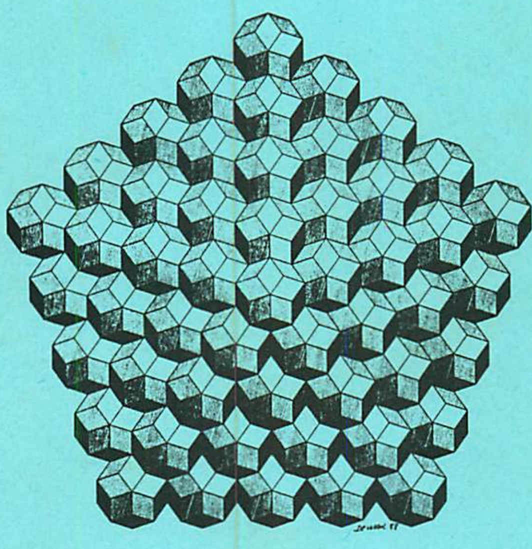
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



Edited by Gy. Darvas and D. Nagy

Buda pest

August 13-19, 1989

hungary

PARTIALLY DETERMINISTIC STRUCTURES

Yu.A.Kravtsov

Engineering Center "Lidar"

Abstract

This presents the basic agreements on randomness as accepted in mathematics and theoretical physics (randomness as the intrinsic probabilistic measure in the axiomatized theory of probabilities; randomness as an algorithmic complexity in the algorithmic theory of probabilities) and in experimental physics (randomnesss as the decreasing correlation, as the absence of a discrete spectrum, as the presence of a local instability or fractal structures, and at last as nonreproducibility, noncontrollability, unpredictability, etc.).

Based on the agreement about randomness as unpredictability, the relations between the observed , Y, and model (predicted), Z, structures were formalized. The formalization leads to the idea of partially deterministic, i.e. partially predictable, structures which is similar to the previously introduced ideas of partially deterministic process and fields [1,2].

The degree of determinism $D(Y|Z)$ of the observation Y relative to the model Z that is defined as a "share" of the model Z in the observation Y, is proposed as a quantitative measure of the prediction quality. Formally, the quantity $D(Y|Z)$ can be expressed as a generalized scalar product $\{Y,Z\}$ which, in a sense, characterizes the projection of Y onto Z:

$$D(Y|Z) = \{Y,Z\} \Big/ \{Y,Y\}^{1/2} \{Z,Z\}^{1/2}.$$

The equality of $D(Y|Z)$ to unity means the perfect determinism (perfect predictability) of Y, using the model Z; a small value of $D(Y|Z)$, as compared with unity, corresponds to randomness (perfect unpredictability) of Y on the basis of the model Z, while the intermediate values of D between zero and unity are interpreted as partial determinism.

In the specific case, when we speak of a system of spatial points, whose position is characterized by the radius vector $\vec{Y}_j$, the operation $\{Y,Z\}$ can be defined as the number of cases where the distance $|Y_j - Z_j|$ is not larger than ε, i.e.

$$\{Y,Z\} = \sum_{j=1}^{N} \theta\left(\varepsilon - |\vec{Y}_j - \vec{Z}_j|\right),$$

where $\theta(x)$ is the unity function: $\theta(x) = 1$ for $x \geq 0$ and $\theta(x) = 0$ for $x < 0$.

Then the quantity $D(Y|Z)$ acquires the meaning of the relative number of coincidences between $\vec{Y}_j$ and $\vec{Z}_j$ within the sphere of radius ε. If $\vec{Z}_j$ is a regular (hypothetic) lattice and $\vec{Y}_j$ are coordinates of atoms in the real lattice, $D(Y|Z)$ can be used as the characteristic of regularity of the lattice. If, however, $\vec{Z}_j$ is not a fully regular (not perfectly periodical) structure, $D(Y|Z)$ characterises the level of mismatch between the structures Y and Z.

References

1. Yu.A.Kravtsov and V.G.Petnikov. Partially deterministic wave fields. Sov.Phys.Doklady, 1985, v.285, N 4, 871-873
2. Yu.A.Kravtsov and V.G.Petnikov. Partially deterministic wave fields in optics. JOSA-A, 1987, v.4, N 1, 11-16