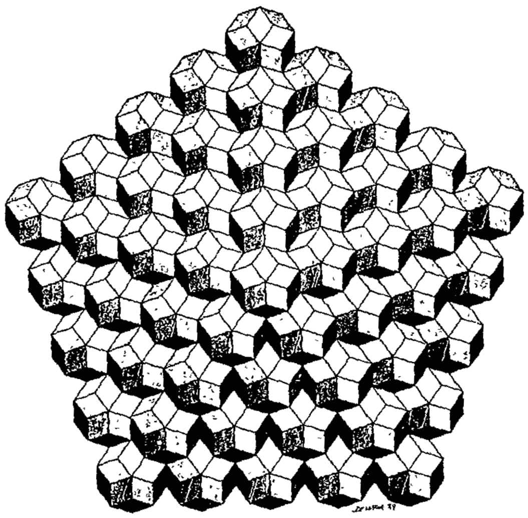
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

II.



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NOTATION AND NOMENCLATURE

Wasma'a K. Chorbachi and Arthur L. Loeb
Harvard University, Cambridge MA 02138, USA

The notation and nomenclature for two-dimensional symmetry groups developed in the context of crystallographic research is not necessarily most suitable for use in an art-historical or design context. There are even differences between crystallographers and solid-state scientists in the manner in which they deal with symmetrical patterns.

Fundamental to crystallographic notation is the concept of a lattice, a collection of all points in a pattern related to each other by translational symmetry. This emphasis on translational symmetry is the result of the translational symmetry of the X-ray beam used by crystallographers to determine the location of the crystal elements: diffraction of the beam by the crystal is a direct result of this translational symmetry of the crystal. Solid state scientists, however, are more concerned with the symmetries of the fields, electrical, magnetic or quantum-mechanical, around each crystal element than with the absolute orientation of these fields. Accordingly, Fischer et al.[1] developed the notion of the lattice complex, a collection of all points related by any symmetry operation. Directly coupled to the concept of lattice is that of unit cell, whereas the lattice complex corresponds to the concept of fundamental region.

Rudolf Arnheim[1] recently criticized crystallographic nomenclature for not distinguishing between what he calls rotational and centric symmetry. In his reply to Arnheim's critique, Loeb[2] surmised that Arnheim refers to the distinction between centers of rotational symmetry located on lines of mirror symmetry on the one hand, and on the other hand those not on mirror lines. The notation in the International Tables (IT) for X-ray Crystallography is contrasted with that of Loeb and Le Corbeiller (L) for three patterns illustrated in Figures 1a, 1b and 1c, demonstrating that the L-notation does indeed make this distinction .

Figure 1: Patterns described respectively as 33'3'', 33'3'', 33~3 (L-notation) or p3, p3m1 and p31m (IT notation)

Rotational symmetry is fundamental in the L-notation; rotational symmetry may exist even in the absence of reflection symmetry, but the coexistence of reflection lines invariably implies rotation (in special cases translation) symmetry. Centers of rotational symmetry (L-nomenclature calls them rotocenters) form lattice complexes called roto-complexes whose symmetry values are determined by a single diophantine equation:

$$k^{-1}+l^{-1}+m^{-1}=1,$$

k, l and m being the symmetry values of the respective roto-complexes. Patterns are classified according to the five solutions of this equation: 1∞∞, 22∞, 236, 244, 333; ∞-fold rotational symmetry amounts to translational symmetry. In the L-notation a k-fold roto-complex in which all centers lie on mirrors is denoted by an underline: k. Distinct rotocomplexes having the same symmetry value are distinguished by a prime: k and k', and enantiomorphically paired rotocomplexes are denoted by a ~: k, k~. Significantly, the IT notation makes no distinction between rotocenters having the same symmetry value but belonging to different roto-complexes; in Design such centers will generally accomodate different motifs, so that the distinction is indeed fundamentally important.

In Design the relative positions of rotocenters and reflection lines make a great deal of difference. In the absence of reflection lines patterns tend to be very, even overly dynamic (Figures 1a and 2a) , and they will exist in two mutually enantiomorphic manifestations. Conversely, when all rotocenters lie on mirror lines, the patterns tend to be static; the best balance is found when some of the roto-complexes lie on mirrors and others are enantiomorphically paired. (Compare, in Figure 1, the patterns 33'3", having no reflection symmetry, 33'3", having all rotocenters on mirrors, and 33~3', having 3 and 3~ enantiomorphically paired.) It is easily shown that such balance is not possible in the 236 system.

