraction, $F = 2\pi\delta R_a$ - cross section, δ - wall thickness.

These three failure modes are shown in Fig. 1a for unidirectional glass-fiber reinforced plastic (GFRP) with the following characteristics: E=50 GPa, σ_c=500 MPa, γ=30 KN/m, R_a=7.5 mm.

For a tube element with optimal geometry the various modes of failure must occur simultaneosly.

Then the rational tube length for simultaneos~~ly~~ occuring two fracture modes may be derived:

from (1) and (2) in the following form

$$l = 2.27(\gamma^2 E^{5/2})/(R_a \sigma_c^{9/2}) \tag{4}$$

from (2) and (3) in the following form

$$l = 2.22(E^4 R_a^{20}/\gamma^4)^{1/16} \tag{5}$$

from (1) and (3) in the following form

$$l = R_a \pi (E/2\sigma_c)^{1/2} \tag{6}$$

Three graphs of these equations for the GFRP are shown in Fig. 1b.

From (4) and (5) the optimal average radius and the optimal length for the given material are:

$$R_{opt} = 1.01 E\gamma/\sigma_c^2 \tag{7}$$

$$l_{opt} = 2.24 E^{3/2}\gamma/\sigma_c^{5/2} \tag{8}$$

For the GFRP R_{opt}= 6.06 mm and l_{opt}= 134.4 mm.

2. Under bending condition each link of a multi-link tube structure is loaded by different stress, and therefore the optimal length to radius ratio and the optimal thickness change from link to link. The optimal thickness is found from (1). It

$$\delta = R-((R^3+ 4M_x/\pi\sigma_c)R)^{1/4} \qquad (9)$$

where R - the external radius, M_x - the bending moment.

The optimization problem for multi-link tubes can not be solved with the thin-walled approximation since the thickness is a parameter of optimization. Therefore it is necessary to receive the exact equation like (5), with the following expression for the moment of inertia of the circle segment:

$$I=(\alpha+\sin\alpha\cos\alpha)(R^4-r^4)/4-4\sin^2\alpha(R^3-r^3)^2/9\alpha(R^2-r^2) \qquad (10)$$

where r - internal radius, $\alpha = \pi/n$ (n - the number of segments, $n\gamma\delta l$ - the work of splitting), and to solve this equation by some numerical methods.

Our numerical research of this model shows that for $\delta/R_a \ll \alpha^2$ the optimal link length l_i decreases with the R_{ai}(average radius of the i-th link) increasing, and for $\delta/R_a \geqslant \alpha^2$ the optimal link length l_i increase with the wall thickness δ_i increasing.

In order to verify the applicability of the proposed criterium of optimization to natural biological structures like a bamboo trunk, we consider the bamboo trunk as multi-link tube structure under bending condition due to wind load. For a fixed radius of the lowest part of a trunk the thickness of the tube was found from (9). Then the optimal first link length was found from the exact equation like (4) for the chinese lantern mode of fracture.

We consider the linear decreasing of the trunk radius from the lower part to the top and assume the following properties of the wood: E = 30 GPa, γ = 20 KN/m, the total length l = 14 m, the first link radius R_1= 0.1 m, σ_c=350 MPa, and the critical bending force P = 8 KN. The results of computer modelling show that the link length varies from 0.14 to 0.2 m, and monotonously increase from the ends of the trunk to the middle (See Fig. 2). Quantatively it is in agreement with the real changing of bamboo link lengths.

CONCLUSION

The considered failure modes take into account the scale effect, and so the optimal dimensions of the tube structures may be found using the criterium of simultaneos mixed-mode failure initiation.

The proposed approach was developed in order to prove that some strong biological structures agree with optimal relations connected with simultaneos initiation of different fracture modes.

This method may be used in the design of tube constructions (e.g. ferms) using unidirectional composite materials (CFRP, GFRP and others).

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Fig. I

Three modes of fracture under axial compression.

a) Stress versus length; b) length versus radius.

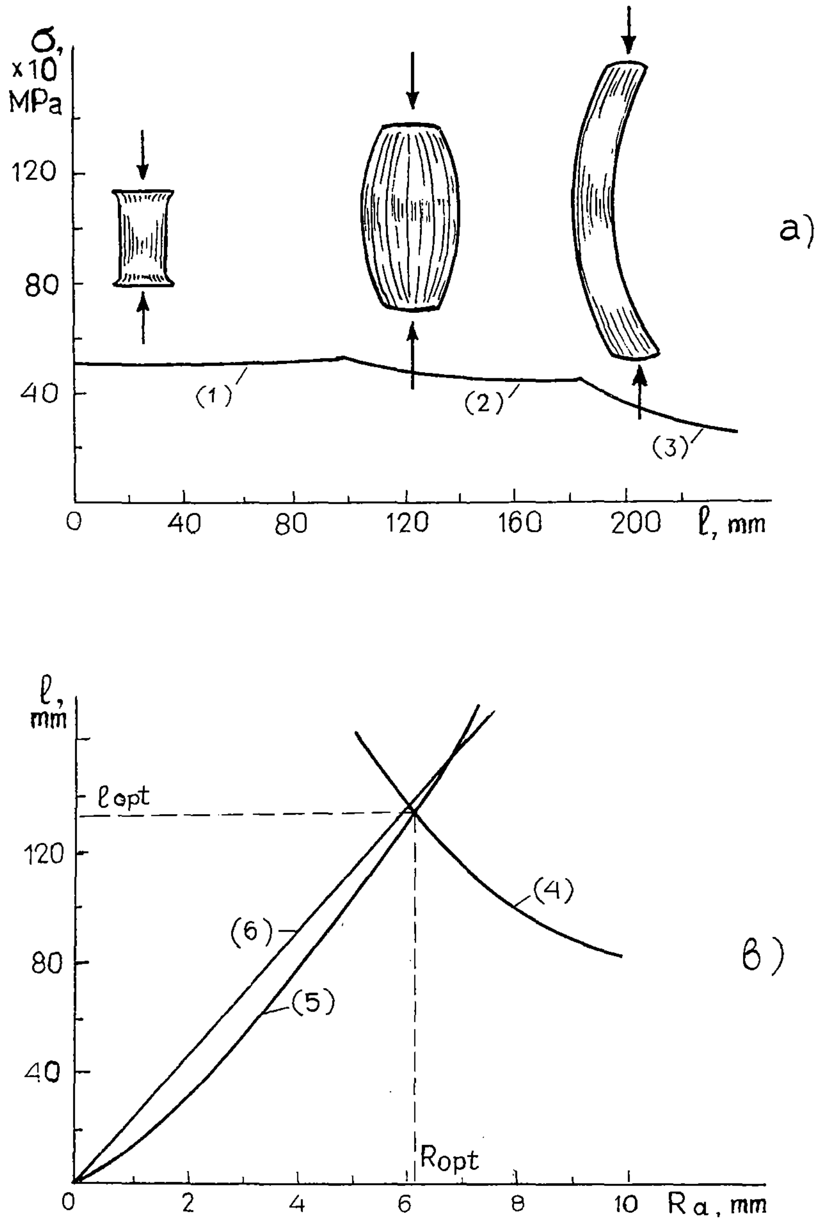
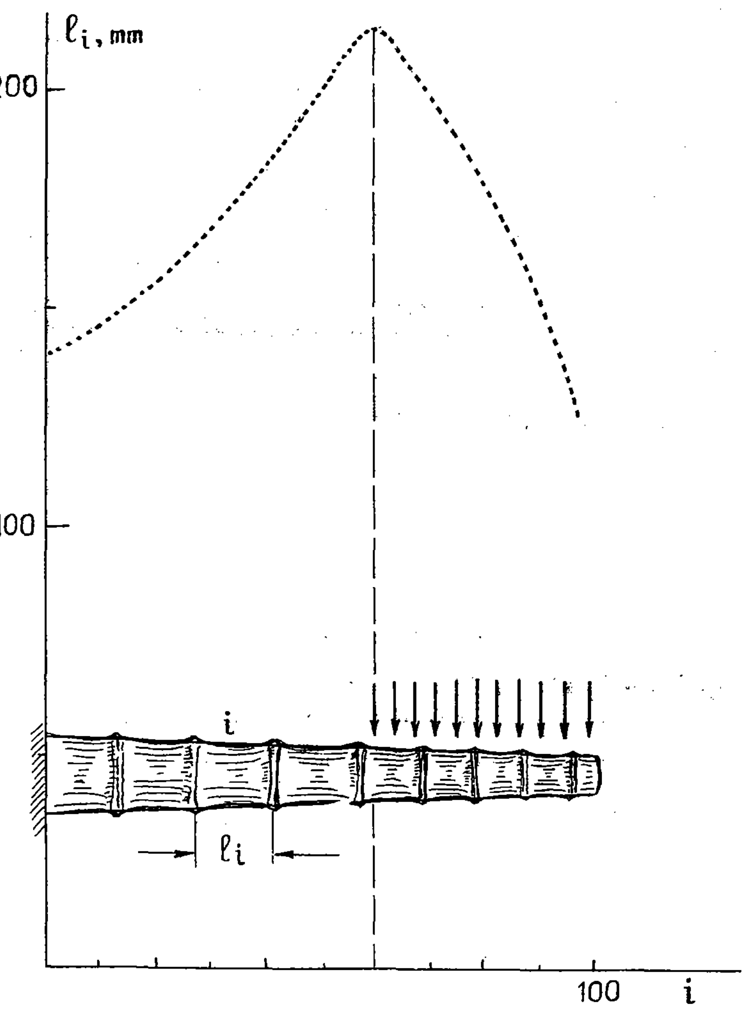


Fig. 2

The link length versus the link number.



THE CRITERION OF SIMMETRY-ASYMMETRY IN THE EVOLUTION OF THE GENETIC CODES

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According to the criterion of symmetry-asymmetry, the triplets of the genetic code can be divided in four classes: 1) In the first class there are the codons which present in the first and third positions the same nucleotide base. For instance, AGA and GAG. 2) In the second class there are codons which present in the first and third position two different nucleotide bases, but both purines, or both pyrimidines. For instance, AGG and GAA. 3) In the third class there are codons which present in the first and third positions two complementary nucleotide bases. For instance, AGT and CGG. 4) In the fourth class there are codons which present in the first and third positions two anti-complementary nucleotide bases. For instance, GAT and ATC.

The criterion of symmetry-asymmetry is very important for the classification of codons. The following arguments plead in this sense: 1) In each class there are 16 codons. The codons of the second class can be deduced from these of the first class by the substitution of the third nucleotide base according to the rule: A ⟶ G, G ⟶ A, C ⟶ U and U ⟶ C.

The codons of the fourth class can be deduced from these of the third class by means of a similar rule to that utilized for the transition from the first to the second class.

2) If we make abstraction from the differences existing between the two purine bases, or these between the pyrimidine bases, the codons of the first and second classes are symmetrical codons, while these of the third and fourth classes are symmetrical triplets. There are 16 pairs of symmetrical codons and 16 pairs of asymmetrical codons. The codons of each pair specify the same amino acid.

In the literature there are mentioned three types of genetic codes. The "universal" code, which is utilized by all living organisms, starting from viruses and ending with man. The mammalian mitochondrial code and yeast mitochondrial code, which are utilized by the respective mitochondria (Tzagoloff, 1982; Grivell, 1983).

From the point of view of symmetry, the code of mammalian mitochondria shows the most harmonious structure, the code of yeast mitochondria is in an intermediate position, while the "universal" code presents the most asymmetrical form.

On the other hand, the molecular mechanisms involved in the processes of translation and transcription present the most simple form in the genomes which utilize the mammalian mitochondrial code and the most complex form in the genomes which utilize the "universal" genetic code (Grivell, 1983).

Taking this into account the code of mammalian mitochondria is probably to preserve the most related structure to the primitive genetic code, while the "universal" genetic code shows the most evolved form. Therefore, the evolution of the genetic codes developed from a more symmetrical form to an asymmetrical structure.

In the code of mammalian mitochondria, in each class of codons there are: a) one codon which initiates the translation; b) one codon which ends the translation; c) different codons for hydrophobic and hydrophilic amino acids.

Therefore, each class of codons of mammalian mitochondrial code could function as an independent genetic code. This fact suggests the hypothesis that during the evolution of the genetic codes a phenomenon of duplication of codons took place. The 16 codons of the first class have generated the 16 codons of the second class. Then, the 16 pairs of symmetrical codons have generated by a new duplication the 16 pairs of asymmetrical codons.

In the genetic codes there is a structural and functional correspondence between the second nucleotide base of the codons and the specific amino acids. If the second nucleotide base is a purine, then the specified amino acid is hydrophilic. If the second nucleotide base is a pyrimidine, the specified amino acid is hydrophobic.

Taking into account these aspects, the following model concerning the origin of the genetic codes can be conceived.

At the origin of life, certain primitive polynucleotides have been achieved by means of some non-biological ways. These primitive polynucleotides have been formed by ribonucleotides, because only the ribonucleotides can be synthetized without the intervention of some specific enzymes (Eigen, 1977).

Despite of the fact that they have been ribonucleotides, these primitive polynucleotides contained thymine instead of uracyl. This substitution was necessary because thymine is more suitable to achieve a direct hydrophobic interaction with the hydrophobic amino acids, then uracyl. For the existence of these primary polynucleotides at the origin of life pleads the fact they are still present like vestiges in the sequences of the actual mammalian mitochondrial genes (Grivell, 1983).

In the structure of the primary polynucleotides there were three principal zones: a) an initial sequence formed by pyrimidine bases; b) an intermediary sequence formed by an alternate succession of purine-pyrimidine bases; c) a final zone formed by purine bases.

The initial zone was necessary for the initiation of synthesis, because the hydrophobic interactions are more suitable to achieve intermolecular connections with hydrophobic amino acids, than the hydrogen bonds with the hydrophilic amino acids.

The intermediate zone was necessary because the polynucleotides having the type PpPpPpP (purine-pyrimidine-purine) can perform easely the transition from a conformation to another one (Felsenfeld, 1985).

The final zone was necessary because it determined the synthesis of a hydrophilic tail at the end of the polypeptide, which was important for the building of different biological membranes.

The codons existing in these three zones have been symmetrical codons (PPP, PpP, pPp and PPP).

According to the data presented in literature, it is probably that the purine bases have selectively connected the hydrophilic amino acids, while pyrimidine bases have fixed the hydrophobic amino acids.

After the fixation of amino acids at the level of the nucleotide bases, some primitive molecular mechanisms have acted and have produced the synthesis of the respective polypeptide. Then, the primary polypeptide has acted on the respective polynucleotide and has induced its replication. In this way, the first structural and functional correspondence between the primitive polynucleotides and polypeptides has been achieved.

The role of the triplets was determined by the fact that the selectivity of a nucleotide base, for certain amino acid was also influenced by the neighbouring situated nucleotide bases (the previous and the next in sequence).

For instance, if cytosine was included between two thymine bases, it had a smaller probability to connect a hydrophobic amino acid, because the thymine bases present a greater affinity for the hydrophobic amino acid than cytosine. Due to this, in the actual genetic codes the triplet TCT specifies a hydrophilic amino acid (serine).

In conclusion, at the origin of the genetic codes an important role had been played by the direct interaction between the nucleotide bases and the amino acids, and by the symmetrical codons.

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THE CYCLIC EVOLUTION OF UNIVERSE AND SYMMETRICAL THERMODYNAMICS

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The second principle of thermodynamics asserts that in an isolated system the energetical transformations develop in such a way that the entropy increases (Katchalscky & Curran, 1965).

The entropy was associated with two essential aspects of evolution: 1) The increase of entropy shows the decrease of the free energy and the growth of the bound energy. 2) The increase of entropy indicates the decrease of order and the growth of disorder (Boltzman, cited by Atlan, 1972).

The application of the second principle of thermodynamics is cosmology has generated some contradictions:

1) According to the second principle, starting from the origin of universe and going to present, the cosmological entropy and the disorder have increased. However, the astrophysical and cosmological data have revealed that the complexity of organization has increased both in microcosmos, as well as in macrocosmos (Weinberg, 1984).

2) From the origin of universe until present, the temperature of cosmos has decreased. This is in contradiction with the second principle of thermodynamics, which indicates that the increase of the entropy is correlated with the growth of the temperature.

On the other hand, the astrophysical and cosmological data indicate that with 15-20 billion years ago, the matter and energy of universe were concentrated in a sphere which had a much smaller radius than the actual radius of universe (Einstein, 1955; Tolman, 1962; Ionescu-Pallas, 1980). Then, at a certain moment the "big bang" was produced and the expansion of universe developed.

According to the theory of generalized relativity, there are two possibilities for the future evolution of universe: a) If the average density of matter in the framework of universe is smaller than a certain critical density (0.5×10^{-29} – 2×10^{-29} g/cm^3), then the expansion of universe will continue for ever. b) If the average density of matter is greater than this critical density, the expansion of universe will stop in the future and it will be followed by a compression phase.

Until now, the average density of matter has not been determined with a sufficient precision to clarify this dilemma (Robinson, 1980; Rubin, 1983; Maddex, 1984; Thuan, 1987). However, the cosmological observations show the existence of non-luminous matter arround the galaxies and the fact that neutrinos probably have a resting mass. In this case, the density of matter in universe is closely situated nearby the critical density.

On the other hand, when the temperature of universe will reach zero absolute degree, because of the expansion of space, the photons will probably acquire a resting mass. In this moment, the density of matter within the universe will surpass the critical density. Subsequently, the expansion of matter will stop and it will be replaced by a compression phase. This possibility suggests the existence of a cyclical evolution of universe.

The cyclical evolution of universe requires the application of the thermodynamics of dual tendencies (a symmetrical thermodynamics).

This thermodynamics is characterized by the following aspects: in the framework of universe two kinds of forces work: a) Forces which support the expansion of matter and energy in cosmos; b) Forces which determine the compression of matter and energy.

During the expansion phase, in the universal space the forces of expansion are predominant, while during the compression phase the forces of compression will be more important.

2) In relation with the expansion forces it is necessary to define a value of free energy and one of bound energy. Some similar values must be introduced in relation with the compression forces. During the expansion phase, the free energy of expansion forces diminishes and that of compression forces increases.

At the limit of expansion, the free energy of expansion will attain its minimum, while that of the compression forces will reach its maximum, and so on.

3) During the evolution of universe, two kinds of order arise. The first order was achieved in the primary nucleus of universe. It was characterized by the division of space in two compartments: a) the sphere of the primary nucleus, which included all the matter and energy; b) the potential space, free of matter and energy, situated around the primary nucleus. I suggest to name this order Planck's order.

The second order will be accomplished at the limit of expansion. It will be characterized by a homogeneous distribution of matter and energy in the interior of black holes and stars of neutrons. The temperature of the universal space will be about zero absolute degree (Dicus et al., 1983). I suggest to name this situation Nernst's order.

4) In relation with these aspects, two kinds of entropies can be defined: a) the entropy of expansion forces (Planck's entropy). b) The entropy of compression forces (Nernst's entropy).

When Planck's entropy attains its maximum, the free energy of the expansion forces is at minimum and the order is that of Nernst's type. This situation will be achieved at the limit of the expansion phase. When Nernst's entropy is at maximum, the free energy of compression forces is at minimum and the order is that of Planck's type. This situation was in the primary nucleus of universe (at the origin of the universal cycle) and it will be again achieved in the end of the universal cycle.

These entropies always develop in opposition with one another.

We may consider the Planck's entropy as being positive and the Nernst's entropy as being negative.

The thermodynamics of dual tendencies (the symmetrical model) is in perfect agreement with the modern data of science. The classical thermodynamics is valid for the analysis of the processes developing in the physico-chemical systems having non-cosmological dimensions. In these systems, the force of gravitation can be neglected, because its interactions are 40 times smaller than the electromagnetic interactions.

From the point of view of the dual thermodynamics, the heat is not a degraded form of energy, because it can be totally utilized in order to produce the expansion of universe by means of the impulses of photons and elementary particles. During the expansion phase, the heat is transformed in potential gravitational energy. During the compression phase, in proportion as the space will diminish, the gravitational energy will be again transformed in heat.

In conclusion, the classical thermodynamics is a peculiar case of the dual (symmetrical) thermodynamics. The classical thermodynamics is valid for the analysis of the small chemical and physical systems, while the dual thermodynamics is necessary for the interpretation of the cosmos and universe.

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SYMMETRY AND APPROXIMATE STUDY OF MULTIDIMENSIONAL STRUCTURES IN NONLINEAR MEDIA

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One of the problems in a study of stationary and other self-similar structures is the solution of nonlinear elliptic equations. Most of the methods being used in this field are approximate and numerical. We shall consider only the equations that have two or more spatially-homogeneous solutions (backgrounds), and the rest solutions are transitions from one background to another. Under such conditions in one-dimensional case application of method of matched asymptotics provides good results. Its generalization can be proposed for multidimensional problems if the symmetrical solutions are of interest.

We have studied the self-similar solutions of nonlinear heat equation $T_t = \alpha\Delta(T^{\sigma+1}) + T^{\beta}$, that satisfy

$$\alpha\Delta y - 0.5(\beta-\sigma-1)\cdot(\xi,\nabla y^{\alpha}) + y^{\alpha\beta} - y^{\alpha} = 0, \qquad \alpha=(\sigma+1)^{-1}, \ \xi=\mathbf{r}/\psi(t) \qquad (1)$$

In one-dimensional case linearization of (1) and matching the solution of linearized equation with asymptitics of $y(\xi)$ when $\xi\to\infty$ ($y\to 0$) provided good approximate solutions. One of the ways to study the multidimensional problem is to construct approximate solutions. For that, like in the Galerkin method, one can select n rays and perform matching only on them. Then approximate solution is a result of interpolation.

From the physical point of view the solutions possessing several axes of symmetry are of interest. Such symmetric approximate solutions and corresponding numerical solutions y have been found. The approximate solutions proved to describe y

qualitatively, and in some cases quantitatively for the very small number of rays. In fact, it is necessary to know the behaviour of y only along two rays, that are the halves of neighbour symmetry axes (the reflection transfers axis into another one, so the behaviour along all axes will be defined). Thus one can construct approximate solutions using only two one-dimensional functions.

To describe the behaviour of the function along the rays, one must know the solution of linearized equation with explicit dependence on the coordinates on the chosen rays. Such solutions can be efficiently received by means of separation of variables technique. In two-dimensional problem there were two coordinate systems where separation could be done (Cartesian and polar systems). The corresponding two classes of approximate and numerical solutions have been constructed. A number of results have been received for 3-D case and for the system of two elliptic equations.

As a result of this work the following hypothesis has been proposed. It is possible, that for the solutions - transitions from one background to another symmetry makes the number of essential degrres of freedom very small. Similar situation occurs in synergetics when the "order parameters" can be defined due to the presence of dissipation. The separation of variables (symmetry of differential operator) enables the approximate investigation of such systems.

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KINEMATIC SYMMETRY AND "HYBRID" INVARIANTS IN A QUASIMONOCHROMATIC LIGHT BEAM

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1. Introduction

The category of symmetry has long been interpretted and used differently and efficiently by the human thought /1, 2/. Not yet formed as a philosophical conception, symmetry served as one of the indications of harmony, order and beauty. Just because this category was among the first to receive the mathematical status: geometric and arithmetic image.

In full measure the conception of symmetry has been embodied in the group theory. Without limiting the number and picture this theory adequately takes account of and discloses with weightly arguments the essentials of the symmetry conception both in the static and dynamic aspects. Its effectiveness resides in matching up the initial symmetries with the laws of conservation,i.e. with invariants of a transformation group.Though the main invariants were obtained long before the advent of the group theory, their real meaning and fundamentality were understood only in the making of the group-theoretical:no matter how we transform the surroundings by a group of transformations there exists a set of values that do not vary as result of these transformations.

At present, based on a more wide and deep understanding of the mathematical aspects of symmetry, it has become possible to construct a new class of the so-called "hybrid" invariants /3/.

2. Approach

In this construction the first prerequisite is a somewhat different kinematic interpretation of the conception of symmetry. As applied to the discussion of a quasi-monochromatic light beam propagating through a linear optically transparent medium in a remote field, a set of possible states of the system forms a two-dimensional complex space C(2) (spinor space from complex enveloping components of the electric vector). By fixing a certain initial state of the beam (system) one finds all other states by making corresponding linear transformations. This type of symmetry serves to "enumerate and classify" all states of the system, and is "kinematic" in this sense /4/.

Let us now consider how the quasi-monocnromatic radiation passes through a linear medium. Since a light beam at an arbitrary instance of time is identified by the spinor $\zeta(t)$ (or cospinor $\tilde{\zeta}^*(t)$) its state on leaving the medium is determined by the finite fundamental representation D(1/2,0) of the group SL(2, C) or is expressed in terms of the matrix of transformation as $\zeta'^{\mu} = U^{\mu}_{\nu}\zeta^{\nu}$.

A product of spinors and cospinors time-spaced by a value τ constructs two-point time formations in a ray. This representation is a mixed spin-tensor construction S with a valence (1.1), which transforms according to

$$S'^{\mu\dot{\nu}} = U^{\mu}_{\varkappa}\tilde{U}^{\dot{\nu}}_{\dot{\lambda}}S^{\varkappa\dot{\lambda}} \tag{1}$$

Analogously, a set of products of n spinors and m cospinors, taken at N = n + m arbitrary instances of time, is transformed according to the expression

$$S'^{\{\nu_i\}\{\dot{\mu}_j\}} = \prod_{\substack{i=1,n \\ j=1,m}} U^{\nu_i}_{\varkappa_i}(\tilde{U}^{-1})^{\dot{\mu}_j}_{\dot{\lambda}_j} S^{\{\varkappa_i\}\{\dot{\lambda}_j\}} \tag{2}$$

Here, {...} is the sign of the set, ν_i, $\varkappa_i$, $\dot{\mu}_j$, $\dot{\lambda}_j$ are the indexes of the i-spinor and j-cospinor. In the conventional symbolic notation the expression (2) takes the form

$$(S')^{N} = D^{n}(1/2,0)\otimes D^{m}(0.1/2)S^{N}$$

3. Invariants

The expression (2) demands averaging because of the presence of random element in the sequence of spinors. The matrix of transformation may be taken outside the averaging sing <...> in the class of stationary transformations. Thus, the representation (2) specifies essentially the transformation of correlation functions $K(\tau_2, \tau_3, \ldots, \tau_N)$ /3/.

In the general case this is a reducible representation, it splits into unreducible representations

$$\begin{array}{lll} D(n/2,\alpha)\ldots\ldots & D(\beta+1,\alpha) & D(\beta,\alpha) \\ D(n/2,\alpha+1)\ldots\ldots & D(\beta+1,\alpha+1) & D(\beta,\alpha+1) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & \\ D(n/2,m/2)\ldots\ldots & D(\beta+1,m/2) & D(\beta,m/2) \end{array} \qquad \beta=\begin{cases} 0, \text{at } n = 2K \\ 1/2, \text{at } n = 2K+1 \end{cases} \qquad \alpha=\begin{cases} 0, \text{at } m = 2l \\ 1/2, \text{at } m = 2l+1 \end{cases}$$

for the specially symmetrized (by the indexes of formula (2)) correlation functions $K'(\tau_2, \tau_3, \ldots, \tau_N)$ which are combined into invariant ensembles similary to the Dicke model for a system of N two-level molecules and represent coordinates of the vector in the corresponding representation space.

If the light source forms the optical process of the chaotic class, then for the description of correlation constraints it issufficient to have second-order correlation functions. But in the case of a nonthermal, partially coherent, radiator there is a cause to use a systematic description of coherence of higher orders; here, the constructed invariant structures may be substantial.

In the constructed space representations the convolutions of vectors with the coordinates of the enumerated correlation functions K' are simplest invariants, or more complex convolutions of tensors composed of the time-spaced components of like vectors. Thus, the ensemble of possible invariants in the light beam propagating through a linear optically transparent medium has a developed hierarchy not only in the sense of their affiliation to one or other order of the correlation functions, but also from the view point of belonging to one or other unreducible ensemble.

We cite as an example the transformation (1) of correlation functions. Using the formulas

$$K_{11}(\tau) = S^{11}(\tau) - S_0(\tau) + S_1(\tau) \qquad K_{21}(\tau) = S^{21}(\tau) = S_2(\tau) + iS_3(\tau)$$

$$K_{22}(\tau) = S^{22}(\tau) = S_0(\tau) - S_1(\tau) \qquad K_{12}(\tau) = S^{12}(\tau) = S_2(\tau) - iS_3(\tau)$$

the components of these spin - tensors can be matched up with points ($S_\alpha(\tau)$, α = 1,2,3,4) of the four-demensional complex space C(4). We can construct from the components of complex four-vectors $\{S_\alpha(\tau)\}$ an invariant coupling equation for the group

$$\det S^{\mu\dot{\nu}} = S_0^2(\tau) - S_1^2(\tau) - S_2^2(\tau) - S_3^2(\tau)$$

In the case of unitary transformations U this invariant decomposes into

$$I_1(\tau) = S_0(\tau) \qquad I_2(\tau) = S_1^2(\tau) + S_2^2(\tau) + S_3^2(\tau)$$

For spin-tensors $S^{\mu\dot{\nu}}$ with Hemitian matrices (that corresponds to τ = 0 in the product $\mathcal{E}(\tau) \otimes \tilde{\mathcal{E}}^*(\tau)$), the representtation (2) is none other than the Lorentz transformation.

4. Remarks

In closing it may by said that the effect of the medium through which radiation propagates may be interpretted as generalized operation of symmetry in a set of constructed invariants. Here, we have essentially a certain structural symmetry: absence of anysimilarity in the small (for a concrete correlation function) and appearrance of symmetry for polynomial formations of correlation functions grouped in a definite manner.

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Dissymmetrization in biological morphogenesis.

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Dissymmetrization is inevitable at all levels of life organization, in evolution and in ontogenesis. At the molecular and subcellular levels the breaking of primary order of liquid crystal organization (for example, arrangement of linear cytoskeletal structures) establishes a system of point and one-, two-dimensional singularities.

At the cellular level the topological equivalence of a cell (in particular, ovum) to a sphere does not mean that a cell possesses sphere symmetry. The cell surface and the cortical layer bear scalar and vector fields: heterogeneous polarized distribution of membrane and cytoskeletal components, biochemical gradients, directed ion currents and electric fields. All the fields disrupt the isotropic symmetry of spherical surface, as the vector field on a sphere inevitably possesses singularities.

Point singularities on spherical surface determine polarization of a cell. During integral polarization of a cell (chemotaxis or another directed movement, capping of somatic cells, ooplasmic segregation of eggs) subcellular components are displaced with respect to an immovable, stationary point in the process of continuous deformation. The existence of the stationary point also disrupts the initial spherical symmetry of a cell. Morphologically peculiar cellular site (biological pole) appears and functions as a source or sink during wave-like movement of subcellular components in the somatic cells or ooplasmic segregation wave in the eggs.

Breaking of primary symmetry in development is inevitable, it is programmed genetically and epigenetically. But cell environment asymmetry determines the localization of singular point and singular line, i.e., cellular polarization axis. In ovum, the orientation of animal-vegetal axis depends on environmental anisotropy during oogenesis i.e. contacts of the egg with cellular and extracellular structures of the ovary. Contact interrelations (cell-cell, cell-substrate), chemical and physical gradients of the environment may determine the orientation of polarization axis in the somatic cells.

Cell divisions during egg cleavage result in a certain pattern of cell contacts on blastula surface. The cell boundary pattern conforms to topological conservation principle; only five homogeneous patterns on spherical surface are possible. So during embryogenesis the cell contact pattern becomes heterogeneous: one or several cells inevitably acquire the greatest number of contacts. These cells become an initiation centre of morphogenetic cell movements during gastrulation. Following morphogenetic movements of cells and of cell sheets in embryogenesis reinforce the disruption of primary egg symmetry.

THE EMERGENCE OF SYMMETRY PERCEPTION AND THE ASYMMETRY IN MORPHOLOGY IN EARLY MAN (HOMININAE):

AN APPRAISAL OF THE EVIDENCE AND INSIGHTS ATTAINABLE BY A FORMALISTIC APPROACH TO THE CONCEPT OF SYMMETRY

Hermann PROSSINGER

Extended ABSTRACT

I

Misia LANDAU (1984) has shown that most accounts of human evolution are morphologically structured like Russian folk tales. In order to avoid such a literary construct, a more objective approach to human evolution must be presented.

Several approaches can be adopted; they can be considered genuinely scientific if they present consistent data and/or predictions.

The two genera of hominids (bipedal walkers) are presented: *australopithecines* and *hominines*. The currently assumed phylogenetic relationship (DELSON 1987) is presented, along with a discussion of the dating of the fossils upon which such a classification rests. The emergence of symmetry perception in early hominines is intimately interconnected with this classification and therefore a careful presentation is important. Biochemical analysis of present *pongids* (chimpanzee, gorilla, orang utan) is related to fossil evidence to further substantiate the evolutionary rates. (PILBEAM 1987)

Fossil evidence is an empirical structure; hypotheses must complement it. A modern approach to viewing humans in their ecological context is presented. In the last few years, paleoanthropology has moved away from presentations reminiscent of LANDAU's description. (Topics such as "Man the Hunter", "Man the Social Animal", "Women the Collectors of Herbs", "Man the Maker of Tools", "Man the social Animal" are becoming rarer in professional circles.) Instead, one attempts to describe the differences that are consequences of the paleoenvironment (FOLEY 1984,1987). Humans interact with their environment, just as all other animals (!) do: changes in microclimate, fauna, flora, drainage patterns, etc. may necessitate a change in diet or in food acquisition. This description attempts at correlating bipedal evolution with mental evolution: the documentary evidence is tools, tool sites, living floors (ISAAC 1977, M. LEAKEY 1971 1976, etc.). A more formalistic, rigorous approach is perhaps desirable. Too many interrelations between environment and early man's reactions are postulated. It is not clear which of such reactions are independent variables, which are consequences of the independent others. A model analogous to a thermodynamic system with a finite reservoir and non-linear feedback is suggested. Population studies in the animal kingdom have been made (MOUNTFORD 1988, HASSELL 1987) and these predict chaotic behavior. Chaotic behavior is nonpredictable in detail, but not random. It is expected that such behavior will be demonstrable for paleoenvironments as well. It seems that the discrepancies in the fossil evidence may be exhibiting such chaos.

II

The analysis of tool sites in East Africa is necessary for conclusions about human mental abilities to be drawn. The quantity and quality of the tools sites are compared and their outstanding characteristics is noted. OLDUVAI, KOOBI FORA, OLORGESAILLE, KILOMBE, ISMILA are certainly very important, because they permit a mathematical investigation of human mental abilities (cluster analysis, symmetry mapping, etc.). The review of the archeological evidence begins with a presentation of stratigraphy, dating, tool abundance and distribution.

The concept of tool culture for such early sites is a very delicate issue. The concepts implied by the nomenclature are reminiscent of classifications made by French archeologists in the previous century for European tool sites that were much more recent. When the nomenclature is extended back into older times, many personal views expressed by archeologists may mask the objectivity of the evidence. Tool cultures at this early stage are named OLDOWAN, DEVELOPED OLDOWAN and ACHEULEAN. The description of these cultures is not independent of tool fabrication and faunal context of tool use. Many new insights into tool use, tool genesis and tool culture distribution (for example: domed KARARI scrapers) have been gained by empirical evidence gleaned from tool manufacture in present times at the original sites (TOTH 1987). Until recently, the debate centered around the use of the large tools; now it has given way to a reappraisal of the importance of the microflakes which had often been considered debris. However, the morphology of the larger tools (for example: handaxes) have now acquired a new importance because their existence reflects the mental abilities of shaping (due to their symmetry), the mental ability of anticipation (due to their collection at places far from their ultimate use), and the knowledge of manufacture (due to the statistics of their material composition).

The classical discussion of whether the OLDOWAN was possible before *man* (i.e. beyond 2 million years ago) is compared with the discussion of whether homo habilis is a valid taxon (JOHANSON et al. 1987). The ACHEULEAN appears quite suddenly in the archeological record; dating the transition and distribution evidence are discussed. The interpretation of the ACHEULEAN as a culture achieved by homo erectus is attractive. The reasons for this preference to other explanations is presented and critically reviewed.

III

The controversy whether the ACHEULEAN and DEVELOPED OLDOWAN coexisted at OLDUVAI is the crucial stepping stone at comprehending the evolution of man's mental abilities. The ACHEULEAN requires much more advanced cognitive abilities. ACHEULEAN handaxes exhibit remarkable symmetries, the axial symmetry being the most conspicuous. The discussion of modern formalistic symmetry concepts is presented: it is an extension of metric analysis of tool artifacts by GOWLETT (1984,1988). It follows that the emergence of the ACHEULEAN and the emergence of symmetry perception are inextricably linked. The emergence of symmetry perception therefore becomes historically datable.

IV

Modern particle physics has coined the term "*symmetry breaking*". The expression is somewhat misleading. It actually means a symmetry present at a higher energy state that is not present at a lower energy state. An analogy for anthropology is developed: mirror symmetries exist in general skull morphology, a detailed metric analysis reveals a "*broken symmetry*" (PROSSINGER, current research). The "*symmetry breaking*" in skull morphology is correlated with the ***asymmetry*** of brain functions. Paleoanthropology has analyzed brain asymmetries in *hominines*, *australopithecines* and *pongids* (HOLLOWAY 1982) and attempted to correlate them with the emergence of mental abilities (HOLLOWAY 1969).

V

We can now perceive how a discussion of ***symmetry*** and ***asymmetry*** in paleoanthropology offers a well-rounded perception of ourselves: the detection of the emergence of symmetry perception enables us to conclude the emergence of other mental abilities that make homo sapiens possible. It is to be noted that the formal concepts of symmetry necessary for such a description are different from the mirror symmetry perceived by early hominines - thereby satisfying FOLEY's postulate of describing our ancestors in ways adequately different from ways of describing ourselves.

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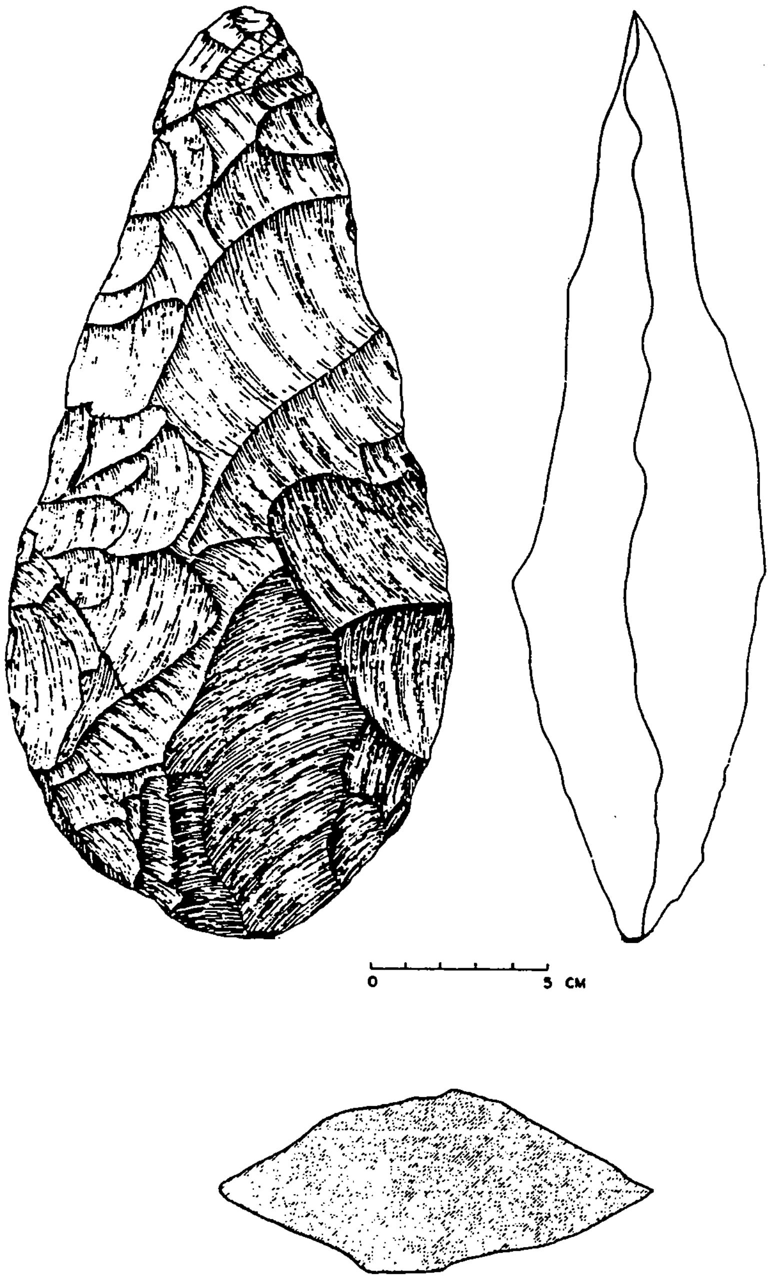


HANDAX from ISMILA / TANZANIA

Music of Colour and Form, Music of Moving Images

by Yu. Pukhnachev

"Plastic music", "music for the eye", "music of colour and form" are notions that have some history. They represent a new art operating with visible melodies thought of as the analogues of the sounding ones.

It was Aristotle who wrote that colours due to their harmony can be related like musical chords. As far back as 1734 a French monk L.Castel created a colour clavecin based on simple correspondence between the notes of the scale and the colours of the spectrum.

In 1910 A.Scryabin finishes his symphony "Prometheus". The upper line of the score is marked "Luce" meanining the part of colour. Cherishing the dream to combine music and colour the composer begins a new symphony but dies prematurely in 1915 and the work is unfinished.

In the recent years colour music becomes more popular and developed. T.Wilfred (USA), A.Laszlo (Hungary), N.Cheffer (France), N.Mc Lareen (Canada) along with the Soviet representatives K.Leontyev, M.Malkov, B.Galeyev, Yu.Pravdyuk, S.Zorin can be named among those who contributed to this development.

New books are published on colour music and practically each of them contains an unhappy statement on the absence of beautiful pieces of the art of musical colour. No masterpieces have appeared yet.

Where do new possibilities lie for overcoming the difficulties the colour music experiences at the present time? One such possibility is to find new development trends based on wide-sense interpretations of the notion of colour music.

Let us draw on the screen two coordinate axes. Our screen will be the set of points of the coordinate plane each point having an arbitrary colour and brightness. Any disagreement in colour between two neighbouring points is an element of form. This statement can easily be proved by contradiction: if the colour of the screen does not change from point to point we will see no image.

So we add the third time axis to the two we already have. To be at home with the situation we arrange the images as a vertical chain of small frames appearing on the screen at consequtive moments of time. We make a pile of those frames and then imagine the pile to stick into a compact bar.

A rising sun against the blue sky. On a film the disc of the rising sun will appear at a still greater height in each of the conse'utive frames. If we then pile the frames up and stick them into a compact bar the discs will make an oblique cylinder inside the blue contents of the bar.

How is the movement represented in such models? Any disproportion in colour between two points arranged one above the other on the time axis is an element of movement. We again can prove this by contradiction: there is no movement on the screen if the colour of the screen is unchanged with time.

Such a treatment of movement is a universal generalization taking into account physical displacement, brightness and contrast variations, i.e. all the changes that take place on the screen. We assume, of course, that every point of the three-dimensional space-time model has an arbitrary colour and brightness.

We now take any other moving object and register the phases of its movement in a series of consequtive frames. Piling up the resulting flat images layer by layer we will obtain a three-dimensional body. We will call thus obtained bodies the images of movement.

When movement is treated in a generalized way rather than as a simple physical displacement general-type images are obtained (multiply-connected, with blurred boundaries, etc.).

We now define the music of colour and form, the art of plastic melodies as the skill of processing and montage of the images of movement.

What technical means can be used to create and reproduce plastic melodies?

We begin with the modern equipment for colour music. Two principal directions can be singled out.

First is to create images of colour music in any possible way and then fix them on a film by any mechanical device. Numerous effects can be achieved but improvisation while reproducing the melody is almost out of the question.

Second is to reproduce the melody directly by the player. The whole process is built on improvisation but the number of devices reacting quickly enough to the tempo of the player's music is rather limited (it can be a flexible film reflecting or refracting the light incident on the screen).

A new approach based on the understanding of movement images will, in the author's opinion, strengthen the merits of the two directions mentioned above and diminish their demerits.

Let an image of a certain movement obtained by any means used in colour music be shown on a screen. Any light-sensitive device can read out the moving image and load it into a computer memory, the way a TV image is recorded on a video. Now the computer memory stores the information on the brightness and colour of every point of the screen in every point of the screen in every moment of time and thus stores the model, or image, of the movement reproduced on the screen.

Certain regions of the three-dimensional model will correspond to certain phases of the movement. We separate out the key movement phases and the corresponding key regions of the image. We then take in every such region a characteristic point whose shift and change of colour would cause a deformation and change of colour the whole region according to a certain program.

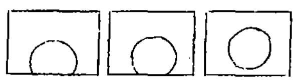
We must now reproduce on the screen the moving image stored. Let the computer, in accordance with the information it has on the image, control the colouring of every point on the screen in every moment of time while the player sets the shifts and changes of colour for characteristic points (and hence for key regions). Then in reproducing the image of movement we will get the same degree of improvisation the pianist enjoys when he hastens or slows down his tapping of the keys while playing from a score.

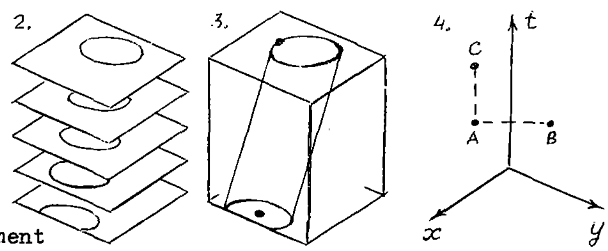
For an example we again turn to the oblique cylinder representing the movement of a rising sun. In such a simple case the configuration of the whole image (i.e. the diameter, height and inclination of the cylinder can be given by only two points. This corresponds to the mathematical description of the cylinder containing six parameters).

Let us imagine the control panel of a plastic music performer the music being considered as a body of moving images. The panel consists of a few screens allowing a lead and demonstrating what will happen on the auditorium screen in 8, 16, 24, 32 and more clock times. In order to correct in advance a certain movement image appearing on a leading screen the performer simultaneously pushes the pedal under the screen and the panel button that addresses the program processing the corresponding image. The image is reproduced in a quick tempo on the right large screen of the panel. In the key phases of the image evolution the performer strikes the key elements of the images on that screen with the fingers of his right hand. The screen is provided with sensing elements and a new position of the characteristic points determining the configuration of the whole image is defined. The whole image is reconstructed by the processing program accordingly. Simultaneously striking certain points of the left large screen (indication colour and brightness) with his left hand, the player determines the new colour of the key regions under reconstruction. If there are no correcting strikes the moving image is reproduced as stored in the computer.

If the plastic melody contains a few movement images their reproduction should better be performed by separate players each having his own panel (the way different musicians of a symphony orchestra are entrusted with different musical instruments). A conductor is responsible for the synthesis of all the images into a single plastic melody.

A variety of melodic trends of plastic music makes the performers play in harmony or even makes them compete when improvising (the competition being the more fascinating since its rules are defined by the conductor and are not entirely clear to the competitors). Such music is related to the traditional symphonic music, to a sports competition and to a scientific experiment. Developing and perfecting it could become a rich and colourful art.





on the fig.4:
colourA $\neq$ colourB: form
colourA $\neq$ colourC: movement

Separate frames showing sunrise (1) are piled up (2) and stuck into a compact cylinder (3). The resulting oblique cylinder is the image of the sun movement. Its dimensions and inclination can be given by only two characteristic points (marked fat). It is convenient to view movement images as three-dimensional bodies in space the x- and y-coordinates lying in the plane of the screen with time as the third coordinate (4).

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PRINCIPLES OF SIMMETRY IN TOWN PLANNING

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The principles of symmetry in the organization of natural landscapes, humanized by man, we can differ at the earliest stages of the creation of human settlements (25000 years ago) and later in the urban space (8000-7000 years B C). In the majority of plans of these settlements we can find a center and an axis of symmetry.

In the Mesopotamian and Persian architecture (3100-2700 years B C) buildings had as a rule rectangular plans. The same we can say in relation to the planning of settlements in the Ancient Egypt, where we can find a collection of symmetric fragments in the majority of town plans.

In the Ancient Greek architecture Plato and Aristotile defined an ideal city planning system as a regular and correspondingly a symmetrical. After Plato the form of a plan of the Ideal City should be a circle with concentric districts (one symmetry axis of an infinite order). After Vitruvi (Caesar's architector) the Ideal City should have an octagonal plan (one axis of symmetry of the 8th order).

Later in the Middle Ages and the Renaissance the plans of the Ideal City had strictly symmetrical structures of a rectangular, octagonal or other polyhedral shape, which could be inserted into a circle or a sphere.

In general the planning of modern cities is based upon the principles of symmetry. While formalizing the processes of town planning in accordance with the modern theory of symmetry we can imagine them in a Cartesian system of rectangular coordinates. Let the axis OX, OY and OZ be the coordinate system axis. If we place any build-up area of a city into the given system of coordinates (we consider a plane city) we can neglect the coordinate OZ. The change of coordinates in the vertical direction OZ in the urban space is considerably smaller than in the horizontal directions. As a result of consideration we recieve one plane (XOY) where the transport ways are symmetry bands together with the build-up areas. They are represented as symmetrical polygonal nets. These are the so-called plane isogones, which can be complete and incomplete. The whole system of town planning is represented as a one-sided plane continuum Thus we can distinguish the following elements of symmetry in the town planning: symmetry bands, symmetry polyhedrons, symmetry tesselations.

