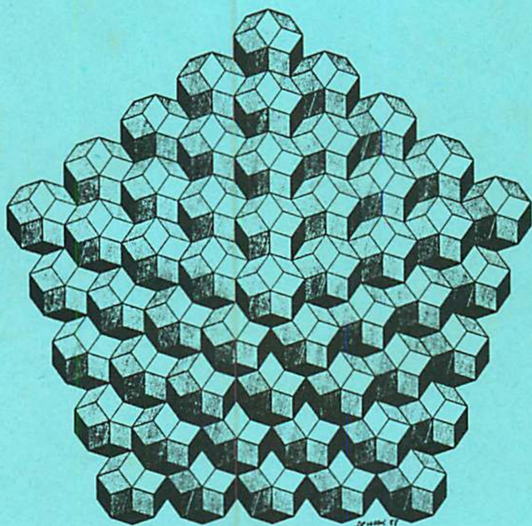
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



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STRUCTURES AND META-STRUCTURES

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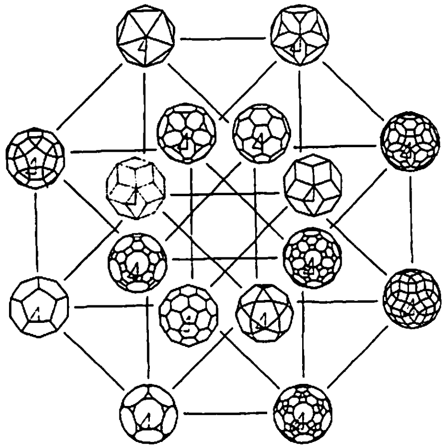
This presentation deals with spatial patterns termed *space structures* , and the patterns underlying these patterns. The patterns-underlying-patterns are termed *meta -structures* (or meta-patterns). This work is based on author's previous work [1,2] which suggests that just as patterns have a structure, as determined by the organising principles of symmetry, topology or geometry, meta-patterns also have a structure which is determined by the same ordering principles. That is to say, meta-patterns have a symmetry, topology, and a geometry, and in this sense both patterns and meta-patterns are self-similar.

Among the meta-structures, the more interesting and useful cases are the higher-dimensional structures, principally n-dimensional cubes or *n -cubes* . In author's previous work, n-cubes have been shown to serve as organising and transformational diagrams for families of regular-faced structures and some of its derivatives, namely, polyhedra, plane tesselations, polyhedral packings, and more recently, infinite polyhedra and non-periodic packings of icosahedral polyhedra. In all cases, the structures and their transformations are characterised by the underlying Boolean logic of the n-cubic space and are governed by higher-dimensional DeMorgan's laws relating unions, intersections and their complements. The structures are correspondingly indexed in binary combinations of 0's and 1's.

In a different field, that of movement studies, this concept has been used by the author to organise Rudolf Laban's movement "efforts" in a hyper-cubic space [3]; we know of this dance-theoretician's work from the better known Labanotation system for dance. In a broader sense, meta-structural ideas hold potential for organising concepts both *within* and *between* a wide variety of disciplines, and form the basis of a new science [4]. Goranson suggests applications in artificial intelligence [5] and Zellweger has used similar ideas for Logic [6].

The concept is illustrated with the example of plane tesselations and polyhedra, though the idea can be extended to higher-dimesnional space structures. The tesselations and polyhedra are organised according to their symmetry into a 2-dimensional lattice where each distinct symmetry occupies a distinct vertex of this lattice and is determined by the complimentary pair of angles of a right triangular fundamental region. Inter-symmetry transformations take place in this lattice plane. If we restrict to mirror-symmetric structures, a family of 16 topological structures are possible for each symmetry from four basic types of transformations within the fundamental region. These 16 are arranged on the vertices of a 4-cube (Fig.1; example shown for icosahedral symmetry), and each symmetry has its own 4-cube associated with it. This extends the meta-space into a hyper-cubic lattice within which the structures transform continuously to one another both within and between symmetries. The edges of the meta-lattice provide direct transformation paths between structures, and higher-level transformation paths are along the face, 3-cell and 4-cell diagonals of each 4-cube.

