Symmetry of STRUCTURE

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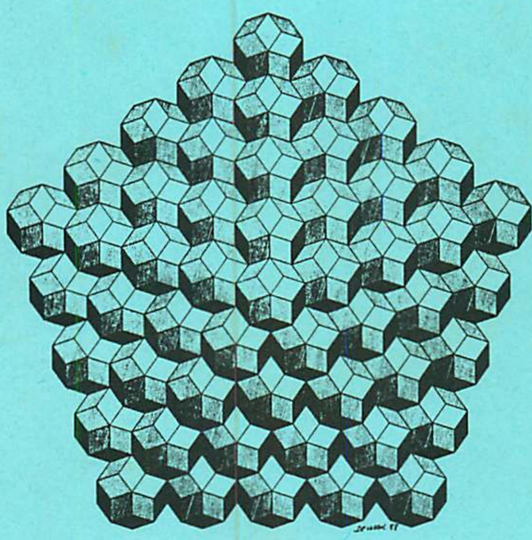
Abstracts

I.



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SYMMETRY AND ASYMMETRY IN THE ACTIVE VIBRATION CONTROL

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Attention to active vibration control problems has increased in recent years. The proposed applications are many complex technical projects, such as robotics, aerospace structures, track/vehicle systems and others. Active vibration control systems using the optimal feedback can reduce the destructive vibration and noise effects on technical and operating characteristics and on the health of people [1,2]. This paper is concerned with important role of symmetry and asymmetry in active vibration control systems design.

1. It is well known that in symmetric, many-degrees-of-freedom linear control systems it is possible to provide the autonomous vibration control of every mode [2]. In this case the modal decomposition simplifies the stability and performance analysis, because every mode control problem can be solved separately, like for a single-degree-of-freedom system. For example, let us consider the vibrating beam with two active vibration isolators (Fig.1) at the beam ends [4].

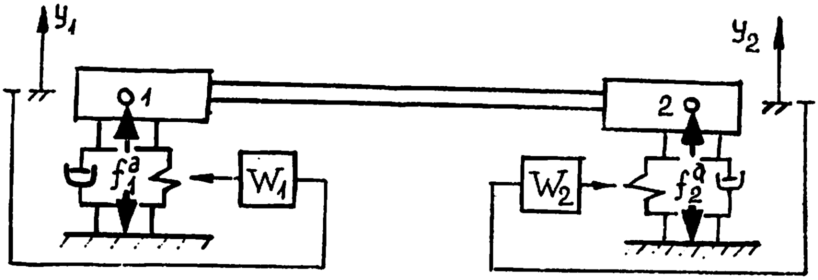

Fig.1

Taking into consideration the n first beam normal modes, the equations of motion are written in control theory terms:

$$(s^2 + \lambda_k\omega_k s + \omega_k^2)q_k = \frac{1}{m_k}\Sigma\varphi_{kj}f_j^a, \quad k=1,n$$

$$f_j^a = W_j(s)y_j, \quad j=1,2 \qquad (1)$$

$$y_j = \sum_{i=1}^{n} q_i\varphi_{ij}, \quad j=1,2$$

where q_k, ω_k, λ_k, m_k are the generalized coordinate, natural frequency, damping and generalized mass of the k -th mode; y_j are the displacements of the beam ends, where the isolators are attached, φ_{kj} is the k -th normal mode, $W_j(s)$ are the trasfer functions for each loop, s - the differentiation operator.

If it is assumed that the beam and the control loops are symmetric after the normalisation ($\overline{m}_k$ is the new generalized mass) one can find all the complex natural frequencies of the system (1) by solving characteristic equations of the form :

$$D_k(s) = s^2 + \lambda_k \omega_k s + \omega_k^2 - \frac{2}{\overline{m}_k} W(s), \qquad k = 1,n \qquad (2)$$

Thus the property of symmetry helps to estimate the critical frequences and feedback gains of the rigid object multi-degrees-of-freedom control systems, by considering single low-order subsystems, which can be described by simple characteristic equations.