In the reality all the symmetrical ideal planning systems are transformed, in general, into a non-symmetrical planning system, but with the symmetry fragments of the build-up.

The main factors, which transform the ideal symmetric systems into a real architectural medium, are: natural landscape forms, variation of the seasonal temperatures, dominating winds, solar radiation and gravity.

Actually, on the one hand, the influence factors destroy partly the ideal symmetric planning system, on the other hand, the same factors promote to create the symmetric forms of the prehistorical intuitive settlement planning. Obviously, many things depend on the balance of influence factors on the planning structures. The topographical forms as mountain heights, dimensions of river valleys and the forms of the coastal planes transform in the greater extent the symmetric systems of planning. We can consider an example - the project by Leonardo da Vinci to straighten the Arno river bed in order to make it an axis of symmetry of the regular polygonal plan of Florence.

As a result of our study we propose the following scheme of the process description going from an ideal to real project of town build-up:

Plan of the town with symmetrical elements → Man influence factors (like a non-symmetrical topography → Actual town build-up with symmetrical fragments

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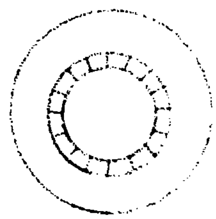
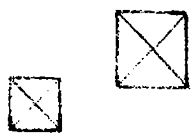
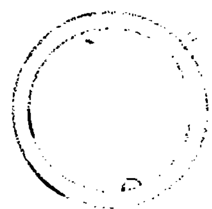
Plan of typical paleolite settlement



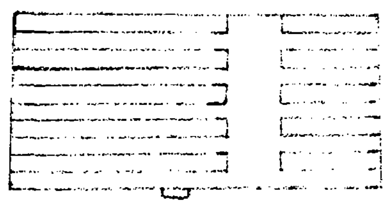
Horesm castle settlements



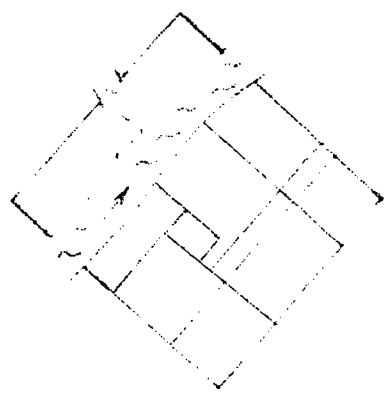
Egyptian pyramids



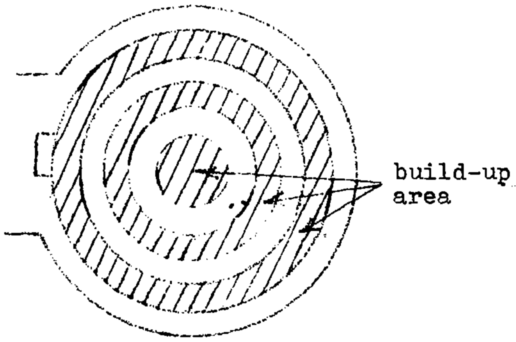
Stownhendge



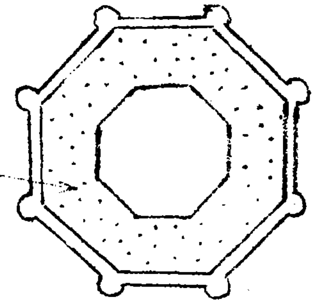
Egypt: Kahun (2000 years B C) fragments of settlement



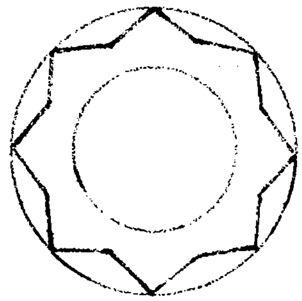
Mesopotamia: Borsippa plan



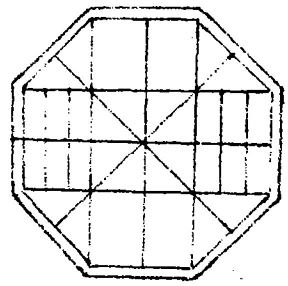
Plato's plan



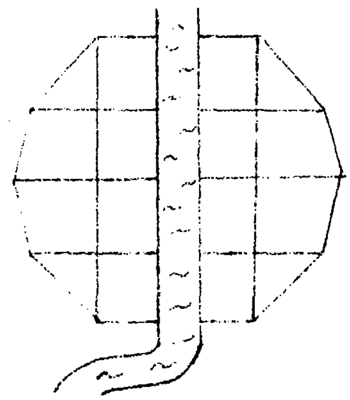
Vitruvi's plan



Averlino's plan



Vasari's plan



Leonardo da Vinci's proposal

BALANCE AND ITS SIGNIFICANCE IN MY DRAWINGS, PAINTINGS, AND TACTUAL EMBOSSINGS FOR THE BLIND

RÉ, Paul Bartlett

10533 Sierra Bonita Ave. NE, Albuquerque, New Mexico 87111 U.S.A.

Slide Lecture: 40 slides: 60-90 min. delivery time

In Fig. 1 are shown the basic shapes used in 18 of the artist's drawings and paintings. In the completed artwork, the regions of the basic shape are subtly shaded or colored. The basic shapes have the following unifying geometrical properties. First, they are all derived from 1 or more closed curves, with possibly 1 or more line segments removed. Second, all line intersections are of 3 types: X-intersections (4 branches formed by 1 continuous line crossing over another); T-intersections (3 branches formed by removing one branch from an X-intersection); or V-intersections (2 branches formed by removing 2 adjacent branches from an X-intersection). Eg, basic shape IV-3 has 2 X-intersections; IV-14 has 4 T-intersections; and IV-15 has 4 V-intersections. (Ré, 1980) All three basic shapes are derived from one closed curve. The derivation of basic shapes from closed curves is significant; it symbolizes the interconnectedness of everything in existence. In sociology, it corresponds to the peaceful, constructive interaction of human beings. In ecology, it represents the dynamical interdependence of all species, including man. It expresses the hope that mankind use thoughtfully the physical and spiritual bounties of nature. The key to this is <u>balance</u>, and balance is a dominant theme of the artist's work. (Ré, 1982) Using a slide of each work represented in Fig. 1, he discusses their balance which is often based upon strict or approximate rotational, reflective, or translational symmetry. Eg, both the basic shapes and completed drawings, IV-3: "Cavern" and IV-14 "Yin Yang", have 2-fold rotational symmetry. And basic shape IV-10: "Mountain" has reflective symmetry about the vertical centerline, but is asymmetrically shaded. Ie, the shading of the V-intersection in the center of the work exhibits a <u>tonal inversion</u>: to the left of the intersection, the shading is darker below the basic shape boundary, but to the right of the intersection, the shading is darker above the boundary. In Fig. 2, this is designated as shading type, V_a. The other type of V shading, V_b, is symmetric. In considering all possible intersection shading types, the artist requires that at each branch of an intersection, one side of the boundary be shaded darker than the other side in order to delineate the boundary. Thus, at V-, T-, and X-intersections, there are respectively, 4, 8, and 16 possible shading types. Fig. 2 groups together in blocks those kinds that are equivalent if the angle between successive branches is changed and/or if a mirror reversal of the whole intersection diagram is made. This reduces the total number of essential shading types for all 3 kinds of intersections to 10. Note that shading types V_b, X_d and X_c have reflective symmetry: X_a has 4-fold rotational symmetry; and all other types are asymmetric. (Ré, 1981, P. 106-7)

In Fig. 1, basic shapes III-4: "Madonna", IV-8: "Inner Joy", III-14: "Blossom", III-16: "Serenity", IV-4: "Longhorn", and III-22: "Embrace" are derived from closed curves that have reflective symmetry about the vertical centerline. In each case, the removal of 1 or more line segments introduces an asymmetry that is crucially important to the work. This balanced asymmetry, or near-symmetry, the artist finds very appealing (much more so than strict reflective symmetry). And shading or coloring can introduce further asymmetry. Eg, Fig. 3 shows the basic shapes employed in 20 paintings with indications of differently colored areas. These paintings contain an 'island' in 1, 2, or 3 colors surrounded by a 'sea' of white. Some islands have an internal lake of white, as in III-4: "Madonna" where the 'occluded arm' is violet, and the 'occluding arm' is royal blue. To maintain a balance, the colors chosen for the 2 'arms' needed to be fairly close in hue, yet sufficiently different so that the boundaries between the two regions were clearly defined. The colors chosen also reinforce the meditative feeling of the work. In other paintings, colors of widely different hues were used. Eg, III-8: "Molecule" employs vermilion for the 2 lobes, and royal blue for the 'owl face'. In the Ostwald color system, these two colors are 7 1/2 standard steps apart in hue. Color triads and complementary colors are 8 and 12 standard steps apart in this system. (Ré, 1981, p. 107-110) Note that the two closed curves from which the basic shape of III-8 is derived both have a reflective symmetry, one about an axis from the lower left to upper right of the work, and the other about an axis from lower right to upper left (approximately) of the work.

Some works in Fig. 1 are quite asymmetric, yet are still very balanced; eg, the painting, III-21: "Listening Ears", and the pencil drawing, IV-7: "Goatscape". IV-7 also has the name, "Swan". During the lecture, the artist will encourage the audience to give alternative names for the works. This enjoyable exercise is based on one of the beauties of abstract art, namely, its multiplicity of referents. When one entity can evoke many ideas, this is one step in unifying diverse cultures. And these different gestalts entail a kind of symmetry. At the end of the lecture, the artist will present a few slides of recent works. One of these, V-19: "Three Swans", has 3-fold rotational symmetry and also has the names "Vortex" and "According to Knot" (note the echo of Gordian knot). The artist will also rotate some of the slides in order to point out other symmetry aspects, such as convex outward and convex inward gestalts.

Since 1979, the artist has translated the basic shapes of his visual art into raised line embossings. These are not only pleasing visually, but can be appreciated tactually by blind people. Using these embossings, he has made both an art book and traveling exhibit of TOUCHABLE ART. (Ré, 1983) The making of embossed translations (a significant word) can be considered a kind of symmetry operation. The artist will illustrate some of the basic shapes in Fig. 1 with both the visual work and the embossed translation.

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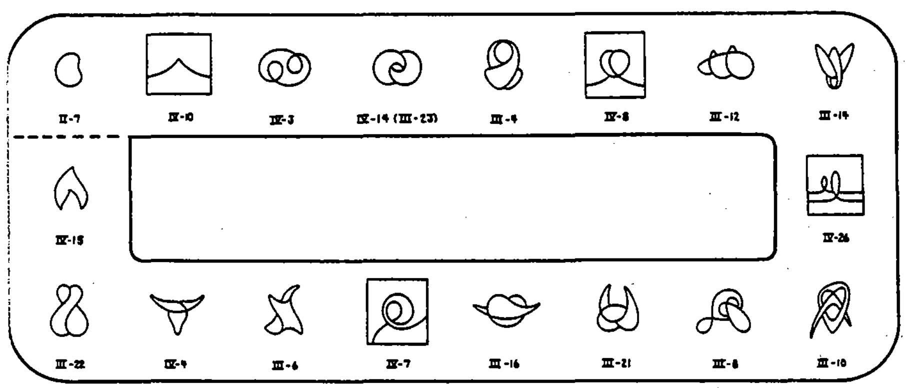


Fig. 1. Progression of Basic Shapes in TOUCHABLE ART: A Traveling Exhibit and Book for the Blind and Sighted.

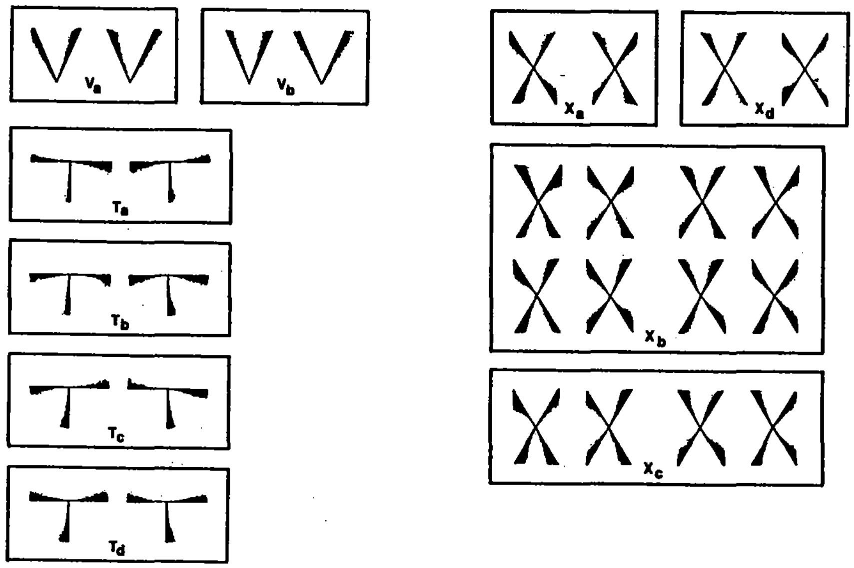
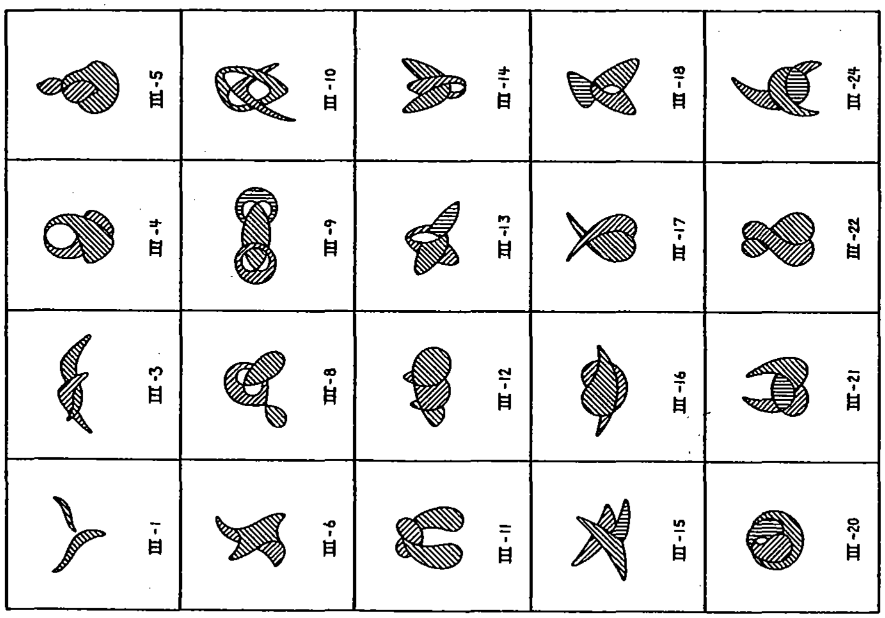


Fig. 2. Diagram of Intersection Shading Types in Pencil Drawings.



(← UP) Fig. 3. Basic Shapes in Paintings with Indication of Differently Colored Areas.

REGULAR SPHERICAL GRIDS OBTAINED WITH THE METHOD OF DEFORMATION OF A SECONDARY NETWORK.

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Geodesic domes, worked out for the first time by B. Fuller, are the form often used to cover wide spanned structures. Geometric parameters of triangular spherical grids approximating the surface of sphere play a significant role in geodesic domes. The grids of this type are determined by means of various methods (T.Tarnai, 1987). Triangular spherical grids should describe as little as possible the curvature of this surface and the lenghts of its individual segments should not be too differentiated while the plane angles included between two adjacent segments should assume the values from the smallest range of their possible but necessary changes. In order that the grid might be determined as a regular spherical grid it should consist of the lowest number of segments of different lenghts. The coefficient η , being the ratio of the longest segment to the shortest segment of a given spherical grid decides about the regularity degree of this grid.

The pictures of the changes of network lenghts brought about by the nodal interstice in metal and interstitial atom (T.Penkala,1977) were the inspiration to work out the method of deformation of a secondary network. The essence of this method consists in the appropriate deformation of a triangular grid in the base plane only to obtain a regular triangular spherical grid after having it re-projected (J.Rebielak,1988). This method employs the properties of the central and orthogonal projections to the plane. Figure 1 presents the scheme of determination of

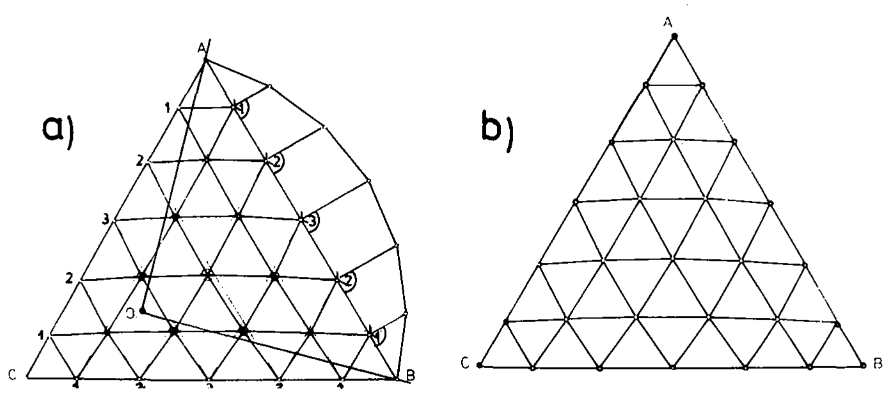

Figure 1.

the deformed secondary network for the principles of orthogonal projection to the plane. The points of division of the great circle to the n-number of equal segments after their orthogonal projection determine a new arrangement of side edges of the triangle ABC. Linking the points of the same ordinal number we determine the inner triangular fields of which the centres of gravity determine the position of nodes of the deformed flat triangular grid. After having orthogonally projected these nodes we get the nodes of regular spherical triangular grids. Figure 2 presents dimention relations of spherical grid formed in the way mentioned above for the division of side edges to n=6 equal segments. Through "a" was defined a length of edge of equilateral triangle ABC. The dimention relations of the spherical grid were determined for the same outer conditions, that is the same distance of the sphere centre from the base plane Z=0,75a , sphere radius R=1,0a and the same dence division of side edges (n=6) using the principles of central projection in the method of deformation of secondary network ; these relations have been shown on the figure 3.

Different values of the coefficient η for the both presented grids can be noticed apart from the different arrangement of the segments of the extreme lengths but at the same density of a given spherical grid (n). Figure 4 presents the course of the changes of the coefficient η values according to the division degree of the spherical triangle sides (n) for two versions of the method of deformation of a secondary network. The curve 1 shows the course of the changes of the coefficient η values for the spherical grids determined by means of orthogonal projection to the plane. The curve 2 shows the increase of the coefficient η values for the grids determined by using the principles of the central projection.

The analysis of the graph on the figure 4 reveals that very dence grids determined by central projection are slightly more regular than spherical grids generated by the orthogonal projection to the plane. The triangular spherical grids generated by the orthogonal projection to the plane are characterized with a favourable degree of regularity for less dence divisions of side edges.

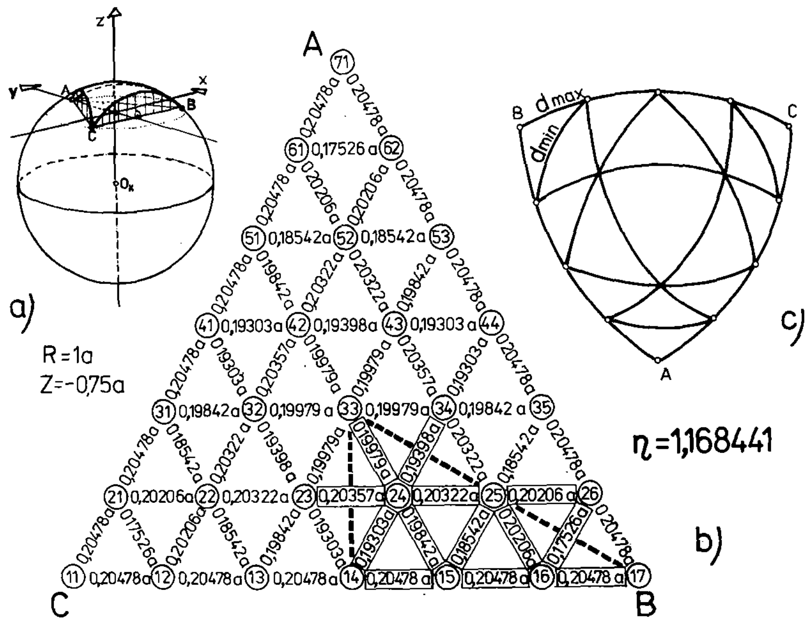

Figure 2.

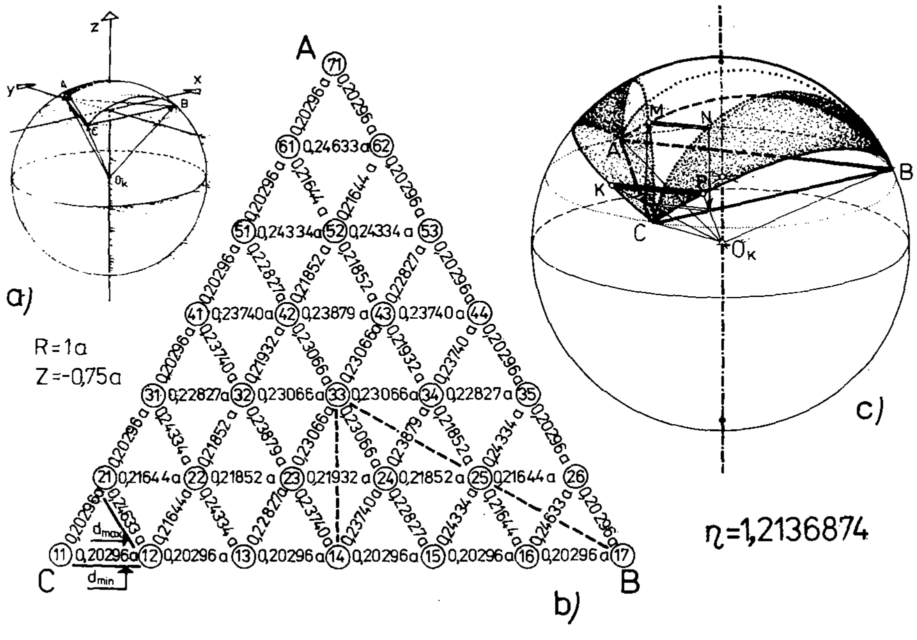

Figure 3.

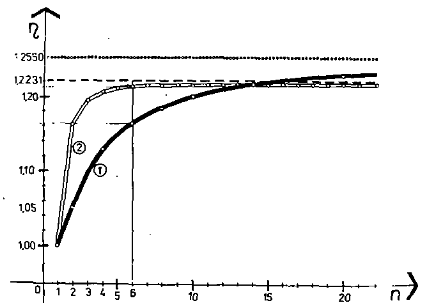

Figure 4.

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Symmetric bracing of one-story buildings with cables and asymmetric bracing of one-story buildings with rods

(Extended abstract)

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Summary: The minimal systems of diagonal cables or rods which make a one-story building infinitesimally rigid were first studied by Bolker and Crapo (1977-79). Some of the remaining open problems were settled in the last four years (Chakravarty, Holman, McGuinness, Schwärtzler and the author). As a byproduct of these investigations we found that if the cables of such a minimal system are all parallel then the patterns of these cables are highly symmetric. This result seems to be somewhat surprising since asymmetric, rather than symmetric patterns arise in case of rods. These observations will be presented. The main tool is graph theory and network flow techniques.

Consider a 1-story building, with the vertical bars fixed to the earth via joints. If each of the four external vertical walls consists of a diagonal, the four corners of the roof become fixed. Hence questions related to the infinitesimal rigidity of a one-story building are reduced (Bolker and Crapo, 1977) to those related to the infinitesimal rigidity of a 2-dimensional square grid of size $k \times \ell$ where the corners are pinned down. Then the minimum number of necessary diagonal rods for infinitesimal rigidity was proved to be $k+\ell-2$ (Bolker and Crapo, 1977) and

Theorem A: The minimum systems correspond to asymmetric 2-component forests (Crapo, 1977).

(In what follows, every graph will be the subgraph of the complete bipartite graph $K_{k,\ell}$; the twwo subsets of the vertex set of $K_{k,\ell}$ will be denoted by A and B with respective cardinalities k and ℓ. A 2-component forest with vertex sets V_1, V_2 of the components is called asymmetric if $|V_i \cap A| \neq \frac{k}{\ell} \cdot |V_i \cap B|$.)

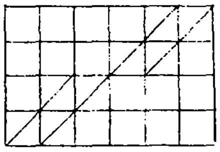
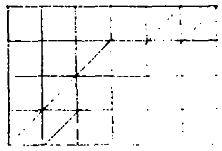
If we wish to use diagonal cables for infinitesimal rigidity, the minimum number of these cables was proved to be

$$\begin{cases} 0 & \text{if } k=\ell=1, \\ 4 & \text{if } k=\ell=2, \\ k+\ell-1 & \text{otherwise} \end{cases}$$

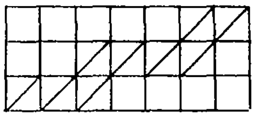
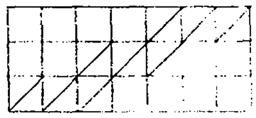
(Chakravarty et al, 1986) and the minimum systems were characterized in a somewhat more technical way only recently (Recski and Schwärtzler, 1989). In a previous stage of our investigations we found the following partial result (Recski, 1988):

Theorem B: Suppose that all the diagonal cables are parallel. Then the system makes the grid infinitesimally rigid if and only if $|N(X)| > \frac{l}{k}|X|$ holds for every proper subset X of A, where N(X) denotes the set of those vertices of B which are adjacent to at least one vertex of X.

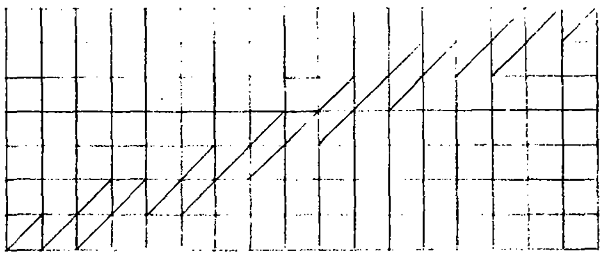
In order to illustrate Theorem A, consider the following two systems. The first one has an infinitesimal motion while the second one is infinitesimally rigid.

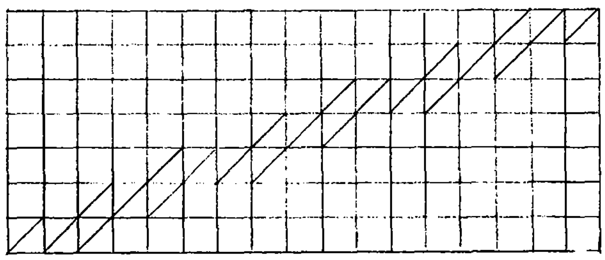
 

The next two systems illustrate Theorem B. Again, the first system has an infinitesimal motion while the second one is infinitesimally rigid.

The first pair of illustrations is not surprising at all; plenty of examples are known to justify the vague statement that "The less symmetric bar-and-joint frameworks are the more rigid". However, the second pair suggests another statement that "In case of tensegrity frameworks symmetry may be advantageous for minimum rigidity". Of course, symmetry alone is not enough; only one of the following two systems is infinitesimally rigid (which one?).





A possible explanation could be that cables prescribe inequalities, rather than equalities, among the shears of the rows and columns, hence each relation $a \leq b$ must be accompanied by a relation $a \geq b$ as well.

Peter Revers:

Aspects of the definition of Symmetry in Taoistic thoughts.

After the physicist Niels Bohr had been knighted on the 17th of October 1947, he chose a highly remarkable coat of arms: it contained the chinese symbol "T'ai-chi", representing the complementary relationship between the opposites Yin and Yang. In the inscription above the symbol one can read "contraria sunt complementaria": Contraries are complementaries": an idea which has indeed influenced the line of thought of natural science. The "T'ai-chi"-symbol itself is based on the principle of rotational symmetry. The comprehension of symmetry in taoistic philosophy though has neither a connection to the mirrored symmetrical structures which we are familiar with, nor can it be ascribed to the european definition of symmetry.

First of all one must clarify that a prevalant definition of symmetry, or even a similar term as it prevails since the 18th century, does not exist in the thoughts of Taoism. Nevertheless if I review aspects of the definition of symmetry this can only be done under the assumption that the definition in its general meaning as proportion or correct relation is understood and removed from all constrictions (especially concrete mathematical deck operations).

A central moment for the tradition in taoistic philosophy is formed from the "congruous effect (tiao or tiao-ho) of Yin and Yang"[1] adjusting harmony. Yin and Yang characterize therefore not only two contrary forms of attributes, nor can they be reduced to the meaning of one opposite or one opposition. Of equal importance is the idea of flowing movement, of permanent alternation, that can be experienced in the figure of rotational symmetry. Thus, the chinese philosopher Chou Tun-Yi (1017-73) was of the opinion in his treatise "T'ai-chi T'u": "The Supreme Ultimate through movement ... produces the Yang. This movement, having reached its limit, is followed by quiescense... and by quiescence it produces the yin. When quiesence has reached its limit, there is a return to movement. Thus movement and quiescence in alternation become the source of the other."[2]

Accordingly a dualistic interpretation of the principles "Symmetry" and "Asymmetry" cannot be brought to accordance with taoistic reflection. One further important aspect for the phenomenon "Symmetry" is explained by the chinese philosopher Chang Tsai (1020-77) "The Great Harmony is known as the Tao. Because in it there are interacting qualities of floating, and sinking, rising and falling, movement and quiescence, therefore there are engendered in it the beginnings of emanating forces, which agitate each other, overcome or are overcome by one another and contract or expand, one with relation to the other..."[3]

Although the strongpoint of the philosophical concept of Chang Tsai is the explanation of the ether, it is nonetheless the emanations of Yin and Yang that form the concrete outward shape of the physical universe. Through this Yin and Yang become categories which take on immediate structural function. Following these indications of taoistic philosophy the tangibility of symmetry thinking can be demonstrated especially in regard to east-asiatic music. Moreover it will

show that on the one hand our understanding of symmetry has found analogous structures, on the other hand phenomena that appear assymetrical according to the perspective of Taoistic thought, through which relations to the principle "symmetry" become visible, will be discussed.
An adequate understanding of symmetry from the angle of Taoistic thought allows a very suitable realization by means of music, because in the dimension "Time" as constitutive element of all changes and all transformations the principle of continual flowing is more likely to be conspicuously expressed.
Symmetry is therefore not confined to an abstract constructive dimension. Furthermore it acts as an equalizer between contrarities (e.g. rest in itself and forward force, static and dynamic treatment) and to that extent a correspondence to the harmonizing effect of the forces of Yin and Yang.

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1 Marcel Granet, Das chinesische Denken, München 1981², s.
2 zit. nach: Fung Yu-Lan, A History of Chinese Philosophy, Vol.II, Princeton 1952, s. 453
3 a.a.O., S. 479

COLOURING REGULAR MAPS J. F. Rigby

This talk will be illustrated in colour with slides and OHP transparencies.

The five regular polyhedra can be blown up like balloons to form five regular maps on a sphere. For instance, the icosahedron is shown in Figure 1, and the icosahedral map (Figure 2) is made up of 20 spherical triangles meeting at 12 vertices; the map has 30 edges. The edges of a map do not need to be straight, and the faces do not need to be the same size; the regularity stems from the fact that each face has the same number of edges (three in the case of the icosahedral map) and each vertex is the meet of the same number of faces (five in the case of the icosahedral map), but we usually try to illustrate a regular map with as much visual regularity as possible. The icoshedron itself, with its 20 plane faces, illustrates the map just as well as the blown-up version on the sphere.

Regular maps on more complicated surfaces are more interesting; they can no longer be illustrated with their faces all of the same size. A famous simple example is the map on a torus with seven hexagonal faces, each face adjacent to all the others: take the rectangle in Figure 3 and glue the edge AE to DH thus forming a cylinder; then glue the two ends of the cylinder together, imagining it to be made of elastic material, so that A, B, C are glued to E, F, G. The resulting torus has a map of seven hexagonal faces meeting by threes at 14 vertices, and the map has 21 edges.

Another way of illustrating this map is by means of Figure 4: instead of cutting out and gluing the basic parallelogram (no longer a rectangle, but the material is elastic) we simply *identify hexagons labelled with the same number*. Think of the numbers as representing seven different colours; we then have, in Figure 4, a regular or "perfect" colouring (Grünbaum and Shephard 1987) of the hexagonal tessellation in a plane, which leads to the map on the torus when we identify hexagons of the same colour.

A word about regularity: if we apply the gluing process to the rectangle in Figure 5, we obtain a regular map of nine quadrangles (or squares) on a torus, but the map obtained from Figure 6 is not regular even though four squares meet at each vertex. The reason for this is that if we stand on square A and take a walk southwards we pass over two other squares (E and J) before arriving back at A; but if we walk eastwards we pass over three other squares (B, C and D) before arriving back at A. We shall not give a rigorous definition of regularity here (see Coxeter and Moser 1980), but this example shows that there is more to the idea than we mentioned earlier.

Figure 7 shows a more complicated regular map: Coxeter's map {4,6|3}. Think of the figure as a solid object; the ten bars have triangular cross-section, and the bars meet by fours in the manner shown in Figure 8. The surface of the solid consists of 30 quadrangles meeting by sixes at 20 vertices, and there are 60 edges (Bokowski and Wills 1988). Can we illustrate this, as we did the hexagonal map on the torus, using a tessellation?

Figure 9 shows a regular tessellation in the hyperbolic plane, composed of regular quadrangles meeting six at each vertex. The hyperbolic plane may seem strange and alarming to those unfamiliar with it; suffice it to say that we can only illustrate it in a distorted manner, using the inside of a circle to depict the entire infinite hyperbolic plane. All the quadrangles in Figure 9 are regular and all have the same size in "hyperbolic reality", but in our picture they appear to get smaller as we approach the boundary. Now, imagine the 30 faces of Figure 7 to be labelled with the numbers 1 - 30; label the faces of Figure 9 so that the numbered faces fit together in the same way as in Figure 7 (compare Figure 8 with Figure 9 where only a few numbers have been inserted). Then the quadrangles of Figure 9 will be labelled with thirty numbers, or coloured with thirty colours, and when we identify quadrangles with the same colour we are led back to the regular map of Figure 7.

Thirty colours is a large number to use. Figure 10 shows a perfect colouring of the same tessellation in only five colours. If we examine it closely we find that the black tiles (for instance) are surrounded in six different ways by tiles of other colours. We say that two tiles are *equivalent* if they are surrounded in the same way; the colouring then contains 30 different types of inequivalent tile corresponding to the 30 numbers in Figure 9, and if we identify equivalent tiles we are led back once again to the regular map of Figure 7.

There are many interesting regular maps; most of them cannot be represented by plane faces in 3-space as in Figure 7. If a regular map has N faces, we can obtain it from a perfect colouring of a regular tessellation with N colours, but we can often reduce the number of colours to a divisor of N in the way just described. For instance, there is a colouring of the tessellation {3,8} in only ten colours that leads to a regular map with 604800 faces. The number of colours can sometimes be reduced even further if we use instead a semi-regular map.

Of more mathematical interest is the fact that these colourings can sometimes be used to investigate the symmetry group of a map. For instance, the colouring in five colours leading to the map {4,6|3} can be used to show that its symmetry group is $C_2 \times S_5$, and from the colouring in Figure 1 we can show that the symmetry group of the icosahedral map is $C_2 \times A_5$. There is a perfect colouring of {3,7} in seven colours, which leads to Klein's map $\{3,7\}_8$ and can be used to show that the symmetry group of this map is PGL(2,7), the group of collineations and correllations of the projective plane of seven points and lines.

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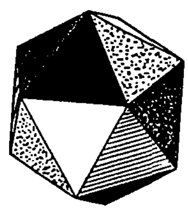


Figure 1

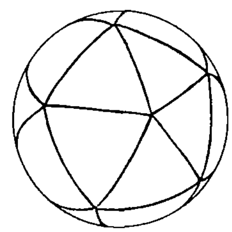
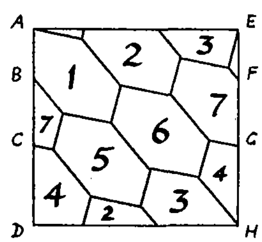


Figure 2



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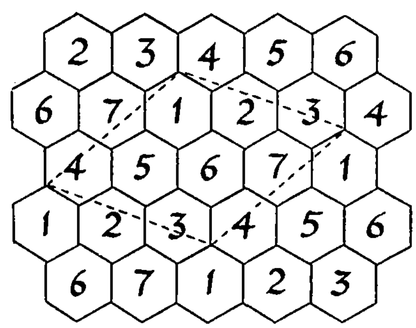

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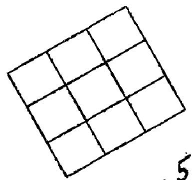

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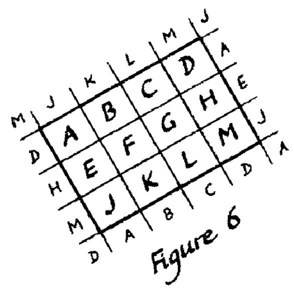

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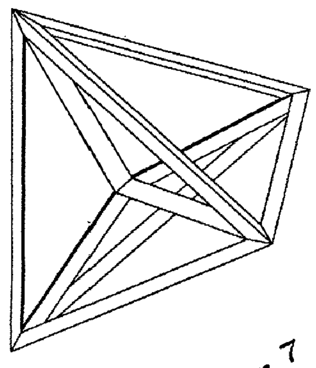

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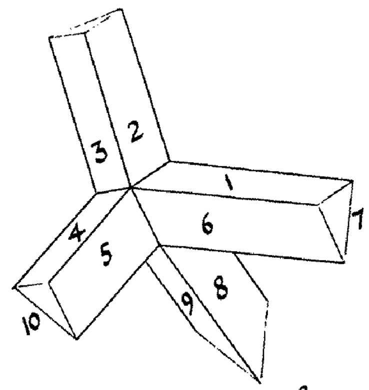

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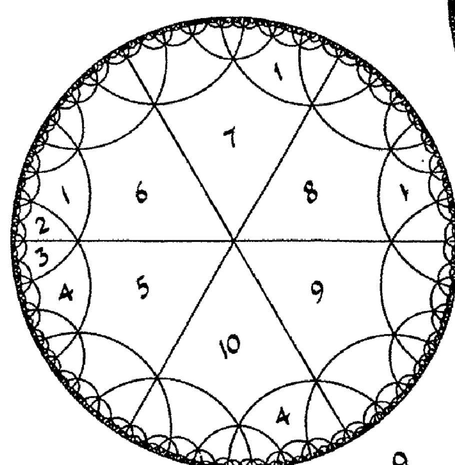

Figure 9

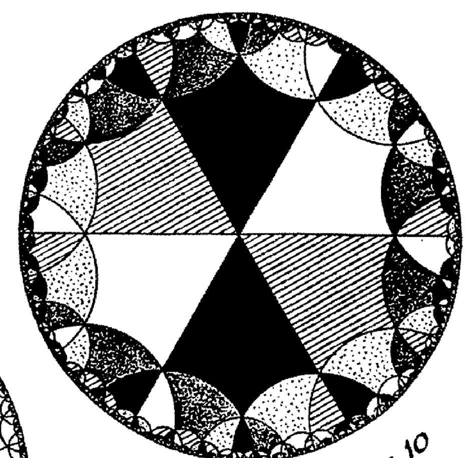

Figure 10

QUASICRYSTALS FOR ARCHITECTURE
The Visual Properties of Three Dimensional Penrose Tessellation

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Penrose tessellation refers to non-repeating patterns made up of only two elements. Although the two-dimensional Penrose tessellation (made up of fat and thin rhombii) has been known for ten years, the three-dimensional case (made up of fat and thin rhombohedra) is only a few years old. This new way to construct three-dimensional lattices has remarkable visual and structural properties, leading to applications in the physics of atomic structure as well as architecture and environmental sculpture.

A non-repeating pattern is an apparent paradox; one thinks that if there is a pattern then there must be repetition, and if there is no repetition there can only be randomness. Penrose tessellation is something inbetween: there is positional order, meaning that given one unit the positions of the others are generated; small regions are repeated elsewhere in the pattern; there are rotations which leave the pattern essentially unchanged, in that unit cells are still oriented in one or another of just a few directions. And yet, the patterns are not periodic-- like an irrational number there is no regular repeat of sequence. (Steinhardt, 1986)

Quasicrystals are now made in physics labs, much to the surprise of solid state physicists who used to think that atoms must either be arranged in a well-ordered regular crystalline lattice or in a highly disordered glass arrangement. Quite by accident D. Shectman and his collaborators discovered in a rapidly cooled sample of an aluminum-manganese alloy properties of both metallic crystal structure and glassy random structure. Moreover these samples, later duplicated by others, had the fivefold (pentagonal) symmetry that had been disallowed for patterns until the discovery of quasicrystals. Once large samples of pure quasicrystals can be made, their electrical and chemical properties can be studied, possibly with quite startling results. (Nelson)

The history of quasicrystals is the development of more and more powerful mathematical techniques to generate them, techniques that allow more and more of their subtle symmetry to emerge.

The Matching Rules Technique, invented and used by Roger Penrose, is a local operation of marking the 1-d boundaries of the 2-d units in such a way that when they are assembled, mark to mark, a non periodic tessellation is guaranteed. This local operation can make no predictions about the position and orientation of units far from the area being worked, and thus can lead to the erroneous assumption that the pattern is more random (less constrained) than it really is. Investigations of the infinite pattern as a whole were accomplished by Penrose by showing that the matching rules imply a system for breaking the units apart into smaller self-similar units in such a way that another, more numerous non-periodic arrangement is made. (Gardner)

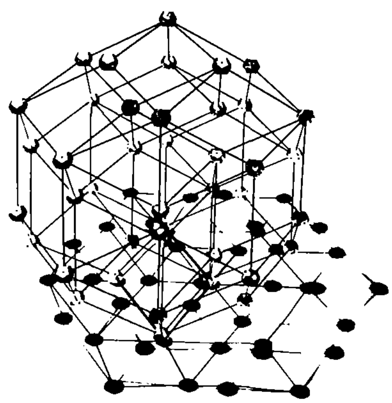
The Projection Method, invented by N. de Bruijn in 1981 requires the construction first of a cubic, thus periodic, lattice, generally in twice the number of dimension of the desired tessellation. This cubic grid is projected, sliced, and projected a second time to obtain the tessellation. (de Bruijn) Although this method requires, for example, the construction of a four-dimensional grid to obtain a two-dimensional Penrose tessellation, there is no need to visualize the four-dimensional grid, and so it is possible to scale up to higher dimensions in a straightforward though cumbersome way. The four-dimensional Penrose tessellation has now been investigated by Elser & Sloane with the projection of an eight-dimensional hypercube. (Elser & Sloane) The projection method is also useful in solid state physics for the easy calculation of the diffraction patterns of particles shot though quasicrystal samples.

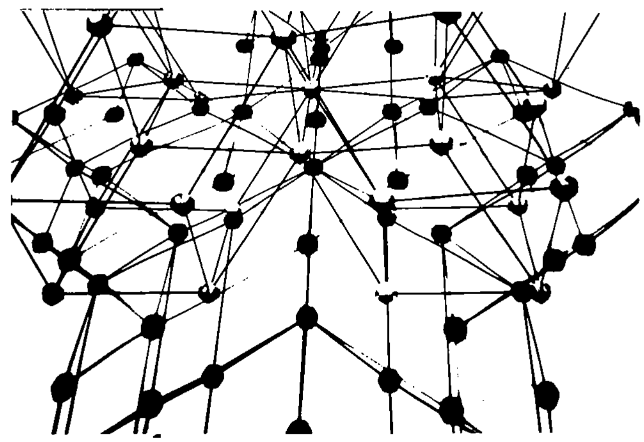
The Generalized Dual Method introduced by de Bruijn developed especially by P. Steinhardt in 1985 is by far the most powerful technique yet devised for the study of quasicrystals. First a grid dual (a dual is a distillation of a pattern into its basic structure) to the final tessellation is constructed, and then the dual grid is filled in with unit cells. This method can generate a larger class of non-periodic patterns including those with arbitrary orientational symmetry and those in any dimension; it can easily generate large patterns; and it provides a more complete description of the patterns. With this method we see how the long-range orientations are intrinsic to the structure, and we have a technique to predict the sequence of position of units. Moreover it is the only method that generates four zonohedra (the four medium-sized groupings of the two unit cells of the three-dimensional Penrose tessellation). They are: the fat rhombohedron, the rhombic dodecahedron, made up of two fat and two thin rhombohedra, the rhombic icosahedron, made up of five fat and five thin rhombohedra, and the rhombic triacontahedron, made up of ten fat and ten thin rhombohedra. It is only these zonohedra that are the full three-dimensional analogue of the Penrose tiling in that only these medium-sized assemblies have matching rules and inflation and deflation capabilities that force non-periodic expansion, although each zonoherdon can be resolved (subdivided) into its composite fat and thin rhombohedra. (Steinhardt et. al. 1985/6) I have discovered matching rules using two fat and two thin blocks that can sometimes generate these zonohedra. If they can be perfected so that the zonohedra, and thus the entire tessellation, are inevitably created, then it would be possible to create a computer program that simulated the growth of quasicrystals using local information only.

Using the powerful dual method, I have written computer programs which generate, rotate and slice 3d quasicrystals, and which demonstrate the visual behavior of these structures when seen from different angles. They have icosahedral symmetry which means that they look like they are made up of squares, triangles, rombii,and pentagons, respectively as they are rotated. It is thrilling to see this structure transmute before your eyes, in real time, becoming one thing and then another, dissolving cells at one place and recreating them elsewhere, becoming at one moment a dense thicket and at the next a transparent lacework-- and all the while knowing that the structure is not really changing, that only a rotation of a fixed, rigid, structure is being observed.

Experiencing these programs supports an original application of this geometry to architecture and environmental sculpture. Rays of light from the sun are parallel, and cast shadows in isometric projection like the two-dimensional projections of three dimensional quasicrystals. Thus it is possible to build structures which visually behave like the computer program I have written. For example, consider Buckminster Fuller's geodesic dome in Montreal. This structure casts a shadow which is an intricate triangular net, and as the sun moves across the sky the triangular net shadow shifts across the floor. If the dome where made up of quasicrystal elements, tessellated according to Penrose matching rules in three dimensions, the shadows would be seen not so much as shifting but as transforming from a pattern of triangles to one of squares, to one of a 2-D Penrose pattern, to one of pentagons, and back again to one of triangles and hexagons. The same effect could be obtained to a lesser degree with a quasicrystal space frame (a flat slab) or a barrel vault or even a spherical cluster is seen from many angles, if for example it were hanging in a atrium space where viewers could be underneath and well as on balconies. (Computer generated engineering studies of these structures is being undertaken to study their structural properties.)

Like other large structures built with a new, and still obscure technology the effects would seem miraculous, like St Chapelle, or the Eiffel Tower, or the Grand Coulee Dam must have been to its first viewers, and still is to some extent for us. To build a crystal cage for the capture of a deity, to build a tower so much higher than any other structure, to fill in a mountain so that the vast American landscape can be pressed into the service of human beings, these are not only magical feats of mathematics and engineering, but in addition they tell us something profound about the values of the people who built them. In the case of the quasicrystal, even on a much smaller scale, we could be allowed the satisfying experience of seeing in complexity and apparent confusion a structure, a symmetry, that is elusive, subtle, flexible yet amazingly powerful. And that image of complexity mastered is something for which our culture yearn, now more than ever.





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THE ASYMMETRY OF THE PROGRESS OF THOUGHT

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The history of ideas shows the building of new systems of concepts from the concepts of the past. The new concepts of scientific theories may be incommensurable with the concepts of their predecessors. Both the sense and reference of some concepts may differ, if compared. But even if concepts are new they have meanings that are analogous to ideas of the past or can be understood in relationships to old familiar ideas that form their context. How particular concepts and theories in science are related to the concepts and theories that preceded them is part of the subject of the history of science, but it is the obvious directionality of both the history of science and the history of philosophy that is the subject of this paper.

The directionality of the progress of thought is an instance of an asymmetry. It is an asymmetry that results from the creativity in the production of ideas. The direction in science is produced in part by the broadening of generalizations as it was described by Baconian induction and in part from the falling of more areas of experience within the purview of proper scientific investigation, following a scheme such as the ordering of the sciences by Comte. Either principle could be used as a criterion for the measurement of progress in science, but neither captures the intrinsic development of ideas, themselves, from one to another. In the history of philosophy the mere accumulation of concepts and theories could be used to measure progress, so that just the existence of more philosophical theories and ideas would count as progress in philosophy.[1] But while this criterion also could be used to measure the direction and the amount of progress in the history of ideas, it is how new concepts develop out of old ones that best captures the asymmetrical directionality of the history of ideas and constitutes the dynamical process by which creativity produces an asymmetry of development within the asymmetry of time itself.

New theories are incommensurable with the ones they replace because the concepts in the new theory are not the same as the concepts in the replaced theory, but the new concepts could not come into being without some conceptual ancestors, perhaps from a different intellectual tradition or from the distant past. Einstein's theories may be incommensurable with Newton's but neither could have come into being without Euclid and Democritus. New concepts develop out of old ones, such as mass from matter[2] but while it can be intelligibly described there is no generalizable pattern in the process and therefore nothing regular or symmetrical with other processes of conceptual development.