Through the addition of three other transformations based on special subdivisions of the



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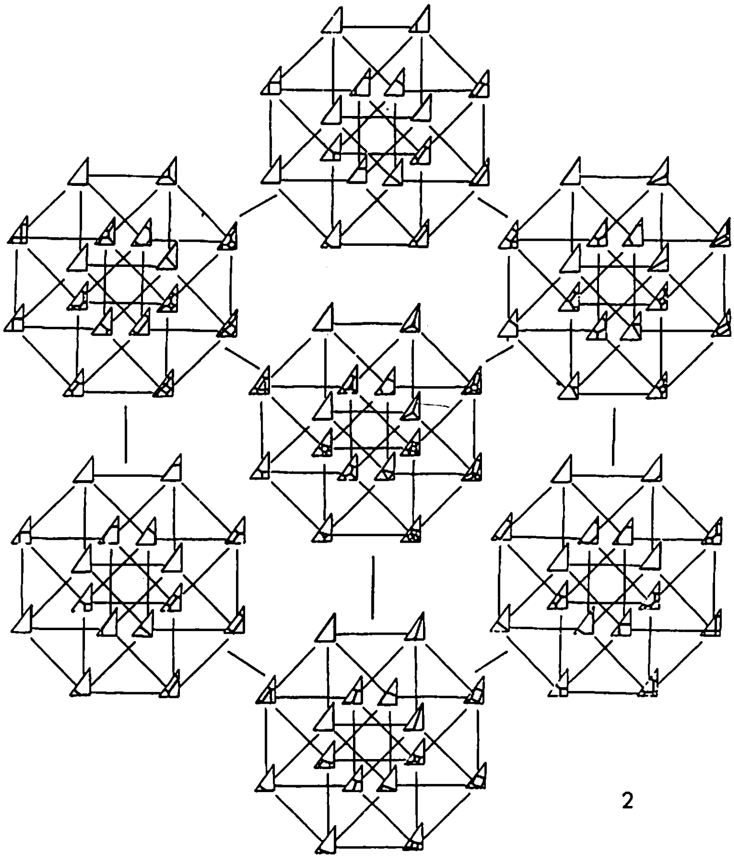

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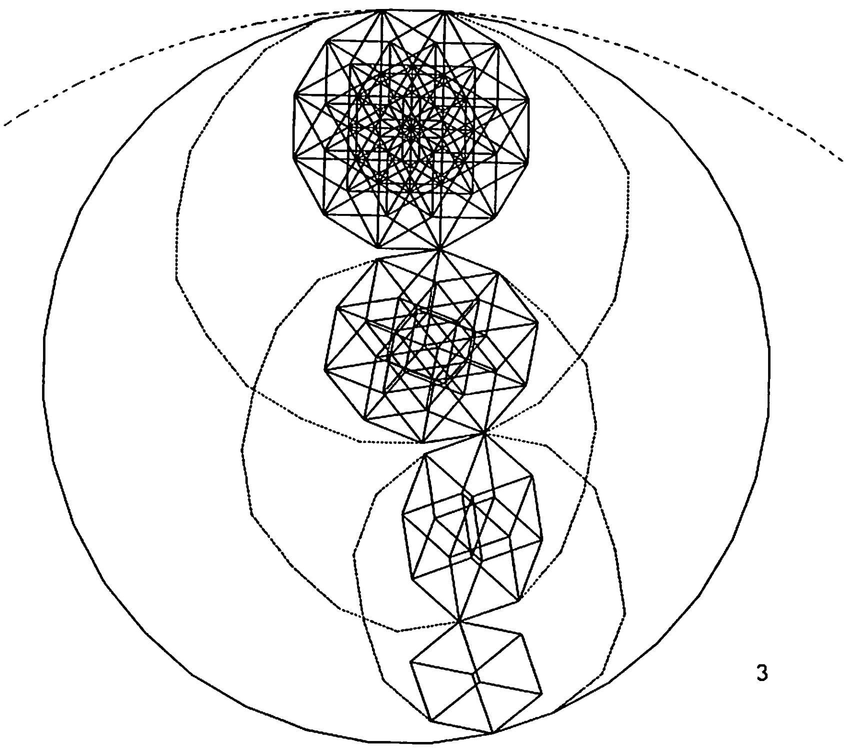
fundamental regions, the 16 structures within each family can be converted into 128 structures which can be organised in a 7-cube space. This space decomposes into sixteen 3-cubes on the vertices of a tesseract, or eight 4-cubes on the vertices of a 3-cube (Fig.2; fundamental regions shown). New transformations (e.g. dualisation, frequency, handedness, etc.) can be continually added, extending the meta-structural space correspondingly into an increasingly encompassing meta-space for defining, generating, and transforming complex space structures. Such a system is open-ended and provides a simultaneous way of classification, generation and transformations of space structures. The open-ended nature of the system makes it more complex than the I-Ching, the Chinese system of changes, which is restricted to 64 hexagrams.