2 The advantages of the symmetric structures are well known in the vibration control theory [1,5]. Only in symmetric structures it is possible to uncouple motions, and solve the problem of vibration control for each type of motion seperately. In this case there are used the principles of mutual compensation, etc. [1,5]. Vibration isolation problem for symmetric structures very often can be solved by the passive vibration isolation. But in practice for some constructions it is very difficult to keep symmetry of the structures under all operation conditions. For example, the mass or inertial parametres of the structure may be modificated during the operation. In this case it may be more advantageous to use the active vibration control not to reduce the vibration, but to make the real structure vibration to be like the vibration of the absolutelly symmetric system. This formulation implies some special methods of control system design,which can be explained using the previous example (Fig.1). Assume that the rigid body is asymmetric. Then for the dynamic symmetry of points 1 and 2 vibration one can find the needed transfer functions ratio $W_1(s)/W_2(s)$ by substitution the expression $|y_1| = |y_2|$ in (1).

In this problem the principles of mechanical vibration parametres control [1,5] are used for the assigning the property of dynamic symmetry to the vibration of real asymmetric structures.

3. The inverse problem can be also solved using the active vibration control i.e. it is possible to introduce the optimal asymmetry into the symmetric structures vibration. This fact can be used for the vibration reduction of the self-excited nonconservative mechancal systems (such as rigid unbalanced rotor shaft, structures subject to flatter, etc.) to increase the dissipation of some low-damped modes by strengthening their coupling with the heavily damped modes.

Consider two neighbouring normal modes in (1). Assume that the first mode ω_1 has the dissipation much lower than the second one ω_2 ($\lambda_1 \ll \lambda_2$). Let $W_1 = k_1$, $W_2 = k_2$,where k_1 and k_2 are some feedback gains. If it is assumed that the original mechanical structure (e.g. beam)is symmetric, one can write the characteristic equation of two coupled neighbouring modes, as follows:

$$[s^2+\lambda_1\omega_1 s+1-a_1(1+\alpha)][s^2+\lambda_2\omega_2 s+\omega_2^2-a_2(1+\alpha)]-a_1a_2(1-\alpha)^2 = 0 \qquad (3)$$

where $a_1 = k_1/m_1$, $a_2 = k_1/m_2$, $\alpha = k_2/k_1$ is parameter, which describes asymmetric control. The equation (3) can be analityically treated by the root loci method. Fig.2 shows the root loci of the equation (3),when the parameter α is varied.In the case of symmetric control $\alpha = 1$ ($k_1 = k_2$) the coupling between two modes is very low - the starting points are marked by stars (Fig.2). When α increases, root loci approach each other,i.e. rich damping mode ω_2 gives some dissipation to the poor one ω_1 . There is an optimal value of parameter α^* (Fig.2), when the coupling between modes reaches maximum, and at the same time there is the greatest value of the damping δ_{max} equal for both modes:

$$\delta_{max} = -0.25(\lambda_1\omega_1 + \lambda_2\omega_2)$$

Equation (3) gives the analytical expression of the optimal parameter α^* , which helps to determine the best control points which provide the maximum coupling connection between the modes.

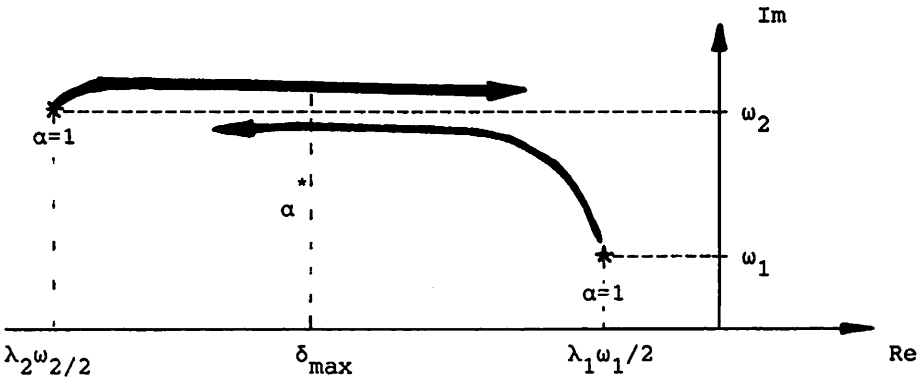

FIG.2

CONCLUSION

The examples show, that it is very useful to understand the role of symmetry and asymmetry when designing the active vibration control systems.

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