Symmetry of STRUCTURE

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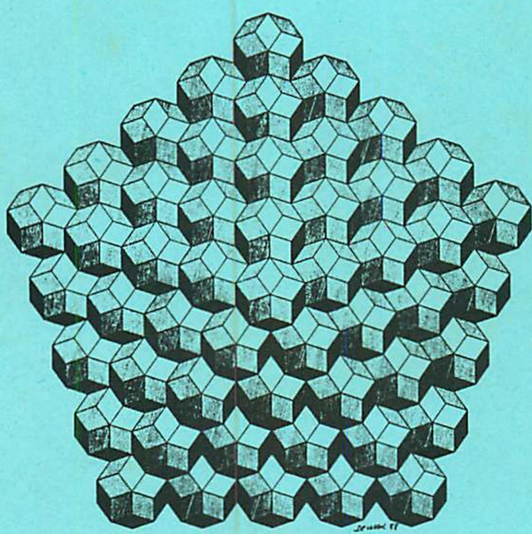
Abstracts

I.



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RESONANT INFLUENCE, SYMMETRY AND SPECTRUM OF STRUCTURES IN NONLINEAR MEDIA

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The formation of structures in nonlinear media is one of the important problems in synergetics. Determination of spectrum of structures enables efficient control of processes in such media.

Symmetry and groups of transformations sometimes play important role for the study of structures. First, the concept of "structure" implies conservation of some properties in time, the existence of some invariants. Sometimes structures can be described by self-similar (group invariant) solutions, that depend on combination of x and t: $\xi(x,t)$. Second, invariance of structure under a finite group of transformation also provides valuable information about solution.

One of the models that we investigated has been proposed in course of study of fast processes in plasma physics. It describes the medium with nonlinear coefficient of heat conductivity and bulk heat source. The distribution of temperature satisfies the equation

$$T_t = \mathrm{div}(T^{\sigma}\nabla T) + T^{\beta} \tag{1}$$

The process of burning (heating of the medium) goes on in regime with peacking, when $T\to\infty$ as $t\to t_f$ at least in one point. Such regimes are intermediate asymptotics of some real processes. Burning in regime with peacking gives rise to the localization of heat in certain region, outside of it T being equal to zero or limited. Temperature growth and localization result in the formation of structures – regions of intensive heating, where the form of temperature profile does not change (fig.1). For initial data of general type the process tends to burning as one or more simple structures with one temperature maximum. It is possible to create more complicated structures with several maxima. They can be considered as unification of a number of simple structures, that are interacting with others and attracting to the common center. The decrease of scales goes on synchronous in the whole structure, its form being described by the solution $T=g(t)\cdot y^{\alpha}(\xi)$, $\xi=x/(t_f-t)^{\gamma}$. Thus, if the initial temperature profile is set in accordance with the function $y(\xi)$ the development of process differs from that in the case of an arbitrary data. Such influence on a medium sometimes is called resonant.

All the concepts mentioned above appeared during the study of one-dimensional problem. In the multi-dimensional case new properties appear – form of the localization region and configuration of maxima. If one uses the concept of complex structure as the set of simple ones then symmetry is important for its existence.

In multi-dimensional case $y(\xi)$ satisfies the equation

$$\alpha\Delta y - 0.5(\beta-\sigma-1)(\xi,\nabla y^{\alpha})+y^{\alpha\beta}-y^{\alpha}=0 \qquad \alpha=(\sigma+1)^{-1} \qquad (2)$$

In our days there are no general methods of its investigation, but the hypothesis of symmetry has enabled us to propose approximate method of investigation. It is based upon linearization of (2) and matching the solution of linearized equation with asymptotics on a number of rays. With the help of symmetry it is possible to choose the directions of the rays and find a proper solution of linearized equation. Then we made an estimation of the number of structures and found some of the solutions $y(\xi)$ numerically. In two-dimensional case there are two classes of structures with maxima situated on the concentric circles and in the sites of the rectangular lattice.

This is not the only problem where symmetry enables efficient investigation of structures. The second example is the two-component medium with trigger properties. The processes in it are described by the system of parabolic equations with cubic nonlinearity

$$W_t = D\Delta W + AW - |W|^2 \cdot BW \qquad (3)$$

Under certain conditions on matrices D, A, B there are two stable solutions W_0 and $-W_0$ (that can be denoted by black and white colors). In one-dimensional case system from an arbitrary initial data evolves to the stationary structure that consists of elementary ones - narrow regions of transition from W_0 to $-W_0$ and vice versa. Two-dimensional case differs substantially. From an arbitrary initial data system usually evolves to homogeneous background - black or white - and no structures appear. Like in the case of heat structures the symmetry of initial data is necessary for the formation of stable stationary structures. The symmetry must be colored - the invariance under reflection (or turning) and mutual change black ↔ white. This approach has enabled to use a number of approximate methods mentioned above and to find several two-dimensional structures.

Two of simple two-dimensional structures are shown in fig. 3. It is interesting that they can be combined to form "parquets", e.g. like in fig. 4.

Thus, the symmetry of initial perturbation is of great importance for the resonant properties, formation of structures of different complexity in a nonlinear medium.

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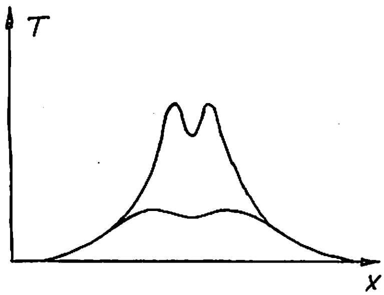

Fig.1 Complex heat structure with two maxima.

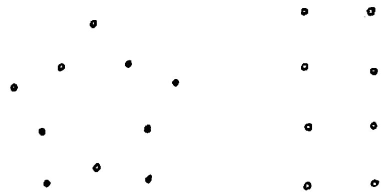

Fig. 2. Examples of configurations of maxima in 2-D complex heat structures.

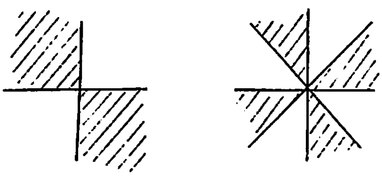

Fig. 3

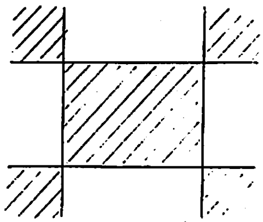

Fig.4