There are two dimensions of the process of conceptual progress that can be distinguished. They might be called longitudinal and transverse, but both are irregular. In the emergence of new ideas in the continuum of time, all the related new concepts in a field do not come into being together from their predecessors nor do they come in with any perceivable rhythmic order in time. How long a unified train of ideas will last, how rapid changes within it will be, how far back in time will ideas reach to be continuous with their predecessors will vary. Conceptual development within as well as among fields of thought appear to be woven together from uneven strands of ideas in time. If time is a derivative of events as the American philosopher, C. S. Peirce conceived it to be, then his description of time as a rope of uneven , successive, twisted structure[3] fits the progress of thought.

If the progress of thought is viewed in transverse section, where a new idea joins its predecessors lacks any pattern. There is no designatable place or pattern of places where a new idea joins those past. There is nothing analogous to a continuation of a family's property passing through the eldest son or some other regular arrangement of offspring, only that perhaps in disanalogy to the eldest son, a perfected expression of complex ideas is often its culmination and continuations grow out from some point around it in a minor strand of the cultural context or by picking up a strand of thought from farther in the past, somewhat as new branches grow out from some apparently random point behind the leader branch.

Although we read parts of the history of philosophy as continuous, from Plato to Aristotle and from Kant to Hegel, Both Aristotle and Hegel begin somewhere else than their famous predecessors, Aristotle in Greek medicine and biology and Hegel from theology school and Greek tragedy. Einstein draws on a different intellectual tradition from Newton. Kepler and Galileo are nearly contemporaries but worlds away from each other in thought. Visual art, once following strict traditions, now exhibits some similarities to the progress of ideas. It has drawn on the forms of modern technology as well as gone back to African and near eastern sources and some of its masters (Picasso, Chagall, Miro, and Klee) cannot be taught or gone on from, so young artists must find another point from which to continue. Not only is the progress in thought an overall asymmetrical process in time, but it is irregular in its temporal strands and in the conceptual location of the sources from which it will develop.

Symmetrical forms may be broken or compounded in ways to produce asymetrical ones, but as C. S. Peirce said, " But everybody can see that symmetrical forms, put together symmetrically, will never make an unsymmetrical form. Why not ? Because symmetry is a special kind of equality. Now equality can be built up out of inequalities; but is (sic) is evident that inequality can never result from a chain of equalities, for if one thing is equal to a second, and this second to a third, the first is equal to the third and you are percisely where you were at the outset." (C. S. Peirce. The New Elements of Mathematics. 821-822) If one begins with symmetry one will end with symmetries in the history of thought. One also tends to complete broken patterns in perception and so complete a symmetrical form. One also searches for identities that persist in time (E. Meyerson. Identity and Reality) thus adding equalities to equalities and therefore not losing the symmetries one perceives but reproducing them through time. But if the creative process in the formation of ideas lacks pattern, giving the history of ideas an asymmetrical directionality and unpredictable lines of growth, then the templates of historical understanding ought to be , themselves, asymmetrical. The understanding provided by the patterns of the past will depend on how the interpreter of the past folds the past back on to itself to produce the pattern. Just as in a Rorschach blot, the symmetry is produced by folding the blot on itself while still wet. Regularities may be produced by the process itself or if the two figures are superimposed without interacting they might reinforce some lines to form a new pattern. The possibilities for creativity of interpretation arise in the idiocyncracies of superimposition and the tolerance for asymmetrical appendages. Thus Hegel's Phenomenology of Spirit is a sometimes intelligibly pattened account with irregularities of detail that threaten to overcome it.

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Extensions of the symmetry notion on geometric objects with one or more colours

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The objects I'll speak about have plane images and the mathematical treatment is minimum. The succesive extensions are presented in a logical order, more than in a chronological one. For more details, there are references (and the references given there). Projection and models will be presented.

1. THE CLASSICAL SYMMETRY (ISOMETRY)

1.1. An isometry is a mapping of the euclidean space $\mathbb{E}^n$ onto itself, which preserves all distances. In the plane (n=2), the isometries are: a) the identity; d) reflection in a line; e) glide reflection (in a line). A symmetry of a set S (of points from $\mathbb{E}^n$) is an isometry which maps S onto itself. The set of symmetries of S forms (under composition) his symmetry group G(S).

1.2. A (plane) motif M_o is a bounded and connected set of points (from $\mathbb{E}^2$). A discrete pattern $\mathcal{P}$ is a (non empty) family M_i ($\subset \mathbb{E}^2$) of pairwise disjoint, congruent copies of M_o, so that the symmetry group G($\mathcal{P}$) acts transitively[1)] on the M_i-s. There are three cathegories of $\mathcal{P}$: a) finite patterns (or rosette); b) strip patterns (frieze, band or border ornaments) with one translation vector as symmetry; c) periodic patterns (wallpaper or plane ornaments) with 2 (in different directions) translation vectors as symmetries.

1.3. If the motif M_o is asymetric (i.e. no symmetry of G($\mathcal{P}$) is its own symmetry), the pattern is called primitive and there are: a) two families of finite patterns - with c_n (the cyclic group) and d_n (the dihedral group); b) 7 types of strip patterns; c) 17 types of periodic patterns. If the motif M_o is symmetric (i.e. there is at least one symmetry of G($\mathcal{P}$) that is a symmetry of the motif also), the pattern is called non-primitive and there are: a) a familly of finite patterns; b) 8 types of strip patterns; c) 34 types of periodic patterns. For the history of these groups of symmetry, their extensions and illustrations see Grünbaum & Shephard (1987), pp.55-56, 218-256.

1.4. A chain $\mathcal{S}$ (or rod pattern) is a rotational, infinite, cylindrical surface C with a marked family $\overline{M}_i$ ($i \in I$) of pairwise disjoint congruent copies of a motif $\overline{M}_o$ (i.e. a bounded and connected set of cylinder points), so that the symmetry group G($\mathcal{S}$)

1) For each pair M_k, M_h of the family M_i, there is a symmetry of G($\mathcal{P}$) that transforms M_k onto M_h.

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acts transitively on the $\overline{M}_1$ - s. Their symmetry groups were studied first in 1929. T.Roman (1969, 1985) classified these in 17 primitive and 19 non-primitive classes, using their plane development.

1.5. A major extension was performed by adding a "marking" to each motif copy. The first : colouring it in black or white so that the symmetries maintain the colour and the antisymmetries change them. Indirectly it was made in 1929-1930. Directly (but remaining fast unknown) in 1935 . The book of Shubnikov (1951) gives a new start to application of antisymmetry (and multiple antisymmetry) not only in geometry and cristallography but also in physics, chemistry, biology by scientists from U.S.S.R., and from many other countries in the 50's and 60's. See the book of Shubnikov & Koptsik (1972 in russian, 1974 in english) and the review article of Zamorzaev & Palistrant (1980, p.231).

1.6. An other acception of colouring, mathematically founded by van der Waerden & Burckhardt (1961) is the $q^{2)}$-colour symmetry for a set S as a pair (σ, π), where σ is a symmetry of G(S), and π a permutation of q indices, compatible with σ ; the compositon law of the colour - symmetry group is: $(\sigma_1, \pi_1)(\sigma_2, \pi_2) = (\sigma_1\sigma_2, \pi_1\pi_2)$. Results are given in M.Senechal (1979), T.Wieting (1982), Jarrat & Schwarzenberger (1981), T.Roman (1970, 1989). Review article: R. Schwarzenberger (1984), Grünbaum & Shephard (1987, pp.463-470).

1.7. All the above mentioned notions and results are for discrete geometric object from 1.2 and 1.4. The continuous (or semi-continuous) patterns as well as the tiling of the euclidean plane will not be tackled, but see Grünbaum & Shephard (1987).

2. THE HOMOGRAPHIC SYMMETRY

2.1. A homographic mapping of the euclidean plane $\mathbb{E}^2$ (without a point 0) onto $\mathbb{E}^2 \setminus \{0\}$ is obtained by one of the elementary transformations defined through the complex function $z \mapsto \frac{az+b}{cz+d}$ (ad - bc $\neq$ 0), i.e.: identity, rotation, homothety, inversion, reflection and their combinations. An ω-symmetry of a set S (of motifs in $\mathbb{E}^2$) is a homographic mapping of S onto itself. The set of ω-symmetries of S forms (under composition) his ω-symmetry group $G_\omega(S)$ if it contains an inversion and his similarity group $G_s(S)$ if it contains a homothety and is not a $G_\omega(S)$.

2.2. The $G_s(S)$ were applied in crystallography by Shubnikov (1960). For subsequent developments see Zamorzaev & Palistrant (1980, pp.240-241).

2.3. A discrete ω - pattern is the homographic transform of

2) r- in [15] , k - in [11] and [2], N - in [10]

a discrete pattern . The finite ω-patterns are the homographic

Fig.1. Fig.2.

transforms of discrete strip pattern segments. The discrete angular patterns are the transforms of the discrete strips patterns. The homographic ω - patterns are the homographic transforms of the discrete periodic patterns. A conic column $\mathcal{C}$ has its plane development (i.e. an angular domain, provided with motifs) the transformed $\mathcal{S}$ chain plane development. Details about this geometric objects see in Roman (1971, 1972, 1985, 1989).

3. THE SURFACE SYMMETRY

3.1. Generalized discrete patterns on surface are extensions of the three cathegories from 1.2. a) Finite discrete patterns on surface[3] are obtained by plasing congruent motifs in congruent cells of a rotational surface segment, included between two planes P, P' orthogonal to the rotation axis. For C one obtains segments of strip patterns; the projection on P of these on C_0, S_0, E, H_1, H_2, P_e is a finite ω-pattern. b) Strip discrete patterns on surface are obtained by placing adequate motifs on a surface segment, the parallel planes P, P' yelding hyperbolic, parabolic or right line sections; c) Periodic patterns on surface are obtained by placing adequate motifs in the cells of rotational C_0, H_1, H_2, P_e (for $z > 0$) , cells resulting by sectioning with a special family of planes orthogonal to the rotation axis and a second family of axial planes; the projection on the plane $z = 0$ is a homographic ω-pattern.

3.2. Isometries of the plane map of a bounded surface and discrete patterns on the surface are illustrated by two exemples: a) for the ellipsoid the map is an open rectangle domain: $u \in [0, 2\pi)$, $v \in (0, \pi)$. The map of a segment of E is the band between the segments $v = h$, $v = h'$. A map of 2 elliptical finite discrete patterns is given in fig. 3 and a map of a periodic discrete pattern on E, for $u = \frac{\pi m}{2}$ (m = 1,2,3), $V = \frac{\pi h}{3}$ (h = 1,2,)

Fig.3. Fig.4.

is drawn in fig. 4; b) for the torus, the map is an open square $u \in (0, 2\pi)$, $v \in (0, 2\pi)$, the oposed sides beeing identical. One defines the Θ-symmetries of the torus; the discrete Θ-patterns: parallel, meridian and helicoidal finite Θ -strips and the periodic Θ-patterns are studied (Roman, 1979).

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3) The considered surfaces are: sphere with centre O: S_0, ellipsoid E, hyperboloids H_1, H_2, paraboloids P_e, P_h, rotational cylinder C, rotational cone with apex O: C_0.

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4. APPLICATIONS

4.1. Illustrations for unicolour and bicolour discrete patterns from roumanian folk art will be projected.

4.2. There will be projections for: colour Escher patterns, colour homographic patterns, colour chains -- also as suggestions for their wider utilization.

4.3. Tires profiles models - as examples of Θ-patterns will be shown.

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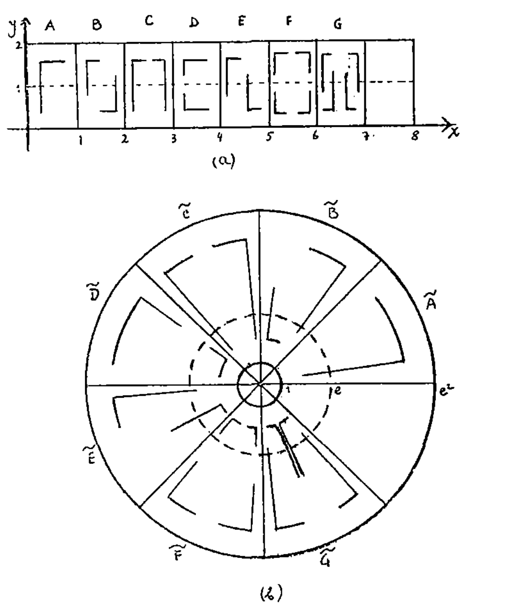
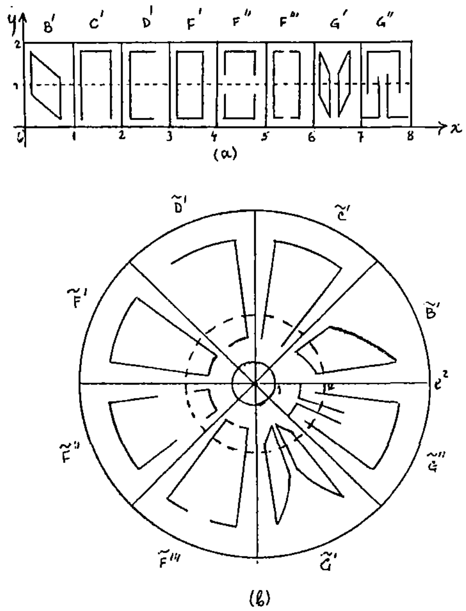


Asymmetric motifs for strip patterns (a) and for isometric ($\tilde{A}$, $\tilde{C}$) and homographic finite patterns (b)

Fig. 1



Symmetric motifs for strip patterns (a) and for isometric ($\tilde{C}'$) and homographic finite patterns (b)

Fig. 2

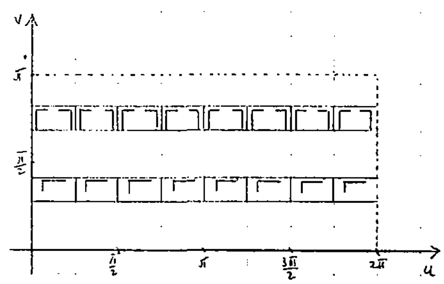


Fig. 3

Two elliptical finite patterns on the ellipsoid map.

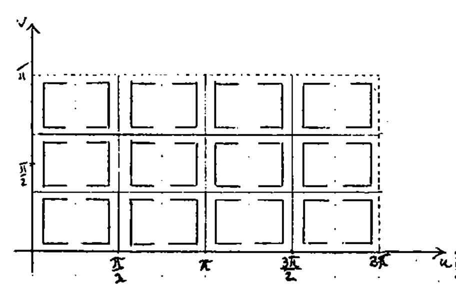


Fig. 4

Periodic discrete pattern on the ellipsoid map.

SYMMETRY IN THE STRUCTURE OF SCIENCE

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Abstract
It is shown that reproducibility and predictability, forming the dual cornerstones of the structure of science, are kinds of symmetry.

Introduction

The structure of science rests firmly upon the dual cornerstones of reproducibility and predictability. Reproducibility is the repeatability of experiments by the same and by other investigators, giving data of objective value. That makes science a common human endeavor. Predictability means that order can be found among the phenomena investigated, from which laws can be formulated, predicting the results of new experiments. (Then theories might be developed to explain the laws.) Predictability makes science our means both to understand and to exploit nature. (Thus nature's irreproducible or unpredictable aspects, whatever they may be, lie outside the domain of concern of science.)

It is well known that symmetry holds an eminent position in science. Examples: spatial symmetry of crystal lattices, temporal symmetry of periodic processes, Poincaré symmetry of relativistic theories, gauge symmetry of elementary particle theories. However, symmetry is of crucial importance in the structure of science itself. Indeed, I will show that both reproducibility and predictability are kinds of symmetry.

Symmetry

In everyday speech symmetry usually means a balance, a repetition of parts, a regularity of form. More precisely and generally symmetry can be said to be invariance under transformation, i.e. the situation is symmetric if there are one or more changes that can be made that nevertheless leave some aspect of the situation unchanged. Consider, for example, a uniform metal equilateral triangle and imagine rotating it by 120° or 240° about its center within its plane. Although a transformation, a change, has been made, the result looks the same and has the same physical properties as the original. Thus our piece of metal possesses symmetry under these rotations with respect to external appearance and physical properties. If the triangle were not uniform or had a corner chopped off, it would not possess this symmetry.

Actually, a system that might possess symmetry may in general be concrete or abstract. The transformations involved need not be geometric, but may involve any concrete or abstract aspect of the system, as may the invariant aspect be concrete or abstract and does not have to be appearance or physical property. But the very least we need for symmetry is the possibility of making a change and some aspect that is immune to this change.

Reproducibility as symmetry

Putting things in terms of experiments and their results, reproducibility is commonly defined by the statement that the same experiment always gives the same result. But what does "same" mean here? No two experiments or results are identical; they will always differ at least in time (repeating the experiment in the same laboratory) or in location (duplicating the experiment in another laboratory) and will differ in other respects as well. So by "same" we must mean "equivalent" in some sense. We cannot even begin to think about reproducibility without permitting ourselves to overlook certain differences involving time, location, and various other aspects of experiments.

Let the difference between two experiments be expressed as a transformation, the change that must be imposed on one experiment to make it into the other. Such a transformation might involve temporal displacement and/or spatial displacement and/or rotation. It might involve putting into motion or bending the apparatus. Or we might change the experiment to measure temperature rather than pressure, for instance. And so on and on.

But not all possible transformations are associated with reproducibility. Let us see which are. Temporal and spatial displacement are obviously included. And the motion of the Earth requires us to add rotations and velocity transformations. To be able to use different sets of apparatus, we need replacement by other materials, other atoms, other elementary particles. Due to unavoidably limited experimental precision we must include small changes in the conditions. And we also need changes in certain other aspects of experimental setups, over which we have no control in practice or in principle.

So we define reproducibility: Consider an experiment and its result, consider the experiment obtained by transforming the original one by any transformation belonging to the above set of reproducibility-associated transformations, and consider the result obtained by transforming the original result by the same transformation. If this transformed result is what is actually obtained by performing the transformed experiment, and if this relation holds for all transformations belonging to the set, we have reproducibility. This is symmetry, as can be seen as follows: Consider a reproducible experiment and its result. Transform it and its result together by any transformation belonging to the set of transformations we associate with reproducibility. The pair (transformed experiment, transformed result) is, of course, different from the pair (original experiment, original result), but there is an aspect of the pairs that does not change under the transformation. This is that the result is what is actually obtained by performing the experiment. Said in other words, this symmetry is that for any reproducible experiment and its result, the experiment and result obtained from them by any transformation belonging to the above set are also an experiment and its actual result.

Predictability as symmetry

Again expressing things in terms of experiments and their results, predictability is that it is possible to predict the results of new experiments. Of course, that does not come about through pure inspiration, but is attained by performing experiments, studying their results, finding order, and formulating laws. So imagine we have an experimental setup and run a series of n experiments on it, with inputs inp_1, inp_2, ..., inp_n, respectively, and corresponding results res_1, res_2, ..., res_n. We then study these data, apply experience, insight and intuition, perhaps plot them in various ways, and, maybe with a bit of luck, discover order among them. Suppose we find that all the data obey a certain relation, R, according to which all the results are related to their respective inputs in the same way. Using function notation, we find that $res_i = R(inp_i)$ for $i=1, ..., n$. This relation is a candidate for a law, res = R(inp), predicting the result res for any input inp. Imagine further that this is indeed the correct law for our experimental setup. Then additional experiments will confirm it, and we will find that $res_i = R(inp_i)$ also for $i = n+1, ...$ as predicted. Predictability is the existence of such relations for experiments and their results.

That predictability is a symmetry is seen as follows: For a given experimental setup consider all the different input-result pairs (inp, res) that have been, will be or could be obtained by performing the experiment. Transform any one of these into any other simply by replacing it. The transformed pair is different from the original one, but the pairs possess an aspect that is not changed by the transformation. This is that inp and res obey the same relation for all pairs, namely the relation res = R(inp). Put in different words, this symmetry is that for any predictable experiment and its result, the experiment and its result obtained by changing the experimental input obey the same relation as the original experiment and result.

Conclusion

Following the definition of symmetry as invariance under transformation, it is shown that both reproducibility and predictability are kinds of symmetry by showing for each the changes that can be made and the aspect that is immune to these changes. For reproducibility any experiment-result pair can be transformed by any of the set of reproducibility-associated transformations. The invariant aspect is that the result is what is actually obtained by performing the experiment. For predictability any input-result pair can be transformed by replacing it with any other pair for the same experimental setup. What is invariant is the relation between experimental input and result. Since reproducibility and predictability are the two most fundamental cornerstones of science, we see that symmetry not only serves within science, but is actually intrinsically involved in its structure and is thus inherent to the very existence of science.

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A dual way of computing - learning from cerebral asymmetry

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Extended summary

Despite the impressive technical and conceptual power of digital/logical computing there has been signes and facts showing its inherent practical and theoretical barriers. Computing structures using artificial electronic copies of a yet oversimplified but analog regular model of biological neuron (instead of the more primitive yes-no logical model) demonstrated striking capabilities in some examples. These types of artificial analog/"neural" circuits or "neural" computing provided an alternative regular analog solution for some problems which resisted the attacks of the digital/logical way of problem-solving of artificial intelligence, although a lot of important questions are open.

Hence, we have two types of computing (processing) paradigms, the digital/logical and the analog/"neural" way which offer different capabilities. In both cases the strong underlying facts are the different models of the biological neuron operations.

In course of learning from the nature to provide better artificial (electronic) information processing systems it has been recently realized by the author that facts and analogies concerning the functional cerebral asymmetry could strongly motivate the introduction of a dual way of computing structure[1]. It is emphasized immediately that it is not the modelling of the cerebral functions, it is simply a one way street from cerebral asymmetry[2] to electronic computing using some facts concerning the qualitative features of nonlinear circuits and operators[1,4,5] and motivated also by the unified view of physical-information-and circuit-aspects of information processing[3]. The aim of this paper is to introduce the dual computing structure in a less technical-mathematical framework.

The basic motivating facts and results are as follows.

A. Cerebral asymmetry[2,6,8]

(i) A representative sample of the different functional processing abilities of the left and right hemispheres (LH-RH) are summarized next (selected for our purposes)

LH	RH
- analytic (breaking into parts)	- holistic (global)
- differential	- integral
- sequential processing and temporal resolution of information	- immediate processing and perception of the parts vs. whole relation
- verbal abilities	- performing abilities
- matching of conceptually similar objects	- matching of structurally (pictures, curves etc) similar objects
- isolate a "shape" in irrelevant background (a surprise in LH performance)	- forming whole "gestalt" from incomplete information
- information ordered in time	- information ordered in space
- events of high rate of change ($<$50 msec)	- events of small rate of change

(ii) The dual memory encoding hypothesis in memory theory says that verbal (name) an pictorial symbols of a notion is represented parallel (a special dual representation is the metaphore).
(iii) Simultaneous processing in some tasks[2,6] (e.g. expert musicians realizing melody and structure jointly).
(iv) Direct (real time) realization and detection of complex inputs (e.g. the "grandmother cells").
(v) The competency level of a given hemisphere for some particular task, i.e. the division of labour between the hemispheres is changing depending on attentional focusing and other factors[8].
(vi) Many neurons are organized in a few layers of two dimensional arrays containing modules which are columns of this organization and these modules have specific functions.

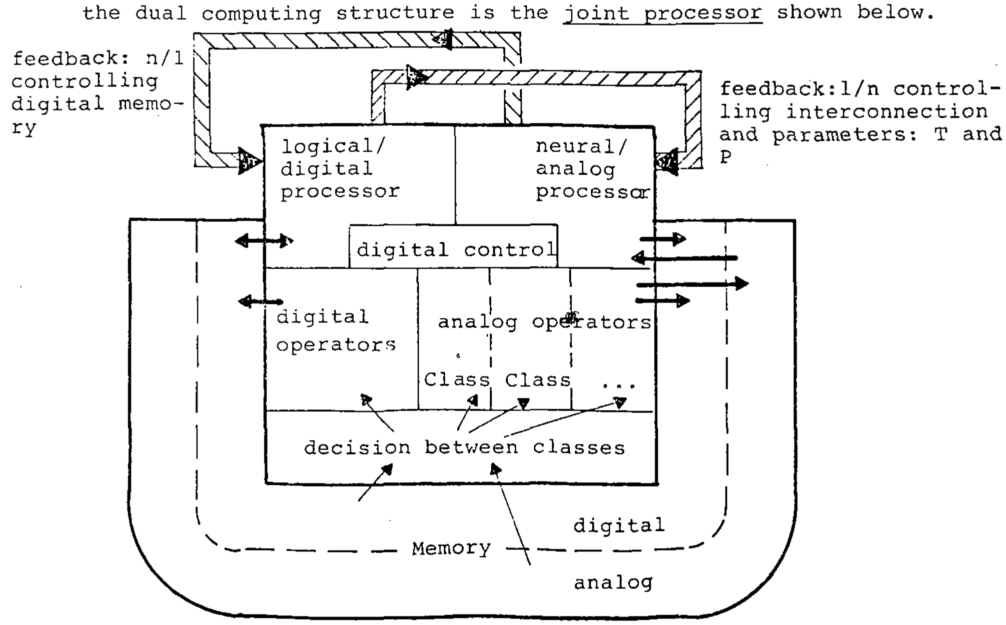
B. Nonlinear circuits and systems
(i) Any n-variable function can be realized by a three layer structure of one variable nonlinear transfer function elements[1].
(ii) Any nonlinear operator is unique except scaling and delay[3].
(iii) An operator with fading memory can be approximated by a nonlinear memoryless operator and delays in a forward structure[4].
(iv) Cellular neural networks are able to consider local and global information processing on a finite 2- or 3-dimensional analog processor grid[7]

The notion of the analog event is important. It is a continuous signal on a finite time segment with some specific storing and processing properties[1]. The digital events are the verbal symbols coded in a 0-1 fashion.

Now, we summarize the dual computing structure. It conists of five main parts including different computing modules.

(i) the memory encoding part (Me)
(ii) the dual (digital and analog) memory (M),
(iii) the digital controller (C)
(iv) the processing arrays (Pa)
(v) the memory decoding part (Md)

The controller (C) is a standard digital finite state machine with an inherent finite memory making unique digital-logical decisions (next state functions and output functions). The memory (M) has a dual structure, a digital and an analog part. All the other three parts have basically three building blocks, the three processors: (i) the digital/logical (for the verbal hemisphere analogy), (ii) the analog/"neural" (for the non-verbal hemisphere analogy) and (iii) a joint processor-pair having the preceding two processors with internal feedback. In this model the application of the analog-events provides real-time, "immediate" detection of some prescribed patterns (templates). The digital processor array is a 1-, 2- or 3-dimensional array of simple digital processors working in systolic/cellular mode (communicating with strict neighbours only) or in connection mode (all processors communicate with all others). The analog processor array is again a 1-, 2- or 3-dimensional array of analog processors (nonlinear amplifiers) connected through weighted paths (feedback or feedforward by e.g. resistors). In the cellular neural networks[7] connection is in a finite processor-distance neighbourhood. The crucial element of the dual computing structure is the joint processor shown below.



The crucial parts are the two feedback paths: the logical - analog "neural" (l/n) and the analog "neural" - logical (n/l) paths. The l/n path controls the elements of the T and P matrices of the feedback modules (or the weight parameters of the forward modules). The n/l path controls the memory contents of the logical/digital processor performing the recursive functions.
Hence, (i) a mutual coupling is introduced between the recursive functions and the algorithmic elements of the neural circuits (extended recursive functions), (ii) real-time detection of standard but complex events, (iii) logical control of dynamic properties of "neural" processors and (iv) dynamic control of logical inference sequences of the logical processors.

Acknowledgements
The deeply motivating discussions with Professor Árpád Csurgay is gratefully acknowledged. The research was sponsored by the Research Fund AKA of the Hungarian Academy of Sciences.

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Ideal and real crystal forms.

The theory of crystal forms is based on concepts of simple forms , i.e. initial, or ideal, and derivative, or false (distorted) ones.

An ideal simple form represents a combination of faces linked by elements of crystal symmetry. The faces should be identical in shape and size (I). The 47 ideal simple forms can be simulated by wellknown educational models. For the majonty of real crystals these forms appear only to be the "quasi-ideal" ones, more or less corresponding to geometric models. For all elements of proper symmetry of a crystal, the complete conservation can be provided in media with sphere symmetry of $\infty/\infty m$ (in the case of thorough and uniform influx of the feeding material to a growing polyhedre). Anyway, such conservation is obtained when all corresponding crystal and medium elements are coincident. The 146 symmetry variations of simple forms (2) are derived from 47 initial geometrical figures after registrating their crystallograohic symmetry. Their models are constructed by means of addition to edges of 5 edge arrow with symmetry of 1,2, m , $1m$, $2mm$ (fig. 1,2 (3). To establish the latter ones on real crystals there should be taken into account face striations, etch figures, etc.

Distorted crystal forms with different dimensions of simular faces are the most widely distributed in nature.

According to P.Curie's idea of symmetry, visually on the crystal there remain only its proper symmetry elements, which coincide with ones of the parent medium (4).

In natural environment there dominate media with point symmetry ∞m of cone (gravitation field). Parallel with the cone symmetry, which gives rise to " pyramidal" forms, the symmetry of plane dyhedre is considered to be a most widely spread. This "dome" symmetry, m , enters the subgroup of the cone symmetry, common for ererything that grows in no-vertical way or moves straightforwardly along the earth surface (4). Description and simulation of distorted forms can be realized using the concept of false simple forms. The latter ones are derived from initial simple ideal forms by means of their symmetry reduction. By this, face numbers and angle magnitudes remain constant, but face and edge dimensions are changed. Such configurations are described as combinations of compound false forms or " subforms". For example, a distorted cube will form " tetragonal prism with pinacoid " or three "pinacoid" combinations. According to Mokievski's idea, derivation and simulation of simple false forms can be perforned using either different colouring of faces of the 47 simple forms (5) or stereographic projects (fig.3). Their total sum will be 430.

Assuming, that the 146 symmetry variations of simple ideal forms are initial, we can obtain the 1263 ones for false forms (6) by sequential reduction of their symmetry (from $m3m$ and $6/mmm$ to 1). Thus, we can distinguish 4 gradations of simple crystallographic forms: two of them being initial ideal (47; 146) and the other two being derivative, or false, (430,1263) ones.

It is interesting to underline the similarity of the abovementioned ratios of 4 sum values to three ones:

146 : 47 = 3,1

1263 : 430 = 2,9

430 : 146 = 3,0

One is struck by the affinity of values 146, 430, 1263 with products:

$47 \times 3 = 141$; $47 \times 3^2 = 423$; $47 \times 3^3 = 1269$.

Respectively, the ratios of false simple forms to their ideal prototypes approach the value 9:

$430 : 47 = 9{,}4$; $1263 : 146 = 8{,}7$.

The solution of the abovementioned ratios requires further investigations. Here, a certain role may belong to the analogy (parallelism) between the sequence of form distortions perfomed by Fiodorov's extensions and dislocations, on the one hand, and the classic hierarchy of category is represented by single system, the intermidiate one by three systems (one extension), and the lowest one also by three systems (extensions and dislocations). All said above confirms the presence of many mysteries that still cover a great part of, one would think, such a clear and trivial field as the simple crystal form theory.

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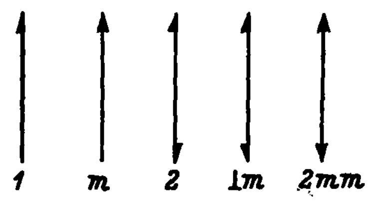
Figures.

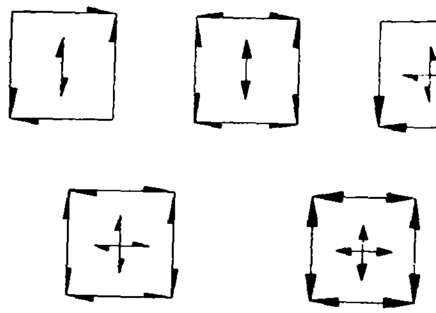
Fig. 1 Five types of finite edge symmetry.

Fig. 2. Faces of 5 cube symmetry variations.

Fig. 3 the 9 combinations of false subforms derived from a cube.



1



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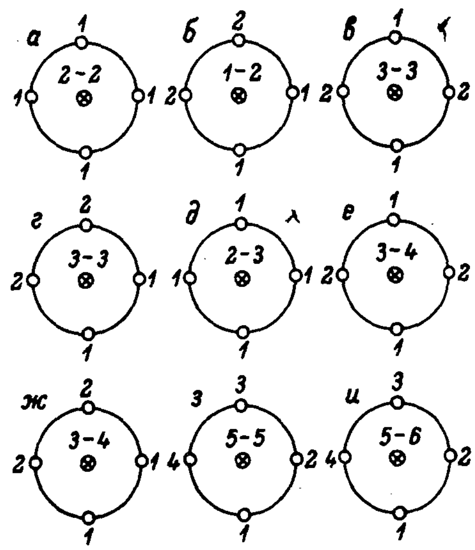

3

SYMMETRY/ASYMMETRY:TWO KINDS OF TIME: AN ARTIST'S VIEW

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This is a proposal for a lecture with slides to demonstrate -- through my own personal work -- how as visual artist -- I gradually, intuitively and, often haltingly, accepted a vanguard position of viewing 20th century art as a search for understanding and visualizing the "Shape of Time".(1) In the process I will demonstrate how the concepts of balance/imbalance: symmetry/asymmetry are fundamental to my understanding of time. I will show how remaining in balance while in motion, requires moving back and forth from symmetrical to asymmetrical modes of feeling and thought. This back and forth motion reveals two aspects of Time -- stopped time and moving time. (2)

Science/art/technology permit us to to record this motion in time, or to stop time. This process of stopping/moving in time can be manually, mechanically, electronically and photronically recorded. Pens, brushes, cameras, copiers, video, computers -- all -- can be tools for recording our perceptions of time/space. With each new tool we alter our perceptions of space/time and in the process we alter our relationships to objects, to people and to our environment.(3)

In the United States, although there has been a shift from the visualization of objects in space to the visualization of objects in time, there has been little, if any, systematic study. I founded the program Generative Systems(4), at the Art Institute of Chicago (1969-1980), precisely to create such studies. In establishing this program I interwove my personal art work with that of the program. In this way I integrated into the teaching process personal/subjective/ unconscious studies with general/ objective/ conscious studies. The last course I created for Generative Systems was called Homography, a foundation course in the visualization of time. Manual, mechanical, electronic and photronic tools were use to record relative ways of visualizing time.

HOMOGRAPHY (man's meaning) was intended primarily as a course in which all tools could be used for art exploration. As it turned out it became a course in THE VISUALIZATION OF TIME. I developed 9 relative ways of visualizing time based on my objective and subjective findings. All 9 ways were described symmetrically, i.e. Pressure/Flow was based on my dreams and my observations of the patterns of pressure and flow in my own hand and machine experiments. Stretching/ Compressing also came from dreams and from observing in my experiments that I could stretch or compress an image by light or heat or even by sound with a telecopier, merely by holding the recording needle down in time as I transmitted an image.

For each symmetrical way of visualizing time, the student was asked to use either manual, mechanical, electronic or photronic tools. Precise problems were set up, which gradually allowed for greater and greater problem setting by the student.

These are 9 relative ways of visualizing time:
(See attached figures 1-8)

1. Pressure/Flow
2. Scanning/Closure
3. Interference/Filtering
4. Internal/External
5. Opaque/Transparent Layering
6. Close Packing/Stacking
7. Stretching/Compressing
8. Metamorphosis/Morphogenisis
9. Synchronicity/Simultaneity

To these one can add other, innumerable ways of visualizing time.

In a sense, with only a gradually dawning awareness, I moved in a direction that (I very late realized) would have been a logical direction for the Bauhaus thinkers. Of these Laszlo Moholy - Nagy, came the closest to understanding the nature of visualizing time. I am now convinced that this was partially a result of his exploration with the technological tools of his time, which was directly related to his holistic philosophy.

Moholy's work with media technology permitted him to move into multi-dimensional landscapes, beyond any other of his contemporaries, more so than the Italian Futurists or the Cubists. Even Paul Klee, whose Pedagogical Sketchbooks became a model for integrating natural structural process into the creation of art, moved primarily from 2-dimensional to 4-dimensional space. It was Moholy who moved from 3-dimensional space to the 4th and multi-dimensional space/time, ie. his light modulator. Only late in my artistic career, during the course of working with new communication tools in the development of Generative Systems, did I come to understand Moholy's work.

My great respect for Moholy's life and work was partially responsible for my enthusiastic response to your Hungarian invitation to submit proposals for an article and lecture. It is in the spirit of Moholy's quest for knowledge that I plan to develop this lecture with extensive use of slides of my own work to balance my words.

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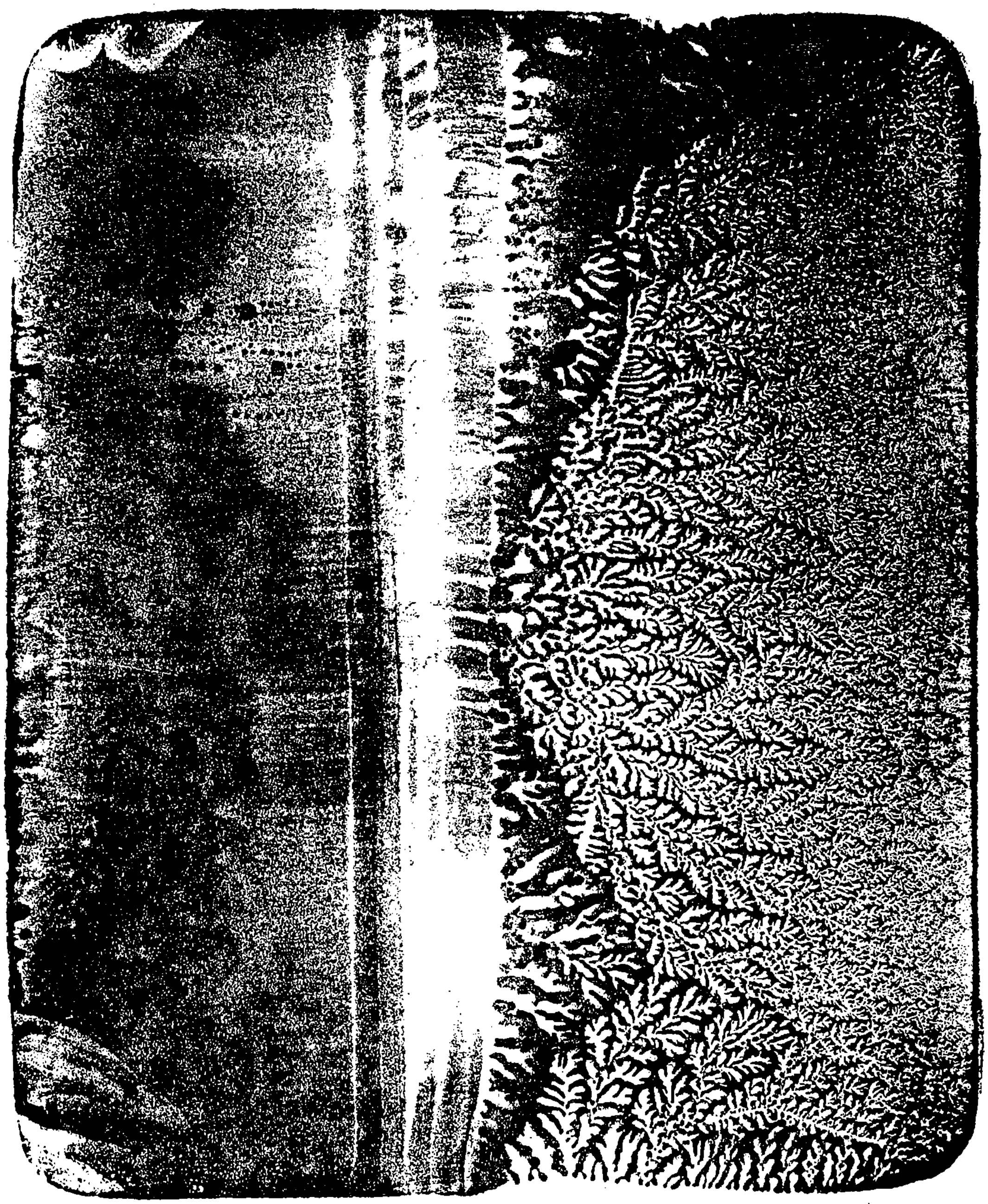
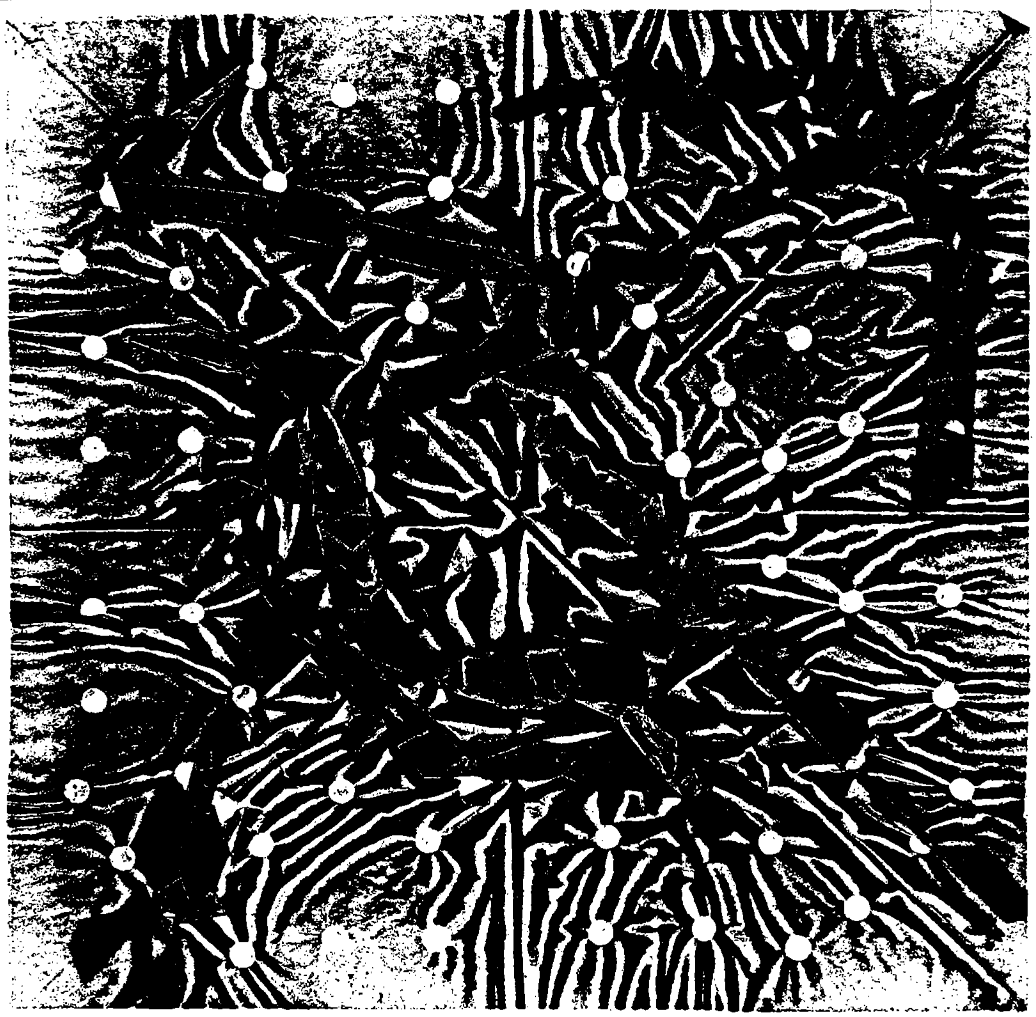
PRESSURE/FLOW FIG. 1



PRESSURE/FLOW

FIG. 2



10.25.79

OPAQUE/TRANSPARENT LAYERING

FIG. 3

Layering lace
Layering linen
Layering eggs
Layering cakes
Layering around
Layering in wait
Lady in waiting
Landy in Lay and Lady

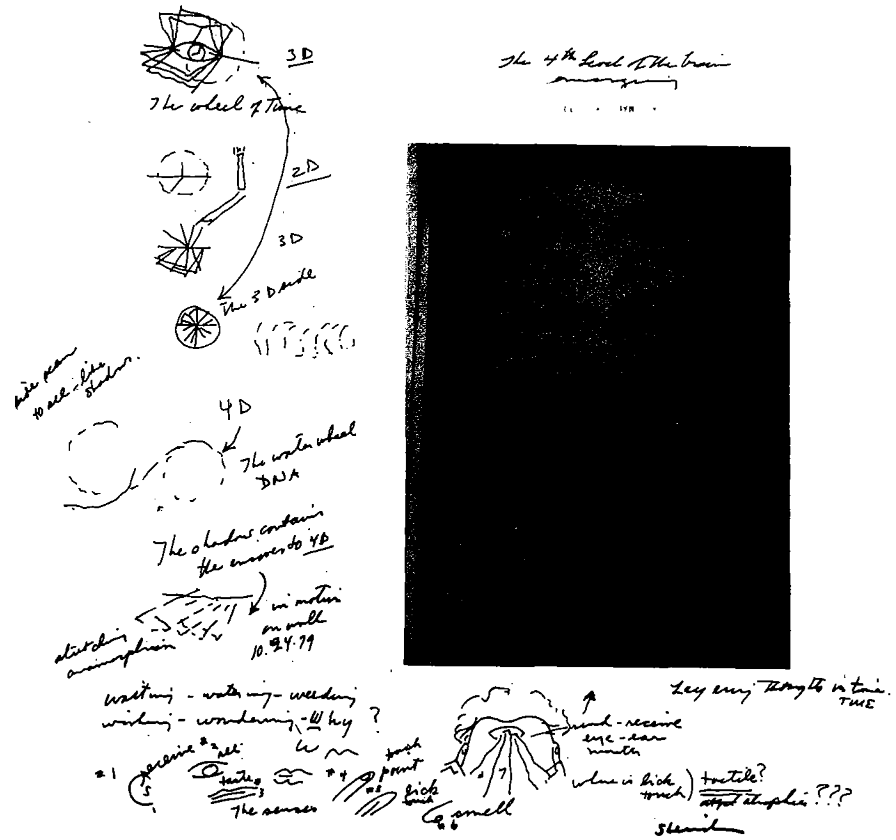


FIG.4
OPAQUE/TRANSPARENT LAYERING

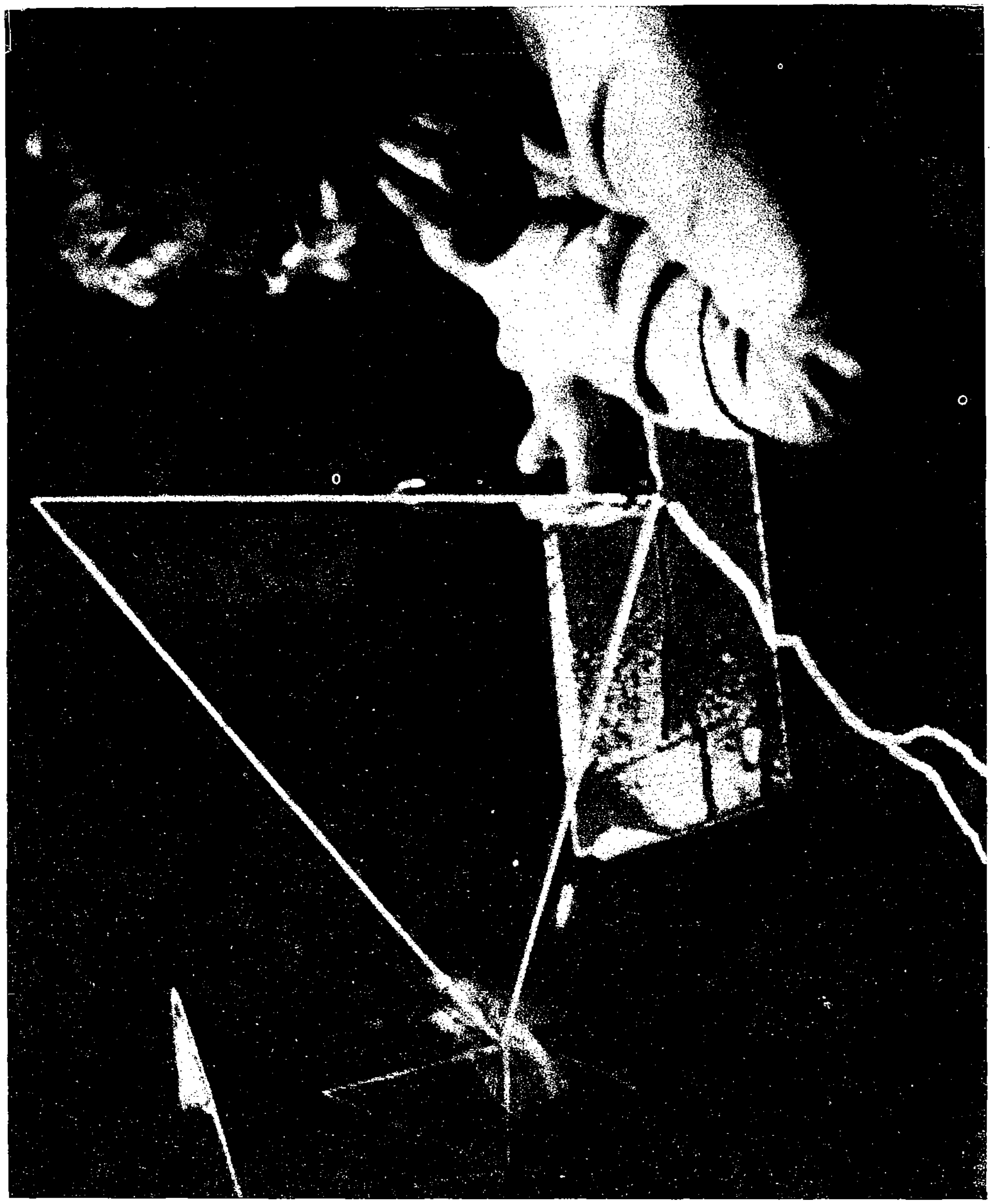


FIG. 5
STRETCHING/COMPRESSING

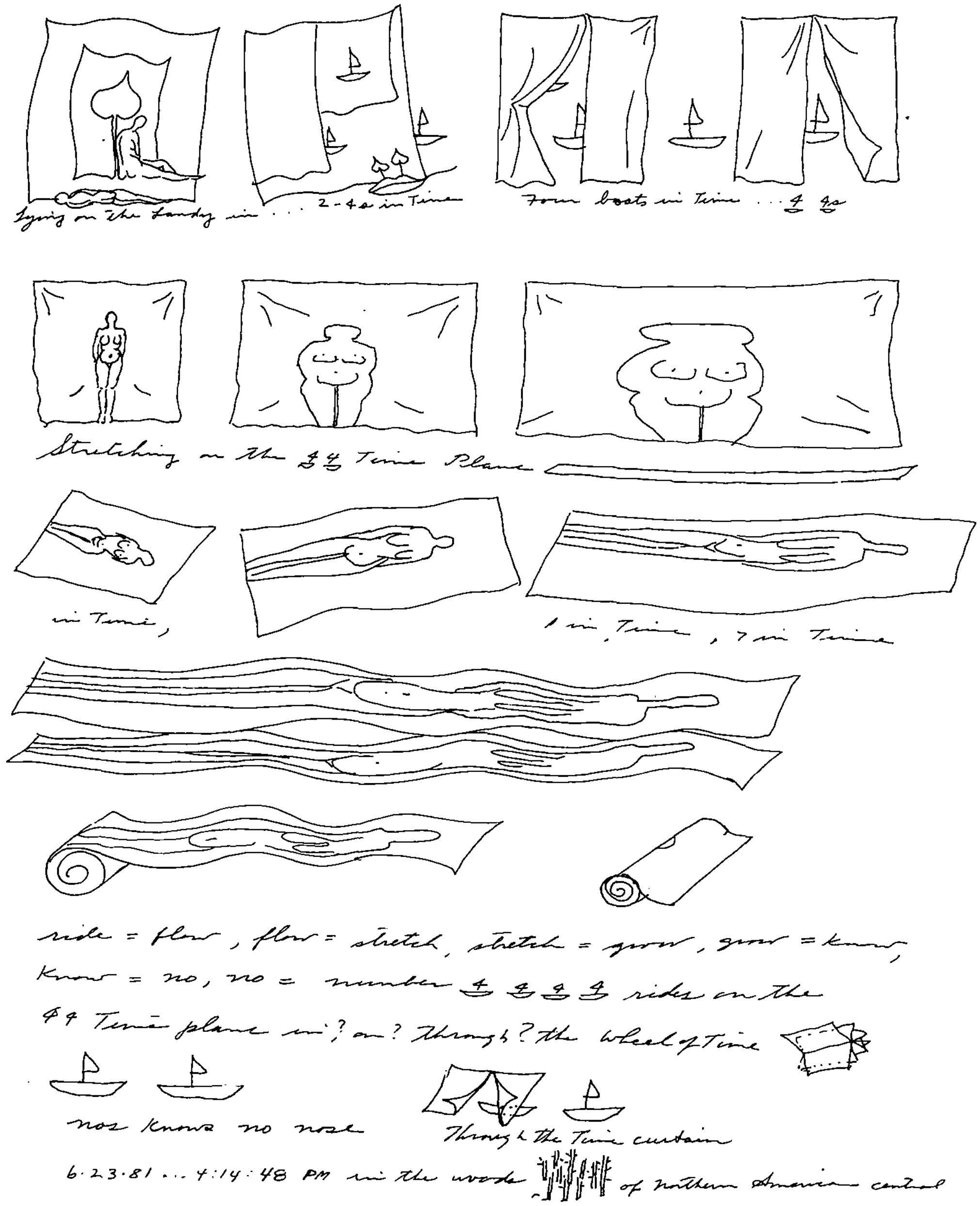


FIG. 6
STRETCHING/COMPRESSING

"Stretching Form in Time" was made by putting the computer graphic system video out of synchronization - The resulting stretched parts were then reassembled much like an out-of-sync image on your home T.V. In this case I used a Cromemco Z-2D computer + CAT 400 graphics and EASEL, Time Arts software . . . .