Further examples contrast the notation 244' with p4, both of which represent the pattern shown in Figure 2a, 244' with p4m (Figure 2b), and 244~ with p4g (Figure 2c). The notation p4 does not tell us that there are three distinct rotocomplexes, one having symmetry value 2, and two separate and distinct ones having symmetry value 4. The notations p4m and p4g have created the mistaken impression that the former corresponds to patterns having only mirror lines, the second only glide lines, when, in point of fact, both p4m-patterns and p4g-patterns contain mirror lines as well as glide lines. The notations 244' and 244~, on the other hand, show that in the former case all rotocenters lie on mirror lines, whereas in the latter only the two-fold rotocenters lie on mirror lines, while the fourfold sets are mutually enantiomorphic.

Figure 2: Patterns described respectively as 244', 244', 244~ (L-notation) or p4, pm and pg (IT-notation)

The five groups having two-fold rotational symmetry only, are denoted in the IT respectively as p2, pmm, cmm, pgg and pmg; only the first of these notations indicates the rotational symmetry. By contrast, the respective L-notations 22'2"2'", 22'2"2'", 22~2'2", 22~2'2'~(g/g') and 22~2'2'~(m/g) show all four sets of rotocenters and their interrelations, and specify the reflection lines to distinguish the two cases which have the same sets of rotocenters.

Above, we noted the L-notation for the patterns of Figure 1. The IT-notation for the first of these (33'3" in the L-notation) is p3; the fact that there are three distinct sets of three-fold rotocenters is not shown. This is unfortunate, because visually it is not always easy to distinguish between p3 and p6 patterns, and the L-notations 33'3" and 236 point up the differences, the presence or absence of 2-fold rotocenters marking the difference. The IT-notation for the remaining two of the 33'3" groups is p3m1 and p31m, but there has been some confusion as to which is which, as there does not appear to be a logical distinction between the two sets of symbols.

In the mid 'seventies one of us (W.K.C.)[4] examined many systems of notation in her search for a suitable language and notation to study and classify Islamic geometrical patterns. She found the ones most pertinent to the arts to be Hermann Weyl's <u>Symmetry</u>, H.S.M.Coxeter's <u>Introduction to Geometry</u>, and A.V.Shubnikov' and V.A.Koptsik's <u>Symmetry in Science and Art</u>.[5] These books expanded on the discussion of symmetry, the second using IT notation, the third including immensely detailed and exhaustive enumeration far beyond the needs of art historians, not being designed to meet the specific needs of artists and designers. After some years of study she found that A.L.Loeb's <u>Color and Symmetry</u> [6], even though initially published as a monograph in Crystallography, presents the language most appropriate for art-historical studies. In contrast to the other symmetry notations the L-notation indicates the symmetry values of all rotocenters, distiguishes between distinct and mutually enantiomorphic rotocenters, and indicates whether rotocenters do or do not lie on mirror lines.

The L-notation was originally designed as part of an explicit program for developing a more sophisticated or linguistically more highly developed language of structure, aiming at precision in the communication of relevant details. The L-notation has been taught quickly and effectively to art historians and designers.[7] At a recent symposium previous students, now Design professionals, who were trained with the L-notation, demonstrated that with use this notation easily becomes vernacular.

In conclusion, then, we would have to say that the IT-notation, based primarily on lattices and translation symmetry, which are fundamental in X-ray diffraction, do not necessarily best serve the purposes of art historians and designers, for whom the L-notation has the advantage of explicit indication of all rotocenters and their interrelationships.

[1] Fischer, W., H.Burzlaff, E.Hellner and J.D.H.Donnay: Space Groups and Lattice Complexes (U.S. Dept. of Commerce, National Bureau of Standards, 1973)

[2] Rudolf Arnheim: Symmetry and the Organization of Form: A Review Article, Leonardo 21, ..-.. (1988)

[3] A.L.Loeb: Symmetry and the Organization of Form: A Rumination on Rudolf Arnheim's Review Article (Leonardo, 1989, in press)

[4] Chorbachi, W.K.: In the Tower of Babel, in Symmetry II: Unifying Human Understanding, ed. I.Hargittai, Pergamon Press. In press,1989).