A possible generalised model (Fig.3) for a system of changes with multiple parameters and hierchichies is a recursive n-cube (or n-cubic lattice) which can be decomposed *downwards* into i-cubes (i=<n) on the vertices of an (n-i)-cube, where each i-cube itself can be decomposed into j-cubes (j=<i) on the vertices of (i-j)-cubes, and so on. In a similar way, the n-cube can grow *upwards* into larger and larger super hyper-cubes. Though the application of such a model to space structures remains to be fully established, aspects of the present work point in that direction. A promising application is in the area of "shape grammar" as supported by our present studies in architectural form-generation.



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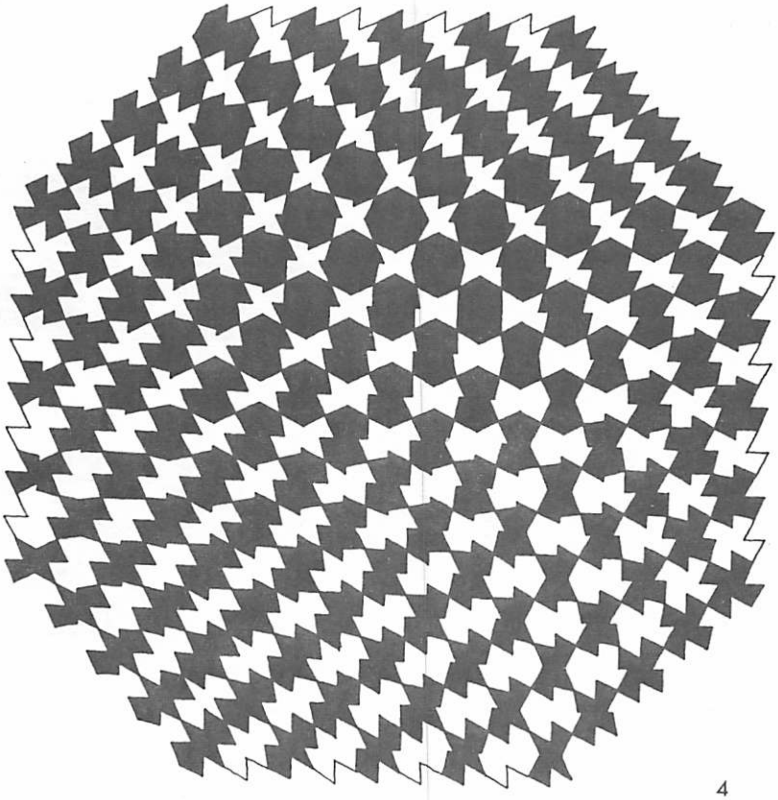
The transformations between structures can be *discontinuous* (discrete or digital mode) or *continuous* (analog mode) , providing two fundamental models for transforming information. The discrete transformations change a structure to another by changing 0's *to* 1's or vice versa in an on-off manner. The analog transformations change a structure in a graded manner, from 0 *through* 1. The latter are gradual, continuous and incremental changes. Both types of transformations provide a natural basis for computer animation of structures. Both provide alternative modes of meta-structural thinking. The discrete 0-1 changes are well-known from the binary system used in computational sciences. The continuous 0-thru-1 changes provide a basis for a new transformational logic.

The concepts presented lend themsleves naturally to computer-animation. Computer-animated continuous polyhedral transformations can be seen in a collaborative project with R.McDermott and Patrick Hanrahan [7]. Continuous transformation of the Penrose tiling can be seen in a joint project with David Sturman [8]. An interesting fallout of such a system of a continuum of transformations is the new class of *hyper-Escher* patterns (Fig.4). These are higher-dimensional analogs of Escher's metamorphic patterns.



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2. Lalvani, H., *Structures on Hyper -Structures* , Lalvani, New York, 1982.
3. Lalvani, H., In : *LIMS News* , Vol.6, No.2, Laban/Bartenieff Institute of Movement Studies, New York, (Dec.) 1983.
4. described by H.T.Goranson as 'meta-morphology' for morphology-underlying-morphology (personal communication), 1987; the usage differs from A.Tyng's earlier usage of the same term for the science of metamorphosis or form changes.
5. Goranson, H.T., see Abstracts (this conference).
6. Zellweger, S., see Abstracts (this conference).
7. P.Hanrahan, H.Lalvani, and R.McDermott, *Sketches of Polyhedra Transformations* , 12 min. color video, Computer Graphics Laboratory, New York Institute of Technology, 1984; Also, see Lalvani and McDermott, Abstracts (this conference).
8. Lalvani,H. and Sturman, D., *Transformations of the Penrose Tiling*, a computer animation video, N.Y.I.T. and M.I.T. Media Lab., 1988.

Credits: Computer image in Fig.3 is executed by Neil Katz.