These hands took about 1 minute to compress and 10 to stretch. . . A hand image 6 stories tall took 4 hrs 40 min

"Stretching and Compressing in Time" 3M VRC remote copier. . . Controlling the time it takes to transmit an image. . .

FIG. 7
METAMORPHOSIS/MORPHOGENISIS

10.25.79

The order of Things

Structure for 21st Century Art

morphogenisis - the mathematics of natural growth

used for natural systems.

Soft systems
Soft ware
soft
Sofa

mouth

Macdonald's

leaf

budding

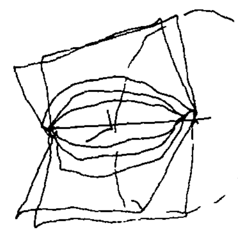
10.29.79

mouth

became

arrow

genetic growth



the clock comes up through the land

budding in motion.

The outside sheet must be the human imposed structure the technology which is not yet a natural system - but bits and pieces - analogous - metaphors for life - living structures - we are still close to the crystal structuring era - except in cloning - areas to study.

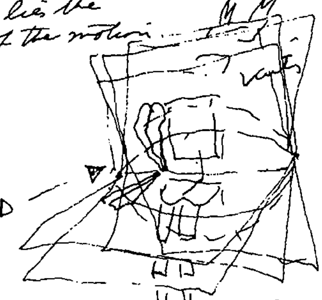
coming up through the ground

coming as a shadow wherein lies the record of the motion

foreshadow

move this way

see (1965 notes)



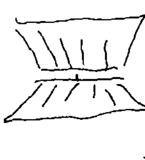
butterfly in motion

4D

side

front

slit-scan is only 2D version of time layering



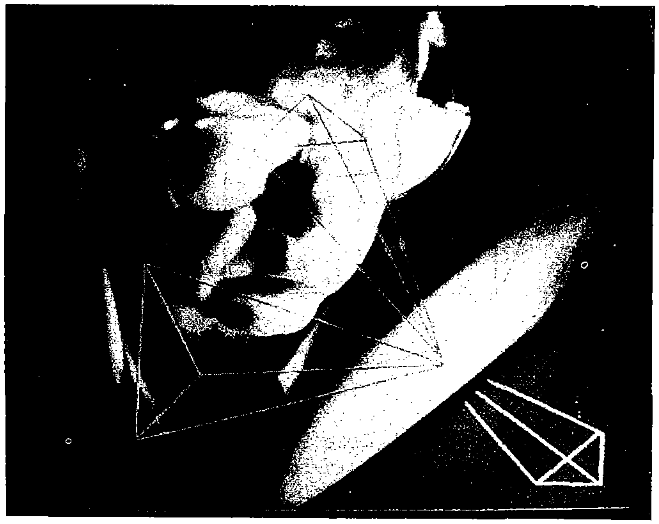
Make 3D

going through the time plane by weaving through the layers. one time layered - map the territory for the voyage through time.

Tripping Through Time

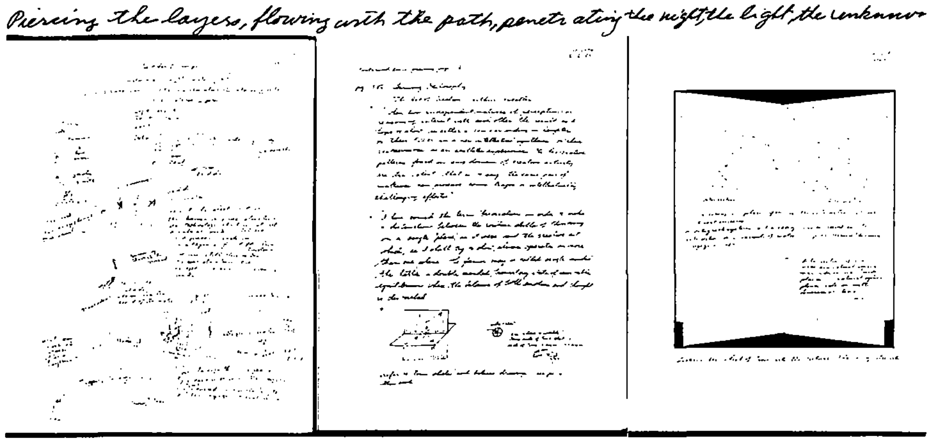
cusp

FIG. 8
SYNCRONICITY/SIMULTANEITY



"Light Plane"...1985 computer graphic...Cromemco Z-2D/CAT 400 hardware, EASEL/Time Arts software...Looking for the other side of Time on a light plane of the Wheel of Time. The night side, the right side, the dayside-the outside...with the fireside, inside/outside...

Piercing the layers, flowing with the path, penetrating the night, the light, the unknown



"Wall Note No. 10" with 3 pages from nightbooks of Oct. 1979 and April, August, 1983.

STRUCTURAL SYMMETRY IN ORGANIZATIONAL DEVELOPMENT
FITTING THE SQUARE PEG IN A ROUND WHOLE

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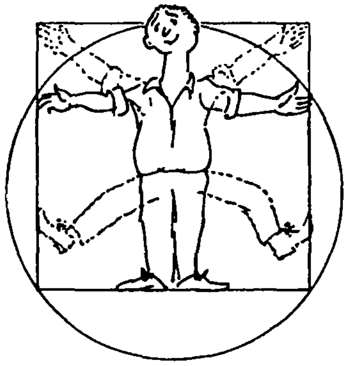

The phrase Organizational Development (OD) has been in recurrent use in the last 25 year or so. It refers to a planned process of bringing change in the culture of the organization both at the system and subsystems levels, subsuming structural changes (Argyris 1964). It is a wholistic approach to restore the dynamism of collective human effort. It is an approach at symmetrization of decisions of various members of the organization which when left unreviewed for long contribute to organizational dissymmetry (Lorsch & Lawrence 1970).

The process of symmetrization is to get the change moving with a dynamic balance in the structure rather than getting it right (in terms of static symmetry), and then reviewing and adjusting the structures and policies as a result of experience (Newman 1973, Balachandran 1987). To describe it in a simpler way, for organizational development, symmetry is perceived and discerned in terms of a dynamic integration among the structural parts of the organization (Kanter 1985).

Symmetry through Organizational Design :

Productive human work requires various resources. The co-ordination of these resources for achieving a purpose is essentially a process of putting order in randomness often expressed as reduction of uncertainty (Brown 1960). This coordination gives the organization a structure.

The function of such a structure is to hold the resources together, put form, consistency, and stability amongst the parts for all of them to be comprehended as a whole. The structure then becomes the regulating mechanism interrelating behaviour of people with each other, with the environment and with organization's objectives (Mintzberg 1983, Melcher 1976). In fact any feature that does the work of delineating, regulating or integrating the relationships among various resources has structural implications for the organizations (Newman 1973).

Many theorists opine that resource integration is a function of organizational design (Argyris 1964, Lorach & Lawrence 1970). Part of this opinion considers deisgn only as one OD approach which aims at keeping the systems consistant with the needs of the organization, the emphasis being put on the grouping of activities; issues of level and spans distribution of power including centralization versus decentralizations and interdepartmental relationships etc. Others would include organizational goals and organization's relations with environment as well in the design. (Maheshwari 1977).

Organizationall Reality & Structural Symmetry :

One of the major difficulties with the organizational structure is the discernment of the form of the structure or what most people would like to visualize as organizational reality. Ignoring the relationships with other resources, most of the time organizational structure is conceived in terms of hierarchy among roles and positions.

Until Elton Mayo (The Human Relations School) came along, structure was perceived essentially in mechanical terms, the symmerty confined to tasks and people. The organization was perceived as a dyad, most reflected in the use of such dyadic terms as employer-employee, line-staff, supervisor-subordinate, and product-function. Other attempts at discerning the structural reality visualized four types of structures; manifest, extant, assumed, requisite (Brown 1960).

The complexity of the structural reality is aggravated by perceived role relationships which in one heirarchical level are conjectured to be at least 6. It is further made complex by the potentiality of the relationships which increase at an exponential rate with increase in the number of members in the organization (Koontz & O'Donnel 1968). Then there are the expectations, preferences, career needs, and political needs of every member with which he relates himself with others (Burns 1969). In other words, perceiving one organizational reality in terms of symmetrical structure is fraught with a lot of assumptions, presumptions and subjectivity (Westerland & Sojostrand 1979).

Asymmetrization : A Constant Factor

Most organizational behaviourists therefore view the structure simply as 'situational' and process'. Activities relevant to the objectives and resources to be used, define the situational structure, and the decision process involved in carrying out those activities define the process structure (Newman 1973).

Decision making, exercise of discretion or making judgment and choice is necessary for coping with uncertain situations. It is also a major area of satisfaction, growth and power for the individual (Jaques 1971). The structural consequences of all decision making cannot always be predicted. Decision making therefore is a factor of constant asymmetrization of structure - a process essential to the dynamics of organization.

Many OD theorists and consultants would specifically add the intuitive and emotional behaviour of people to the process structure, often adding to the complexity of the situational structure as well (Zoll 1974, Argyris 1964). For example, individuals are constantly comparing themselves with others to assess the extent of asymmetry (in wages, status etc. Patchen 1961) between themselves and others.

In fact, any symmetrical configuration that stabilizes creates a perception of power (vested interest) in some, and feelings of powerlessness an others. This starts the cycle of powerless trying to make themselves more powerful, and the powerful trying to conserve or increase their power in order to defend their vested power - a process leading to periods of dissymmetry in the structure.

Many have therefore viewed OD as a process of power equalization, (Galbraith 1983, Mintzberg 1983), an approach shifting from organizational design to individual sensitization (Roger 1969, Argyris 1978).

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Abstract of

Beyond Symmetry, Asymmetry:Peace Within and Without

Ramesh Sheth & Madhuri Sheth

Symmetry enjoins structure and connotes space. Scientific revolution has emphasized understanding through constructs of intellect - essentially space concepts. This has led to extreme geometrization of human being and the universe. In consequence, it has not only resulted in an imbalanced development of the human mind on a vast scale, but has deprived man of the experience of reality that unites him with the universe. It has thereby missed the comprehension of the fundamental creative principle, and has underrated the human potential for variety of creative methods.

The Indian seer, ancient or contemporary, realized an undifferentiated supreme consciousness, a state that does not admit any structure. He conceived differentiated universe of matter and energy, as Purusha & Shakti, emanating from this supreme consciousness not as opposites but complementary principles. These are the basics of original symmetry brought about by a spontaneous wave of creative urge, 'Spanda'.

Human choice is also rooted in 'Spanda'. It is explained by the theory of Guna Dynamics (in fact applicable to both

microcosim and macrocosim), of the three substantive attributes of Sattwa (Symmetry), Rajas (Asymmetry) and Tamas (Dissymmetry). These states are in dynamic evolution from Tamas (Dissymmetry) towards Sattwa (Symmetry) and beyond. Human awareness, if restricted to differentiation and structure, does not realise the potential beyond Symmetry. This potential may involve non-cognitive methods of communication and creativity which could reduce the conflicts within and without.

The Number and the Form in the animate nature

Joseph Shevelev, U.S.S.R.

The advent of body of living objects and the advent of its form is an integral and indivisable event: neither the body nor its form could arise separately. The integral description of this event modeling the advent of the "space of being" (the form) as well as the energetic process of becoming would give us the mathematical law of harmony. The vectorial geometry opens the clue to this problem.

In this report I shall speak about the growth at the elementary level, i.e. about the basis of formgiving. In May 1979, I noted that a straight-line segment divided in the ratio of the golden section ($\phi^0, \phi^{-1}, \phi^{-2}$) and a right-angled triangle of the geometric progression ($\phi^0, \phi^{1/2}, \phi^1$) display the identical interrelation of their parts ($1, N, N^2$) thus representing the same variable triangle. The interrelation between two variables (two sides of the triangle) is governed by a quadratic law, the third quantity remaining constant. It bekame obvious that a straight-line segment divided in the golden ratio is a contracted vectorial triangle. The vector in its twofold nature characterized by direction and quantity represents the movement in terms of both the space (the straight-line segment) and the energy (the force). The observation of other states of that vectorial triangle revealed the unknown symmetry group reproducing the forms of various objects of animate nature featuring the regeneration of individual being cycles. Examples are found in such diverse objects as the apple, the egg, the mollusc shells, the capsule (cranium) containing the brain of mammals. These symmetries are described by the equation $\vec{N}^n = \vec{N} + \vec{1}$, where N and 1 are modules, with $n = \pm 2^{\pm 1}$. The coincidence of indicatrices with contours of the real natural forms is not a random effect. The growing point of an organism is coincidentally the centre of the geometrical construction or the centre of polarity. The cluster of the living matter which enables the process of formgiving is a geometric point, whereas the form of any living object is a bounding surface of the space of expansion attained at the moment by the growing point. The space is the body; the body is the space. The law of squares represents the essential polarity of the elementary event of growing: the k-fold change of the distance alters the surface of a sphere by a factor of k^2.

The following vectors are assigned to the terms $N^2, N, 1$ of the above equation: the singular potency of the point of origin ($\vec{S}$), the vector of the exterior force factor or the potency $(\vec{U})$ and the resultant vector $(\vec{R})$. We can see the dichotomic structure of formgiving. 1. Two kinds of symmetry are being generated: the rounded "female" forms are produced under the prevailing influence of S, the expressed vertical ("male") forms - where U prevails. 2. In S-symmetries, the resultant indicatrix R follows the program $\dot{S}$ (the mechanism of conservation); in U-symmetries, the indicatrices *R* describe the new forms (the mechanism of variability). 3. There are the plus-symmetries presenting the directly proportional casual relation, and the minus-symmetries for the inversely proportional relation. In the plus-symmetries, the value of expansion in orthogonal directions is determined by the golden number $\phi^{\pm 1} = 1,6180339^{\pm 1}$, whereas in minus-symmetries it is a function of the binary golden number constituted by a pair of numbers $\varphi^{\pm 1} = 1,4655712...^{\pm 1}$ and $\Phi^{\pm 1} = 1,7548777...^{\pm 1}$ appearing invariably together. The number $\phi^{1/2} = 1,2720199...$ outlines the space of the symmetry of similarities; it may be considered as a structure based upon the bond $\sqrt{5}$ or $\sqrt{2}$. The number $\sqrt{\phi}$ defines the angle of intermolecular bonds of water which serves as a base for the life and genetics. Moreover, it forms the helix of the Nautilus shell, its curved contour being defined by a set of fundamental constants: 2, ϕ, e, π; it comprises the proportional relations of the equal-tempered musical scale as well as those of the historic architecture. In U,S-symmetries, the golden numbers ϕ, φ, Φ are the radius vectors characterictic for values of expansion. The parameters of curves in their critical points reveal the numbers $\sqrt{1}$, $\sqrt{2}$, $\sqrt{5}$, $\sqrt{7}$. Of a special importance are the numerical values of expansion along the vertical. Their interrelation and relationships with vertical displacements of geometric centres of doublets and triplets in reference to the point of origin are expressed by numbers comprising a row ob basic physical constants (e.g. spin, magnetic moment and rest mass of proton, neutron and electron). The above observations made by the author help to define and to model the concept of WHOLENESS.

The whole - as it pertains to biological structures - is a singular UNITY generating all its parts in succession from ONE base. Let us denominate this initial base as the number OMEGA ($\omega^{\pm 1}$). Then the wholeness at the elementary level is a succession of numbers making up the UNITY in the most simple and natural way. Assume that each subsequent number within the structure of the UNITY is the result of successive multiplication of the base ($\omega^{\pm 1}$) by itself. At the same time we assume thereby the self-regeneration (or replication) of the initial number $\omega^{\pm 1}$:

$$\omega_n^{\circ} = \omega^{\pm 1} + \omega^{\pm 2} + \omega^{\pm 3} + \dots + \omega^{\pm n}$$

We shall consider now, of all the unities $\omega^{\circ} = 1$, one attaining the maximum extent when $n \to \infty$ and the maximum contraction when $n = 2$.

1. From the above equation, it follows when $n \to \infty$ that the base of wholeness, the number $\omega^{\pm 1}$, equals $2^{\pm 1}$ (the dichotomy).

2. When n = 2, the same equation shows that the base of wholeness is the number $\omega^{\pm 1} = \phi = 1,6180339\dots^{\pm 1}$ (the golden number).

Case 1 ($n \to \infty$): it is nothing but mathematic generalization of the growth manifesting itself as unlimited potency to seize the space under conditiones of a finite and confined system (the UNITY).

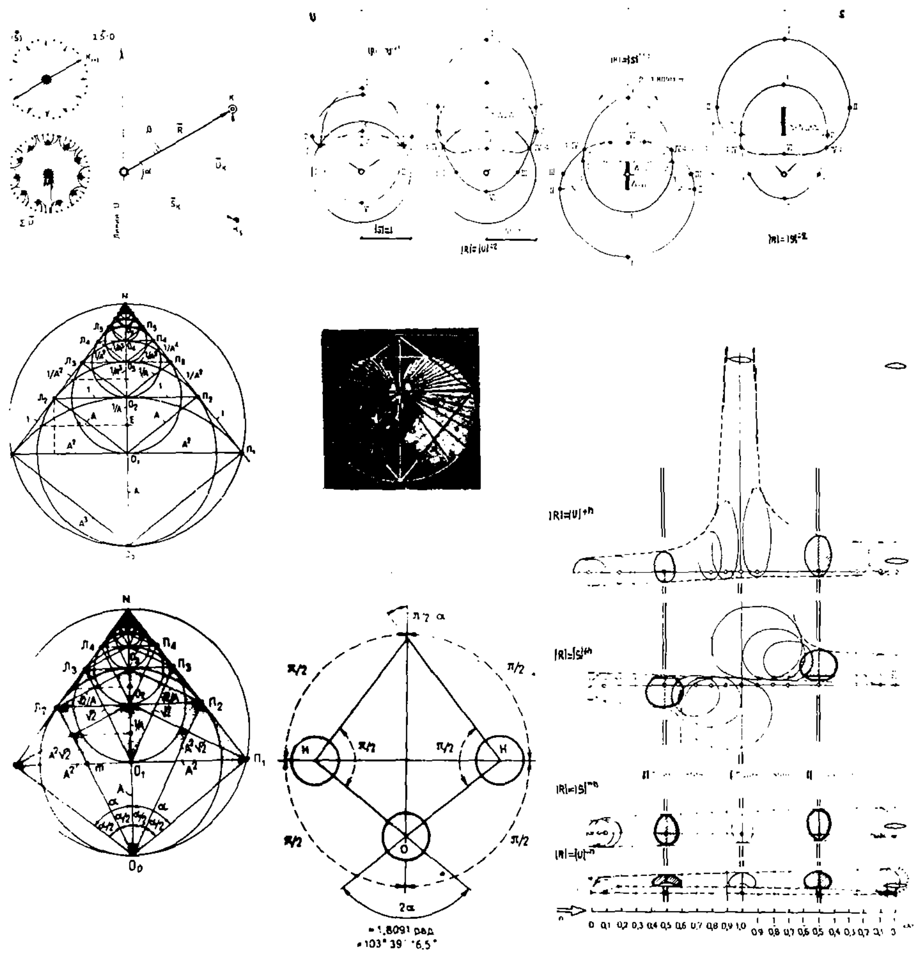
Case 2 (n = 2): a complementary structure, dichotomic by its nature, is revealed.

Thus we learn from the algorithm of wholeness that the potency of growing, essentially, is the dichotomy ($\omega_{n\to\infty}^{\pm 1} = 2^{\pm 1}$), whereas the essense of any dichotomic structure is the golden section ($\omega_2^{\pm 1} = \phi^{\pm 1}$). In actuality, the plus-dichotomy ($1/2^{+1}$) in the nature is division of a cell in half. It is the mechanism of building up the INDIVIDUAL living structures, The minus-dichotomy, on the contrary, represents the conjugation of two sexual cells (the male and the female ones) into ONE; it is the advent of life, the event responsible for the growth of life as the WHOLE both in its continuity and discreteness. As it is widely demonstrated today, the intricate structures are governed by bonds of the golden section.

The principle of the dichotomy suggests the second step of the investigation aiming at the algorithm of wholeness of the second generation structures. Let us establish it by adhering to the Hamilton's principle of least action. It inherits the substance of the algorithm of elementary structures and, coincidentally, opens the way to the diversity of forms of the animate nature: each subsequent number contained in the structure of UNITY is derived by multiplication of the foregoing one by itself in accordance with the given rule.

To obtain TWO extremely contracted unities of the second generation, it is necessary, firstly, to double the number of elements of the structure (now n = 4) and, secondly, to replicate the initial base $\omega^{\pm 1}$, the parts of the whole grouping around it. The algorithm of the wholeness constitutes the groups: (a) the binomial $\omega_2^{\circ} = \omega^{\pm 1} + \omega^{\pm 3}$ where $\omega^{\pm 1} = 1,4655712\dots^{\pm 1} = \text{♀}^{\pm 1}$, and (b) the trinomial $\omega_3^{\circ} = \omega^{\pm 1} \pm \omega^{\pm 2} \pm \omega^{\pm 4}$ where $\omega^{\pm 1} = 1,7548777\dots^{\pm 1} = \text{♂}^{\pm 1}$.

Such is the logical model of wholeness of biologica structures ›incident with the vectorial model of formgiving. It shows also that ıe binomials of plus- and minus symmetries, i.e. the numbers ϕ and , are interrelated by the number $\pi/3$. The energy and the space; the ichotomy and the golden section; the orthogonal and hexagonal crys-al systems as opposed to the animate nature objects exhibiting the olden numbers in orthogonal and hexagonal sections only: they all re bound together, in such a manner making up the indivisible whole-ess.



SYMMETRY IN INTERACTIVE ART

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In this paper I will provide a psychological framework for understanding and appreciating my interactive art works which are themselves artistic statements of perception and cognition.

Before developing these ideas, several distinctions are necessary. First, the discussion will be limited to visual art and visually based art. By visually based art I mean works such as installations and events which might involve other senses besides vision, but which have significant visual components and are rooted in the visual experience. Second, while all art involves an interaction of the viewer with an object or event, interactive art makes explicit this idea and requires the viewer to become more involved than the detached contemplation often associated with the aesthetic experience. In my own work, movement by the viewer activates photocells or other sensors which then, through a computer interface, change the sound or visual environment which is the art object.

The basis of the psychological framework is the symmetrical relationships between forms of stimulus energy outside of the viewer and the mental models of those stimuli in the viewer's mind. A mental model is defined as the viewer's conceptualization or representation of the interactive art piece and the way it works. The axis of symmetry in this case lies at the interface between the viewer and the external world, as shown in Fig. 1.

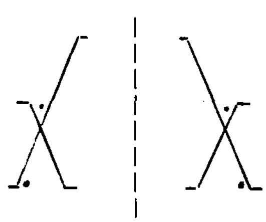

Fig. 1. Schematic diagram of the symmetry between the structure of the external energy and matter on the right and the mental model of that stimulus on the left.

Since the specific types of energy and matter in the internal and external representations are quite different, the symmetry is in the correspondence not only between the representational information of the objects such as the shape of a tree or the redness of a piece of rope, but also between the formal underlying structures of the two representations, such as the vertical or the horizontal orientation.

To make the discussion more concrete, I will describe an installation that I was commissioned to do in 1987 for an outdoor public arts festival. Fig. 2 is a sketch of the piece.

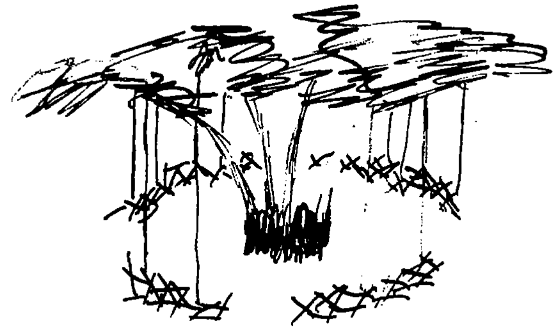

Fig. 2. Sketch of interactive art work, Mayfair Network

The central element was a willow tree, 35-40 feet high. Surrounding the tree was a ring of about 80 logs piled together about 3 feet high, with several gaps in the ring so that viewers could enter and walk around inside the piles of logs. Strands of red rope were hung from the tree over the logs and were draped among them. Inside the ring there were photocells connected to a small computer that could generate sounds in eight speakers - four in the tree and four among the logs. I envisioned it both as a playful piece of art and as a metaphor on the nature of our perceptual experience. Viewers inside the logs and ropes were generating the sounds as they activated the photocells by their movements. In this way they created their own inner nonspatial world while looking out over the logs and through the red rope at the spatial "reality" of the external world. Some immediately either accepted it as a playful art work or got an intuitive sense of the inner-outer world metaphor. However, the dominant initial response was to try to figure out how the piece worked, and then experience it as an art work. Some viewers discovered the photocells and so understood the mechanism. Many others, however, developed the idea that pulling on the red ropes caused the sounds and even told friends to go over and make sounds with the ropes.

Clearly the work was about the symmetry between inner and outer worlds. Recognition of this symmetry added to the understanding and appreciation of the piece as art. But can we go further and try to understand more about the nature of the structure of these worlds and the symmetrical relationships? Can a mental model be a work of art? How do these phenomena fit with other work on mental models?

Norman (1983) described a number of characteristics of mental models based on work with devices such as calculators, cameras, watches and aircraft. To use his terms, these models are "incomplete..., unstable..., unscientific..., and parsimonious"(p. 8). Furthermore, they "do not have firm boundaries" (p. 8). The terms, incomplete and unscientific, certainly describe some of the mental models developed for Mayfair Network. These models were based on the casual and uncontrolled observation that pulling on the ropes was often followed by sounds. The people assumed that there was a causal relationship, without testing the alternate hypothesis that people inside were generating the sounds. They developed "superstitous behavior" from the sequence of events and persisted in it even when it did not always happen. Several people did not seem to believe me when I explained about the photocells. The pulling model was more parsimonious and simpler than photocells and computers. Based on these informal observations we might assume that some of the salient features of the art work which were symmetrically mapped into the mental model were representations of the red rope and the contingency between the pulling action and the sound. I would consider this mapping as part of the work itself.

Going one step further we can ask about the type of mental model that might be operating. One candidate is the strong analogy model (Young, 1983) which emphasizes the role of familiar devices in understanding new devices. Applying this to interactive art, we might want to know to what extent Mayfair Network was viewed as analogous to an art work or to a piece of playground equipment. Both seemed to be operating in different people although the conditions which produced these different models was not altogether clear. I will be expand on this discussion with other art work and other models.

I will also discuss the need to incorporate structural properties of interactive art works into mental models, based on Arnheim's theory of visual vectors and balance (e.g. Arnheim, 1983; 1988). The information presented above was primarily representational about what was perceived to be there: trees, rope, etc. Structural information, such as symmetry within the work, the roles of vertical, horizontal and diagonal lines in creating visual balance, etc. are all factors that are important means of communication between the artist and the viewer. They would therefore seem to be involved in creating mental models.

In summary, I have tried to show that in order to fully appreciate interactive art it is necessary to understand the nature of mental models. Viewing interactive art as a form of symmetry between the art object and the mental model may help us to understand these processes and to design better art. In addition, this unique conceptualization provides an added layer of meaning to the aesthetic experience. The models are part of the total art work as a process piece, and they live on after the physical object is gone.

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THE NUMBER OF DIVISIONS OF A FLAT SURFACE INTO EQUAL-SIZED ELEMENTS. CONTINUAL INTEGRAL

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The problem of counting the number of divisions of a two-dimensional surface into preassigned equal-sized elements is considered. If it is impossible to cover the surface with the given elements, this number goes to zero. In the present report the continual integral for the number of divisions is constructed and then used to formulate the criterion for the equality to zero of the sought-for number of divisions.

The lattice of nodes is constructed on the two-dimensional surface. The number of nodes is sufficiently large such that each element has the same even number of nodes, and if these elements consist of nonconnected regions, the number of nodesin each region should also be even. The requirement for evenness follows from the method of construction of the continual integral for the number of divisions. Each region or element is represented as a set of nonoverlapping dimers which cover two nearest nodes, and a separate weight is assigned to each dimer.

In the paper /I/ the continual integral is constructed for the number of possible dimer coverings of the surface with allowance for the assigned weights:

$$Z=\int\{dV\}\exp\sum_{ij=1}^{M,N}\left(-a_{1,i,j}V_{i+1,j}V_{ij}(-1)^{j}-a_2V_{i+1,j+i}\cdot V_{ij}(-1)^{j+1}+a_{3ij}V_{ij+1}V_{ij}\right.$$

where ij - are coordinates of the node at the lattice, V_{ij} are the Grassmann variables, is the weight allocated to the $d_{3,i,j}$-the dimer with the ij-coordinates. Using this integral as a generating function, one can construct the continual integral for the given elements. Really, each element was represented as a set of nonoverlapping dimers. The corresponding pairs of Grassmann variables commutate with each other which permits each element of division be brought into correspondence with the product of Grassmann variables in the continual integral for the number of possible divisions:

$$Z = \int \{dV\} \exp \sum_{i,j=1}^{M,N} \sum_{f} W_f \prod_{s=1}^{2m} V_{i+\Delta s,f,j+\delta s,f}$$

where 2 m is the number of lattice nodes per element, W_f-is the weight allocated to the element of a given type.

From the equality to zero of the continual integral over the Grassmann variables it follows that there exists a transformation

$$V_{ij} = \sum_{a,b} \omega_{ijab} U_{a,b}$$

that leads the integrand containing the Grassmann variables V_{ij}, to the expression with a smaller number of variables $U_{a,b}$; the integral over these latter integrals will be equal to zero by definition /2/. The above consideration enables us to formulate the criterion for the equality to zero of the number of divisions:

If there exists a set of numbers ω_{ijab} such that

$$\sum_{\{p\}} \sum_{ij=1}^{M,N} \sum_{f} \omega_f \prod_{s=1}^{2m} \omega_{i+\Delta s,f,j+\delta s,f;\,\{a,b\}} = 0$$

the division of the surface into given elements is not possible.

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SYMMETRY IN THE INDIAN CONTEXT - A critique

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EXTENDED ABSTRACT

"Symmetry" is a word that has acquired over the centuries a variety of connotations and dimensions in a wide diversity of setting. The countries with a vibrant cultural heritage like India can boast of nuances of "Symmetry" of its own; Indian ethos has shaped, through the centuries, in evolving facets of symmetry. While in some distant past, notions on symmetry in the Indian context carved out in a distinctively original manner, it is only, in recent times, one finds a fairly close confluence of such notions with those that have a contemporary character. The purpose of this presentation is to glean through the Indian past, aspects of symmetry without being bothered that the relevant terminology was somewhat different and also to bring to the fore how such evolving wielded tremendous influence may be somewhat unknowingly and unconsciously on formats that contemporaneity and posterity might warrant for.

Indian culture, steeped as it is in religion, could hardly find a medium for its expression other than those religious in nature. Indeed, the essence of religion being dominantly a way of life, there used to be manifestations of intellect and creation largely through media of structures e.g. temples and even practices associated with this which one may now a days look askance at, for being too ritualistic in nature. This becomes obvious even through a cursory glance; even an apparently mundane religious ritual may smack of micro-existence of forms and structures, fairly symmetrical in nature, which if blown up succinctly, could emerge into respectable macro-forms and structures. For example, the altar which used to be a regular feature in the Vedic times and often even now, as a forerunner of religious activities, if scanned critically, does exhibit in no uncertain terms a symmetrical character. Here was a symmetry that could be consummated elsewhere in many other facets that would, indeed, go to the total make up of the activity. Often, symmetry *per se* would be a pervading theme. Any symmetrical structure would necessarily call for reflections about an entity, a line or a plane and the necessary concommittants are parts that are, in this kind of parlance, described as the image of one of the other or vice versa. The Indian structures e.g., temples are replete with architectural designs that seek to build upon symmetry; as a matter of fact, a

ritual grid would unfold itself, through definite proportions in respect of measurement without losing the aesthetics. One can cite copious examples cutting through diverse regions of India, architectural forms that reflect in a profound manner thought-processes, intrinsically symmetrical in nature.

The science of Sulvas (the rope - geometry) which formed the crux of geometry in ancient India is full of forms that speak of symmetries in abundance. The unity in the midst of diversity a much-talked about Indian theme is found, on critical assessment, to be built into symmetrical forms. While one may not find replicas of symmetry of ancient Greece or elsewhere of the Western World in the Indian setting, the bid for perfection in the design of symmetry continued and even continues today to be unabated. Here also, measurements, based on apposite and meticulous proportions, seemed to afford methodologies for the purpose of construction based largely on symmetry. The locations of the deities, in respect of their hierarchies, were often made out in symmetrical forms and there is a host of such examples in the Indian situation. The Indian sarees (dresses for ladies) even today have borders that exhibit symmetry in abundance, which strictly speaking is a translation symmetry, which, in turn, is also ~~been~~ brought out with a highly aesthetic version in decoration that go with religious ceremonies. The expanding format of symmetry does necessarily bring about complexity but the processes of embellishment at every stage is hardly glossed over. The Konaraka of Orissa, Kajuraho in Madhya Pradesh bear ample testimony to this. Symmetry is found interwoven with texture of life, even at the risk of being dubbed as amorous or lussous. Symmetry does not remain as something banal but grows out of inessentials again with the aid of proportions in respect of measurements. Holistic yearning holds way. Thus, in any kind of exercise on bringing out symmetry, there is a back and forth mobility between reality and extra-reality, i.e., transcending the reality bordering often candidly on divinity; The fusion achieved thereby may appear to be enigmatic but it does reflect a remarkably sustainable evolution. The Indian view of symmetry is not necessarily symbolic in the artistic sense, as often invoked in the western world. It is fully embedded in the Indian thinking which seldom draws a demarcation between what we find in reality and what we should have beyond reality. Reality has thus in the Indian setting an analytical continuation, to borrow over from mathematics - from which symmetry, to Indian practitioners over the ages, could be hardly immune.

Coming to developments at present, there is no need or to read into or to graft into, the modern concept of symmetry in the Indian setting. Indian setting, has developed moorings of its own which feel in conducively well with contemporary notions of symmetry. Indian efforts to look for geometrical forms through dissections even to the extent of shapes with fractional dimensions, bear close proximity to what have coming to be known as fractals. Sulava science is full of ample illustrations of this sort. As already mentioned, the exercises that make use of dissections in the geometrical arena, would seek the basis on notion for symmetry. Transformation geometry, a spin-off of thought-processes on symmetry has thus acquired a dinstinctive character in the Indian context. Thus, in brief, the search for symmetry as practised in India, apart form being an intellectual activity, is a reflection in a wider, canvass, on the Indian ethos and culture and built up through the centuries and continuing to respond to the urges and imperatives of the present.

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THE INVARIANTS OF THE SELFORGANIZATION IN THE OBJECTIVE OF THE DIALECTICAL LOGIC

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The human projects which ignore the great laws of nature bring misfortune alone. The present-day reality fraught with the threat of ecologic collapse is regrettable illustration of this thought, which is going back to Heraclite yet.

The great laws of nature pointed out by the thickers of the past are, first and foremost, the laws and principles of the dialectics which govern the world harmony and stability of its structures. The level of modern knowledge makes it possible to define them concretely in the methodological field of sinergetics which, being a bearer of an idea, plays the part of the scientific outlook.

The principle of splitting the entity is known to be the essense of the dialectics. Splitting of the entity into two measurable opposites, $E \rightarrow E_{\nu} + E_{\mu}$, is a general universal mechanism underlying structural genesis at all levels of matter organization.

The nature is full of the splitted entities, it is possible to say that the nature consists of these wholes everywhere: kynetic and potential energy, gravitations and electromagnetism, North and South poles, fauna and flora, eucariots and procariots, male and female of animals, poikilotermal and homoiotermal organisms, regional and branchy sectors of the economy, basic and turnover funds, economically active and passive population, production sphere and service and so on.

How does the process of optimum proportioning of splitted entities-self-organization as self-harmonization in biological, geological, ecological, economic and other reality structures - is realized? The large scale of the entity (universum) may be taken as unity. Then, within its boundaries, the measures of the interacting opposites obey the conservation law: $\nu + \mu = 1$. By virtue of the internal connection of the entity, one of the measures is subordinate being the order parameter: $\nu = f(\mu)$. Such is the sinergetic reality of the splitted entities /1, p.61-62/. According to Gegel, the law of measure development is the power laws /2, p.431/: $\nu \sim \mu^1, \mu^2, \mu^3$ ... The combination of two relations gives the generator of invariants of the development and self-organization of natural systems: $\mu^k + \mu - 1 = 0$. For the integral powers, exactly such a situation obeys the simplicity principle and thus is alone taken into consideration, the development invariants or the knots of the linear measure accurate to the 3 decimal places are 0.500; 0.618; 0,682; 0.725; 0.755; 0,778; 0.797; 0,812... Being referred to as the generalized golded strong-point sections, they express the optimum proportion of the parts achieved by the whole or universum as a splitted entity in its self-organization process. These points are the attraction attractors of the integral characteristics of systems on the scale of their relative values.

The invariant values of the measure indicate a natural development process not burdened with pathology. The pathologic nature of this process and the state of the developing system might be judged about from one more evolutional series: 0.570; 0.705; 0.741; 0.767; 0.788... It is produced by the same generator but with the power at a maximum distance from the integral values,i.e. 3/2, 5/2, 7/2... : $\mu^{k+1/2} + \mu - 1 = 0$. The values of the integral function of the whole equal to one of these "antiknots" or disharmony points at the scale of relations would indicate the degeneration or degradation, stress or climax, destructurization or disharmonization process undergone by the system. They are the pushing attractors.

From the point of view of a science we are interested in such splitted entities, which have universal all-theoretical sence and are at the same time the summarizing characteristics of the studying material. Such are the dynamic and stochasticaly laws of the universe, symmetry and asymmetry, order and chaos, uniformity and diversity. Entropia is used as the measure of chaos, disturbances, diversity. The redundance is the measure to organize the whole, the limitation of the diversity, "monolitically" structured matherial:

$$R = 1 + \sum_{i=1}^{n} p_i \log_n p_i$$

The Prigogin's principle works when there are states which are situated far from equilibrium, that is to say, when the densities of the structural components of the whole are equalized: $p_1 = p_2 = \dots = p_n = 1/n$. According to this principle the production of entropia dH/dt in a given system, which is connected with the current value of the produced measure H, the other words, in computation per unity of the value this characteristic, production of entropia reaches minimum level just in the stationary states of the system:

$$\frac{d}{dt}\left(\frac{I}{H}\frac{dH}{dt}\right) = 0.$$

This, obviously, is valid for the redundancy too, the production of the redundancy reaches maximum level under this states. That is why the modules of both constants $\frac{I}{H}\frac{dH}{dt}$ and $\frac{I}{R}\frac{dR}{dt}$, according to the simplicity principle /3/, should be distinguished on the basis not more, than an integer multiplier. It is so, because the relative increase or changles in both measures should be commensurable with each other. This is the prerequisite of the harmonization of the components of the splitted entity. Here comes, that $H = R^k$.In the combination with the law of preservation $H + R = 1$ we have a equation: $R^k + R - 1 = 0$. The redundancy values, coinciding with one of invariants 0.500, 0.618, 0.682... characterising the organizational measure and lewell of limitation which is preserved in the variety system, show, that there is a structural harmony in the system. This harmony is more favourable for the full realization of its possibilities. The transition between such states is performed by a loap /4/.

Adducing the example we shall remark the words of J.Kasti: "When there are such parameters which macke the systemic shift from one attractor to another the catastrophes take place" /5, p. 140/, and the words of Y.Demek: "The invariants which come one instead of another, are representing the stages of the evolutionary process" /6, p. 203/. It is known that biosphere here is selforganizing system.

Ex.: Chemical changes in the planet atmosphere (in parts of a unit) are the genetic reasons of the biosphere catastrophes:

Epoch*	Nitrogen	Oxygen	Argon	Carbon dioxide	Neon	Helium	R
A	0.67	0.32	0.008	0.002	0.00002	0.00001	0.618
B	0.74	0.25	0.008	0.002	0.00002	0.00001	0.654
C	0.78084	0.20948	0.00934	0.00031	0.00002	0.00001	0.683
D	0.81	0.18	0.01	0.09	0.00002	0.00001	0.705

*A - Mesozoic era; B - Transition period: Mesozoe-Cainozoe; C - Nowadays; D - Future forcast: the climax of Technogen.

The percentage of oxygen in the Mesozoic era (32%) was concluded according to the analysis of the air from the amber dated to 80 million years ago /7/. Both in the Mesozoic era and Nowadays there are correlating values of the measure of the organization R: 0.618 and 0.683 correspondingly. These states witness about the structural harmony of the whole. In the transition period after Mesozoic and before Cainozoic eras, there was, for an obvious reason, transference the measure R through the disharmony point 0.654. This fact can be confirmed by the amber specimens from the time horizonts, at that very period the dinosaurs and the other giant reptiles died out.

As a result of burning of all resources of the organic fuel, extracted from the Earth, the oxygen percentage in the air may fall off to 18%. At that period the measure of the organization R reaches the point of disharmony 0.705 which is lying on the knot measure line. At that period there will be the climax of chaosogenity in the biosphere, new life conditions under which the animals couldn't survive. It is under the question too, if there will be a place for a man on the Earth.

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Symmetrical Structures in the Ancient and Medieval Turkic Poetry

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A theory of the origin of poetry, created by the eminent Russian philologist A.N.Veselovski, who also coined the term of "historical poetics", is based on a wide-ranging study of the Eurupean, Asian and Australian folklore, as well as the Graeko-Roman, West European and a section of Oriental literatures. According to this theory, poetry had emerged from a single primitive culture and gradually developed into a distinctive art. Eventually, the distinctive kinds of verbal art, epics, lyrics and drama, evolved from the primitive syncretism of folk poetry. The primitive poetry was linked with ritual. At first, it performed extra-aesthetic functions, and was recited in a chorus. The language of poetry was evolving on the basis of "psychological parallelism", which produced the main elements of the figurative speech: comparisons, metaphors and poetic symbols.

The "psychological parallelism" was also the basis on which the symmetrical structures emerged in poetry, because poetry is distinguished by lexical, phonetic, morphological and syntactic parallelism at all levels of poetic language. Rhythm is based on symmetrical recurrence of strong and weak (long and short, stressed and unstressed) units of verse forming the metrical arrangement of feet in two or more lines of poetry. Rhyme is a phonic symmetry at the ends of lines. It has two function, the rhytmic function (stresses the rhythm of verse) and the stanzaic function (combines the lines of poetry into stanzas). Poetical figures are symmetrical structures based on various types of symmetry, including rhythmic symmetry, rhymic symmetry and semantic constructions symmetry. These general definitions are applicable to the theory of poetry of any nations. However, in various cultural traditions they have distinctive features explained by phenomena which transgress the limits of poetry and literature as a whole.

The history of evolution of poetic forms in the ancient and medieval Turkic literatures offers a striking example to above thesis. The ancient Turkic literature, found over a vast territory of Central Asia, Southern and Western Siberia, includes poetry in the runic, Manichaean and Uighur script. These are either specialized religious texts of prayers ang hymns, or texts of more common usage - epitaphs and epic (historico-heroic) poems, created from the mid-8th century (although the Turkic poetic tradition undoubtedly emerged earlier) to approximately the 13th century. The ancient Turkic literature belongs to the pre-Islamic period of history of the Turkic peoples and reflects the religio-cultural environment (pre-shamanic, shamanic, Manichaean and Buddhist) within which it was evolved. The language of the ancient Turkic poetry was based on specific linguistic norms because, as is known, any system of versification (the rhythmic and phonic arrangement of verse) emerges on the basis of phonetic laws of a specific language. In the ancient Turkic poetry, phonic arrangement of verse is constituted by the alliteration system. It is a verse with symmetrical rhythmic and euphonic structures at the beginning of the lines. In contrast with anaphora, which only is a means of poetic decoration in any national poetry, the ancient Turkic alliteration system constitutes a regular device for stressing the rhythmic pulse of the verse (word or horizontal alliteration) and for combining the lines into stanzas of various length (vertical or stanzaic alliteration). The alliteration system, fixing the symmetrical rhythmic structures at the beginning of the lines was compulsory for the ancient Turkic poetry.

The beginning of conversion to Islam of the Central Asian Turkic peoples in the mid-10th century opened a new era in the development of their culture and aesthetics. With the spread of Islam and Islamic culture in Central Asia, Asia Minor, Transcaucasia and the Volga Region, the Turkic peoples adopted the Arabic writing and language of the Qur'an and aquainted themselves with the classic Arabic and Persian literatures. The creation of the classic Turkic poetry started in the 11th century. Now, this poetry was formed on the basis of aesthetic ideas which were different from those of the ancient Turkic poetry. It was explained by the spread, in the Turkic milieu, of the Arabic and Persian poetics, including the theory of meters called 'aruz', the theory of rhyme and the theory of poetic figures.