[5] Hermann Weyl: Symmetry (Princeton University Press, Princeton, NJ, 1952); H.S.M.Coxeter: Introduction to Geometry (John Wiley & Sons, Inc., New York, NY, 1961); A.V.Shubnikov and V.A.Koptsik: Symmetry in Science and Art ,transl. G.D.Archard (Plenum Press, New York, NY, 1974)

[6] Loeb, A.L., op. cit.: Notation and nomenclature introduced by Le Corbeiller and Loeb at the Congress of the International Union of Crystallographers at Rome in 1963

[7] Loeb, A.L.: A Studio for Spatial Order, Proc.International Conference on Descriptive Geometry and Engineering Graphics, Fiftieth Anniversary Symposium of the Engineering Graphics Division of the American Society for Engineering Education, 13-20 (1979)

Figure 1a

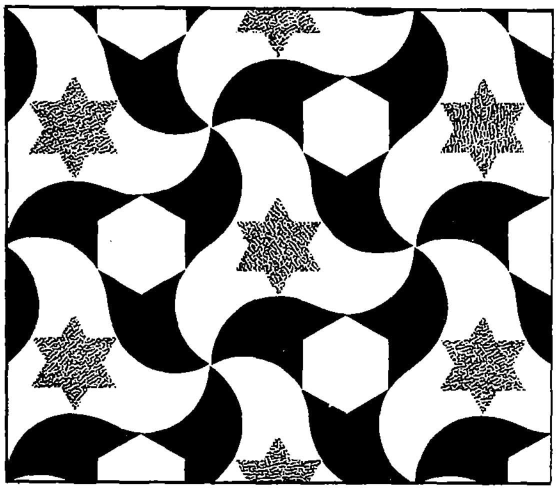

Figure 1b

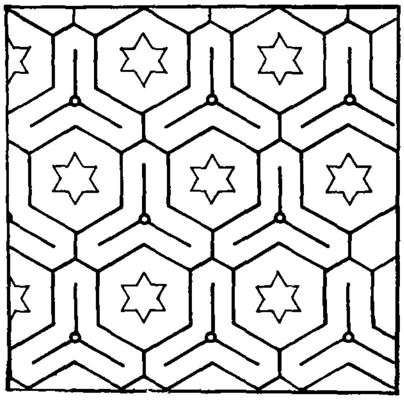

Figure 1c

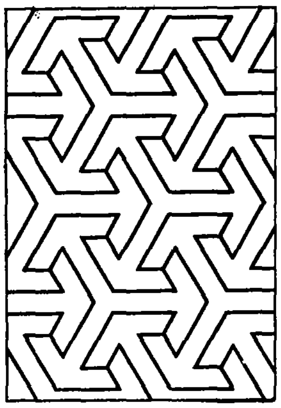

Figure 2a

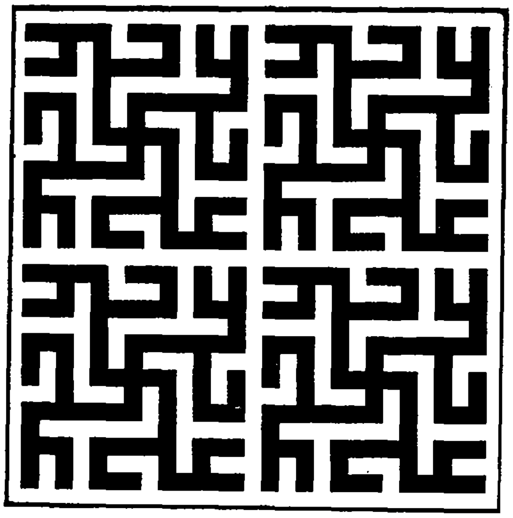

Figure 2b

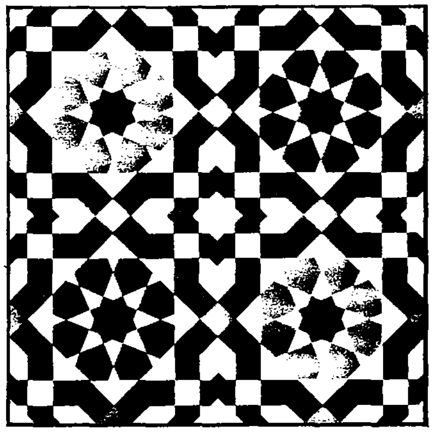

Figure 2c