With adoption of aruz (the quantitative versification), the Turkic poetry was obliged to turn from alliteration system to rhyme at the ends of the lines, because any type of versification is based on the correspondence between the rhythmic and the phonic arrangement of the verse. For this reason, the ancient Turkic tonic-temporal versification (the irregular recurrence of the lines, with the unequal number of syllables, balanced by the period of utterance) combined with the alliteration system, was in the classic poetry replaced by rigidly fixed aruz meters and the exact rhyme at the ends of the lines.

It was difficult to write poetry by laws of versification created on the basis of Arabic and Persian languages. For this reason, beginning from the 11th century a series of devices were created to adapt the aruz to the Turkic language. Eventually, the balance was established between the intrinsic properties of the Turkic language and the laws of poetics borrowed from the alien linguistic and cultural tradition. Therefore, the shift in fixing of the rhythmic symmetry of verse by the phonic symmetry from the beginning of the lines (alliteration system) to the end of the lines (rhyme) was explained by ideological changes occuring in the Turkic culture.

In dealing with the replacement of one system of versification by another system during the transition from the epoch of the ancient Turkic literature to the epoch of the classic Turkic literature one notices the increasing number of symmetrical structures in poetry. The Arabic-Persian theory of poetic figures, the number of which is great, is very elaborate and formalized. There are semantic (verbal), phonic, and graphic (based on the beauty and distinction of Arabic characters) poetic figures. There is usually a definite correspondence between poetic figures and rhythmic structures of the verse. Some of the figures are designed to stress the melody of verse by sound similarities (phonic symmetry). Murā'āt an-nazīr, for example, is called the "double symmetry". This semantic figure uses the names of uniform things or similar notions.

The classic poetry seems to be encased in a shell of symmetrical correlated structures. This impression is created by the entirety of poetic figures, types of rhyme and radīfs, as well as rigid fixation of meters in aruz. In the ancient Turkic poetry, symmetrical rhythmic and phonic structures are arranged in a relatively free manner; for example, the alliteration at the beginning of the verse may be extended to two, three or more lines, or be repeated every other line at random, according to the writer's plan. In the classic poetry, in turn, there is a compulsory correspondence between the type of rhyme and the laws of genre (masnavi, ghazal, etc.).

The elaborate theory of poetry, created by several generation of the Arabic and Persian scholars, and adopted by the Turkic literature, shows the attitude of Islamic culture to poetic language. The creation of poetry by the laws of this theory suggests that profound changes occurred in the psychology and aesthetic ideas of the Turkic peoples.

For this reason, the symmetrical structures of the Turkic poetry mark cultural epochs in the history of the Turkic peoples.

THE SYMMETRY OF THE QUINCUNCX

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In his seminal book Hermann Weyl makes just two tantalising references to "Thomas Browne's quaint account in his Garden of Cyrus (1658) of (...) 'quincuncial' symmetry".

During the early 1970's, as a sideline to my main research, I looked for examples of the aesthetic use of this relationship and found only two. The first was the familiar "knight's move" in chess, the second was in certain paintings by the Swiss Constructivist painter Richard Paul Lohse.

I explored some of its implications in a small number of paintings and drawings begun at that time, and this research was carried much further in many subsequent works, including relief constructions, by my (then) pupil Richard Bell.

Five is, in any case, a notoriously difficult number to accomodate aesthetically and the symmetry of the quincuncx is also quite distinct from the D5 (pentagonal) symmetry to be found, for example, in some compositions by Poussin.

Some of these problems will be introduced using slides and diagrams.

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SYMMETRY OF THE EARTH SURFACE AND SOIL SPACE STRUCTURES.

1. The aim of the present work was to carry out the cartographic analysis of the Earth surface and soil space structures using the symmetry theory. Accordingly, it was necessary to present the topographic map, a mathematic model of the Earth surface, which has a form of continuum of horizontal lines, as discontinuum. For this purpose the continuum of topographical map isohypses was transformed into the discontinuum of new relief plastics maps by means of additinal lines (morphoisographs). These lines were drawn according to Menier, they follow the normal and go to the horizontal lines, uniting the meanders of the same curvature. The identity of the original (isohypsic) and modified (morphoisographic) forms of the Earth surface is proved by the presence of invariants.

2. Indivisible systems consisting of two symmetric subsystems convexities and concavities are shown in the relief plastics map (Fig. 1). With convexities painted one can get a twocolour model of topographical or soil space having symmetry properties. These models possess interesting varieties of symmetry similar to those of antisymmetry.

3. The relief plastics maps have been compiled for the vast territories of the USSR including all the natural regions and zones. It allowed to find the differences in the symmetry of pattern structures in every significant geomorphological region, type and subtype of soils, as well as the peculiarities of their periodical recurrence. The soil and rock composition was not taken into account as

in crystallography while studying the symmetry of patterns. It is important to calculate the number of possible forms of the Earth surface which exist in nature, to determine their geometrical peculiarities and to find the symmetry of their spatial-temporal relationships as well. Thereafter it is possible to study the correlations of natural forms of the Earth surface taking into account the composition of rocks and soils. So, the correlations found allow to examine on the symmetry basis the properties of rocks and soils as well as a type of mineral deposits.

4. Polygonal, curvilinear and branching treelike structures are given in the relief plastics maps and soil maps (Fig. 2). Any relief is based on a treelike pattern appearing in the weakened (fatigned) zones of the Earth crust and having a morphological basis in the form of a hydrographical network. The treelike patterns are classified according to the symmetry of borders, they also have the forms of open and closed graphs (Fig. 2 D,E). Their rounded "crowns" form circles, ellypses, spirals. They are situated at the nodal points of the plane crystallographic lattice (Fig. 2 B) and their spatial arrangement has a lot of symmetrical, polygonal and other structures of various hierarhical levels (Fig. 2 C). The forms are inserted into each other thus forming the symmetry of similiarity.

The Earth surface is a complex spatial and temporal arrangement of symmetrical (and assymmetrical) forms (Fig. 2 A). The Earth crust block surfaces of different age are sometimes exposed at the day (apparent) surface and therefore their original relief as well as the weathering soils and crusts of the same time period can be studied structurally.

The relief of different age has specific patterns of symmetry. Long - term evolution of the planet resulted in topological changes of the relief patterns. The direction of these changes can be found by analysing strange attractors. Soils, the youngest natural bodies of the Earth, follow the relief borders. This indicates that the soil space formation is going on still actively. If the time of origin of any soil series or relief is known,the arrangement of the Earth crust symmetrical forms of different age can be drawn on the map. The time when one form is transformed into another is indicated by dislocations manifested morphologically.

5. The analysis of the symmetry-dissymmetry of the Earth surface forms allows to conclude that the patterns of the relief and soil space can be characterized as dissipative structures during geologically long period of time: they change periodically their forms and spatial correlations between them depending on the quantity and quality of energy obtained. The relief patterns like living beings increase and dicrease their dimensions, are divided or merged, forming rectilinear or curvilinear lines in different directions. These changes are not casual and are likely to be purposeful.

Probably, as a result of fighting against entropy the Earth has developed a special mechanism for receiving, accumulation, transformation and redistribution of energy due to topological adaptation of the most mobile structures of the relief and soil space to economically more profitable energy consuming forms, e,g. spirals.

CONCLUSIONS. It is suggested to construct cartographic models of topographic and soil surfaces on the mathematic basis instead of ordinary maps depicting relief and soil peculiarities. These models based on the symmetry theory can be applied to establish the regularities of systematic-structural correlations.

FIGURE LEGENDS.

Fig. 1. Comparative estimate of sketching on location (A), topographic map of the same locality, the relief forms of which are presented by the curves of the horizontal lines (isohypses) (B), relief plastics map, where the same locality is given in another form (the convexities are shaded).

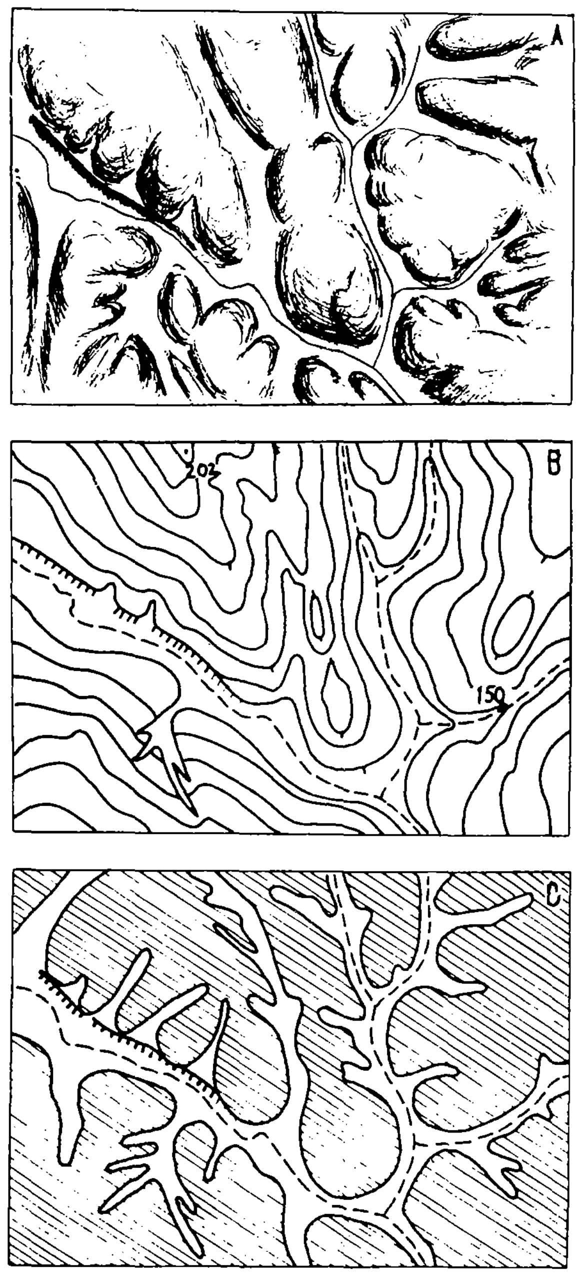
Fig. 2. New relief plastics maps show the structures of the Earth surface and soil space.

A - Vortical structures of the relief and soil space on the territory of the Urals and Kazakhstan.

B - Hexagonal structures of the Earth surface and soil space on the territory of Moscow Region. The picture of optical diffraction (Furier transformation) obtained from the relief plastics map in the Institute of Crystallography USSR Academy of Sciences.

C - Spatial patterns of various types and subtypes of soils in the USSR obtained by means of the relief plastics map compiled on the basis of optico-structural analysis.

D, E - patterns of soil space in the vicinity of Pushchino, Moscow region, drawn according to the relief plastics map at a scale of 1:50 000. D-soddy podzolic, E-grey forest soils. The patterns differ in the form of treelike structures.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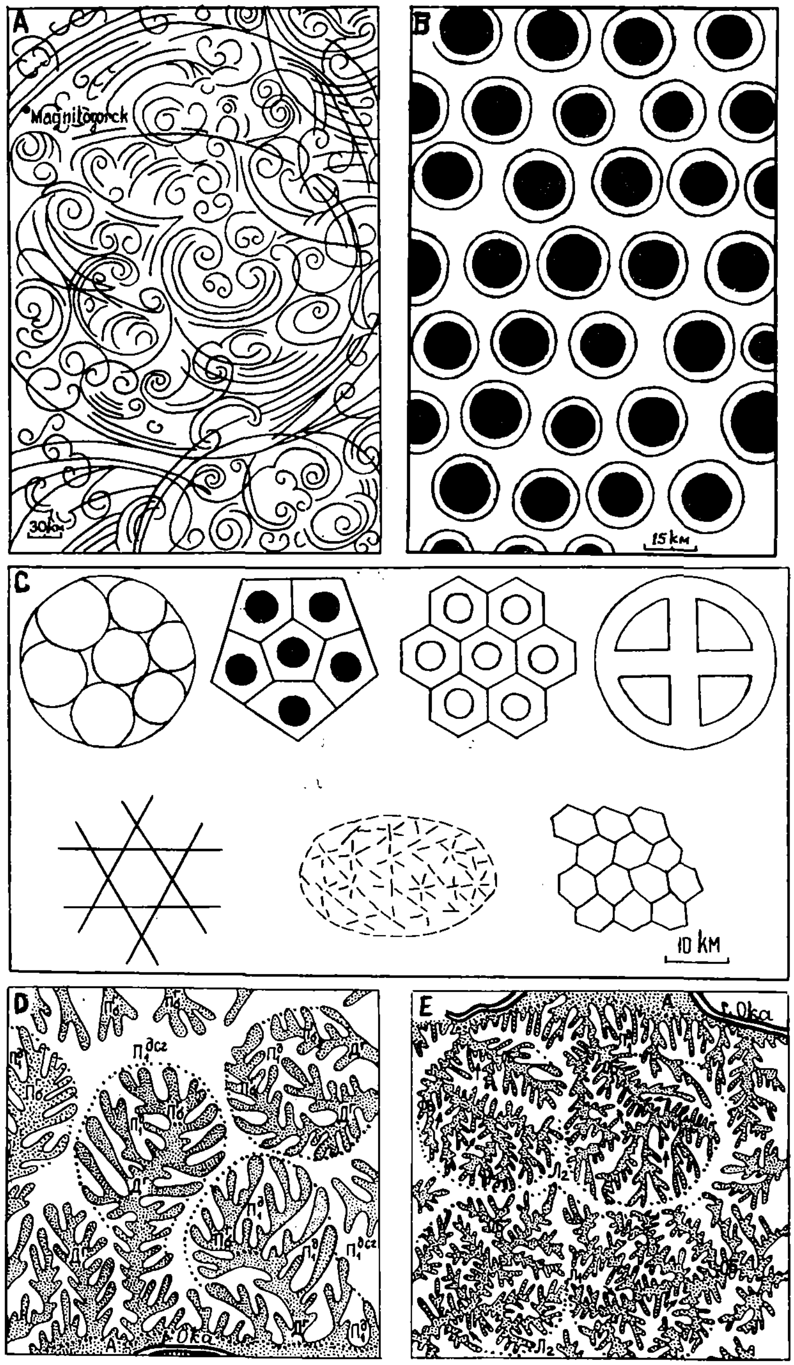
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SYMMETRY AND MOLDAVIAN MELOS

(measure, scale, mode)

The order of musical language forming has been searched for since antiquity. At that time such elements as sound and its composition, intervals, scales and their typology, tonality, ways of modulation and the rules of melody construction were being developed. They all refer to some extent to symmetry. However, the term itself does not appear in musical treatises of that epoch. Instead such concepts as rationality, proportionality, parallelism repetition, etc. are used. The term symmetry actively penetrates into musicology in the second half of the nineteenth century.

By now musical science has achieved certain results in revealing the symmetry in the structure of musical form, harmony, polyphony, rhythm, melodics and other means of expression. However, they are inadequate and sometimes contradictory. This may serve as an explanation of the fact that none of the existing musical encyclopaedias has the article on symmetry in music.

Measure(taktus) belongs to the number of main categories involved in this problem. Originally it meant a unit of measure. Later it was provided with power properties characteristic of musical language which can not be referred to a unit of measure. We mean the so-called metric times and the ways of distribution of various kinds of accents. Measures introduced alien elements into the natural melodic flow."Hard" and "light" times were more often missing or placed regardless of the rules of measure theory. Due to this reason the second function of the measure, such as the carrier (accumulator) of melodic power properties was rejected in the previous century at least in musical ethnography. The first function that measures and makes melody writing and reading easier causes no objections. But while writing down a melody from a phonogram various measures(bars) are chosen even by higly qualified experts. Which of the bars is the optimum? The solution of the given problem lies on the way of searching for invariant elements of the rhythm of the three syncretic arts: poetry, melos, and dance. They were first mentioned in Aristoxeni "Rhythmicorum elementorum" (IV B.C.). That is chronos protos(primary time) and its derivatives: chronos disemos, chronos trisemos, and chronos tetrasemos, long undeservedly forgotten by musicology.

Oscillographic analysis has corroborated the presence of the above mentioned time categories common for syllabic rhythm of the singing folk poem and kinetics of folk dance in Moldavian folklore. Their approximate average value is 150, 300, 450, 600 milliseconds. These values are changed in live performance but only within the range of zonal nature of the tempo and rhythm of the given genre. They form a symmetric group the basis of which is arithmetical progression: 1.2.3.4. (correlation of elements).

Various combinations of these times lead to the generation of invariant formations at a larger structural level. Thus, symmetrical binary formulae: chronos disemos + chronos disemos and chronos tetrasemos + chronos tetrasemos are the bases of Moldavian folk dance rhythmics of "Syrba" and "Khangul" respectively. The other binary formula: chronos trisemos + chronos disemos is the basis of "Khora" folk dance. It illustrates the symmetry (equality) of two proportions in golden section when the relation of the whole to a larger patr is equal to that of a larger part to a smaller one, or vice versa; this can be roughly shown in a fraction 3/2 (2/3). Isochronous and one and a half (hemiolios) correlations are combined in ternary formula: chronos disemos + chronos disemos + chronos trisemos, presenting the rhythmics of "De zestre" wedding dance.

The above-mentioned formulae, three binary and one ternary, are the smallest measures which most naturally dismember the melodies of the respective genres. Their values are determined after the quantity of primary times (chronos protos). Depending on the last note designation, a quaver (♪) or semiquaver (𝅘𝅥𝅯), for four rhythmical formulae we will get the following fractions: 4/8, 5/8, 7/8, 8/8 or 4/16, 5/16, 7/16, 8/16. This is a mathematical expression of measures as symmetrical structures.

There are song genres not correlated with dance. They comprise such recitative forms as "bochet" (mourning over the deceased), ballads and "doinas". They cannot be divided into bars. They have a different form of symmetry which appears at a larger structural level; the subject, song poem, rhyme, strophe, etc., which are more or less described in literature.

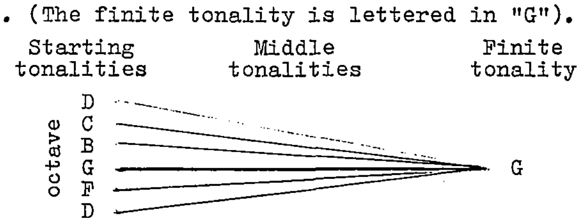
Another important aspect in the theory of musical language causing differences in folk-lorists' opinions is the problem of scale and mode. The absence of complete and clear differentiation comes from lack of knowledge of the symmetric nature of these atructures.

Thus, the scale is defined as the sequence (upwards or downwards) of all the sounds used, for example, in a song melody.

This definition is of one-sided character and is not correct in general. It does not alow for the fact that it is the function of the tone that serves as an organising element but not the tone itself as an acoustic factor. With this in view a musical scale is not a pure sequence of tones, but that of tone functions. In a melody tone functions often change. For example, in a folk song after modulation from the major key into parallel minor key one and the same tone material in the first key forms one sequence of functions in the other key it forms another sequence. Such an approach to these structures turns the scale into a symmetrical group , where every interval-function is a sum or a difference of the other two. The number of tonal transitions in a song (up to five in a Molavian song) is equal to the number of symmetry groups,i.e. scales. Up to now it was considered that there is only one scale there.

The symmetrical nature of a scale of any structure gives rationality to music space and turns it into a category of people's musical thinking. Synoptical comparison of over a thousand Moldavian folk songe showed that all of them were grouped around six main tonal systems. Each of them has two invariant elements, i,e, the starting and the finite tonality. The arrangement of the middle tonalities in a melodic strophe varies from one song to another forming a series of congeneric formations, the whole complex of which is named by us as a m o d e p a r a d i g m.

Taking as a basis an interval separating the starting tonality of a song from the finite one we can get the following names for each of the six mode paradigms: subfourth, subtonal, of the same name, third (three semitones), fourth and quint, as shown in Fig. I. (The finite tonality is lettered in "G").



The peculiarity of the system consists in the fact that a melodic line developing in any of the six mode paradigms is sure to come to "G" point of octave golden section. Each of mode paradigm tonalities presents an intonational sphere with a number of standard melodic formulae that were historically created in the melos of the given ethnic unity. They are often interchangeable and that is a condition for variability, on the one hand, and the way of symmetry manifestation at the level of syntax formation functions, on the other.

Mode paradigms were formed, as seen from Fig. I., on the stages of anhemitonic scale representing as any other musical scale a group of symmetry. From here follows the symmetric nature of the whole of the tonal system of a folk song.

The system of the above-mentioned rhythmical structures and mode paradigms is the basic element of genetic foundation of the Moldavian melos kept in the people's memory and passed on from one generation to another, orally, just as verbal language.

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THE MECHANICAL MIRROR-IMAGE: AUTOMATA AS DOPPLEGÄNGERS

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Jean Eugène Robert-Houdin, who founded modern stage magic in the second half of the nineteenth century, once retreated to an isolated cottage for eighteen months in order to make a special automaton--a self-moving image of a human being. Before becoming a magician, Robert-Houdin had been a highly skilled clockmaker who often had occasion to repair very elaborate clocks and automata; nevertheless, he claimed in his memoirs, his technical skills were stretched to their limit by the task of creating a mechanical figure that could write and draw in a realistic manner. For the figure to be truly lifelike, it had to replicate not only human actions, but also human appearances--a requirement that Robert-Houdin at first found difficult to meet. The face of the automaton was critical to the success of his illusion. It was vital that the features be expressive and individual.

Robert-Houdin rejected the heads that he had commissioned from two different artists as too wooden or vacuous before he decided to take the task into his own hands. His first effort at sculpting without a model was unsatisfactory; though the face he made had regular (dare I say, "symmetrical?") features, it lacked character. Needing a model in his isolation, he turned to a mirror and beheld the character he sought. Even though he says that he studied his reflection carefully as he carved, he insists that when the automaton was complete, he was surprised that he had "unconsciously produced an exact image of myself." His surprise turned to wonder as he set the automaton in motion. Tears came into his eyes, he tells us, when he asked it "who is the author of your being?" and saw his creation "fix an attentive glance on the paper" while its arm, until then "numb and lifeless," began to write his own signature.

Now this is a very curious story, regardless of whether or not it is true (and we can be fairly sure that Robert-Houdin, a consummate showman, wrote his memoirs with more concern for their effect than for their veracity). The magician tells us that he deliberately set out to copy his own image in the mirror, yet once he had completed the work, he expresses surprise at its resemblance to himself and tells us that he has captured his likeness "unconsciously." Once he sets the automaton in motion, he finds the figure coming to life before his eyes. His subsequent elation is so great when the figure produces his own signature that he makes it sign his name "a thousand times."

Where does Robert-Houdin's surprise come from? Why does he suggest that the capture of his own image was an unconscious act? How can we appreciate the significance of the moment in which the automaton takes on Robert-Houdin's sign of identity, his signature, in an act that simultaneously signifies that Robert-Houdin himself has been transformed into a god-like figure as the creator of a new being?

In Freud's sense of the term, the moment is "uncanny," as the familiar suddenly appears in an unfamiliar light and the unfamiliar takes on an aura of familiarity. At the moment when Robert-Houdin completes his work he suddenly recognizes it, as though he were seeing himself from the outside. His experience is similar to one that many of us have had when we turn a corner or glance around and glimpse an oddly familiar stranger. Suddenly, we recognize ourself; then on second take the image resolves into nothing more than an unexpected reflection. Houdin's experience is more intense, however, because the autonomy of the image persists beyond that first, uncanny instant, as though his image had been removed from the mirror altogether and brought to life.

In this respect, Robert-Houdin's story is very similar to a number of novellas and short stories, particularly popular during the Romantic era, in which the main character must cope with the consequences of being suddenly dispossessed of his mirror image, or, in some cases, his shadow, only to discover that the image has acquired a separate existence in his own world. The image is confronted as a second self, which often becomes a rival for love and for status in the world. In many of the stories, the main character becomes so obsessed with his second self that he can no longer cope effectively with the world. In most cases he is reduced to trying to destroy the image, only to find that in doing so, he has destroyed himself.

These stories belong to the same genre as that of the double or dopplegänger. Various literary, psychological and anthropological studies of the double have discussed the double's relations to ideas concerning the human soul, our attitudes towards death, and the problem of narcissism in human relations. In this paper, I will be discussing the ways in which the history of automata exhibits the themes of the double. I will demonstrate how certain automata might be considered to be images or doubles of their makers. In particular, I will focus on the social and cultural implications of constructing mechanical mirror images of ourselves. I will be arguing that such independent doubles are potentially disruptive in social terms, unless we assign them to special categories that fall outside ordinary social-structural patterns. The automaton, then, functions most often as a liminal figure in human culture, marking the places where we must cope with the juncture of two different states of being.

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Klára Széles

Symmetry and Structure in Art

/In the work of art and in the oeuvre, in aspect of interpretations/

A work of art /literary and another work/ can be approaches in several ways. The possibilities, of course include various methods of professional analyses from the point of view of linguistics, semantics, psychology, cultural history etc. In any case the symmetry /assymetry, dissymetry/ plays an important role.

1. Symmetry /assymetry: gramatically, - in literal sense and figuratively -; rhetorically, rhytmically, semantically.
   " " in images, metaphores, paralellism, repetitions, motifs, archetypes and mythological elements and compositions.
2. The hypothesis is: all kinds of symmetries enumerated are realy connections /internal and external connections; connections in an work of art; connections between an individual work and an individual oeuvre; between these and the historical, cultural continuity etc.
3. These connections properly speaking correspond to some, certain networks, systems.
4. These networks, systems /of symmetries/ are essentially implications.
5. The term: implication /implying and being implied/ is
   a./ in the sense that it may condense the characteristic elements of entire lifework and

suggests the governing principle of the pattern that oeuvre has taken, the principle of its functioning;

b./also seted in the sense that there are open or hidden implications with reference to the way the work in question and the entire oeuvre fit into the whole of literary /artistic/ and cultural history;

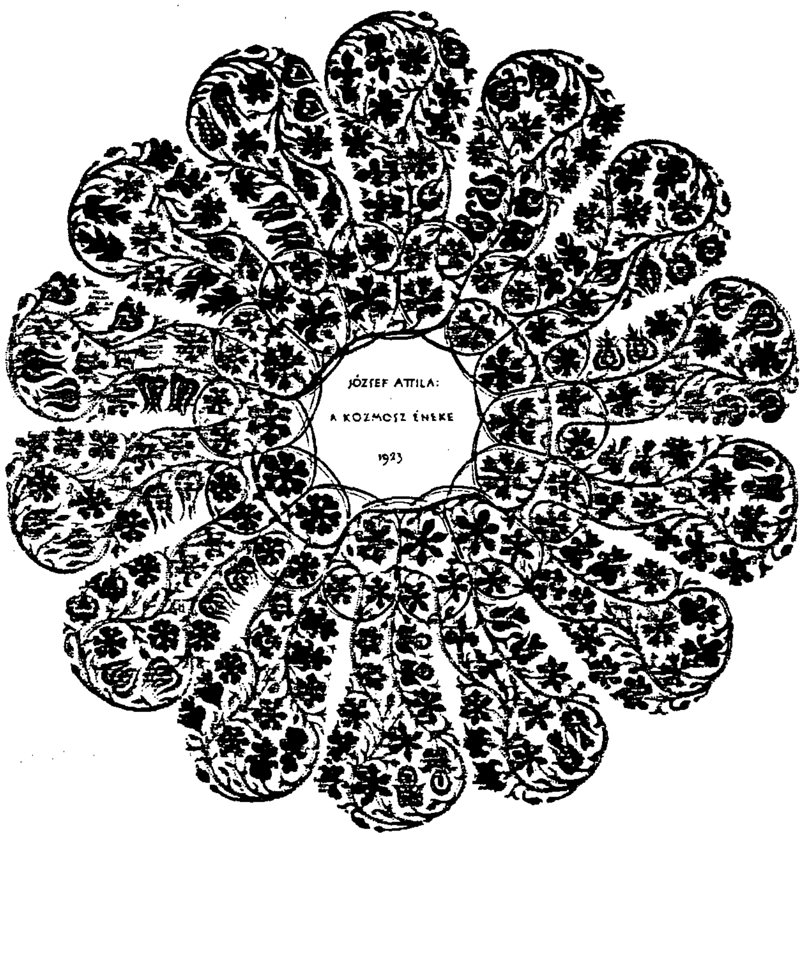
c/ In this way we find microcosmic and macrocosmic references which are intrinsic to the text /or another work of art/ and also detect implications which lie outside of it, implications which are part of the linguislis plane and hence which are only suggested by it; we come across the incidence of synchronous or diachronous associations ~~xxx~~ each either in a static or dynamic manner.

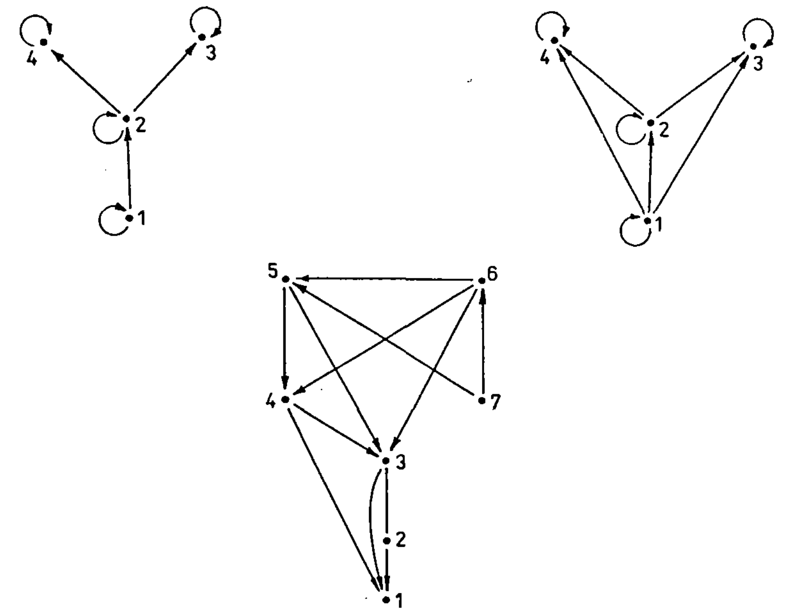
d/ Thus implication /implying and being implied/ is an integration of various simmetries /assymetries/.

In conclusion: the exposition will be the unravelling of the symmetries and symmetrical systems that is at least latently there in the work /in the ouevre etc./, the presentation of the system or relations that make up the texture of the work /the ouevre etc/. Regardless of their methods and approaches, good interpretations of good works reflect as an "imitational/imitative activity" /R. Barthes/ - the laws that that call to life works of art and make them function as such.

This order of ideas is an continuation my former studies: "An attempt at setting up a model for literary analysis" in: filológiai Közlöny, Literatura, Essays in Poetics.

II. Lotz János: József Attila: A kozmosz éneke:





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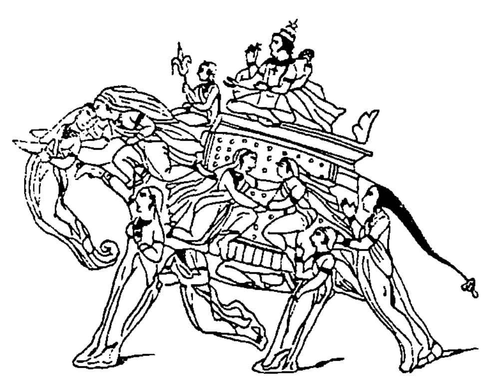

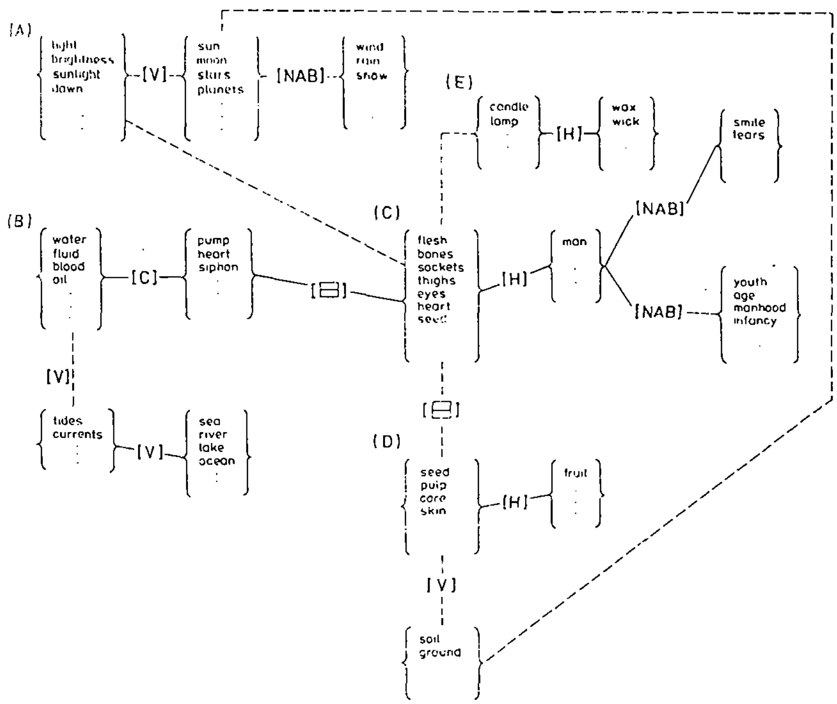

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SOME PROPERTIES OF ALGEBRAIC SURFACES WITH SYMMETRIES

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Let G group be finite and irreducible in substantial m-dimentional Euclidean space E^m. The group is generated by orthogonal reflections. N is a number for all hyperplanes with a common point of intersection, their reflections belong to G. I^G is an algebra of group G invariants.

Polinomials below are invariant to G,

$$\theta_{2t} = \sum_{k=1}^{N} \eta_k^{2t} , \quad t \geq 1 , \qquad (1)$$

where $\eta_k = 0$ are normalized equations of symmetry hyperplanes; θ^G is algebra of such invariants. Actually there are some known different methods for obtaining $I_{\eta_i} (i=\overline{1,m})$ generators in algebra I^G (Coxeter, 1951; Flatto, 1978; Ignatenko, 1980, 1984). Ignatenko V.F. found that not all generators of even degrees can be represented by (1) if $G = F_4 , B_m , D_m$. And the problem arises for finding generators $\theta^G \neq I^G$.
With its solution a number of results (Ternovsky, 1983, 1988) has been obtained which is represented in the following table :

G	ℓ	d_i
$[N]$	2	2, 2N
A_3	3	2, 4, 6
A_4	5	2, 4, 6, 8, 10
A_5	10	2, 4, ..., 18, 30
B_{12}	$24 \leq \ell \leq 144$	2, 4, 8, 10, ..., 48, 54, ...?
D_4	9	2, 6, 8, 10, 12, 14, 16, 20, 24
F_4	5 .	2, 8, 12, 18, 24
E_6	7	2, 6, 8, 10, 12, 14, 18

where ℓ is a number for all generators in algebra θ^G, d_i $(i = \overline{1,\ell})$ are their degrees; in B_{12} case only a number of all generators has been found.

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ANCIENT MEXICO, A WORLD OF ORDER; BUT ALSO A WORLD OF SYMMETRY?

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Paul Kirchhoff, the eminent investigator of ancient Meso-america, wrote in his unedited notebooks:

"Ancient Mexico is a world of order, in which everything and everybody has his proper place .... All things have their place because thus it has been prophesized. Architecture and calendrics are structuring principles: the calendar is a two-fold structuring principle, with time and with space. These cultures do not know chaos."

We found the orderly structure of ancient Mexico in the reality not only in the architecture but also in the orientation lines in the landscape, in the planning of towns, ceremonial centers and of settlements with their surrounding fields. The four solstitial points were of fundamental importance. They provided the basis for the conception of cosmological space. This concept is represented on the symmetrical day sign "olin", (see Fig.1), very clear designed in Olmec times but also to see in the center of the Aztec Calendar Stone. The solstices explain the

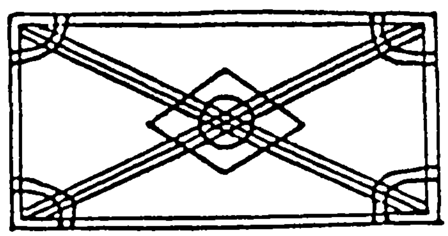

Fig. 1: Olmec engraving on a stone celt from La Venta, Tab. Mexico, identified as a complex representation of the cosmic ideogram (after Koehler 1982, fig. 4).

clockwise deviation of about 25°, that orientation lines show from our usual cardinal directions, see the ground plan of the pyramid and town of Cholula. There are other deviations as for example at Teotihuacan with $16\text{-}17^{\circ}$ and the Templo Mayor of Tenochtitlan and the ground plan of the city with 7°. Although other lines had been measured in the pre-Columbian architecture, their total number was too small to draw unambiguous conclusions. Another method was more successful, the measurement of the churches, i.e. the buildings succeeding pre-Hispanic ceremonial structures (Fig. 2).

The frequency distribution permitted to identify a sequence of angles with regular intervals of $4\text{-}5^{\circ}$: 2°, 7°, 11°, 16°, 20° and 25°. There might have existed an underlying angular unit of 4.5° or 5^{g}, i.e. 1/20 of a right angle. We see the geometry of an angle observation system for the Central Mexico area, based on the sunsets in the summer half-year (Fig. 3). This is <u>the order of space</u>. The geometry of the system did not seem to be an sufficient explanation for the orientations observed. So the attempt was made to find an orientation calendar, the <u>order of time</u>.

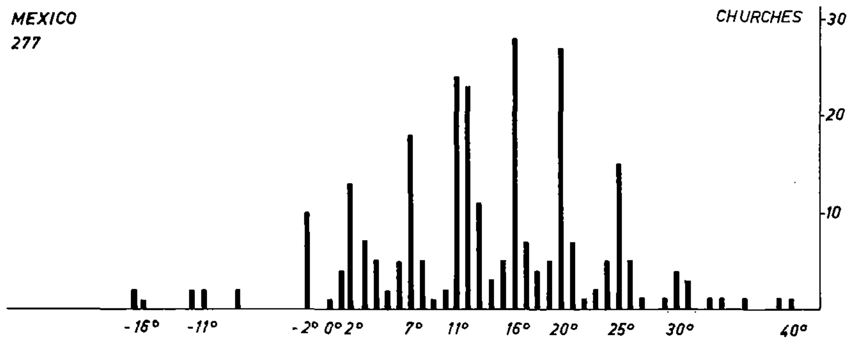

Fig. 2: Histogram showing the distribution about astronomical east of the axes of 277 churches in the Mexican basin.- From Tichy 1982, p.65.

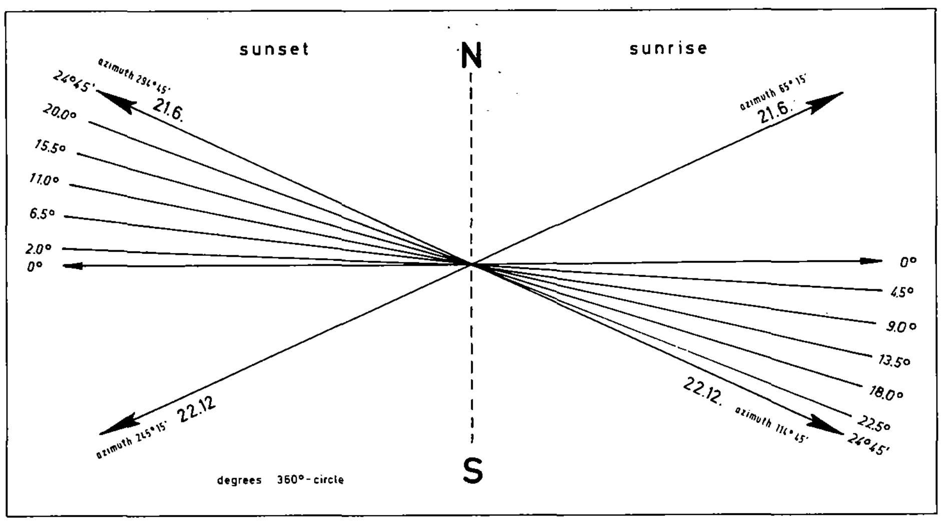

Fig. 3: Diagram of the Mesoamerican Orientation System. The alignments are in accordance with the hypothetical solar orientation calendar with 13-day weeks. The clockwise deviations from East represent the "Winter Sunrise Maya Type" and the deviations from West the "Summer Sunset Centralmexico Type".

There remaines the possibility of a fixed, i.e. a solar calendar according to the known mexican calendar which contained 18 periods of 20 days and 5 days rest. But that permits to explain only two of the direction lines. The calendar does not include the frequent direction of 11-12° nor the zenith passage and solstitial directions.

The direction lines found to be frequent in the Maya area we ordered to their numerical sequence. There is an angle series of 1°, 5°, 9°, 14°, 18°, 22°, also with regular intervals of 4-5°. These values are rarely found outside the Maya area.

Now raised the question of the intervals in days at which these angles occur at the horizon as solar positions. It was sufficient to consider the data for the declination of the sun. The result was more or less 13 days. We know a 13 day week, used in ancient Mexico and there existed also a calendar with 13-day periods but in form of a rotational calendar. A fixed calendar with 13-day periods had to be checked. The year beginning has been placed on the day of the winter solstice. This makes the calendar valid for the whole of Mesoamerica.

What are the properties of the hypothetical orientation calendar? It is very close to the actual motion of the sun. The periods coincide exactly with the solstices and with the days between the solstices, the socalled mid-year days. There exists also a coincidence for the zenith passage days at the latitudes of 15° and 21° n.L. It contains the important days of the agrarian year and it contains all the direction lines between the solstice points which are frequent enough.

The question raises, if there is a strong symmetry in the structure *of the geometrical scheme and of the observational* calendar. We see the symmetry in the course of the sun with the varying values of the declination, the distance from the celestial equator representable with a sine curve (Fig. 4). Where

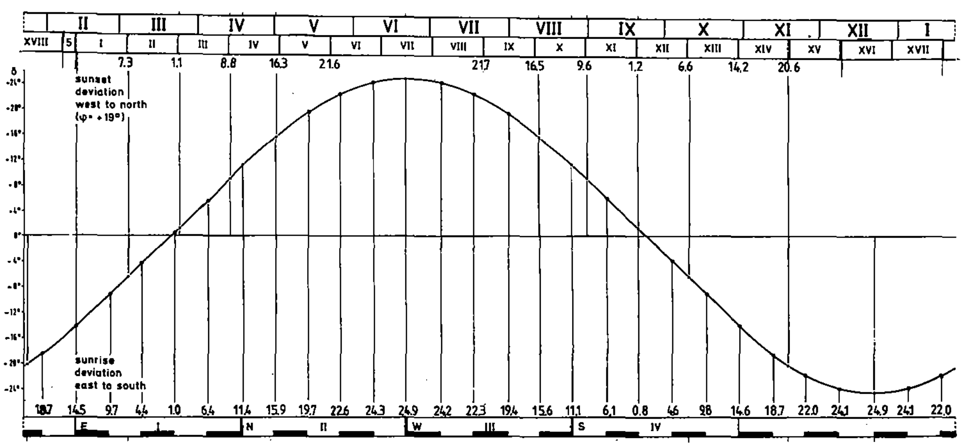

Fig. 4: Hypothetical solar orientation calendars with 20-day periods (above) and with 13-day weeks (below). Vertical: Declination of the sun. Horizontal: Deviation of azimuths on the horizon of 19° N.L.

the ascending and descending branches are more or less rectilinear there we see the coincidence of 13-day periods and angle values of declination and at the same time the angle values of azimuths in the horizon. But the sequence of the direction values is different in the winter half-year and in the summer half-year. In the winter half-year we find the sequence of the Maya area and in the summer half-year the sequence of Central Mexico. The reason for the asymmetry is the unequal motion of the earth around the sun. On January 2 the earth is in her nearest position, the Perihel, and on July 2 at the greatest distance, the Aphel. In the winter half-year the motion is faster than in the summer half-year.

Symmetry and asymmetry we have for that reason also in the angular series and in the orientation calendar. The great order of the mesoamerican world is based on the solar symmetry and asymmetry observed in pre-Columbian times.

Symmetrical structures of great importance we see in the spezialized assemblages of maya buildings able for astronomical observations. There is the group E of Uaxactún/Guatemala, but also Central Mexico has in Xochicalco a structure of this kind in the group C and D with the "Estela de los dos glifos" in the center. The axis is oriented to the sunsets of the midyear days. Other orientation lines allow the observation of sunrises and sunsets at the solstice and zenith passage days and other important days of the calendar. Such symmetrical structures give expression to the unity of the order of time and space given by the course of the sun.

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Duality, Nonlinearity, Asymmetry and Symmetry in Lattice Dynamics

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In my study of lattice dynamics, I have been helped by the ideas such as duality, nonlinearity, asymmetry and symmetry. I would like to present how I met these ideas, how they led me to some discoveries, and to see that many theories can be related by these ideas. I was first led to (1) the concept of duality which relates different one-dimensional lattices with the same frequencies of normal modes. In turn, it led me to (2) the discovery of integrable nonlinear lattice with asymmetric interaction potential. Recently I am interested in (3) a lattice with asymmetric potential and asymmetric kinetic energy (symmetric with respect to kinetic and potential energy terms), which describes asymmetric time sequence, such as stochastic process, chemical reaction, and some environment problem.

1) One-dimensional dual lattices

One of the problems I was interested in lattice dynamics was the effect of impurities on the normal mode frequencies (spectrum) of a linear lattice. We sometimes met different lattices wih the same spectra. In this connection, I found the following duality theorem [Toda 1965]:

If we replace masses by springs, and springs by masses, we can obtain another lattice with the same spectrum as that of the original lattice, if certain relations are satisfied between the masses and the force constants of the springs.

These lattices can be symbolically as

A) .....K U K U .....

B) .....U*K*U*K*.....

with kinetic energy terms K,K*, and potential terms U,U*:

$$K_j=\frac{p_j^2}{2m_j} \qquad U_j^*=\frac{k_j}{2}(s_j-s_{j+1})^2$$

$$U_j=\frac{k_j}{2}(x_j-x_{j-1})^2 \qquad K_j^*=\frac{1}{2m_j^*}r_j^2$$

We note that the kinetic energy is symmetric with respect to the momentum p_j or r_j, and the potential energy is symmetric with respect to the relative displacement x_j-x_{j-1} or s_j-s_{j+1}. The condition of duality (equivalence) of two lattices A and B is

$$\ldots\ldots m_1k_1^*=m_1^*k_1=m_2k_2^*=m_2^*k_2=\ldots\ldots$$

The simplest equivalence of dual lattices can be achieved by the replacement

$$\frac{1}{m_j}=k_j^*, \quad \frac{1}{k_j}=m_j^* \ ,$$

$$p_j=s_j-s_{j+1}, \quad x_j-x_{j-1}=r_j \quad .$$

2) Nonlinear lattice

The idea of the dual lattice was extended to nonlinear lattices [Toda 1966]. For simplicity we consider a uniform lattice with the kinetic energy term $K_j=p_j^2/2m$, and the potential energy term

$$\hat{U}_j=\phi\,(x_j-x_{j-1}).$$

the corresponding dual lattice has the potential energy term $U_j^*=(s_j-s_{j+1})^2/2m$, and the kinetic energy term

$$\hat{K}_j^*=\phi\,(r_j)$$

^ means ninlinear. $\hat{K}$ is a kind of kinetic energy with a mass which depends on the momentum r_j of the lattice B. We see that the lattices A and B symbolically expressed as

A) $\ldots\ldots K\ \hat{U}\ K\ \hat{U}\ K\ \ldots\ldots$

B) $\ldots\ldots U^*\hat{K}^*U^*\hat{K}^*U^*\ldots\ldots$

are equivalent (behave the same).

Because the lattice B seemed simpler than the lattice A, I worked on B, and was able to find an integrable lattice [Toda 1967].

This lattice has the asymmetric potential

$$\phi(r) = e^{-p} - 1 + p$$

It was found that this nonlinear lattice is perfectly integrable [Lunsford and Ford 1972, Hénon 1974, Flaschka 1974].

3) Lattice asymmetric in time and space

Recently I am interested in a lattice with asymmetric kinetic energy as well as asymmetric potential energy. This lattice can be expressed symbolically as

$$\ldots\ldots \hat{K}\ \hat{U}\ \hat{K}\ \hat{U}\ \hat{K}\ \ldots\ldots$$

The dual lattice is the same to the original lattice (self-dual). Motion in this lattice is asymmetric in time and space. That is, the motion to the right is different from the motion to the left. This strange behavior comes from the asymmetricity of the kinetic energy term.

Further, it turned out that this lattice is equivalent to a system considered by Kac and Moerbeke [Kac and Moerbeke 1975] as a model for a certain stochastic process, and to a special case of the Lotka-Volterra model [Lotka 1956, Volterra 1931] for cofl-icting species, or environment problems.

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ROTARY SHADOWS FROM THE p-DIMENSIONAL HYPERSPACE

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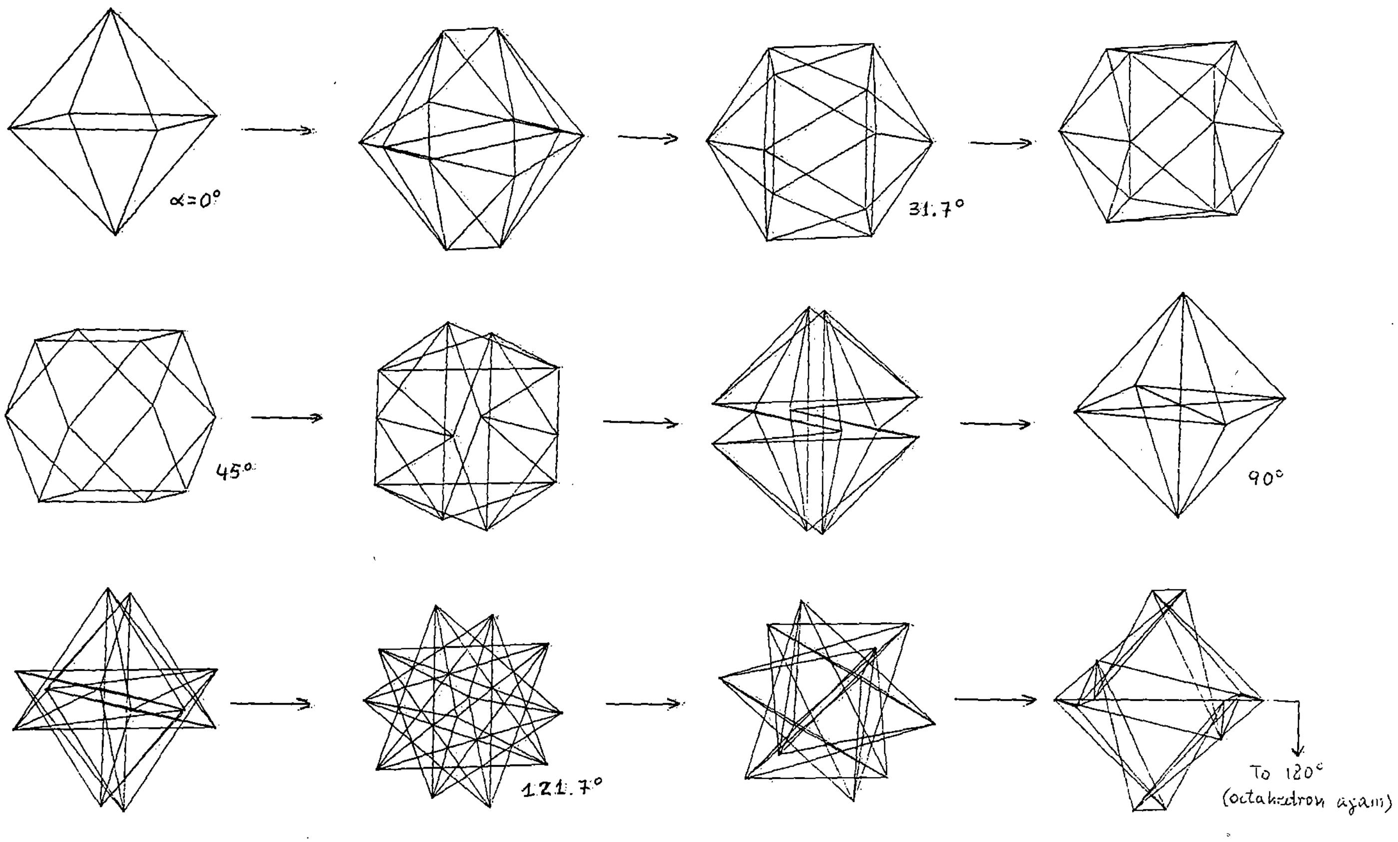
In a recent paper, we found some continuous evolutions that connect the radial skeletons (center to vertices directions) of basic polyhedral forms [1]. Afterward, taking into account the works of Hadwiger and Coxeter [2] about hypercrosses shadows falling on the ordinary spaces E^2 and E^3, we have extended our above mentioned work in order to connect in a continuous way the icosahedral and cubic symmetries and orders [3,4].

Here, we use our "rotary shadow method" [4] to generate two appealing geometric evolutions. We begin finding variable vectors half-stars in a rotary subspace E^n which represent the orthogonal projections of half-crosses defined in the hyperspace E^p, p>n. So, according to the theorem of Hadwiger [2], we start from variable half-stars with p vectors which preserve the equation

$$\sum_{i=1}^{p} v_{i\mu} v_{i\gamma} = \frac{1}{n} \delta_{\mu\gamma} \sum_{i=1}^{p} v_i^2 \quad ; \mu,\gamma = 1, \ldots, n \quad , \tag{1}$$

where $(v_{i1}, v_{i2}, \ldots, v_{in})_{i=1,\ldots,p}$ are the components of the p


α=0°
31.7°
45°
90°
121.7°
To 180°
(octahedron again)

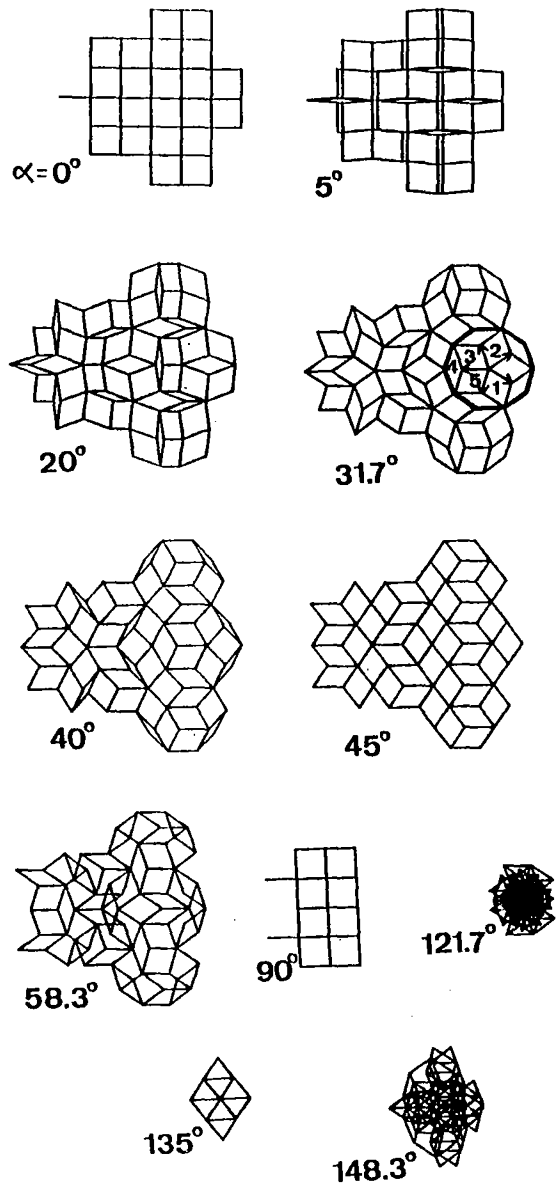


Fig. 2

vectors $\{v_i\}_{i=1,\dots,p}$ in the rotary subspace E^n.

The α-variable half-star $v_1=(c,-s,0)$, $v_2=(c,s,0)$, $v_3=(0,c,-s)$, $v_4=(0,c,s)$, $v_5=(-s,0,c)$, $v_6=(s,0,c)$, where $c=\cos\alpha$ and $s=\sin\alpha$, preserves eq.1 (with p=6 and n=3). By drawing an adequate set of edges which join the vertices of this α-variable star, the octahedron metamorphosis can be seen. Fig.1, where star vectors are omitted, schematically shows this cycle which connects in a continuous way some notable polyhedra (octahedron, icosahedron, cuboctahedron and small stellated dodecahedron)

The α-variable half-star $v_1^*=(c^2,-s)$, $v_2^*=(c^2,s)$, $v_3^*=(-s^2,c)$, $v_4^*=(-2sc,0)$, $v_5^*=(-s^2,-c)$ preserves eq.1 (with p=5 and n=2). When $\alpha=31.7174°$, this half-star coincides with the five pentagonal directions of a Penrose tiling [5]. When α varies, the half-star and the Penrose tiling also evolve in a continuous way. Fig.2 shows an evolutionary cycle of a Penrose tiling patch. This cycle contains a singular state of extreme folding or reversible collapse ($\alpha=121.7174°$).

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PROPOSAL FOR LECTURE (ABSTRACT)

"THE PREHISTORIC ROOTS OF A HUMAN CONCEPT OF SYMMETRY"

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The development of a human concept of symmetry is traced from the earliest bipedal hominids through the evolutionary progression to anatomically modern humans Homo sapiens sapiens. Examination of the material culture of wild and (especially) captive chimpanzees suggests a very primeval concept of balance and symmetry, and suggests that the biological foundation for this cognitive trait is quite ancient; it is believed that humans and common chimpanzees had a common ancestor between five and ten million years ago, and that modern chimpanzees may exhibit more of the primitive ancestral cognitive characteristics.

Evidence from the earliest stone age (the Oldowan) shows that hominids between 2.5 and 1.5 million years ago, including Homo habilis and Homo erectus had mastered the basic principles of flaking stone by percussion, including the intuitive geometric understanding that an acute (less than ninety degree) angle was required to strike off flakes with a stone hammer. Although many of the "core tool" forms (e.g. choppers, discoids, polyhedrons, core scrapers) can be arrived at during flake production, with little or no attention to core form, there may be a rudimentary concept of bilateral symmetry in some of these core forms as well as in retouched flake elements (flake scrapers, awls.) The most symmetrical form occasionally found in the Oldowan, the spheroid (an almost perfectly spherical battered stone) is probably not the result of intentional fabrication but rather a by-product of its long-term use as a hammerstone.

Between 2.0 and 1.5 million years ago there is also evidence of preferential right-handedness in early hominid tool-making populations, based upon analysis of flaked stone material. This evidence, combined with palaeoneurological evidence from endocasts of fossil hominid skulls, suggests that reorganization of the hominid brain by this time that may imply a stronger lateralization of the left and right hemisphere than is presently observed among non-human primates.

The circular stone structure in Bed I of Olduvai Gorge, Tanzania, approximately 1.8 million years old, appears to be a compositional feature constructed by early hominids; this precocious feature has no similar archaeological analog for the next 1.5 million years.

At approximately 1.5 million years ago, the same time of the emergence of Homo erectus in the fossil record of Africa, there is a technological shift towards the production of large artifact forms, often made on large flakes struck from boulder cores, called picks, handaxes, and cleavers. These forms are the hallmark of the Acheulean industrial stage, which lasts for over a million years in Africa and spreads to Eurasia (along with the first hominids to populate Eurasia) about one million years ago.

These Acheulean forms, ranging from 1.5 million years ago down to approximately 200,000 years ago, show some development in sophistication and craftsmanship through time (although stylistic norms are highly variable.) These is a strong sense of bilateral symmetry even in the earliest forms, which becomes more defined in later Acheulean occurrences, especially between 500,000 and 200,000 years ago, the time of evolutionary transition from Homo erectus into archaic forms of Homo sapiens.

Other possible signs of a concept of symmetry in this period include engraved bones with simple geometric designs, exhibiting a conception of parellelism and equidistant spacing.

Beginning approximately 200,000 years ago there is a technological shift towards flake tools, often struck from prepared core forms, which is called the Middle Palaeolithic in Europe, the Middle East, and North Africa, and the Middle Stone Age in Subsaharan Africa and East Asia. These technologies exist in some places less than 35,000 years ago. This is the time period of forms of Archaic Homo sapiens, including the Neanderthals (Homo sapiens neanderthalensis) of Eurasia, and also marks the emergence of anatomically modern humans (Homo sapiens sapiens), perhaps 100,000 years ago or more, in Africa.

Bilateral symmetry can be seen in many of the stone artifacts produced during this time period, including unifacial and bifacial points, later handaxe forms, prepared cores and flakes, lanceolates (in Africa), and tanged elements (North Africa.) In Eastern Europe the first examples of bone projectile points are also found.

Other signs of a symbolic sense include an polished plaque and engraved pebble pebble from Tata, Hungary, as well as engraved bones from La Ferrassie in France and Bacho Kiro in Bulgaria.

Compositional features of symmetry in this technological stage include the Neanderthal "cemetery" at the French rockshelter of La Ferrassie, in which a series of earthen mounds were organized in a geometric pattern, and the paklaeolithic structure of Moldova in the Ukraine.

The Later Palaeolithic (called the Upper Palaeolithic in Europe) is associated with <u>Homo sapiens sapiens</u> or anatomically modern humans. It is assumed that these individuals possessed the same basic brain structure, cognitive and symbolic abilities, and linguistic capacities that characterize extant humans today. The earliest representational art, in the form of paintings, engravings, and sculpture, as well as an abundance of "abstract" (geometric) designs are common during this period, especially in certain geographical regions.

Bilateral symmetry is especially evident in the Gravettian "Venus" figurines of Western and Eastern Europe, carved zoomorphs throughout this period (and especially in the Magdalenian phase) stone, bone, and antler tools, and the stone, bone, and ivory foundations of hut structures.

This paper stresses the fact that the artifactual manifestations of symmetry became more numerous, defined, complex, organized, and stylized through time. This may be correlated with the evolution of the human brain over the past three to four million years, with an ever-increasing pattern of symbolism in the form of imagery, thought, and communication.

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SYMMETRY IN PAUSKARA SAMHITA MANDALAS

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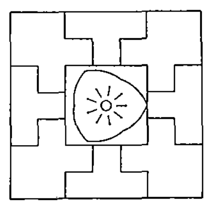
The concept of an ordered, symmetrical and stable, yet evolving cosmos is expressed symbolically in diagrams variously in Indian ritual art. The diagrams called *Mandalas*, *Yantras*, *Chakras* - as well as many other motifs are representations of some aspect of the cosmic process. Concentricity and symmetry are common characteristics of these diagrams.

Universally inherent in man's consciousness, the *mandala* has continually appeared in his constructions, rituals and artforms. Primarily used as meditative tools, the symbolic syntax of *mandalas* reveal a 'universe-pattern ' of the totality of existence, a synthesis of hierarchical, apparently heterogeneous planes of existence. This synthesis allows one to discover the underlying unity of the world and at the same time become aware of one's own destiny as an integral part of the world.

The use of *mandalas* in worship is the central theme of *'Pauskara Samhita'* - which is one of the three main canonical texts of the *Pancaratra* doctrine - a Vaishnavite religious sect in India. This text dates back to 3rd Century A.D. - and describes in great detail the technical construction of *mandalas* to be drawn for the purpose of *' Mandalaradhana'* (*Mandala* worship) as part of the initiation and progress of the spiritual aspirant seeking admission to the cult, with the final aim of attainment of *moksa* (liberation).

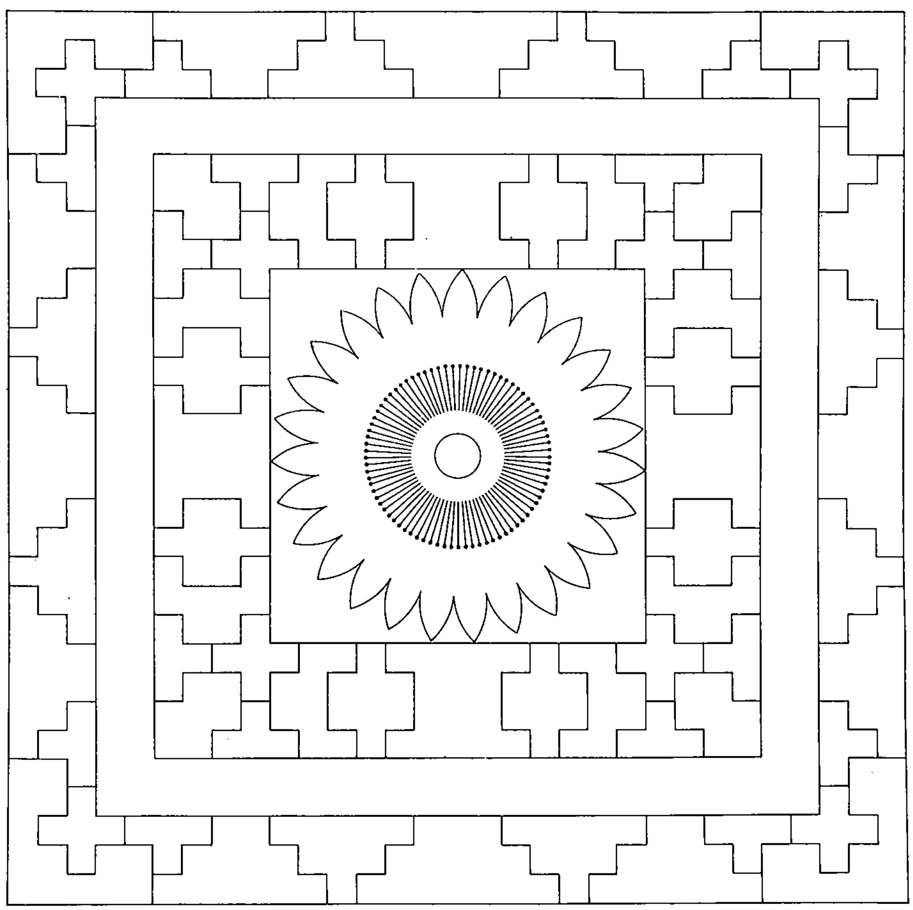


Bhadra (The first mandala of 7x7 = 49 square units)

The *Pancaratra* religious sect is still one of the most important religious sects in the South India. However, the elaborate procedure of *'Mandalaradhana'* as an independent institution as described in *Pauskara Samhita* has been relegated to the background in the course of centuries and forgotten.

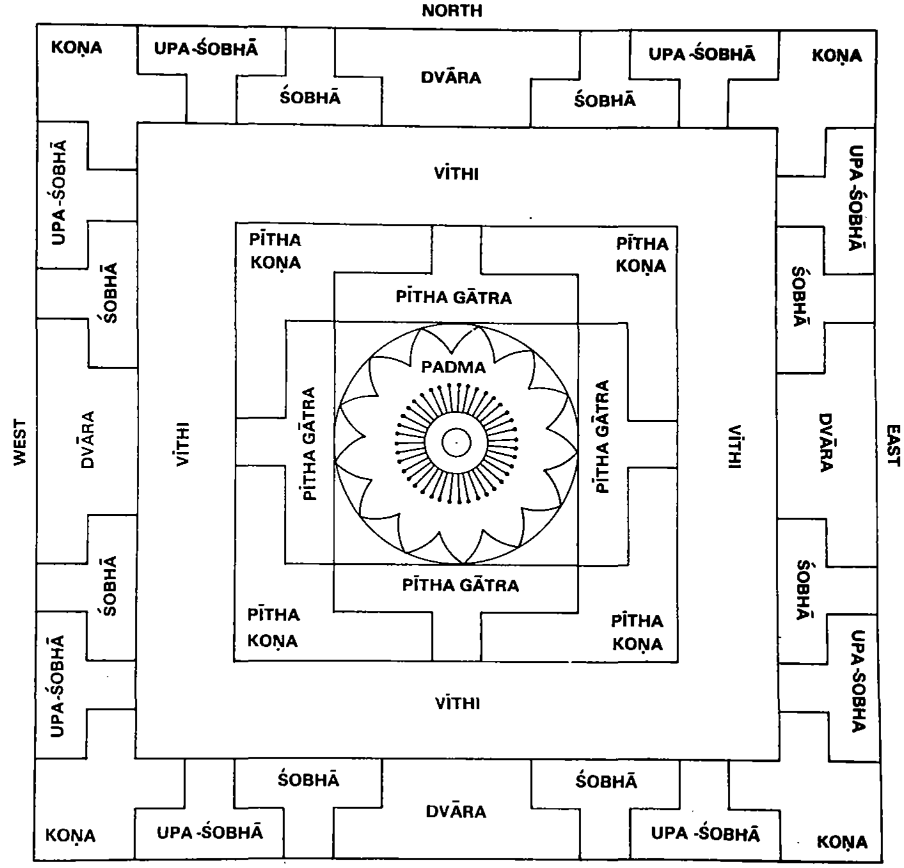
The series of 25 *mandalas* described in *Pauskara Samhita* reconstructed with the help of original textual description by Dr. P.P. Apte and K. Trivedi at Industrial Design Centre, Bombay; represent an important addition to *mandala* literature, as the date of *Pauskara Samhita* makes them among the oldest available description of *mandalas*.

Pauskara Samhita provides a description of four pattern-set classes of *mandala* designs, prescribed for a four year progressive course of spiritual graduation. The four classes are called : 1. *Padmodara* or lotus-hearted; 2. *Anekakajagarbha* or multi-lotus 3. *Cakrabja* or lotus-surrounded by circle; and 4. *Misra-cakra* or of complex-wheels.



Paramananda (The twenty-fifth mandala of 31x31 = 981 square units)

The first class has a lotus at the centre and this category gives twenty-five *mandalas* beginning with *Bhadra* of 7x7=49 square units and increasing by arithmetical progression up to *Paramananda* of 31x31 = 961 sq.units. The central lotus correspondingly grows in number of petals from 3 to 27 also by arithmentical progression. Apart from religio-spiritual aspects and the symbolism; the sketching, colouring and the architectural implication of these *mandalas* are of great interest. From architectural point of view, one may evaluate these diagrams as potential ground-plans of 25 patterns of temple-structures as many of the words used to describe *mandala* components are same as those used in temple architecture.



Component parts of the mandala as described in *Pauskara Samhita*

Also of interest are the verbal descriptions of geometrical techniques involving construction of square-grids, division of square units, equilateral triangles, the technique of concentric circles, drawings of spokes of wheels and the lotus designs with various number of petals and filaments, budding and blooming petals - all based on geometrical devices.

It is proposed to introduce and illustrate with colour slides the various aspects of this series of 25 *mandala*s, showing the evolution of design and structural pattern, and the various symmetries - of colour and of structure, present in these *mandala*s.

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SYMMETRY AND THE ORGANIZATION OF THE "SPACE" OF CARDIAC CYCLE STRUCTURES IN MAMMALS

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The symmetry principle acquires more and more methodological signification in modern biology (Urmantsev, 1974, Urmantsev, 1986). In this connection a exposure of invariants and transformation laws of biological objects is of great importance since only existence of a certain group of transformations and simultaneously the conservation of certain ratios during these transformations enables one to reveal symmetry.

The "space" of cardiac cycle structures for different species of mammals under exercise has been used as a biological object for the symmetry investigation. The heart performs its function due to change of the contrary and mutually complementary activity states of myocard such as tention (systole) and weaking (diastole). Every structure of the cardiac cycle (SCC) can be considered as a system including systolic and diastolic values of some cardiac activity j-parameter and their sun.

The temporal, volumetrical, mechanical and coronary-flow SCC's representing the most considerable biophysical parameters of the cardiac activity have been analyzed (Tsvetkov, 1984,1985). The temporal SCC consists of durations of systole, diastole and the cardiac cycle. The volumetrical SCC includes the volume of the expulsed blood, the volume blood retained in the ventricles and the end diastolic volume of the ventricles. The mechanical SCC represents the mean systolic and mean diastolic pressures in the aorta refering to the duration of the cardiac cycle and the mean pressure. The coronary-flow SCC includes the mean systolic and mean diastolic coronary flows refering to the duration of the cardiac cycle and the mean coronary flow. The "space" of SCC's is totality of the temporal, mechanical, volumetrical and coronary-flow SCC's for different species of mammals under exercise.

Generalized composition of the SCC's "space" (Tsvetkov,1985) is:

$$0.382(\sqrt{\delta})^{k_j+1} a_j W_i^{b_j} + 0.618(\sqrt{\delta})^{k_j-1} a_j W_i^{b_j} \cong (\sqrt{\delta})^{k_j} a_j W_i^{b_j}, \quad (1)$$

where on the left the systolic and diastolic values and on the right the summary value are presented.

On the algebraic expression (1) W_i is weight of some i-animal, k_j, a_j, b_j are the constants for different species in mammals corresponding to the j-parameter. The value δ presents changes of blood supply of an organism at some level of exercise with respect to the "golden" regime of that one approximately corresponding to the organism at rest. For any animal $\delta = \overline{Q(\nu)}/\overline{Q(\nu_{gs})}$ where $\overline{Q(\nu_{gs})}$ is the heart output at the "golden" heart rate ν_{gs} when the ratios of the durations of systole, diastole and the cardiac cycle is analogous to the proportion of the golden section. $\overline{Q(\nu)}$ is the heart output at heart rate ν corresponding to the fixed level of an exercise. For all animals δ =1-4 when the exercise varies from the state of rest to the maximal exercise value.

In the law (1) the influence of weight W and levels of relative blood supply of organism δ on the cardiac cycle structures is presented and the role of the golden section ("golden" numbers 0.382 and 0.618) in the organization of the SCC's "space" is reflected as well.

The structure of cardiac cycle representing the j-parameter of cardiac activity will be designated as j-SCC. In the law (1) has been the symmetry of j-SCC's series in mammals some level of relative blood supply of organism (e.g. δ =1.8) that is analogous for all animals. It is obviously that every level of relative blood supply δ has their "own" structural invariants for j-SCC

$$n_{ST}(\delta) = 0.382\sqrt{\delta}, \qquad (2)$$

$$n_{DT}(\delta) = 0.618/\sqrt{\delta}, \qquad (3)$$

where $n_{ST}(\delta)$, $n_{DT}(\delta)$ are ratios of systolic and diastolic values of j-parameter to their summary value at the level of relative blood supply organism δ , respectively.

The symmetrical transformation of individual j-SCC's of i-animal into the analogous SCC's of other animals within $W_{min} \leq W \leq W_{max}$ and $1 \leq \delta \leq 4$ follows the law

$$C_j = (W/W_i)^{b_j}. \qquad (4)$$

Proceeding from the laws (1), (2), (3) and (4) the invarience in the j-SCC's composition of the multitude of mammals in limits the exercise variation $(1 \leq \delta \leq 4)$ is obvious.

The "evolution" of the compositions of the temporal, mechanical, and coronary-blood SCC's in mammals with exercise variation is connected with "evolution" of invariants $n_{ST}(\delta)$ and $n_{DT}(\delta)$. These invariants are connected with "golden" number 0.382, 0.618 and the values δ are analogous for all considered kind of SCC's (Fig.1).

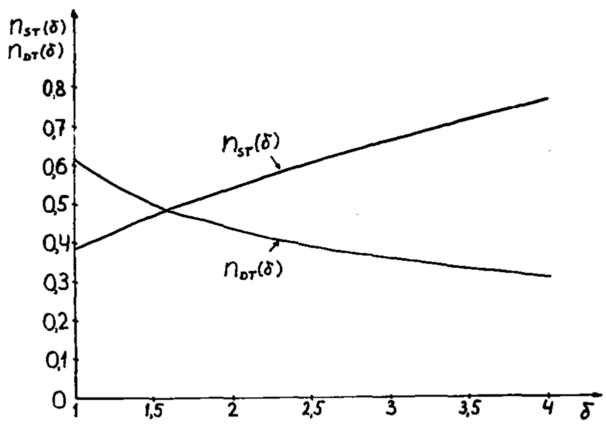

FIG. 1. The invariants of SCC's in mammals under exercise

This phenomen indicates that these invariants are <u>universal</u>. Thus, the golden section is a kind of "technological recipe" of SCC's composition. The properties of the golden section have found extensive applications in the sphere of Nature organization and in the sphere of the creative activity of the men: architecture, painting, music, mathematics, technique etc. (Bochkov, 1974; Petukhov, 1981; Stakhov, 1984; Soroko, 1984; Dubrov, 1987). Hence, there is no doubt about the existence of series of symmetrical objects in the Nature and the art the foundation for the invariance of which is formed by the golden section and its properties.

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SYMMETRY OF DEVELOPMENT

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The substance of one of the section of e v o l u t i o n i c s or the general theory of development /Urmantsev,1988a/, which has been advanced by the author within the framework of his own version of the General System Theory /GST(U)/, is presented.

S y m m e t r y. According to the law of system symmetry in GST(U), "each system is symmetrical at least in one respect". Symmetry means a property of a system "S" to coincide in features "F" after modifications "M". This definition is at the same time a difinition of an abstract equality. Therefore symmetry is equality, and equality is symmetry. The same is obvious for properties of equality relations namely "reflectivity", "symmetry", "transitivity", since these properties are equivalent to three group axioms, i.e. about the neutral element, inverse elements and closeness of a group with respect to itself. The above considerations permit us to treat the history of development of symmetry concepts as the history of discovery of non-trivial equalities and of their investigation concepts. Therefore it is not by chance that four axioms of group theory (one of the set of possible mathematical symmetry theories /Koptsik, 1988/) are at the same time statements about four different symmetries and about four different equalities. This means that any group is symmetrical. Therefore, to reveal the group nature of a system means to reveal its symmetry.

According to the GST(U) law of systemness, "any object is an object-system and any object-system belongs to at least one system of objects of its kind" and, according to the system symmetry law, any system is symmetrical. Hence reality as a whole, both objective and subjective, must obey the law of system symmetry. Reality includes all forms (kinds) of structure, all forms (kinds) of existence (space, time, motion), all forms of alteration (four basic, i.e. identical, qualitative, quantitative and relative forms plus eleven of their combinations by two, three or four), all forms of evolution /four basic, i.e. evolutionary identical (stasigenesis), evolutionary quantitative (quantigenesis), evolutionary relative (isogenesis), plus eleven of their combinations by two, three or four/, all forms of action (two, one and zero lateral) and all forms of relations /conrelative (concordant) and disrelative (discordant)/ of the matter.

This leads to various symmetries and permits one to consolidate the knowledge of different types of symmetry by means of a new general system and philosophical category - "forms of symmetry matter".

D e v e l o p m e n t. It is possible to reach essential progress in cognition of any object "sigma" if we present it as an object-system and reveal in it all the manifistations of systemness (symmetry in particular) which are postulated by GST(U). But to represent any object as an object-system it is necessary to reveal its "primary" elements (i.e. elements which are considered as a minimal unit at the given level of investigation), relations of unity (connections among the elements which make them a whole) and the laws of composition (conditions which restrict the relations of unity).

With respect to development it is appropriate to regard the following consideration.

The primary elements of development are a) the bearers of development A, B, C, D,... which are the objects and at the same time the "results" (phases, stages) of development; b) the forms of development (see above) by means of which something is transformed into some other thing, c) the sources of development which interact positively, negatively or neutrally and act on the bearers of environmental factors. The first stage of consideration of development as a development-system answers such questions as: what develops? into what does it develop? via what evolutionary transformations does it develop? why does it develop?

The relations of unity are a) relations of synchronous and diachronous order of bearers and forms of development; b) relations of unilateral determination of the present by the past and of the future by the present (but not vice versa); c)relations of interaction, unilateral action and mutual non-action among the factors on the one hand, and of the factors and the bearers of development on the other hand; d) probabilistic relations of "polymonovariance" which involves numerous possibilities but only a single developmental form being realized at each moment. The main result of the second stage of consideration of development as a development-system is answering the question, how and in what graph form is the development realised.

The law of composition are prohibitions and permissions connected with a) fundamental physical laws, general and particular system laws (including the laws of conservation), b) action of natural selection at all stages of evolution of all forms of the matter; c) specific "construction" of developing systems ("environment"; "bearers of development"; two, one and zero lateral connections among them), that permits only a limited set of mutual transformations; d) an achieved level of development which although is altered by the new generation of development bearers in turn affects the conditions of existence and routes of modifications of this generation; e) limited set of modifications and developmental forms and an even more limited set of conditions of their realisation at each moment. The main result of the third stage of representation of development as a development-system is the answering of the question, according to what laws does the development occur.

S y m m e t r y   o f   d e v e l o p m e n t. As applied to development, the system symmetry law transforms into a law of evolutionary system symmetry according to which "any developmental system is symmetrical at least in one relation".

The correctness of this assertion is proved by the construction of the following Caylay mathematical groups: 1) symmetry of developmental bearers of different orders; 2) forms of development and modification (or evolutionary and non-evolutionary system transformation) in both cases of the 8th, 16th or 64th order; 3) evolutional and non-evolutional system antitransformations (i.e. "+" and "-" types of system transformations); in each case they are of the 27th, 81th, or 729th order; 4) progressive, isogressive (one level), regressive transformations of 3rd order; 5) progressive, isogressive antitransformations; in each of three cases - 3rd order; 6) inner and outer sources of development; in both cases the 3rd order; 7) actions (two, one and zero lateral actions among bearers, among factors or among bearers and factors) and relationships (con- and disrelative realized in the course of two, one or zero actions); in both cases the 9th order.

It is remarkable that groups, subgroups, invariants of non-evolutionary system transformations and antitransformations (modificational forms) are isomorphic with respect to the groups, subgroups, invariants of evolutionary system transformations and antitransformations (developmental forms). This permits any form of modification to be adequate only to a corresponding particular form of development: stasigenesis to the identical form, isogenesis to the relative form and so on. Each form of modification and the modification in general may be considered as limit in a case of reduction or "embryo" of the corresponding developmental forms and of develop ment in general. It can be said that development in general consists of the embryos.

From the law of evolutionary system symmetry the following laws are inevitable:

a) the law of system contradictioness of development, according to which "any system of development has a subsystem of contradiction-systems, i.e. the subsystem of relations of unity and "struggle of opposites";

b) the law of system non-contradictioness of development, according to which "any developmental system has a subsystem of noncontradiction-systems";

c) the symmetry of the subsystems themselves in at least one relation.

Both laws trivially follow from the recognition of obligatory symmetry of the group nature of any developmental system at least in one relation and also from the obligatory presence in such systems of n contradictional relations between mutually opposite elements (postulated by the first law) and m non-contradictional relations between mutually non-opposite elements (postulated by the second law) of its mathematical group.

Moreover the group considerations prove that in general case

$\underline{m} \gg \underline{n}$, i.e. the number of non-contradicting relations is far greater than that of contradicting ones.

What is the contribution of non-contradicting relations to the objective reality? Like contradicting relations the non-contradicting relations play the role of sources or driving forces of development sources. All the considerations mentioned above are obviously confirmed by the history of scientific, esthetical and philosophical knowledge, the progress of which has been achieved not only due to (but often even in spite of) gnoseological contradictions, but also to gnoseological non-contradictions, i.e. the collaboration and unity of supporters of various paradigms. Relations of non-contradictioness are of a fundamental significance also for the working out of logically non-contradictory theories of natural objects, of society and thinking.

The last assertion (c) is proved by the author by building some relevant mathematical groups, namely of non-contradictory relations of 2nd and 3rd order, of contradictory relations of 2nd and 3rd order, as well as of contradictory and non-contradictory relations of 3rd order. In its turn these conclusions have led to the concept of mutually opposite and mutually non-opposite contradictions and non-contradictions, but under certain conditions identity or intertransformations of such contradictions may occur.

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ON MAPPING THE EQUILIBRIUM EQUATIONS OF A CLASS OF THIN SHALLOW SHELLS WITH VARYING CURVATURE TO CONSTANT COEFFICIENT EQUATIONS VIA LIE GROUPS

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INTRODUCTION

The symmetry of a structure is in essence a form of invariance of this structure under some type of transformations. The acceptance of such an idea and its consistent application seems to be the only way a sense or a feeling for the existence of symmetry to be developed into a well-build theory.

The investigation on symmetry associated with partial differential equations (p.d.eqns) in the above sense is pioneered by Sophus Lie. Recently his theory has been significantly advanced notably by L.V.Ovsiannikov and it is known now as group analysis of differential equations (see Ref.[1]). In principle this theory is based on the concept of invariance of p.d.eqns under continuous (Lie) groups of transformations.

In this paper an application of the above mentioned theory is presented. The eqilibrium eqns of one class of thin elastic shells with varying curvature are considered and it is shown how the symmetry of this eqns can be utilized for their solving.

FUNDAMENTAL EQUATIONS

Let the surface F in the three-dimensional Euclidean space $\mathbb{R}^3$ be given by the eqn

(1) $$z = f(x,y) = (x^2+y^2)^{-2}[A(x^2-y^2)+2Bxy],$$

where A and B are real constants and (x,y,z) is a Cartesian coordinate system in space $\mathbb{R}^3$. The coordinate variables x, y will be used further as Gaussian coordinates in F.

Let us consider now a thin elastic shell with constant cylindrical rigidity D and thickness h and let its middle surface coincide with some part of the surface F.

Since the surface F is evidently asymptotically flat, i.e. $f \to 0$ when $x^2+y^2 \to 0$, then for an appropriate number R the following estimates are valid

$$(\partial f/\partial x)^2 \ll 1, \quad (\partial f/\partial x)(\partial f/\partial y) \ll 1, \quad (\partial f/\partial y)^2 \ll 1$$

in the domain $\Omega = \{(x,y) \in \mathbb{R}^2 / x^2+y^2 \geq R\}$.

Hence, in terms of Donnell-Vlasov theory (see e.g. Ref. [2]) the shell under consideration can be treated as shallow in the domain Ω. Then, in linear treatment, the system of its equilibrium eqns can be written in the following form

$$(2) \quad \begin{aligned} &D\Delta\Delta w - b_{11}\partial^2\phi/\partial y^2 + 2b_{12}\partial^2\phi/\partial x\partial y - b_{22}\partial^2\phi/\partial x^2 = q(x,y), \\ &(1/Eh)\Delta\Delta\phi + b_{11}\partial^2 w/\partial y^2 - 2b_{12}\partial^2 w/\partial x\partial y + b_{22}\partial^2 w/\partial x^2 = 0, \end{aligned}$$

where w is the transversal displacement function, ϕ is the Airy's stress function, E is the Young modulus, q is the external transversal loading, $\Delta \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$ and $b_{\alpha\beta}$ $(\alpha,\beta = 1,2)$,

$$(3) \quad b_{11} = \partial^2 f/\partial x^2, \quad b_{12} = b_{21} = \partial^2 f/\partial x\partial y, \quad b_{22} = \partial^2 f/\partial y^2$$

are the components of the curvature tensor of F, approximated in the manner accepted in the shallow shell theory. Accordingly to formulae (1) and (3), the explicite form of components $b_{\alpha\beta}$ is

$$(4) \quad \begin{aligned} &b_{11} = -b_{22} = (x^2+y^2)^{-4}[A(x^4+y^4-6x^2y^2)+4Bxy(x^2-y^2)], \\ &b_{12} = b_{21} = (x^2+y^2)^{-4}[4Axy(x^2-y^2)-B(x^4+y^4-6x^2y^2)]. \end{aligned}$$

The complicated form (4) of the coefficients $b_{\alpha\beta}$ makes the integration of system (2) quite problematic. A considerable progress in this direction could be achieved however by using the symmetry of this system of eqns.

FORMULATION OF THE PROBLEM

When a system of p.d.eqns with varying coefficients is being solved it is very helpful to be known whether there is a transformation of its independent and dependent variables so that after rewriting the system in the new variables its coefficients to become constant. Indeed, if the system could be transformed in this manner, then each one of the well known methods for integration of systems with constant coefficients may be used for its solving.

In Ref.[4] Bluman shows that when the full symmetry group of a given linear homogeneous p.d.eqn with varying coefficients is known then a definitive and constructive answer of the above question can be found. Following his approach in general outline we analyse this problem bellow for the system (2).

Obviously, the answer of the problem in question does not depend on the form of the right hand side of system (2). In view of that it is naturally the simplest system of this form, i.e. the homogeneous one, to be considered.

SYMMETRY GROUP

Let us denote by S the homogeneous system of eqns of form (2). In Ref.[3] it is shown that the full symmetry group of system S is two-parameter Lie group of transformations in the space $\mathbb{R}^4(x,y,w,\phi)$ of the independent x, y and the dependent w, ϕ variables of system S. The infinitesimal generators of this group are

$$(5)\quad \begin{aligned} X_1 &= (x^2-y^2)\partial/\partial x + 2xy\partial/\partial y + 2xw\partial/\partial w + 2x\phi\partial/\partial\phi, \\ X_2 &= -2xy\partial/\partial x + (x^2-y^2)\partial/\partial y - 2yw\partial/\partial w - 2y\phi\partial/\partial\phi. \end{aligned}$$

TRANSFORMATION OF THE FUNDAMENTAL EQUATIONS

Consider the following transformations of the variables of system (2):

$$(6)\quad \begin{aligned} x' &= f_1(x,y), \quad w' = W_1(x,y)w + F_1(x,y)\phi, \\ y' &= f_2(x,y), \quad \phi' = W_2(x,y)w + F_2(x,y)\phi, \end{aligned}$$

where f_α, W_α and F_α $(\alpha = 1,2)$ are arbitrary, but sufficiently smooth functions in the domain Ω. Evidently only the transformations of form (6) provide linearity and homogeneity of system S after passing to the new variables. That is the reason only transformations of this special type to be considered.

If there exists a transformation of form (6) such that the infinitesimal generators of the symmetry group of system S take on in the new variables the form

$$(7)\quad X'_1 = \partial/\partial x', \quad X'_2 = \partial/\partial y',$$

then after rewriting sytsem S in this new variables its coefficients become constant and vice-versa. This conclusion is a consequence from the following well known general group property of the linear homogeneous systems of p.d. eqns: if a linear homogeneous system of p.d.eqns has constant coefficients then this system admits the group of translations in the space of its independent variables and vice-versa.

On the other hand in passing to the new variables the infinitesimal generators of the symmetry group of system S transform (see e.g. Ref.[5], p.45) as follows

$$(8)\quad \begin{aligned} X'_\alpha = {} & (X_\alpha x')\partial/\partial x' + (X_\alpha y')\partial/\partial y' + \\ & + (X_\alpha w')\partial/\partial w' + (X_\alpha \phi')\partial/\partial\phi' \quad (\alpha = 1,2), \end{aligned}$$

Comparing the right hand sides of (7) and (8) and substituting (5) and (6) in the obtained expressions we work out the following overdetermined linear system of first-order p.d. eqns

$$
(9)\qquad
\begin{aligned}
&(x^2-y^2)\partial f_\alpha/\partial x + 2xy\partial f_\alpha/\partial y = \delta_{\alpha 1},\\
&2xy\partial f_\alpha/\partial x - (x^2-y^2)\partial f_\alpha/\partial y = -\delta_{\alpha 2},\\
&(x^2-y^2)\partial W_\alpha/\partial x + 2xy\partial W_\alpha/\partial y + 2xW_\alpha = 0,\\
&(x^2-y^2)\partial F_\alpha/\partial x + 2xy\partial F_\alpha/\partial y + 2xF_\alpha = 0,\\
&2xy\partial W_\alpha/\partial x - (x^2-y^2)\partial W_\alpha/\partial y + 2yW_\alpha = 0,\\
&2xy\partial F_\alpha/\partial x - (x^2-y^2)\partial F_\alpha/\partial y + 2yF_\alpha = 0,
\end{aligned}
$$

where $\alpha,\ \beta = 1,2$ and $\delta_{\alpha\beta}$ is the Kronecker delta symbol.

Combining all the above we are able to formulate now the following result: the system of form (2) can be transformed to constant coefficient system by transformation of form (6) if and only if the corresponding functions f_α, W_α and F_α satisfy system (9).

It is easy to show that the functions

$$
\begin{aligned}
&f_1 = -\,x(x^2+y^2)^{-1}, \quad f_2 = y(x^2+y^2)^{-1},\\
&W_1 = F_2 = (1/4)(x^2+y^2)^{-1}, \quad W_2 = F_1 = 0,
\end{aligned}
$$

satisfy system (9). The corresponding transformation of form (6) is

$$
(10)\qquad
\begin{aligned}
&x' = -x(x^2+y^2)^{-1}, \quad w' = w(1/4)(x^2+y^2)^{-1},\\
&y' = y(x^2+y^2)^{-1}, \quad \phi' = \phi(1/4)(x^2+y^2)^{-1}.
\end{aligned}
$$

Rewriting system (2) in the variables given by (10) we get

$$D\Delta'\Delta'w'-A\partial^2\phi'/\partial y'^2-2B\partial^2\phi'/\partial x'\partial y'+A\partial^2\phi'/\partial x'^2=q(x'^2+y'^2)^{-3},$$

$$(1/Eh)\Delta'\Delta'\phi'+A\partial^2 w'/\partial y'^2+2B\partial^2 w'/\partial x'\partial y'-A\partial^2 w'/\partial x'^2=0.$$

where $\Delta' \equiv \partial^2/\partial x'^2 + \partial^2/\partial y'^2$.

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Does a factlike origin of time asymmetry violate the validity of the time symmetric conservation law of energy ?

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1. The factlike origin of the second law of thermodynamics.

Attempts to base asymmetry of time on the second law of thermodynamics are numerous although this law is the most controversial one established in the 19th century. This controversy is due to the paradox which is implied by the second law. Indeed, the time asymmetry of irreversible behaviour cannot be reconciled with the time symmetry of the laws governing the mechanics of the underlying elementary particles.

Even the most fundamental approach of reconciliation of the two types of time symmetry is recently published by Prigogine in 1980 (Prigogine, 1980). He assignes a micro status to entropy and establishes a new complementarity between reversible and irreversible evolution.

Reversible evolution is identified by mechanical observables while he identifies irreversible evolution by thermodynamical observables. We queried Prigogine's assertions (Verstraeten, 1987, 1988, 1989). Particulary we argued against Prigogine's concept on micro entropy density operator on philosophical as well as physical grounds.

Moreover, we emphasize the factlike base of the second law of thermodynamics. To give evidence for this assertion we return to Carnot's arguments which generated Carnot's General Axiom. According to this axiom the Carnot engine produces work at maximum efficiency and the coefficient linking the absorbed heat and the produced work is the same for all bodies. The Carnot process does not depend solely on the system itself, yet the surroundings (furnace and refrigerator) are also involved. A Carnot process for a body B may be defined as a simple cyclic process which units heat at the one and only one hotness h^- and absorbs a positive amount of heat at the one and only one hotness h^+, which is hotter than h^-. Hotness h corresponds to a corresponding real value of temperature ϑ so that for the intrinsicely ordered hotness h_1 and h_2 with ordering $\succ$ which we read as "hotter than" :

$$h_1 \succ h_2 \leftrightarrow \vartheta(h_1) > \vartheta(h_2) \qquad (1)$$

Consequently the Carnot cycle contains two isothermal branches corresponding to the emission and absorbtion of heat, and two adiabatic branches.

According to the latent heat theory, the transmitted heat dC during the time interval dt is written in differential form.

$$dC = Q\,dt = \Lambda_V dV + K_V d\vartheta \qquad (2)$$

Λ_V : the latent heat or transmitted heat at constant temperature

K_V : the specific heat or transmitted heat at constant volume.

According to Euler's theorem Q has an integrating factor f locally, where f is continuous and positive. Applying Euler's theorem on (2), the time derivative of a function $\mathcal{H}_f(V,\vartheta)$ is given by

$$\dot{\mathcal{H}}_f = Q/f = (\Lambda_V \dot{V} + K_V \dot{\vartheta})/f \tag{3}$$

When f is identified with the temperature ϑ we reach Clausius' expression for the entropy. As long as the Carnot cycle is conceivable in terms of volume, temperature, latent heat and specific heat, so will be the entropy.

For a Carnot cycle with a furnace and a refrigerator of which the temperature differs only by an infinitesimal amount, the produced work ΔL is

$$\Delta L = F(\vartheta + \Delta\vartheta, \vartheta)\, C^+ \tag{4}$$

with $C^{\pm} = \oint \frac{Q \pm |Q|}{2}\, dt$

(the contour integral is integrated along the isotherms and adiabats of the considered Carnot cycle). As already mentioned F does not depend on the particularity of the body. When we use the universal form $F = J\,\Delta\vartheta/\vartheta$ (Truesdell-Bharatha, 1977) and the expression (3) for the entropy :

$$\Delta L = J\,\Delta\vartheta \oint \mathcal{H}_\vartheta \tag{5}$$

Then

$$\Delta L = J(\vartheta^+ - \vartheta^-) \oint d\mathcal{H}_\vartheta \tag{6}$$

or

$$\Delta L = J(1 - \vartheta^-/\vartheta^+)\, C^+ \quad \text{if} \quad C^+/\vartheta^+ = C^-/\vartheta^- \tag{7}$$

(Applying the theory of latent heat for the isothermal absorbtion)

When $\int_{\vartheta^+} d\mathcal{H}_\vartheta > C^+/\vartheta^+$ then the produced work is less than the produced with maximum efficiency and hence there is a dissipation. This results in the second law of thermodynamics

$$\oint d\mathcal{H}_\vartheta \geqslant 0 \tag{8}$$

The universality of F or J in (4) is an immediate corollary of the principle of the non-existence of the perpetuum mobile extended to the domain of thermodynamics according to Mach (Mach, 1986). So any engine can produce either at maximum

.../...

efficiency or at less efficiency. But by the fact of the non existence of the perpetuum mobile, equation (8) can never be $\oint d\mathcal{H}_\vartheta < 0$ because then the efficiency should be higher than that of the Carnot cycle. Hence the non decrease in entropy is factlike and not lawlike.

2. Does the factlike origin of the asymmetry of time generate a paradox ?

The second law of thermodynamics encouraged some scientists to search for a lawlike foundation of the asymmetry of time and the arrow of time (see for a good survey Grünbaum, 1973,VIII). The first section makes it clear that they were searching invain as this law is just a fact and so is asymmetry of time. A complete and concise survey of the factlike foundation of time asymmetry is given by Costa de Beauregard yet we assigne our approach a particular position in the scope of the mentioned factlike foundation. Indeed, nearly all factlike approaches are based on statistical arguments. Therefore we refer to Reichenbach's branch systems (Reichenbach, 1956), which are branched off the mother system. The entropy of the branchpoint is in most non equilibrium cases very low so that entropy can only increase. In most equilibrium cases the entropy is non decreasing. Grünbaum formulated an ameliorated version of Reichenbach's branch systems. He disconnected the link made by Reichenbach between local time asymmetry of a branch system and the time asymmtry of the Universe (Grünbaum, 1973, p 260). However we query both approaches because they are so little "physical factlike". This concept means that one event govered by physical laws is assigned a physical significance by its actuality only. A physical event is actual when it can be measured according to a physical experimental method.

.../...

Branch systems are not actual, yet they are mental constructions which permit to predict the future of all germane systems with some probability. As Grünbaum (P 253-260) and de Beauregard (chapter 3.4.2) decided outright, branch systems do not permit retrodiction because there is no system before the branch off. The non existence of the system means that it is not actual but we go further : even after the branch off the system is not actual because nobody can tell us how the system is sufficiently defined. Moreover, how can we physically determine the adjective "most" ? By taking a sample at a particular point of time ?
Did the above mentioned authors realize that the particularity of that point of time was only generated by the mental decision that the asymmetry of time was the product of a mental fact not by a physical fact. ? Nevertheless this foundation of the time asymmetry has one advantage : it does not violate the validity of any physical conservation law which presupposes time symmetry.

Our approach of time asymmetry is physical factlike because the non existence of a heat engine with an efficiency to produce work exceeding the Carnot engine is physically relevant if and only if there is an actual physical fact either to support the above mentioned principle or to violate it by an opposite fact.

Does such physical fact generate a paradox between the involved time asymmetry and the time symmetry which is a prerequisite of the first law of thermodynamics ? We examine this problem in the third section.

3. Violation of the first law ?

The first law of thermodynamics essentially links mechanical quantities (mass, action on the environment), which are spatially localizable at any point of time to heat which is not localizable in space. The latter is only known by the mechanical work into which it can be converted. Consequently the validity of this law is controlled directly or indirectly by mechanical quantities which make part of the scope of the mechanics of time symmetric processes. Moreover, another set of time symmetric processes is involved, namely the set containing all reversible processes which identify the body or the set of bodies of which the energy is conserved. With Bridgman (Bridgman, I961,p I22-125 and p I88-I9I) we prefer to use the term recoverable and irrecoverable processes instead of reversible and irreversible processes. It is clear that the processes to recover the identity of the examined body are different from the processes controlling the validity of the first law. In the opposite case the first law would be just a definition of the system and could not be falsified. Hence there should not be a first law.

The time of the mentioned recoverable processes is symmetric. Indeed, the time symmetry is a prerequisite for identification and conservation, because onehas to be able to compare the body at yhe two extremities of the time interval wherein the processes are engineered.

This prerequisite does not beware the body for simultaneous irrecoverable processes. So is a breakdown of the equality (3) the cause of a heat-work conversion at a rate less than the maximum Carnot rate and consequently the system does not return automatically to its initial thermodynamical state.

This phenomenon is purely factlike because we can only formulate necessary conditions of this evolution and not sufficient ones (equation 7). But it does not prevent us to control the validity of the first law, because this irrecoverable evolution evolues in the scope of thermodynamics while the body is defined in terms of recoverable processes. Upon this definition the termal properties are subjoined.

We conclude that the first law of thermodynamics is an extension of the similar law in mechanics so that there is also accounted for non localizable forms of energy. The second law is no restriction of the first law yet a restriction on the efficiency of the heat-work transition only. The first law is general for all bodies while the restriction of the second law imposes constitutive restrictions for evolutions of particular bodies for which no sufficient reason exists. This local anisotropy of time evolution of the thermal properties of the body does not contratict the time symmetry of the recoverable processes which identify the body as long as these processes reduce the actual state of the body to the initial fiducial state. The thermodynamics adduces information to the body yet the additional information produces only necessary conditions for the internal evolution of the body. Hence we conclude that the factlike origin of the asymmetry of time involves a gain of information about a particular body but a loss of knowledge about its internal organization.

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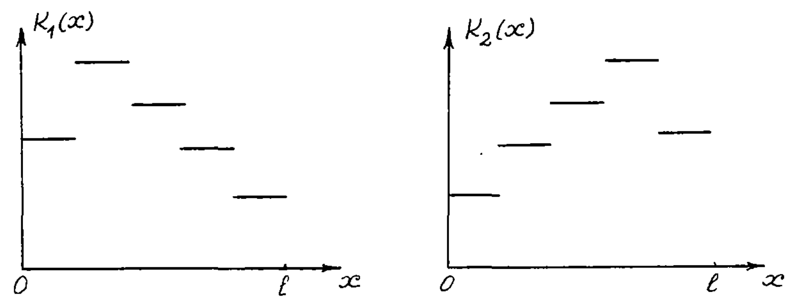
We have to determine function $k(x)$, given by the set n, k_j, x_j, $j=1,\dots,n$, and having the physical meaning of hydroconductivity coefficient in geology or relation of heatconductivity to heatcapacity of structure in the theory of heattransform. Let us consider the value $\ell > 0$ (ℓ is stable), function $\mu(t)$, which is known, and additional boundary condition

$$u(x,t)\big|_{x=0} = \varphi(t), \quad t > 0.$$

The set of solutions of the formulated inverse problem in the case of function $\mu(t)$ beeing smooth, consists of two elements $k_1(x)$ and $k_2(x)$, axisymmetrical to $x=\ell/2$. The illustration of such case is given in the figure:



So, the analysis of experimental data on the basis of formulated model is associated with preferability of the only one of two symmetrical solutions to be taken as a corresponding one.

This question is solved by taking into account the additional assumptions of the character of the solution $k(x)$ we are looking for (that is: monotonous, restricted on some segments and etc.)

Just the same problems, connected with the analysis of various symmetrical configurations are typical for the whole class of hyperbolical type equation problems, describing wave processes of deformation and dynamic fracture of fiber reinforced composites. These wave processes form the basis for acoustic emission phenomenon modelling – effective method of nondestructive control, used for diagnostics of composite materials and constructions.

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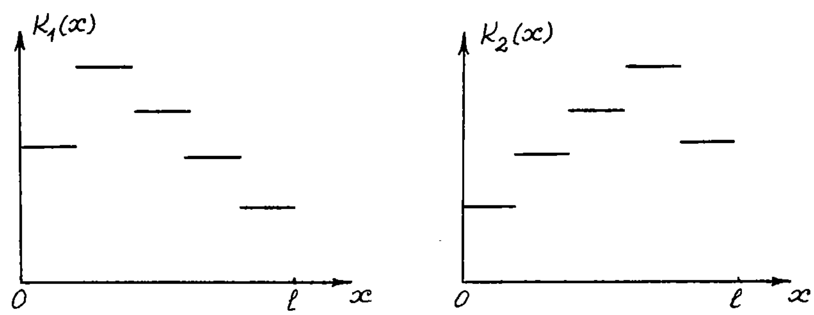

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Some experiences and problems dealing with symmetry for children - from very young to old !

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Among the many experiences that children can have both in school and at home are the ones that are indicated below. Details, more examples of each, and other problems will be given at the session.

Reflection symmetry: Experiences and problems using paper.

What will you see when the paper is unfolded?

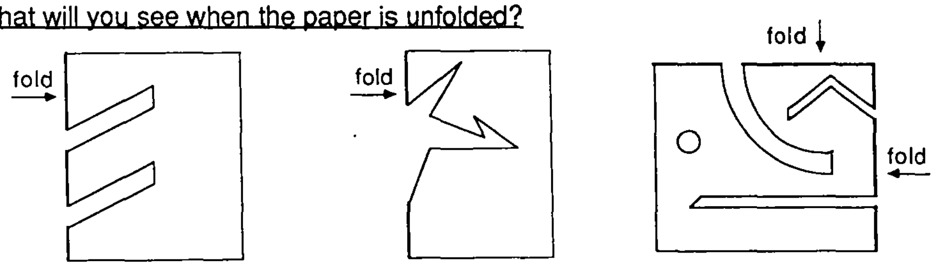


Fig. 1

Predict what you will see when you unfold the paper

Very young children are good at predicting what simple folded cut out pictures, such as half a leaf, will look like when unfolded. Less simple ones challenge even older children. Difficulty can be increased by folding the piece of paper twice or more.

Which shapes can you make by cutting a folded piece of paper?

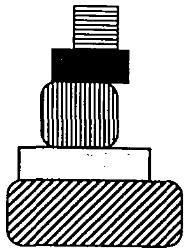
Which geometric shapes can you make by cutting a piece of paper that has been folded once? Which can you do in more than one way? Two ways? Which could you make by cutting a piece of paper folded more than once? These cutting problems can lead to discussion of some of the simple geometric properties of these shapes.

Experiences and problems with reflection symmetry using mirrors.

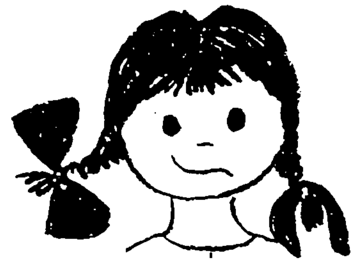
Changing and making pictures with one mirror.

Young children are challenged when asked what they can make from given pictures by using a mirror and also when they are asked to make specific things.

Children are also encouraged to let their imaginations roam as they use the mirror to see many different strange and wonderful shapes. They are often very excited when they make new mirror pictures appear and freely describe what they see .



from "Make a Bigger Puddle Make a Smaller Worm"



from "Look at Annette"

Fig. 2

a. Can you build a tower that has 8 blocks? 7 blocks?
b. Can you make the face smile? Look sad? What else can you see?

Mirror Cards and Mirror Puzzles.

I created Mirror Cards during 1963-5 and tested them with children of all ages. They consist of two types of problems. The first is "Which of several pictures can you change, by use of a mirror, to match one given pattern?" The second kind asks the reverse. Given a particular pattern, which of several others can you make from it by use of one mirror. The Mirror Puzzle Book contains puzzles of this second kind. Included are many patterns that are impossible to match.

The fact that a picture has reflection symmetry is not enough to guarantee that it can be reproduced from the GIVEN pattern .

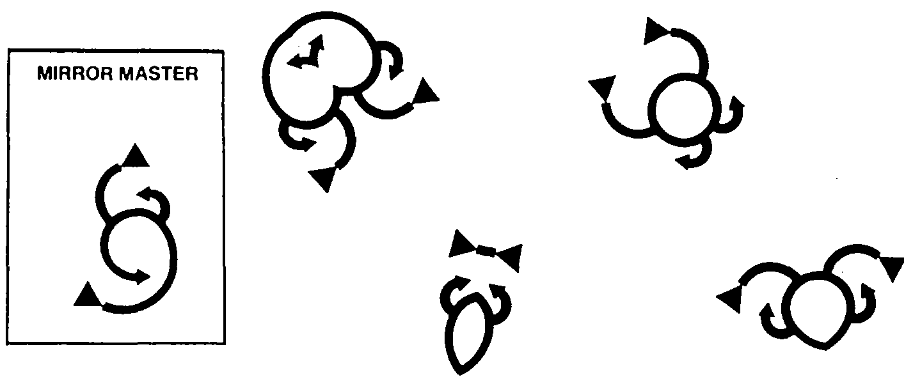


Fig. 3

Which of the patterns shown in Fig. 3 can you make by using the mirror on the pattern called the MIRROR MASTER? The patterns may be in different orientations.

Experiences with half silvered mirrors.

Students can be asked to build a design with blocks, or to draw a design in front of a half silvered mirror. They are then asked to build behind the mirror, without looking into the mirror, what they think the mirror image is.

They can check the correctness of their predictions by looking through the half silvered mirror.

Play and problems with two hinged mirrors.

Use the hinged mirrors. Place one coin between them to see 4 coins, or 6 or 5. After a while students predict how far to open the mirrors to see say 6 or 8 coins. Explore what you see when the object itself does not have symmetry. Move the object closer to one mirror - closer to the other - into the middle. What do you notice? Draw one line and use it to create a regular 4 sided, 5 sided 6 sided figure. (Regular polygon means that the polygon has equal angles and equal sides). Can you make a 6 sided polygon that is not regular? Can you make star shapes?

What happens with two mirrors is sophisticated. Experiments can help one understand what is going on.

The Kaleidoscope can be a topic on its own and I expect there is someone who will talk on different types.

Experiences and problems dealing with rotation symmetry.

One way to introduce rotation symmetry through a problem is to ask "Are there ways of cutting a square into two congruent halves other than the usual ways? Activities can lead children to find out the secret for drawing more interesting halves of squares. Children can get an intuitive feeling for point symmetry and how it differs from reflection symmetry as they discover the secret of drawing such halves.

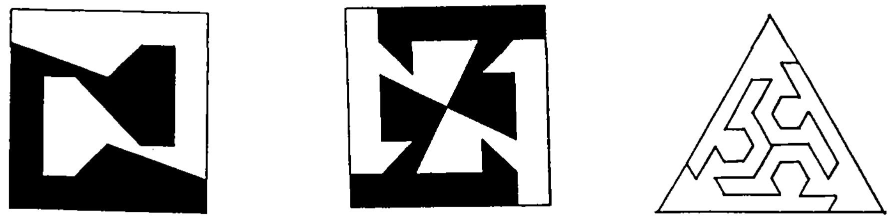


Fig. 4

Making more interesting congruent parts

They explore 3 fold or 4 fold or other rotation symmetry when they explore unusual ways of cutting an equilateral triangle into three congruent parts or a square into four congruent parts and in general a regular n sided figure into n congruent parts.

Work without mirrors that leads to rotation and reflection symmetry.

Other activities, starting with visualizing shapes in 3 and 2 dimensions can leads to work in symmetry. Such motivated problems suitable for school children may also be discussed.

Symmetry in our surroundings.

Children can be encouraged to look for, draw or photograph symmetric images that they find in their surroundings--be they natural or man made. I recently photographed man made symmetric objects - hubcaps of carwheels in the U.S.A. and coalhole covers in streets in England. They lend themselves to much study of plane symmetry. Every region in the world will have its own special symmetrical objects.

What might children be learning when they work with problems of the type described?

I will discuss this in my session, but let me say here that even people who know all the rules of reflection are not always correct in their predictions and are sometimes surprised. Work gives students experience with and intuitve knowledge of reflections, rotations and the difference between them. They learn about geometric concepts and the properties of geometric shapes on an intuitive level. This will be discussed in the session.

Some comments on the indicated activities.

In all of the activities we start with explorations and problems and not with definitions as so often is done in mathematics classes. In all the mirror work, the children can predict what they will see and immediately check their predictions. If not correct, they can amend their solution by moving the mirror. This work may be the first time that children encounter problems whose answer is 'it cannot be done'. It is often the children who are labelled 'poor' at school work who excel in this kind of visual work. Mirror puzzles can be made very easy so that very young children can do them and they can be made so hard that even some members of this conference will be challenged !

Play and problems with mirrors forming prisms and pyramids can be done by children but I expect will be covered by others and so I have not included it.

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SYMMETRY AND EXPERIENTIAL PARADOXICAL WHOLENESS

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Concepts of symmetry evoke both formal and experiential associations which are deeply connected to epistemology. The corresponding connections of the two associations seem to be "symmetrical". On one hand, in the case of mathematical and physical symmetries for example, epistemological principles lurk behind the definitions and the demonstrations or proofs of symmetries. On the other hand, "symmetry" can be the direct sensation of knowing: symmetry being knowing. One may intuit, for example, a wholeness as a "symmetrical" gestalt in which distinguished poles suddenly merge or collapse to a unitary identity.

This talk outlines relations across these associations. Symmetry is approached within a class of problems treating compared distinctions. The form of these problems is followed across a range of related topics, from annihilation to wholeness and epistemology. The talk draws from my paper "The Breadth of Symmetry" in the conference book Symmetry in a Kaleidoscope. This oral presentation will concentrate on describing experiential and epistemological aspects of the associations and modelling them within the structure developed in the written paper.

The initial starting point is a game played with a simple system which consists of a dissymmetric pair of binary spaces marked by distinctions (e.g. left/right on/off etc.). The game derives from a consideration of a calculus of distinctions (Brown, 1979). It involves considering what happens when the two terms of a distinguished pair are moved across the distinction using various possible "crossing rules".

Examples of crossing rules are superposition, transposing mirror image, etc. The basic question is, for an "observer" who is in one of the spaces, what are the possible results of this transformation? The scheme considered, and the results entering our discussion are summarized in the table (on the second page of this abstract). For example, as listed in the table, annihilation is the result of the game played so as to be consistent with the formal logic of distinctions.

Important to this presentation, illusion and paradox can be cast in the form of the game and bring epistemology to a central position. That is, they force the observer to examine the interaction of his process for determining valid knowledge with the identification of the thing known. Different answers reached in this examination indicate that tests for "reality" epistemologically establish "biased" realities.

TABLE

Nature of Distinction*	Crossing Rule	Effect
I. existential:		
exclusionary	superposition	annihilation
configurational	symmetry operation	transformational identity (symmetry)
identificational	interchange	identity (absolute)
II. epistemological:		
identificational	interchange	illusion
identificational	interchange	conditional paradox
identificational	interchange	self-referential paradox
III. distinction generator:	simultaneous	paradoxical wholeness

*The existential category denotes that the distinctions are seen as associated to objects, conditions themselves. The epistemological category denotes that the distinctions draw direct attention to methods of knowing.

For example, both measurement and perception can be argued to be processes for establishing knowledge, but they can yield different results. Historically, in classical physics, perceptual-space was distinguished from measure-space, and taken to be subjective or unreal, thus reducing paradox to illusion. Today, paradox re-arises as the problem of the object-measure, or object-epistemology wholeness.

Consideration of this epistemological role in "existential" or "self-referential" paradoxes suggests the concept of what I will call a "distinction generator" which generates the named distinctions (spaces) as well as the resolution of the parts of the paradox in the spaces themselves. The distinction operator appears inherently paradoxical and infolded.

From the phenomenological perspective, the generator appears as a self-contradictory beast if proposed as existential in a singular "real" space. However, it satisfies (is the solution to) the logic paradox of "A is not A", and it can be seen as a paradoxical wholeness and "seed state" which is conceptually complimentary to dissymmetrical distinction.

The structural character of resolving paradox at this level provides an interesting description of experiencing knowing (experiential epistemology). Examples used to illustrate this will be drawn from the perceptual dynamics involved in understanding equated concepts (Warren, 1986a) and from the experiential paradoxical collapse of dualisms descriptive of certain states reached in meditation. The experiential "wholeness" and "symmetries" claimed of mystical experience appear logically connected to the experience of formally symmetric states.

In the case of understanding equated concepts, a cognitive block may break and the equated terms make sense in a (sudden) dynamic convergence and interpenetration of the distinguished (but equivalent) concepts into a new configuration. The dynamic aspect of this experience has much of the quality of experience associated with visual perceptual forces such as discussed by Arnheim (1969).

Modelling the process of this collapse is offered by suggesting "awareness" is a self-reflected state consisting of a dissymmetric function with two terms (denoted "emate" and "icate") which paradoxically "interpenetrate" each other (Warren, 1986b). In normal perceptual cognitive levels these two terms are hinted at by such paired words as "intuitive" versus "rational" and "sensation" versus "structure", and "of" versus "in".

The root level of awareness, at the level of an "awaring" or distinction generator, may be perceptually approachable as an experiential "seed state" (Hayward, 1987). Such Vedic statements as "I am That" and the Buddhist Void appear consistent with this form.

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THE DUALISTIC SYMMETRY BETWEEN PLANE- AND POINT-BASED SPATIAL STRUCTURES.

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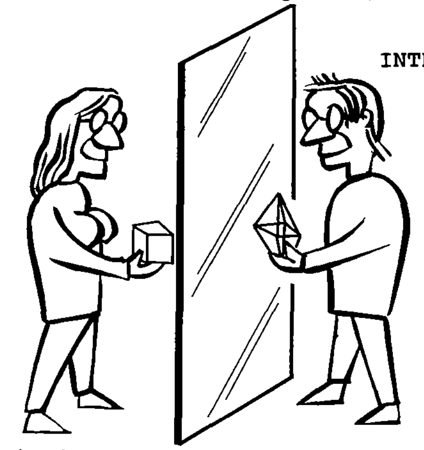

Fig.1

INTRODUCTION

The 3-D structural system based on points and their lines of connection (i.e. trusses, space frames, braced and lattice structures etc) are very well known as efficient and material-ecomomic structures. This, the most basic structural system, creating only axial forces (tension and compression), turns out to have a counterpart, which is its diametrical opposite, its symmetrical mirror image, its polar unambiguous dual system -the plate structure-. This structural system, based on plane thin plates interconnected by shear resistant "lines of support", has traditionally been regarded as a secondary kind of structure, mainly stabilizing buildings for wind forces, but it can with the knowledge presented here, be dealt with in a mere complex and direct way as the lattice structure. It might seem odd to relate the unrefined plate structure to the highly sophisticated lattice structure. However, it has turned out that between these two types of structure there is a connection so fundamental, that they are mutual geometrical and statical duals. To the author's knowledge it is the first time the concept of genuine dualism and complementarity well established in physics is introduced into statics. In fact this theory explains in a very simple way, not only the statical behaviour of all five Platonic polyhedra, but of any arbitrarily plane facetted surface. As the space for this abstract is very limited, only a concentrate of this dualism will be described, and references for more detailed information are given. To make the analogy as clear as possible, the dual qualities will be put directly opposite to each other in the following. The part dealing with lattice structures is generally elementary, and it is included merely to emphasize the analogy to the plates.

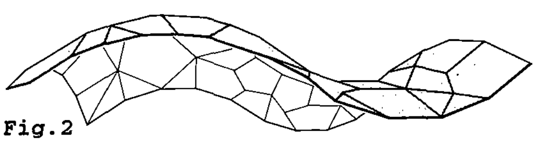

Fig.2

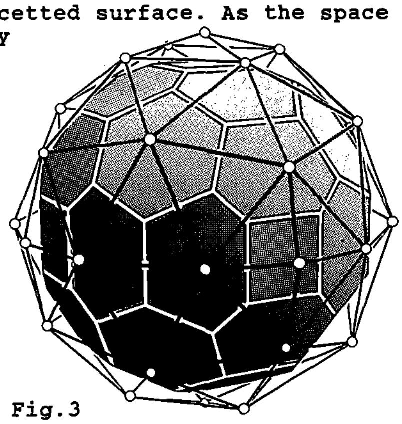

Fig.3

SUBSTANCE OF THE DUALISM

Basic considerations and definitions

Statical basis

LATTICE	:	PLATE
Nodes	- Defined with three coordinates -	Plates (*)
Bars interconnecting the nodes	:	Lines of support interconnecting the plates.
	Transfer of only	
axial forces	:	shear forces
	between the	
nodes	:	plates
	To achieve spatial structural stability for an additional	
node	:	plate
	an extra number of	
3 bars	:	3 lines of support
	is necessary	

This basic minimum relation is of course very well-known for lattice structures, but obviously this is not the case for the plate structures. As it is seen, nodes and plates may be interchanged if bars are changed to lines of support and axial forces to shear forces (Wester, 1984 & 1987a).

(*) A plane is defined by the coordinates to the terminus of its position vector (the vector from origo to the nearest point on the plane of the plate). This point is called a "plane-point", while the point of the node is called a "point-point".

Geometrical basis

A geometrical type of net, containing all the active elements mentioned for both kinds of structure is the spatial 2-dim. net (fig.2), consisting of the following elements: a) Planes. b) Lines of intersection between two planes only. c) Vertices as points of intersections between three or more planes. This kind of net also follows Euler's theorem on relation between the number of faces, vertices and edges in space.

Combination of requirements for stability and topology

If the relations mentioned for basic stability for spatial structures and topology are combined, the result is quite surprising. If a spatial net of the type shown in fig.2 & 4 is either unlimited or singly connected like a simple polyhedron, its statical behaveour is unambiguously related to the geometry of the net (Wester, 1988b).

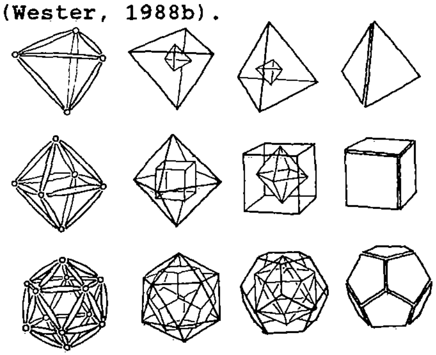

Fig.4

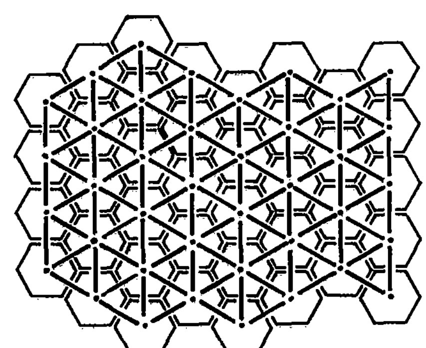

Fig.5

LATTICE : PLATE
In this case it turns out that the net consisting of
triangular facets : 3-way vertices
can only be stabilized by acting as a pure
lattice structure : plate structure
and it will be just stable i.e. a statical and geometrical
determinate structure. Hence no active element
node or bar : plate or line of support
can be removed without losing the general stability of the
structure. The inactive elements in the net
the planes : the vertices
may be removed leaving the general stability unchanged.
In this case the just-stable structure alone consists of
nodes and bars : plates and lines of support
which give the structure the appearance of being totally
open : closed
and the forces in the structure are
concentrated : distributed
in lines and points : along lines and in planes

An example of dual polyhedral structures are shown in fig.3. The above considerations applied to the five Platonic polyhedra divides them into three groups, each with a stable lattice and a stable plate appearance, following exactly the well-known geometrical dualism (see fig.4). These relations may be extended to arbitrarily facetted polyhedra. A polyhedron, which is neither purely triangulated nor 3-way vertexed, may achieve stability by co-operation between the lattice part and the plate part of the structure, in such a way that so-called buffer forces may be transferred between bars and identically positioned lines of support (Fig.6). This effect means that a difference in axial force at the two ends of a bar results in the same difference between the shear forces at the two sides of the adjacent line of support.

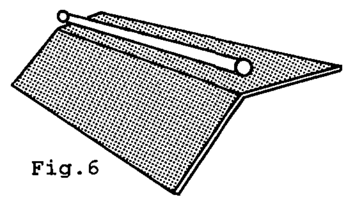

Fig.6

Dualism between exact geometry and statical analysis

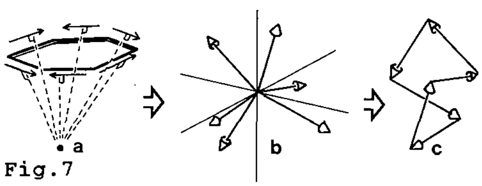

Fig.7

The dualism described so far at the level of topology and stability, may be extended to the level of exact geometry and statical analysis. The core in this relation is equalizing the equilibrium of forces to the equilibrium of moments (Wester, 1987a & 1987c). This exact dualism is based upon the simple fact that a spatial, closed vector polygon, where the vectors are one-way directed through the polygon (fig.7c), describes an unambiguous equilibrium of force vectors on a node (Fig.7b) as well as moment-vectors of the forces on a plate around a point outside the plane of this plate(fig.7a). In the latter case, fig.7b represents the moment vectors on origo. The two situations are so alike that it cannot be determined from the vector polygon alone (fig.7c) whether it describes an equilibrium of force vectors or moment vectors. It is further evident that the two systems cannot be mixed as the equilibrium requires that the whole polygon consists of either forces or moments. If these considerations are continued and put into equations, the whole thing ends up in very basic relations:

- A plane-point is polar to its dual point-point, i.e. the extension of their connection line hits origo, and the product of their distances to origo is an arbitrarily chosen constant.
- The ratio between an axial force and its dual shear force is a simple geometrical relation (the length of the bar over sinus to the angle between the vectors from origo to the nodes in the bar).
- The stiffnesses are related by equalizing the virtual work performed by the axial force in the bar and the shear force over the line of support. The dual ratio between the two is the square of the ratio for the forces.
- The dualism is valid for any 3-D plate and lattice structure.
- The "Euler-Number", the sign of the Gaussian curvature and the level of redundancy remain unchanged during the dual transformation.

<u>Some more dual relations</u>

LATTICE	PLATE
Node with N bars.	Plate with N lines of support.
N points in the same plane.	N planes through the same point.
Triangular mesh.	3-way vertex.
Right-hand sign rule for forces.	Left-hand sign rule for forces.
Visible deformations ie changing the length of its vector.	Invisible deformations ie rotation of a plane.
Invisible def. ie rotation of a node.	Visible def. ie changing the length of its position vector.

CONCLUSIONS

The present theory is seen to create a unification as well as a symmetry between the most basic, archetypal kinds of structure based on axial and shear forces. The correlations between them are general and may be used for any 3-D system. The symmetry is so basic that it fits on many levels, from general considerations of the nature of antagonism, over suitable pedagogical explanations on spatial general stability, over simple rules for design and general analysis of structures, e.g. several biological structures, up to an operational tool for numerical statical analysis. The theory has of course initiated a great number of interesting, only partly clarified, problems some of which seems to include great visual qualities.

ACKNOWLEDGEMENTS

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THE SYMMETRY OF STRUCTURE IN THE GENERATION OF POLYHEDRAL LATTICES

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In defining a systematic process, it is always difficult to decide whether to begin with the whole system and break it down into its primitive elements or to begin with the elements and demonstrate how they generate the whole. I have taken advice from Piaget (1968, p. 141): "Genesis is simply a transition from one structure to another, nothing more; but this transition always leads from a `weaker' to a `stronger' structure; it is a formative transition. Structure is simply a system of transformations, but its roots are operational; it depends, therefore, on prior formation of the instruments of transformation rules and laws."

I have composed three first generation tables of eighty polyhedra each. A look at their general organizational differences leads to their elemental instruments of formation. I think of polyhedra as internally regulated forms poised in a process of evolution rather than as permanent individual structures. Even though the forms in these tables define conventional polyhedra, the assemblage route to their formation is not a conventional method of construction but a tranformational process of generation.

Slide 1. Three different periodic arrangements of the same 80 polyhedral lattices.

	circumferential	radial
a.	20-20	20-20
b.	40	40
c.	canine: 50	bovine: 30

Angular Polyhedral Lattices

The 80 lattices, generated from elemental units, manifest as lines and junctions (vertices) I call foundation sutures. The foundation suture elements contain the equatorial junction valences that join the hemispheres of a polyhedral lattice. A second structural element I call caps provides the mechanics for polar closure. These two formational elements together, sutures and caps, form a template. The template, an irreducible reservoir of pairs of structural elements, contains the instruments of formation. There are two species of foundation sutures, canine and bovine, and two kinds of capping operations, circumferential and radial. When these elements are symmetrically coordinated, they generate the 80 polyhedral lattices.

Slide 2. Elemental template.

The process for coordinating the sutures and caps is one of simple numerical symmetry. The elements correspond with each other in a fundamental 2, 3, 4, 5 and 6 unit evolution. Twist establishes the up and down (z) axes of symmetry. Those lattices that twist upon their axes of symmetry are called primary (p) and those that untwist are called secondary (s).

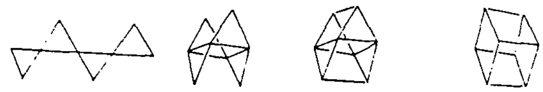
Slide 3. Demonstration of the process of assemblage with the simplest lattice in each domain.

a. canine tetrahedron:

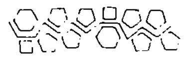
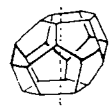
2-D suture 3-D ring (p) capped (s)

b. bovine prismatic octahedron:

2-D suture 3-D ring (p) capped (s)



Slide 4. Analysis of R.E. Williams 6-fold betatetrakaidecahedron.

caps  hybrid suture 

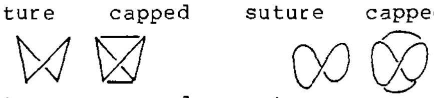
Curvilineal Polyhedral Lattices

Conventional polyhedral lattices may be converted to curvilineal forms via this form generating method. The curvilineal forms are topologically equivalent to the angular lattices because the vertices are conserved in the process of conversion and, consequently, even though their shapes change drastically, the facial areas and vertices remain in the same locations as in the conventional angular polyhedra. The straight lines of the foundation sutures become twisted loops and the straight lines of the caps become arcs.

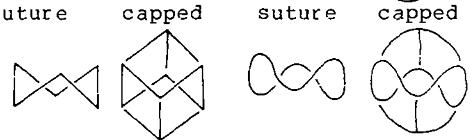
Slide 5. Conversion from angular to curvilineal.

suture capped suture capped

a. tetrahedron:



b. hexahedron: suture capped suture capped



Loop Generation

There is yet another formational level where conventional polyhedral lattices are transformed to 3-D rings of twisted loops. These configurations are not, however, topologically equivalent to the conventional polyhedral lattices because all of the vertices are absorbed in the lineal conversion process. The primitive neutral form of the looping configurations is an unknotted linear ring that evolves from simple to complex in accordance with the number of twisted loops that it acquires. The twisted loop pathways are based upon the same geometric coordinates as conventional lattices, but the lineal orbits are continuous and junctionless. The unique attribute contained in the loop dynamics is in the capability to produce both exostructural and endostructural modes.

Slide 6. Two phases in the loop transformation.

a. the five Platonic angular loops
b. table of curvilineal twisted loops

The elements for the loop configurations are single loops and double loops. Single loops twist right or twist left individually and double loops twist right and left simultaneously.

Slide 7. Loop dynamics.

a. single and double loop elements
b. exostructural and endostructural gymnastics

Summary

Schooled in the arts but, over the years not finding enough either in the history of art or in the contemporary forms of art to hold my attention, I turned to reading about the scientific approaches to structure. This is a reversal of the experience of Smith (1981, p. 358) "I have slowly come to realize that the analytical quantitative approach that I have been taught to regard as the only respectable one for a scientist is insufficient. Analytical atomism is beyond a doubt an essential requisite for the understanding of things, and the achievements of the sciences during the last four centuries must rank with the greatest achievements of man at any time: yet, granting this, one must still acknowledge that the richest aspects of any large complicated system arise in factors that cannot be measured easily, if at all. For these, the artist's approach, *uncertain though it inevitably is, seems to convey more meaning.*" In my schooling, I was allowed to miss the analytical approach altogether. The greatest impact upon my thinking as an artist came later and was the discovery of the Einstein-relativity/Bohr-quantum theory dialogues. This has projected me into a new heretofore unforeseeable level of possibilities.

Slide 8. These slides show some of my information sources plus some of the forms of art that I am now producing.

a. natural patterns containing the foundation suture and orbital loop elements.
b. collages with combined natural materials.
c. sculptures based upon information found mostly in the junction patterns of natural systems.

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THE PRECEDENCE OF GLOBAL PROCESSING IN SYMMETRY PERCEPTION

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It is possible to devise two-dimensional symmetry patterns exhibiting reflection, and rotation, which can be studied at two structural levels of the patterns. Firstly, there is the global structural level, defined by the two-dimensional point group specifying the whole pattern and, secondly, there is the local structural level, defined by smaller two-dimensional point groups contained within the pattern. Hence there are two sources of symmetry information which could be utilized by the human visual system in the perception of symmetry. Utilization of information at the global structural level of the pattern implies that the perceptual processes are organized so that they proceed from global structuring of a pattern towards a more detailed analysis of its component parts. On the other hand, utilization of information specified at the local structural level of the pattern, implies that the perceptual system builds up a pattern from an analysis of these component parts.

Recent research has provided evidence for both local and global precedence in perception. Evidence for the precedence of global processing originates in the demonstrations of the Gestalt psychologists, who proposed that the nervous system is organized so as to respond to the 'Gestalt' or overall configuration of a stimulus pattern (Kohler, 1929; Wertheimer, 1958). Similarly, Navon (1977) has argued for global precedence by proposing that global processing is completed before a more local analysis occurs. Recently, however, Kinchla and Wolfe (1979) have shown that the precedence for global processing shown by Navon, shifts to a precedence for local processing as the overall size of the stimulus is increased. Evidence for more locally directed processing comes from Pomerantz and Sager (1975), who showed that subjects are better at ignoring global rather than local forms. Yet, other results show that neither level can be completely ignored (Boer & Keuss, 1982; Hoffman, 1980; Miller, 1981).

The following study tests the hypothesis of global precedence in perception against that of local precedence, by investigating the relative contribution of symmetry information available at the local and global structural levels of the pattern, to the degree of symmetry perceived. It is postulated that if global processing has precedence in perception, then the degree of perceived symmetry will be specified by symmetry information available at the global structural level of the pattern. If, however, local processing has precedence, then the symmetry information available at the local structural level of the pattern will specify the degree of symmetry perceived. Four different two-dimensional symmetry patterns were created from bilaterally symmetric sequences of differently sized white dots placed against a black background. The first pattern was created by repeating a single such sequence of 16 dots, in both the horizontal and vertical directions, thereby forming a two-dimensional pattern having a four-fold rotation axis and four mirror lines intersecting at its center (4mm). This symmetry structure defined at the global level, is consistent with that of the smaller two-dimensional point groups comprising this pattern, which also have the classification, 4mm (see Figure 1). The second and third patterns were created from two different bilaterally symmetric dot sequences, each of which was comprised of 8 dots. In the second pattern, these sequences extend in the horizontal and vertical directions, thereby forming two-dimensional point groups, half of which have two mirror lines intersecting at the two-fold rotation axis (2mm) and the other half have four mirror lines intersecting at the four-fold rotation axis (4mm). These point groups were reflected about the horizontal and vertical axes to form a larger two-dimensional point group having the classification, 4mm (see Figure 2). In the third pattern, the order of occurrence of the bilaterally symmetric dot sequences used in the second pattern was reversed for the sequences extending in the horizontal direction. The result of this was to produce two-dimensional point groups which are the same as those in the preceding pattern, but which are differently arranged relative to each other. Thus the reflection of these groups about the horizontal and vertical axes produced a larger two-dimensional point group having the classification, 2mm (see Figure 3). Thus both the second and third patterns specify the same symmetry structure at their local structural level, but in both instances this symmetry structure is not consistent with that defined at the global level of the patterns. The fourth pattern was created by combining the bilaterally symmetric sequences of dots used in the preceding patterns. The result of this was to produce two-dimensional point groups having two mirror lines intersecting at the two-fold rotation axis (2mm). These groups were reflected about the horizontal and vertical axes to form a larger two-dimensional point group having the classification, 2mm (see Figure 4). Thus, as in the case of the first pattern, the symmetry structure defined at the global level of this pattern is consistent with that defined at its local level. It is, however, different from that of the first pattern, and is the same as that defined at the global level of the third pattern.

These four patterns were shown to 28 subjects who were asked to rank them for symmetry, from the least symmetric to the most symmetric. It was predicted that the first two patterns or the last two patterns, which have the same globally defined symmetry structure, would be perceived as equally symmetric if global processing has precedence in perception. On the other hand, if local processing has precedence, then it would be the second and third patterns that would be perceived as equally symmetric, as these patterns have the same locally defined symmetry structure. Results showed that the patterns which differed significantly from each other were those having the same locally defined symmetry structure and those specifying a different symmetry structure at the global level. The patterns which were not differentiated from each other on the basis of perceived symmetry were the first two patterns and also, the last two patterns. These results support the hypothesis that global analysis of a pattern precedes a more local or fine-grained analysis. In addition, they show that it is symmetry information available at the global structural level of a pattern that is utilized by the visual system in the perception of symmetry. In the realm of art, there is an apparent intuitive awareness of this. A work by the Op artist, Dieter Hacker, provides a good example. In his "Cubes" (1963), the symmetries defined at the local structural level of the composition are perceived secondary to the global effect of rhythmic movement created by the reversible figure illusion (see Figure 5).

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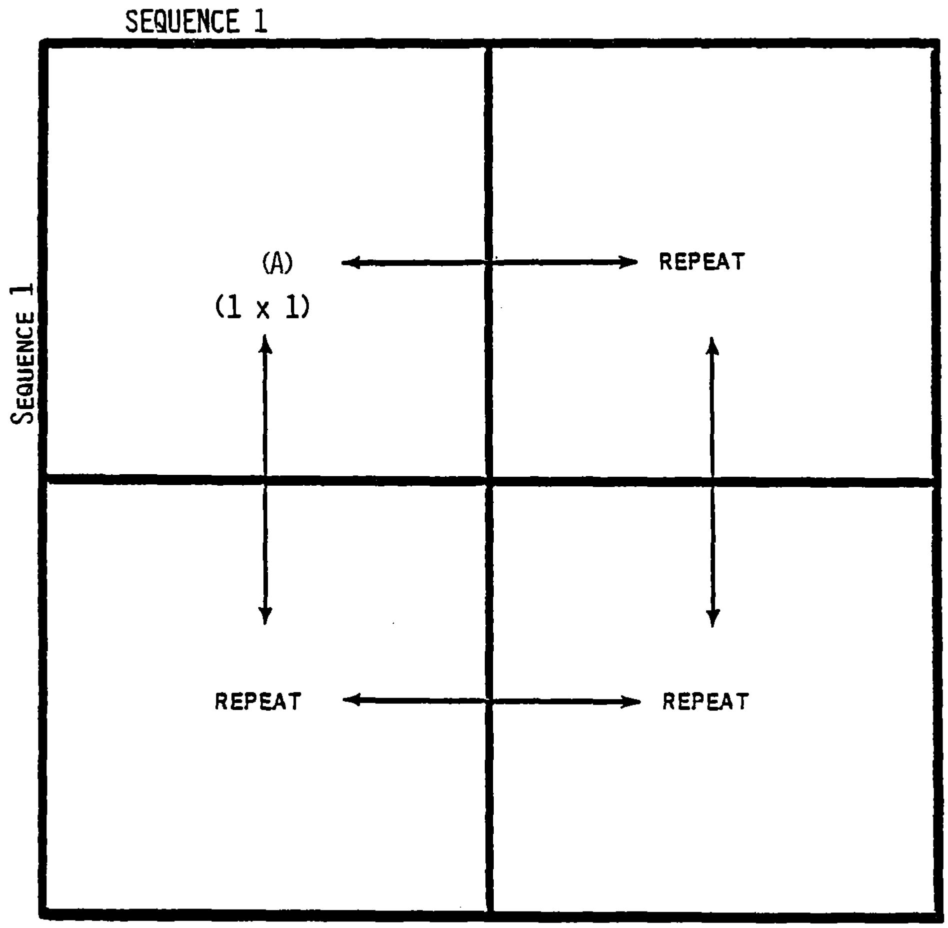
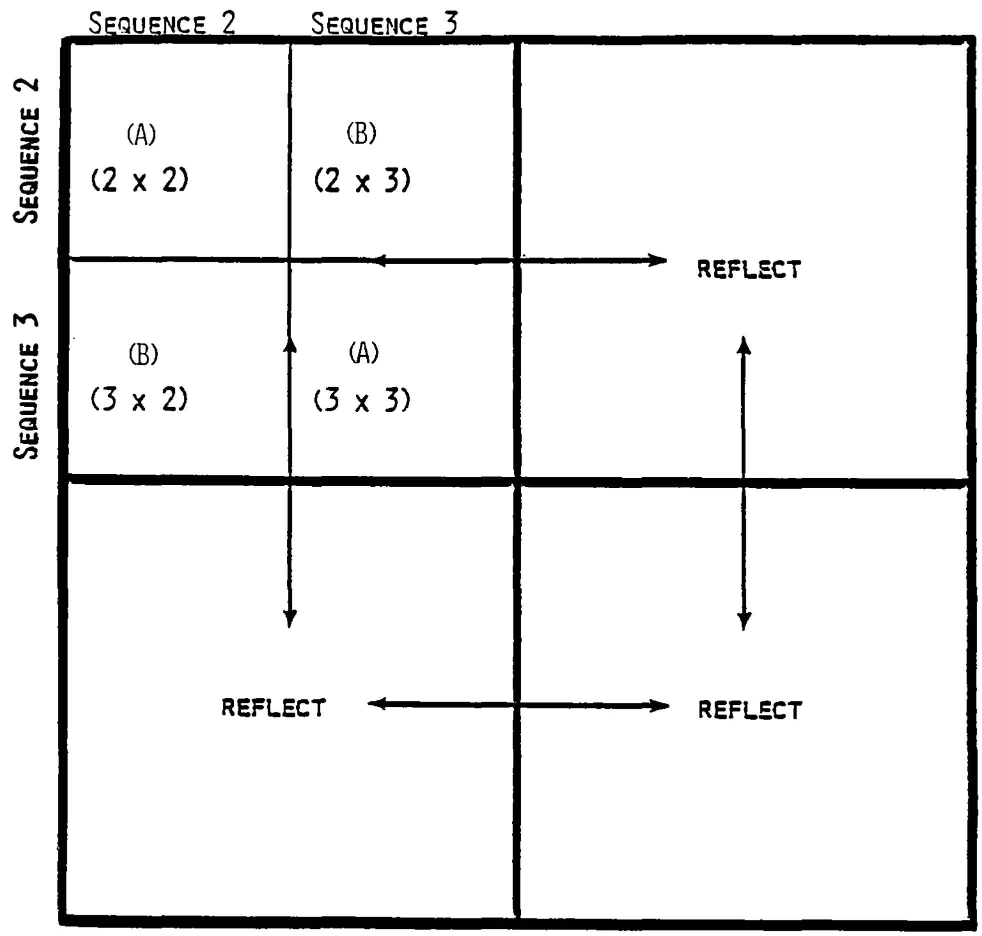
FIGURE 1



(A) LOCAL SYMMETRY PATTERN HAVING A FOUR-FOLD ROTATION AXIS AND FOUR INTERSECTING MIRROR LINES AT ITS CENTER (4MM).

THE GLOBAL SYMMETRY PATTERN HAS THE SAME SYMMETRY STRUCTURE AS THE LOCALLY DEFINED PATTERNS (4MM).

FIGURE 2

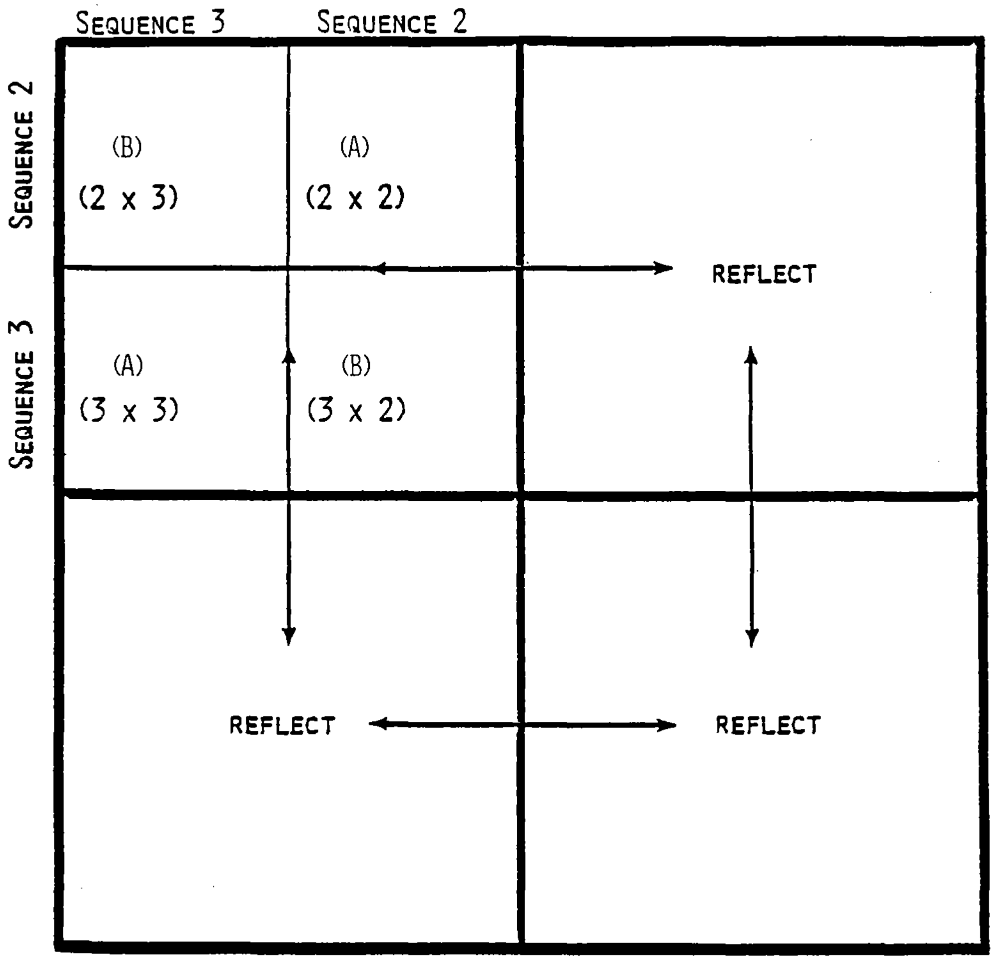


(A) LOCAL SYMMETRY PATTERN HAVING A FOUR-FOLD ROTATION AXIS AND FOUR INTERSECTING MIRROR LINES AT ITS CENTER (4MM).

(B) LOCAL SYMMETRY PATTERN HAVING A TWO-FOLD ROTATION AXIS AND TWO INTERSECTING MIRROR LINES AT ITS CENTER (2MM).

THE GLOBAL PATTERN HAS SYMMETRY 4MM.

FIGURE 3

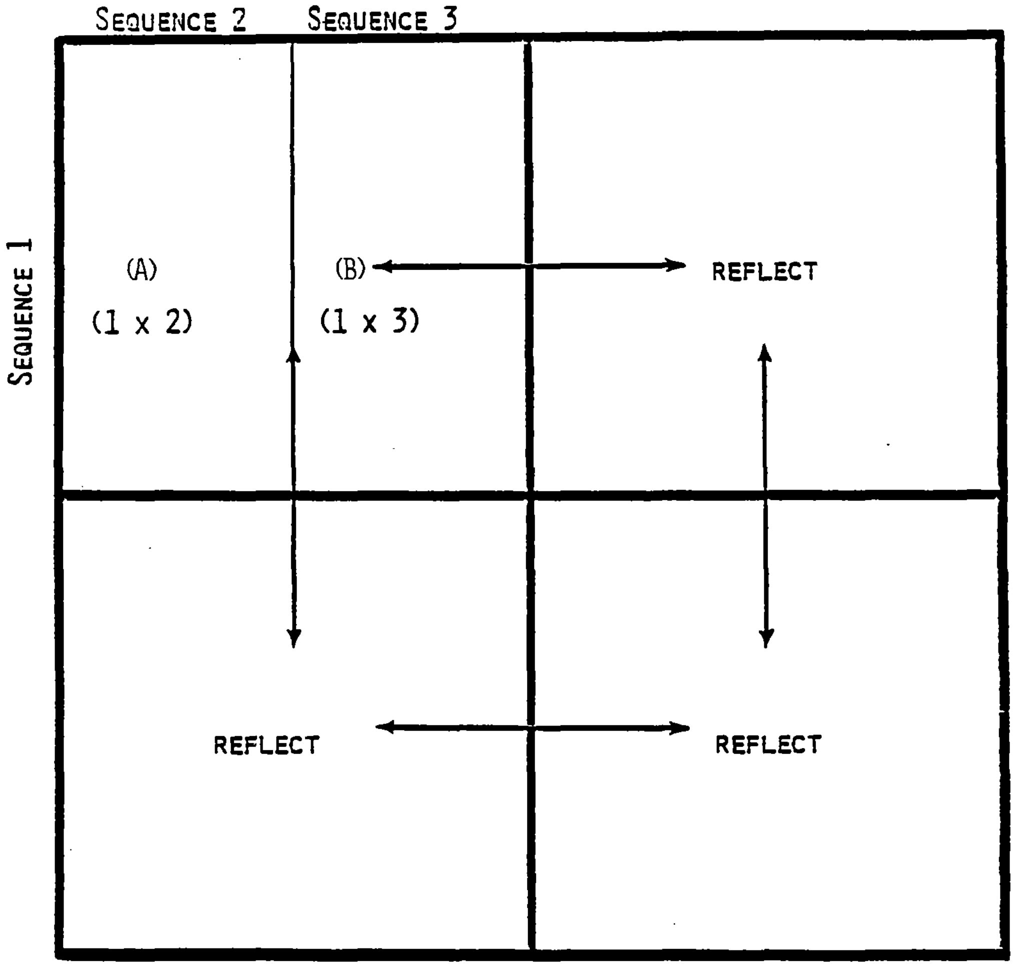


(A) LOCAL SYMMETRY PATTERN HAVING A FOUR-FOLD ROTATION AXIS AND FOUR INTERSECTING MIRROR LINES AT ITS CENTER (4MM).

(B) LOCAL SYMMETRY PATTERN HAVING A TWO-FOLD ROTATION AXIS AND TWO INTERSECTING MIRROR LINES AT ITS CENTER (2MM).

THE GLOBAL PATTERN HAS SYMMETRY 2MM.

FIGURE 4

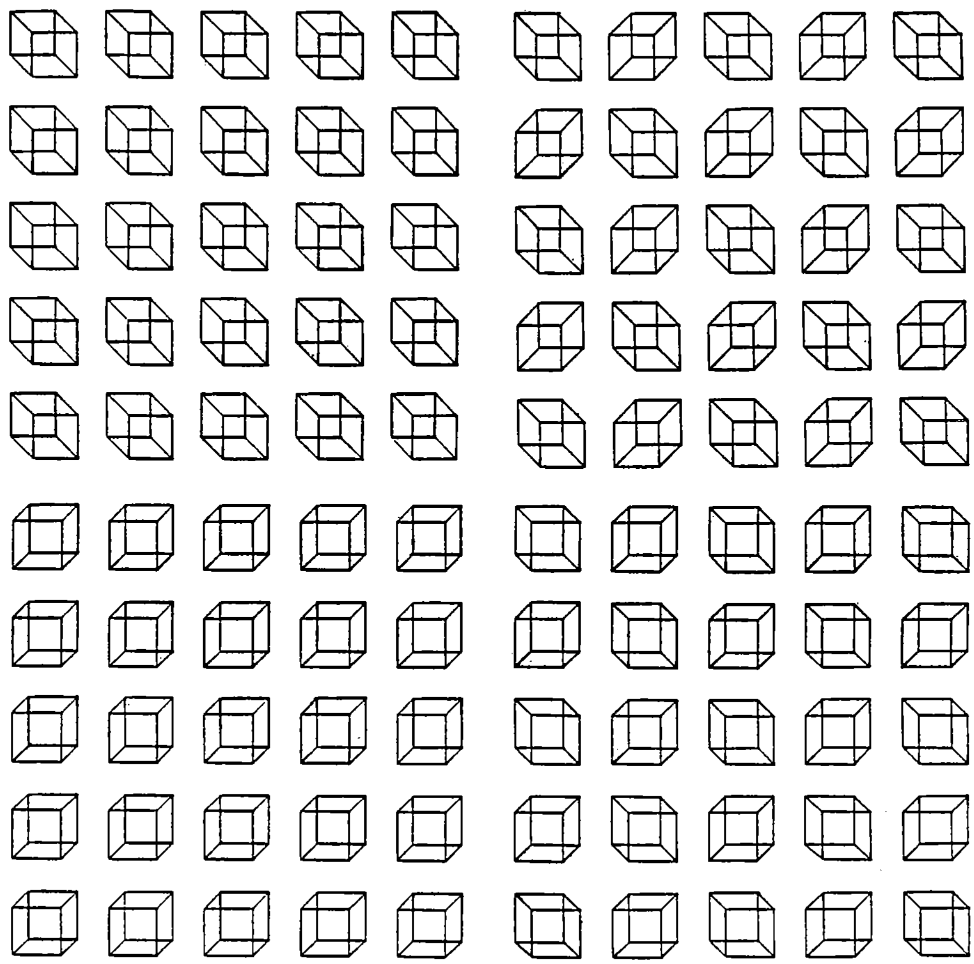


(A AND B) LOCAL SYMMETRY PATTERNS HAVING A TWO-FOLD ROTATION AXIS AND TWO INTERSECTING MIRROR LINES AT THEIR CENTERS (2MM).

THE GLOBAL PATTERN HAS SYMMETRY 2MM.

FIGURE 5

DIETER HACKER "CUBES" 1963, SERIOGRAPH 25 x 25 IN.



SYMMETRY AND THE ARCHAEOLOGY OF MIND

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Abstract

The nature of symmetry as a pattern imposed on artifacts has changed significantly over the course of human evolution. The symmetry concepts employed by modern humans have no antecedents in the conceptual repertoires of our nearest relatives, the chimpanzees. It is clear that they must have evolved.

Modern humans not only produce symmetrical patterns, they use symmetry as a principle to transform patterns. Leaving aside for the moment an explanation for this phenomenon, let us first examine some examples.

The Shipibo Indians of Peru use a distinctive style to decorate textiles, pots, and so on. It employs a few simple motifs and rules to transform those motifs. Most of these rules are rules of symmetry, bilateral and rotational. The motifs are not symmetries but the rules transform them into symmetries. Each individual artisan employs these rules differently, yielding idiosyncratic styles within the general style (Roe 1980).

Ban Chiang pottery from northeastern Thailand presents a similar situation, though in this case the example is from recent prehistory (first millennium B.C.). Ban Chiang potters used several rules of symmetry to fill the decorative field on their pots. The most favored rules used combinations of longitudinal and transverse reflection, sometimes combined with rotation. Once again, the motifs are fairly simple, mostly curved lines and loops. It is the symmetrical transformation that supplies interest and complexity (Van Esterik, 1979).

Comparable rules of symmetry can be found in house building in colonial Virginia, U.S.A (Glassie 1975). The tradition of house building was imported, primarily from England, in the 17th and 18th centuries. In it, certain basic structural elements, including basic room dimensions, window size, etc., are combined using basic rules of assembly. Many of these rules are rules of symmetry and, occasionally, asymmetry. For example, if an exterior was has a central door, it must have an equal number of windows on either side. Once again the elements are simple and are "added" together using rules of symmetry.

These three culturally separate groups of modern humans employ symmetries as a transformational principle in producing patterns of material culture. If not a universal, it is certainly very common and suggests that the modern human mind turns easily to symmetrical transformations.

Symmetrical transformations are unknown for modern chimpanzees, our nearest relatives. Indeed, symmetrical patterns are almost unknown. Thirty years ago much publicity was given to paintings produced by captive chimpanzees. Early interpretations made claims for a sense of "symmetry"

or "balance" in the compositions. However, recent controlled experiments have failed to find any evidence for a sense of symmetrical composition in chimpanzee drawing (Boysen et al. 1987). The only consistent symmetry produced by chimpanzees occurs in the construction of nests in the wild. Chimpanzees pull in nearby branches and weave them into a platform. The result is a radial symmetry. However, it is almost certainly an unintended consequence of the biomechanics of the task and is not a cognitive competence.

In sum, modern humans not only produce symmetrical patterns, they use symmetry as a principle of transformation. Our nearest relatives, chimpanzees, do neither. When, how, and why did this quirk of human thinking appear? Archaeology presents a record of the evolution of the concept of symmetry. Most of this sequence consists of patterns imposed on stone tools. Stone tools are, unfortunately, not an ideal medium.

Stone is a relatively intransigent medium whose qualities present problems in interpreting artifactual symmetries. For example, some types of stone used for artifacts have cleavage planes that affect the nature of stone fracture. Moreover, the simple physics of stone fracture occasionally produces symmetries. As a consequence, it is possible to have symmetrical patterns that are not clearly the intention of the prehistoric artisan. These must somehow be factored out.

There is also no assurance that prehistoric artisans used their most sophisticated spatial concepts, including symmetries, when they made tools. They could well have used more complex symmetries in realms that are archaeologically invisible. This is the problem of "minimum necessary competence." Because of it we risk underestimating the abilities of prehistoric people.

Despite these methodological caveats, archaeologists can, in fact, document a sequence of development. I will describe artifact symmetries at four points in prehistory: 2 million years ago, 1.2 million years ago, 300,000 years ago, and 15,000 years ago.

Artifacts indicate that by 2 million years ago our ancestors still used the spatial concepts typical of apes. In other words, there is no evidence for a concept of symmetry. Tools from this time period appear to have been manufactured with little or no attention to overall shape. Only shape of edge appeared to interest these early hominids (Toth 1985). In modifying edges, the hominids used relatively simple spatial concepts: proximity, boundary, and order.

By 1.2 million years ago there is evidence for a concept of symmetry, but it is a relatively primitive kind of symmetry. Many of the early tools termed bifaces have a crude bilateral symmetry. Indeed the symmetry is often so rudimentary that one is tempted to argue that the symmetry idea belongs only to the modern archaeologist! Nevertheless, on some of the bifaces one lateral edge is almost certainly a reflection of the other. Such a symmetry does not require the euclidean concept of congruency but does require the topological concept of reversal of order and some notion of two dimensional, overall shape. At this time there are also remarkably round artifacts termed discoids and spherical artifacts termed stone balls or spheroids, suggesting a concept of radial symmetry.

The use of a symmetry concept, however rudimentary, places these 1.2 million year old hominids beyond the range of ape spatial performance. However, the symmetry concept is far from modern.

By 300,000 years ago, and probably considerably earlier, stone tools present patterns of symmetry that include euclidean congruency. Congruency requires conservation of amounts of space, not just similar patterns. In addition to congruency, these artifacts often have bilateral symmetry in three dimensions, not just two. It appears that by 300,000 our ancestors could conceive of and execute symmetrical patterns that match in sophistication the basic symmetrical patterns produced by modern artisans.

While we do not yet have evidence for symmetry as a principle of transformation we do, by 300,000, see intentional "violations" of symmetry in the form of fine asymmetrical artifacts and "S-twists" in the profile of the lateral edge. These suggest that the concept of symmetry is in fact more elaborate than simple "reflection of congruency."

By 15,000 years ago, which is virtually the present by stone age standards, we have evidence of symmetry as an organizing principle in media other than stone. In Franco-Cantabrian cave art, the figures do not appear to have been placed randomly in caves but arranged according to principles of composition, some of which are elaborations of a symmetry principle. Similar symmetries can be found on mobiliary art of the same period. However, even by 15,000 years there is no good evidence for symmetry as rule of transformation in the sense that we encounter it in, say, Shipibo textiles. This is puzzling because in most other respects the material culture of this time period appears modern (though not western, industrial, of course).

The sequence that I have described is extremely coarse. Nevertheless, it does document the appearance, perfection, and then elaboration of symmetry as a pattern on artifacts. It remains to examine what implications this sequence has for understanding human evolution. The development of concepts of symmetry, while in and of itself interesting, may have been linked to other developments in the evolution of mind. Two of especial interest to me are the evolution of intelligence and the appearance of "transformational rules."

In my work (e.g., Wynn 1989) I have used Piagetian theory and the geometry of stone tools to assess the intelligence of early hominids. The complete lack of symmetry concepts in the material culture of chimpanzees and two-million-year-old hominids suggests that both fall within the earlier "symbolic" substage of preoperational intelligence, Piaget's second major stage of intellectual development. Apes generally test at this level so the lack of symmetry is consistent with other aspects of their behavior. Traditional interpretation grants two-million-year-old hominids greater intelligence than apes. Lack of symmetry is corroborated by other aspects of culture, however, and it appears that traditional interpretation of our early ancestors hay have been too optimistic.

The rudimentary symmetry of 1.2 million-year-old artifacts suggests that these artisans used intuitive preoperations and were therefore demonstrably more intelligent than apes. More importantly, it indicates an intelligence much less centered on ego than that of symbolic preoperations.

The congruent symmetries of 300,000 year old bifaces required concrete operational intelligence, Piaget's penultimate stage. Especially telling are the symmetries in cross-section, which require reversibility and conservation to conceive and execute. These are hallmarks of operational thought. Evidence for formal operations, Piaget's final stage, is virtually impossible to document from material culture of any age. Artifactual symmetries require only concrete operations.

In sum, symmetry helps us trace the development of hominid intelligence from an essentially ape grade at two million to essentially modern by 300,000.

Symmetry may also help document the evolution in cultural complexity in another respect - the complexity of conventional forms (content being unaccessible archaeologically). The material culture of two-million-year-old hominids, like that of apes, presents no arbitrary forms. Shapes of tools appear to have been tied to immediate tasks at hand. By 1.2 million we have the appearance of a conventional form, in the guise of rudimentary symmetry on artifacts. There appears to have been no overriding mechanical reason for the shape and, moreover, it was a community standard or convention, not an idiosyncratic production of one artisan.

By 300,000 conventional, arbitrary form is well-defined. Indeed, fine symmetries as standard patterns probably appear by 600,000 years ago or so. But by 300,000 symmetry as a convention begins to lose its monolithic strangle hold on form. We see aesthetically pleasing violations, in the form of asymmetries that were clearly intentional and S-twists in the profile of lateral edges. All this suggests that symmetries and by implications other conventional cultural systems, were much more dynamic than before.

Symmetry as a transformational rule is common in modern culture (see earlier discussion). We cannot document the appearance of this development until very late in prehistory; indeed not until after 10,000 years ago.

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EVOLUTION OF SYMMETRY IN MINERAL WORLD

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Symmetry is a universal property characteristic of all natural phenomena and objects and simaltaneously in some aspects specific for each of them. Symmetry is particularly pronounced in the mineral world which elements, i.e. crystals, "shine with symmetry", as the great crystallographist E.S.Fedorov put it. That is why symmetry has been used for a long time to characterise as well as to identify minerals and to prognosticate their properties. Moreover, theoretical and methodical fundementals of the theory of symmetry have been formed on the basis of mineral crystals study within the mineralogical science.

In mineralogy main objects to be investigated in terms of symmetry have always been mineral individuals,i.e. crystals. Methods of the theory of symmetry have in fact never been applied to characterize more complicated systems composed of minerals (mineral aggregates, rocks, fault blocks of the crust, geospheres, cosmic bodies etc.), although A.E.Fersman, A.V.Shubnikov, I.I.Shafranovsky, A.Mackay and other crystallographists drew attention to the fact that such an appliance would be very promising. This weakly investigated field of mineralogy became an object for our research (Yushkin, 1984; Yushkin et al, 1987).

Investigations showed that complex mineral systems are characterized by different values of summary symmetry which is typomorphic, alias reflects genetic nature of objects and changes regularly in the process of mineral systems evolution.

We have introduced the concept of crystallosymmetric structure of complex mineral systems (CSS) which we understand as a proportion of minerals of different crystallographic symmetry in their composition. The means of CSS investigation is crystallosymmetric analysis (CSA) which consists in establishing statistic parametres of minerals distribution on the ranks of the symmetry system (Yushkin, 1985). CSA procedure is rather simple and consists in compiling tables containing percentage proportion of minerals belonging to different symmetry ranks and deducing a number of summary values including symmetry index.

The symmetry index is calculated according to the formula:

$$I_s = \frac{1}{6} \sum_{R=0}^{6} P_R \cdot R,$$

in which P_R is the percentage of occurence of minerals of the given rank R, i.e. of the given crystal system (R=0+6: triclinic - 0, monoclinic - 1, orthorhombric - 2, trigonal - 3, tetragonal - 4, hexagonal - 5, cubic - 6); P_2 = 100%. The symmetry index varies from 0 up to 100% towards the increase of the percentage in the polymineral system of highly symmetrical minerals. If a system is presented by mineral species of the triclinic crystal system only, then $I_s = 0$, if of purely cubic system, then $I_s = 100\%$.

The following totality of basic parametres characterizing CSS of the system to investigate has been developed on the basis of CSA in general terms: parametres of mineral distribution on symmetry categories, crystal systems, point groups (crystallographic classes); symmetry indices, informational entropy of crystallosymmetrical structure H_s. These criteria can be calculated on the constitutional (proportion of the number of mineral species) as well as on the concentration (prportion of the volume of substance of different species) basis.

Each geological system composed of minerals is characterized by a definite CSS which finds its expression in constant parametres of mineral species distribution on the ranks of the symmetry system (categories, crystal systems, point groups). We suggest that this conclusion formulated by I.I.Shafranovsky as the basic law of mineral symmetry statistics (Shafranovsky, 1983, p.66) should be called Shafranovsky's law. The latter presupposes that each mineral system can be distinquished by characterizing symmetry parametres. Resulting from CSA carried out on a considerable amount of mineralogical data we have developed certain parametres to characterize the lithosphere and other geospheres of the Earth as well as the Moon's lithosphere, various types of meteorites, large regional structures such as mineralogical provinces, different mineral deposits, mineral complexes (Yushkin, 1985; Yushkin et al., 1987).

A number of common tendencies in the CSS change has been established.

In direction from abyssal to subsurface zones of the Earth, i.e. to the lithosphere, the process of constant and definite decrease of the mineral substance summary symmetry is taking place alongside with the replacement of minerals of the cubic system, when those of orthorhombic and later monoclinic, triclinic and other crystal systems become dominant. Consequently, the value of the symmetry index also decreases (Fig.1).

Similar tendencies in the CSS change are established by the analysis of the evolutional row of cosmic objects: meteorites (chondrites) → the Moon → the Earth which can be regarded as the main stages of iur planet's development, i.e. meteoritic → "lunar"(regolith-nuclear) → modern. Monotonous decrease of summary symmetry of substance (I_s const) from chondrites (62.50%) through the Moon's lithosphere (55.67%) to the lithosphere of the Earth (42.56%) is of particular interest. It reflects a general evolution tendency. Concentrational value of summary symmetry changes in the same direction: $I_s = 33.58 \rightarrow 29.00 \rightarrow$ [illegible]. The number of mineral species of elementary crystal systems grows abruptly in direction from meteoritic, in particular chondritic substance towards lunar and earthly substance, symmetry entropy of the systems also increasing.

Duration and complexity of the evolution of the mineral deposits, mineralogical provinces as well as energy needed for their formation, correlation of endogenuous and exogenuous factors of mineral formation, role of organisms and organic matter are established exactly enough with the help of crystallosymmetrical indications (Yushkin, 1985).

Basing on the investigation results we have formulated the law of geological evolution of mineral systems' crystallosymmetrical structure: in course of cosmic and geological evolution their crystallosymmetrical structure undergoes complication and qualitative change which manifests itself in the increase of entropy of all symmetry characteristics as well as in the replacement of the dominant role of minerals of cubic and orthorhombic systems to monoclinic; decrease of the mineral substance symmetry occurs alongside with still high and even increasing external morphological symmetry of the Earth and other cosmomineral bodies.

Methodical approach described here can be applied not to the mineral world only, but to any other objects containing cristalline matter (chemical and metallurgical products, synthetical systems, objects of human material culture etc.).

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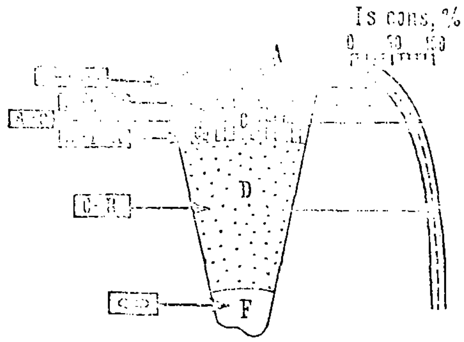

Fig.1 Change of parametres of the Earth's mineral substance crystallosymmetrical structure with depth. Geozones: A - the crust, B - upper mantle, C - intermediate mantle, D - lower mantle, E - external "plastic" core.

Crystal systems: TC - triclinic, M - monoclinic, R - orthorhombic, T - tetragonal, C - cubic.

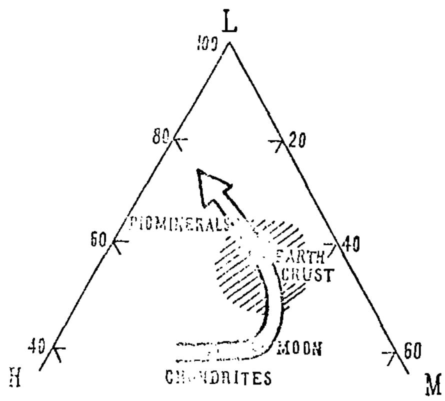

Fig.2 Direction of the change of crystallosymmetrical structure in the row: chondrites - Moon's lithosphere - Earth's lithpspere - biominerals.
Symmetry categories: H - high (cubic system), M - middle (trigonal tetragonal and hexagonal systems), L - low (triclinic, monoclinic and orthorhombic systems). Shaded areas shows the field of figurative points of the mineralogical provinces of the Earth.

Symmetry and the Crystallography of Logic

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The history of logic has been divided into three periods. The "traditional" period starts with Aristotle. The "algebraic" period starts with Boole. The "logistic" period starts when Russell successfully rescues the ideas of Frege. In what follows I will be going back to the middle period, back to the time of Boole, Schroeder, and Peirce.

More specifically, what follows will focus on a direct continuation of two of the main ideas that were emphasized by Peirce. In reference to logic, especially to what is called the propositional calculus. he understood the importance of symmetry. In reference to notation, he understood the importance of iconicity. But he did not go far enough. What follows will show that, when we start with a custom-designed notation, it is an easy step to go from symmetry-iconicity to the crystallography of logic.

In 1902 Peirce devised three iconic notations for the 16 binary connectives (and, or, if, etc.). Before we push his work a small notch ahead, let us look at a simple example in binary logic. It expresses the duality of "and" and "or," as in (1) and (2). A mental grasp of (1) and (2) can be maintained in three ways. Mem-

(1)	(A and B)	≡	Not(Not-A or Not-B)
(2)	(A . B)	≡	N(NA v NB)
(3)	N(A . B)	≡	(NA v NB)
(4)	N(A v B)	≡	(NA . NB)

orize it, derive it from something else that has been memorized, such as we see in De Morgan's laws [(3) and (4)], or work it out again and again, each time coming back to the truth table method.

See Table I, where (2) the duality of "and-dot" and "or-vee" has been worked out three times. Standard form has been used in (a), where the four 2-place entries (TT,TF,FT,FF) line up in a vertical column. Negation acts on A, on "or", and on B, so that it changes the truth table for "or" (A TTTF B), in such a way that it matches the truth table for "and" (A TFFF B), thereby justifying the presence and the validity of the equivalence sign.

Why is it a poor practice to place (TT,TF,FT,FF) in a vertical column? Because this arrangement is not sensitive to the equivalence relations between the connective relations between (A,B). In other words, it does not give us direct notational access to second-order relations. Instead, it forces us to memorize these relations, or work them out again and again, as in (a).

Table I

	1.	2.	3.	4.	5.	6.
	(A,B)	(A B)	(A V B)	(NA,NB)	(NA v NB)	N(NA v NB)
(a)						
	T T	T	T	F F	F	T
	T F	F	T	F T	T	F
	F T	F	T	T F	T	F
	F F	F	F	T T	T	F
(b)						
	FT TT	F T	T T	TF FF	T F	F T
	FF TF	F F	F T	TT FT	T T	F F
(c)						

<u>Symmetry</u> <u>form</u> has been used in (b), where (TT,TF,FT,FF) now stand in the quadrants of Cartesian (A,B) coordinates. Rule 1 (R1) says that when N acts on A, as in NA, the 4-fold truth table for "or" (TTTF) is <u>flipped</u> from left to right. R2 says that when N acts on "or" itself (Nv), all positions in the 4-fold are <u>mated</u>, reversed, or counterchanged (FFFT). R3 says the when N acts on B, as in NB, the 4-fold (TTTF) is <u>flipped</u> from top to bottom.

Any order of (R1)(R2)(R3), that is, all six permutations of these rules, now called <u>flip-mate-flip</u>, can be applied to the "or" side of (b). When R1 and R3 act at the same time, the 4-fold for "or" (TTTF) is doubly flipped (rotated 180 degrees) (FTTT). This activates the pattern of symmetry transformations found in a Klein 4-group (K4). In (b), the mate (6.) of the rotate (5.) of the 4-fold for "or" (3.) repeats the 4-fold for "and" (2.); again, the equivalence sign has been justified. This activates the pattern in the 8-group known as (C2 x C2 x C2), also called (C2)3.

What makes (b) so much better than (a)? It is the ability of (b) to remind us that we have an acute world-wide need to come up with a much better notation. This notation should embody, abbreviate, and participate in the same symmetry transformations that are displayed in the truth table changes in (b). In other words, we need a notation that can do a dance called <u>flip-mate-flip</u>.

Now we are ready for the <u>logic</u> <u>alphabet</u> <u>form</u> used in (c). Start with a 1-stemmed d-letter, when this shape is placed in four positions (p,b,q,d), and likewise for the same four positions of a 3-stemmed h-letter (h, ᖾ, ᖹ, ɥ). A s(T)em stands for (T)rue. The d-letter is an abbreviation for (A and B) and (A TFFF B). It has a stem in the upper-right quadrant of the 4-fold (T---) and

Table II

(???)	1 o	2 p	3 b	4 q	5 d	6 c	7 u	8 s	9 z	10 n	11 ɔ	12 h	13 μ	14 ᖹ	15 ч	16 x
OOO	o	p	b	q	d	c	u	s	z	n	ɔ	h	μ	ᖹ	ч	x
OON	o	b	p	d	q	c	n	z	s	u	ɔ	μ	h	ч	ᖹ	x
NOO	o	q	d	p	b	ɔ	u	z	s	n	c	ᖹ	ч	h	μ	x
NON	o	d	q	b	p	ɔ	n	s	z	u	c	ч	ᖹ	μ	h	x
NNN	x	h	μ	ᖹ	ч	c	u	z	s	n	ɔ	p	b	q	d	o
NNO	x	μ	h	ч	ᖹ	c	n	s	z	u	ɔ	b	p	d	q	o
ONN	x	ᖹ	ч	h	μ	ɔ	u	s	z	n	c	q	d	p	b	o
ONO	x	ч	ᖹ	μ	h	ɔ	n	z	s	u	c	d	q	b	p	o

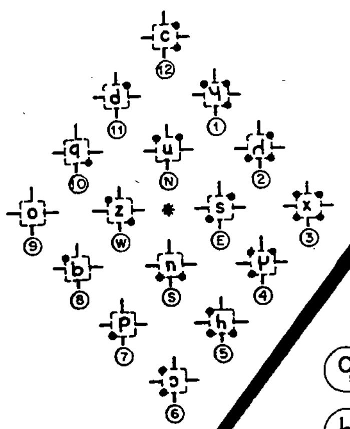
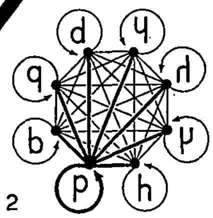


Fig. 1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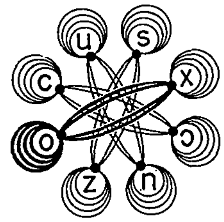
Fig. 3

no where else (-FFF). Likewise. the h-letter stands for (Not-A or Not-B) and (A FTTT B). This 4-fold has three stems (-TTT). And so forth, for a full set of 16 letter-shapes, when each of them is placed inside of an all-common standard square and when all of them are placed in the 4-by-4 clock-compass in Fig. 1.

Now for what happens in (c), in isomorphism with (b). The symbol for the mate (d) of the rotate (h) of "or" (ч) is the same as the symbol for "and" (d). Notice that this is like what happens for De Morgan's laws, which are both cases of "split duality." In (3), the mate of "and" and the rotate of "or" both become an h-letter. In (4), the mate of "or" and the rotate of "and" both become a p-letter. Also compare and contrast the up and down columns of Table I, with respect to (a), (b), and (c).

We have come to the moment when all of the letter-shapes, built to exist at several levels of symmetry-asymmetry, will find themselves trapped in the same 8-group frame of symmetry transformations (C2)3. Let the asterisk in (A * B) stand for <u>any</u> of the 16 letter-shapes, and let O stand for the <u>absence</u> of negation. See that N can act on only three places in (A * B): at NA, at N*, and at NB. This brings us back to flip-mate-flip and all combinations of (R1)(R2)(R3): OOO, OON, NOO, NON, NNN, NNO, ONN, and ONO. When all of these negation triplets act on all of the letter-shapes, we arrive at the (8 x 16) table of transformational negation in Table II. Please realize that, when the flip-mate-flip rules are applied in Table II, they are rich enough to absorb (2), (3), and (4), along with the 125 cells not mentioned.

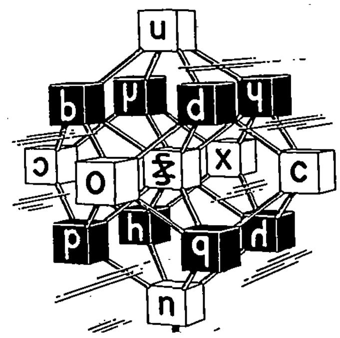

Fig. 4

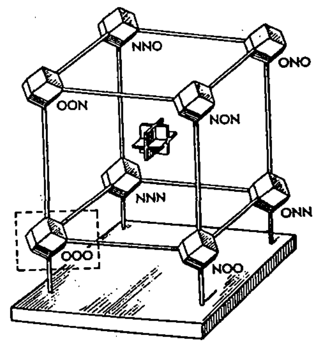

Fig. 5

More about relations between relations. When flip-mate-flip acts on the eight, tall, odd-stemmed letter-shapes along the top of Table II, it generates 64 cells (a half-table) that obey the network of cubic symmetry relations in Fig. 2. When flip-mate-flip acts on the eight, squat, even-stemmed letter-shapes, it generates 64 cells that obey the octahedral symmetry relations in Fig. 3. Remember that an ordinary rhombic dodecahedron is the interpenetration of a cube and an octahedron, such that the black vertices in Fig. 4 absorb the symmetry relations in Fig. 2, and likewise for the white ones and Fig. 3. Also realize that, more accurately, Fig. 4 is a shadow rhombic dodecahedron. A Boolean 4-cube of letter-shapes has been shadowed into 3-space. After that, when a Boolean 3-cube of negations triplets, in the form of three mutually perpendicular mirrors, acts on Fig. 4, it generates the 8-cell of logical garnets in Fig. 5. This symmetry model absorbs all and exactly all of the 128 cells in Table II.

More, there is much more! We have only touched on the logic of two atoms (A,B). No mention has been made of a whole family of hand-held models that can be constructed for (A,B). This approach can also be used for three atoms (A,B,C), for n atoms (A,B,C . . n), for an extension into 3-valued logic, and especially for a consideration of the symmetry structures that are activated by compound atomic forms. In general, the crystallography of logic becomes especially interesting when all of this is extended into n-dimensional geometry, at least enough so that we will be able to activate complex structures, including hyperstructures, when they are themselves located on hyperstructures.

My first example (2) is very simple. Hyperstructures on hyperstructures can become very complex. In brief, the logic alphabet is not only robust enough to stay alive at both extemes. There is also much more in between that has not been mentioned.

SYMMETRY IS PROPERTY OF MATTER

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In a broad sence, the word "symmetry" implies that objects and phenomena contain an element of p e r m a n e n c e, an element which is i n v a r i a n t with respect to certain transformations. This meaning is most often associated with the invariance of geometric figures or natural objects when their equal parts are transposed through rotation and reflection. In other cases, "symmetry" means the invariance of some phenomenon in relation to displacement or "reflection" in time. Sometimes both "spatial" and "temporal" invariance considered together. The symmetry science has being developed historicly differently depending on particular interest of different branches of knoledge. Later became clear that, on the final account, the symmetry is the reflection of the properties of real World, the properties of space and time.

The fundamental laws of conservation which reflect certain invariansy are connected with the gomogenety of time, gomogenety and isotropy of space. The reality of certain phenomenon in the nature, in its turn, is determined first of all by the symmetry of the matter in general, by the symmetry of space and time in particular. From this position the understanding of symmetry have to go not from the phenomenon to the Nature, but from the Nature to the phenomenon.

For the time being everyone feels the necessity for unifining different branches of the symmetry knoledge developed in different time and such unification has allready been developed. Its theoretical ground is the mathematics, the group theory and its experimental graund is based on the properties of the matter.

In our papers (Zheludev, 1983, 1987a, 1987b) the applications of symmetry in different branches of natural sciences are considered: in the electromagnetic phenomena, the physics of high energy, the solid state physics, the physical crystallography, the tensor calculation etc. We use here the knoledge of point symmetry of the geometric images representing the symmetry of space and time. They are: sphere without symmetry planes (pseudoscalar) and polar vector for time; sphere with symmetry planes (scalar) and axial vector for space. Time is anticentrosymmetric (operation of space inversion $C = \bar{1}$ is changing the "sign" of time) but space is centrosymmetric(Fig.1).

In addition to the conventional symmetry its different generalizations are considered: antisymmetry (Shubnikov, 1951), magnetic symmetry (Landau and Lifshits, 1982, p.188) and complete symmetry (Zheludev, 1960, p.346; Fig.2). The most effectiv happened to be the complete symmetry which is based for the begining on equal treatment of space and time but finaly confining to the phenomena satisfining the i n v e r s i o n o f t i m e (operation $C = \bar{1} = T$). Time has two signs but space just one (expanding Universe). The symmetry of space and time is described by group PT-4 (Fig.3).

It is shown (Zheludev, 1987c) that the symmetry of all real phenomena is confined to four rules: the rule of scale, the rule of right (left) hand, the thumb rule and the gyroscope rule. To understand this one have to consider the phenomena described by the simplest tensor relationships

$$\underset{-}{P_i} = \underset{+}{a_{ij}}\underset{-}{Q_j} \qquad (1)$$

T

$$\begin{array}{llll} & H = & a_{ij} & S_j \\ T & + & + & + \end{array} \qquad (2)$$

$$\begin{array}{lll} & H = & [NS] \\ T & + & ++ \end{array} \qquad (3)$$

$$\begin{array}{llll} & H_i = & A_{ij} & P_j \\ T & + & - & - \end{array} \qquad (4)$$

$$\begin{array}{llll} & Q_i = & A_{ij} & S_j \\ T & - & - & + \end{array} \qquad (5)$$

$$\begin{array}{lll} & H = & [FP] \\ T & + & -- \end{array} \qquad (6)$$

[a_{ij} is polar, A_{ij} is axial second rank tensors; (3) and (6) are vectors multiplications; P, Q, F are polar vectors; H, N, S are axial vectors]. All these relationships satisfy the operation of time inversion T (=C = $\bar{1}$): the right and the left sides of the relationships after this operation have the same signs what means that describing by them phenomena are real.

One can combine Eq. (1-6) in four groups representing four rules of symmetry. Some of them are well known.

The s c a l e r u l e is satisfied by phenomena described by Egns. (1) and (2). One vector is transformed into another equivalent one through a scalar either by increasing or descreasing it but preserving its direction. The simplest examples of such phenomena are electric polarization of a medium under the action of an inducing field D, magnetization of a medium in a magnetic field, the motion under the action of external forces etc.

The r i g h t- (l e f t-) h a n d r u l e described by Eqn. (6) is well known. This rule governes such phenomena as the Hall and Magnus effects, propagation of electromagnetic waves, etc.

The mutual orientation of vectors in the Magnus effect obeys the following rule - a real rotation corresponds to the rotation of the "force" vector (not changing its direction upon operation of time reversal R) towards the flux vector in a way to make the angle between them smaller. In order to determine the direction of the motion of a conductor with a current in a magnetic field one may use the right-hand rule. In this case, the extended fingers of the right hand indicate the direction of electron motion (which is opposits to the direction of the current). The sign of the North pole of a magnet (perpendicular to the palm) is determined in such a way that if we look from the side of the pole from which magnetic force lines outcome, the electrons forming electric current move in the clockwise direction (the direction of the current is anti-clockwise). In this case the conductor with a current moves in the direction indicated by the thumb of the right hand. Note that in both cases, irrespectively of the phenomenon nature, the mutual orientation of all three vectors in the above considered phenomena is described in the complete symmetry by group $\underline{mm2}$ and in the conventional symmetry by group m.

The t h u m b r u l e determines the direction of the linear motion of a rotating screw [eqns.(4) and (5)]. A right-hand screw proceeds into the body if it is rotated in the clockwise direction, whereas the left-hand screw, being rotated in the same direction, in unscrewed from the body. The right-hand screws thus correspond to the mutual orientation of polar and axial vectors depicted in Fig.3.

It is worth noting that sometimes the thumb rule is used to determine the direction of the magnetic field of a direct current. First of all, in this case the directions of the current and the magnetic field are mutually perpendicular (and not parallel as is determined by the thumb rule). Sometimes the well known concept on magnetic field lines outcoming from the North pole and closing at the South pole is used. The direction of these lines is taken to be the direction of the magnetic field often denoted by a polar vector. In the actual fact, the magnetic pole is axial and its "direction" (the pole) is uniquely determined by the current direction in magnetic Ampere turns if we look at the North pole, we see that electrons in such turns (i.e., in solenoids) move in the clockwise direction, if we look at the South pole, electrons move in the anticlockwise direction.

The g y r o s c o p e r u l e is applied to the phenomena described by relationship (3). As is known, if to tilt a rotating gyroscope (apply a force to it), it starts rotating about the third axis which is normal to the rotation axis and the axis of rotation provided by the applied force. For a gyroscope fixed at the center of gravity, the application of such a force gives rise to a moment and its precession which may also be interpreted as a rotation of the gyroscope about the third axis. The phenomenon as a whole is described by three mutually perpendicular axial vectors two of which change its sign upon operation R and the third one (the moment of a force) does not change its sign. The gyroscope rule also describes such phenomena as electron paramagnetic and nuclear magnetic resonance.

In complete symmetry the configuration of three mutually perpendicular axial vectors is described by the centrosymmetric group $\bar{3}\underline{m}$ (in the conventional symmetry it is described by group $\bar{3} = C_{3i} = S_6$).

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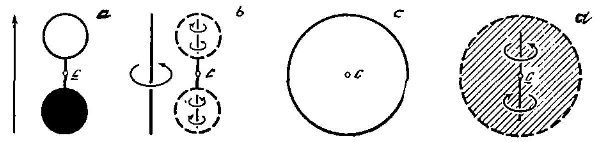

Fig. 1

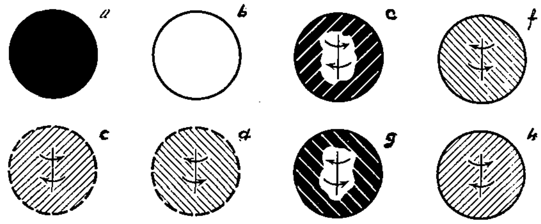

Fig. 2

Bases				Irreducible representation	Operations and characters			
Polar vector	Axial vector	Scalar	Pseudoscalar		1 $(TP)^2$	$\bar{1}$ T	$\underline{\bar{1}}$ P	$\underline{1}$ TP
—	—	—	—	A	1	1	1	1
$\pm A_{xy}$			A	B	1	−1	1	−1
	$\pm a_{xy}$	a		C	1	1	−1	−1
$\pm A_{xy}, \pm a_{xy}$				D	1	−1	−1	1

Fig. 3

CAPTIONS TO FIGURES

Fig. 1. Geometric images of scalars and vectors and their complete symmetry (see below). a - polar vector and two scalar sphere "opposite" sign, complete symmetry $\infty/\underline{mmm}$; b - axial vector and two pseudoscalar sphere having opposite signs of enantiomorphs, complete symmetry $\infty/\underline{mmm}$; c - scalar sphere (operation $\bar{1}$ for it is symmetry operation, but operation $\underline{1}$ converts it into sphere of opposite "colour". d - pseudoscalar sphere (operation $\underline{1}$ for it is symmetry operation but operation $\bar{1}$ converts left sphere into right one).

Fig. 2. Simplest geometric images of complete symmetry. a,b - scalar spheres (+) and (-); c,d - pseudoscalar spheres left (+) and right (-); e,f - left "black" and right "white" spheres; g,h - right "black" and left "white" spheres.

Fig. 3. The characters of irriducible representations and the basis invariants of group PT-4.

SYMMETRY IN EPISTEMOLOGY AND THE SOLUTION OF THE LIAR PARADOX

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Introduction

A basic epistemological observation, fundamental to all science, is the two-fold structure of our world. The world appears to us to be split into two parts, sense percepts and thought concepts. The seeming incongruity of these parts has led to many theories how they are related. In the process of finding the relationship, one or the other part is usually eliminated. The concept of symmetry sheds new light on this philosophical schism. In particular, it turns out that concepts and percepts are *symmetric* in that they are one-sided representations of a unity, a unity which is achieved through cognition.

The long sought-after solution of the Liar paradox demonstrates the effectiveness of this approach to epistemology.

Some remarks on epistemology

In order to demonstrate the symmetric nature of concepts and percepts, we need to distinguish them clearly and then show how they are related. We become aware of percepts without any conscious activity - they just happen. They introduce themselves without explaining their how and why. Concepts, on the other hand, are pure relationships. They are in essence "what connects", i.e. their very nature is connectedness. Concepts are the laws that constitute relationship.

Concepts and percepts are *symmetric* in the sense that they are distinguishable but not separable from each other; they are parts of an underlying unity. Concepts provide what percepts are lacking: they provide relation between isolated facts. It can be shown that this symmetric structure of our world view is not caused by the world itself but rather is brought about by the human being who observes this world. If the world did not appear to us in this symmetric form, then there would be either no need or no possibility for real knowledge. As soon as this situation is realized, cognition becomes a process of restoring the unity which has been split apart by the human being. In terms of the concept of symmetry, through the process of cognition the underlying unity of concepts and percepts is unveiled, unveiled through rational analysis and subsequent synthesis. The result of this process is what is called *reality*. Since this process results in the unity of two parts - concepts and percepts -, it may be called the *law of epistemological symmetry*.

In what follows we apply this idea of symmetry to the analysis of the Liar paradox while characterizing the logic which forms the underlying structure of *any* kind of reasoning.

The Liar paradox

The most common form of the Liar paradox is the following: "Epimenides, a Cretan, says: 'All Cretans are liars.' " The question arises: Is Epimenides lying or not lying ? Both assumptions lead to a logical contradiction. The usual conclusion is that somehow our language cannot express truth consistently. Thus it seems as though the Liar paradox is a purely semantic problem which can be solved only by constructing a consistent truth-concept within a formal language.

In my opinion, the failure of the many attempts to solve the Liar paradox lies in the fact that concentration has been directed toward the logical and semantical rather than the epistemological qualities of the paradox. I wish to show that the paradox cannot be derived, and solved, without referring to the epistemological symmetry referred to above.

To begin with, I shall discuss the Liar paradox in a form which is adopted from P. FINSLER (1925). Let us assume that we are concerned with the number 4 and that we are trying to give a conceptual definition of this number. The following will suffice: 4 is the smallest positive integer unequal to 1, 2, or 3. This definition as a conceptual entity has to be distinguished from "4" or "four", which are *symbolic representations* of the underlying concept, namely the number 4.

However, the above definition can itself be interpreted as a symbolic representation. In order to make this explicit, we write in a box:

1, 2, 3. *x* is the smallest positive integer which is not written in this box.

Somewhat surprisingly, this yields a paradox: If we assume x equals 4, then x must be unequal to 4, say 5; conversely, if x is bigger than 4 then it must be equal to 4. Hence *x* is equal to 4 if and only if it is not equal to 4.

Analysis of the paradox

A moment's thought shows that there is no *logical* error in the derivation of the paradox, that is, no violation of any law of logic. However, we produced a contradiction by reflecting on the conceptual content of the definition and its relationship to the symbolic representation within the box. The *result* of this reflection is the classical contradiction in logic. But this contradiction cannot be derived without leaving the conceptual realm. The reflection on the symbolic representation (within the box) is only possible *after* having *perceived* these symbols through our senses. What appears as a logical contradiction is nothing else than a conflict between the intended conceptual content (namely the definition of the number 4) and its symbolic representation written within the box. In itself, neither the definition is contradictory nor is the symbolic representation deficient syntactically. But the latter does not represent the former.

This becomes clear if we do not write the sentence "*x* is the smallest positive integer which is not written in this box" in the box itself, but, for example, write it somewhere else. Then there is no paradox.

Methodological reflections

It appears that the analysis of the Liar paradox depends on a proper discrimination of different points of view, namely the *conceptual* point of view and the *perceptual* point of view. The unity of these two points of view is called the *epistemological* point of view.

Without reflecting about the facts given by some kind of perception, the Liar paradox cannot be derived. This means that without changing from a conceptual level to an epistemological one and back, the paradox will not arise. Let us see how this applies to the original version of the Liar paradox mentioned in the *Introduction*. In order to derive the well-known contradiction, we need to ask: What is Epimenides *really* doing ? If he is *really* lying, then he contradicts himself in saying: "I am a liar". But the reflection on what someone is *doing* in *reality* is certainly not a purely conceptual one. Without access to some kind of perception, we could not even talk about what Epimenides is really doing.

This becomes evident if we ask ourself: What am I *really* thinking if I say: "I'm lying"? *Observation* and not speculation is needed to derive and solve the paradox. What needs to be observed (and actually *is* observed) in the latter case is my own thinking process.

We have shown that the Liar paradox is neither conceptual nor semantical but epistemological. Without some kind of perception involved, there would be no paradox. Although the resulting contradiction is purely logical, its derivation is definitely not a purely conceptual matter and hence, strictly speaking, not within the realm of logic (see below).

It should be noted that by its very nature, our method of analysis applies not only to all forms of the Liar paradox but to any paradox which is not based on some kind of violation of the laws of pure logic.

Some remarks on logic

It should be clear by now that logic in our sense is more comprehensive than what is commonly understood by symbolic logic or even formal logic in the classical (Aristotelian) sense. Logic comprizes everything purely conceptual and tells us nothing about the existence of an object. In our terminology, symbolic logic is a representation of some kind of formalized logical rules and objects; by its very nature it cannot encompass accurately the whole realm of logic.

In order to become an object of our thinking process, any object has to be observed in some way. Consequently, reflecting about logic cannot be conceptual in the strict sense. Hence, anything we say about logic cannot be the result of purely conceptual (or, for that matter, logical) speculation but rather involves observation, namely the observation of our own thinking process.

We only mention here that B. RUSSELL's so-called set theoretic paradox can be analyzed in much the same way as the Liar. Given these results, I see no substantial argument against the objectivity of our thought concepts. It should be noted that this is, in essence, a result of an approach to epistemology which recognizes concepts and percepts as symmetrical components of a substantial unity.

Bibliographical and historical remarks

The first attempt to solve the Liar paradox along the lines indicated above stems from PAUL FINSLER (1925). He argues on behalf of common sense and classical absolute (Aristotelian) logic. The system of pure logic proposed by BRUNO VON FREYTAG-LÖRINGHOFF (1955) comes very close to FINSLER's conception of absolute logic. GEORG W. F. HEGEL's *Wissenschaft der Logik* is the most comprehensive attempt to describe the conceptual realm (that is, the content of logic) in its entirety from the point of view of the dialectic structure of concepts. WERNER A. MOSER (1985) pointed out, that a clear distinction between logic and epistemology is indispensable for a systematic analysis of the paradox and its subsequent solution. Such a distinction which does not depend on any kind of previous knowledge or unreflected assumptions was provided by RUDOLF STEINER in his thesis (1892).

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