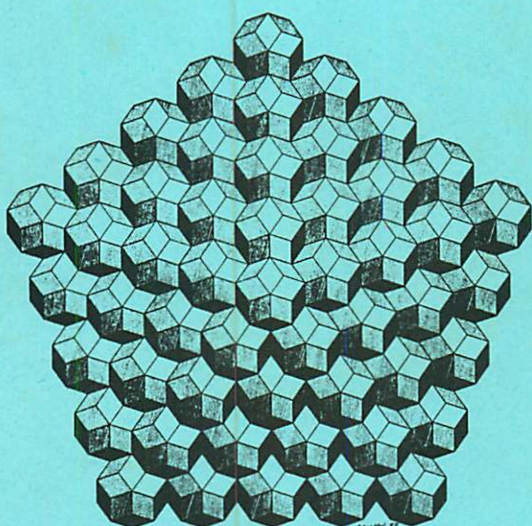
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



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ON THE HISTORY AND ARRANGEMENT OF THE CLASSES OF CRYSTAL SYMMETRY

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In most of the contemporary crystallographic works the 32 symmetry classes are usually arranged in a tabular manner fitting the crystal systems and type of symmetry. From a historical point of view it is interesting to trace the main trends of their arrangement.

The 32 crystallographic point groups have been described by Frankenheim in 1826 (Burckhardt, 1984) and confirmed partly or fully later on independently by Hessel, Bravais, Möbius, Gadolin, Curie, Fedorov, Minnegerode, Schoenflies and Wulff. The first table of the classes of crystal symmetry is considered to be that of Tschermak in 1905, accepted by Becke in 1926 and other authors (Yushkin et al., 1987).

A tabular arrangement, as a sequence only of the crystal systems has been suggested by Gadolin in 1867 and by Fedorov in 1891. The combination of crystal systems with type of symmetry arranged in a tabular way has bean applied by Schoenflies (1891) and later in a supplement of the Russian translation of Groth (1897). This procedure branched in the next years through using both sequence of crystal systems and type of symmetry. Still later differentiation followed chronologicaly. Trends in Figure 1 show symmetrical branching according to the crystal systems (A - triclinic, M - monoclinic, O - orthorhombic, T - trigonal, Q - tetragonal, H - hexagonal and C - cubic). On the left hand side there are A → C transitions and on the right hand side C → A transitions with an exception in the center. It is obvious that the A → C transition has been prefered by most crystallographers. In this branch an interesting diversion has been suggested by Shubnikov et al. (1940), viz. cubic system vertical fitting the columns of types of symmetry.

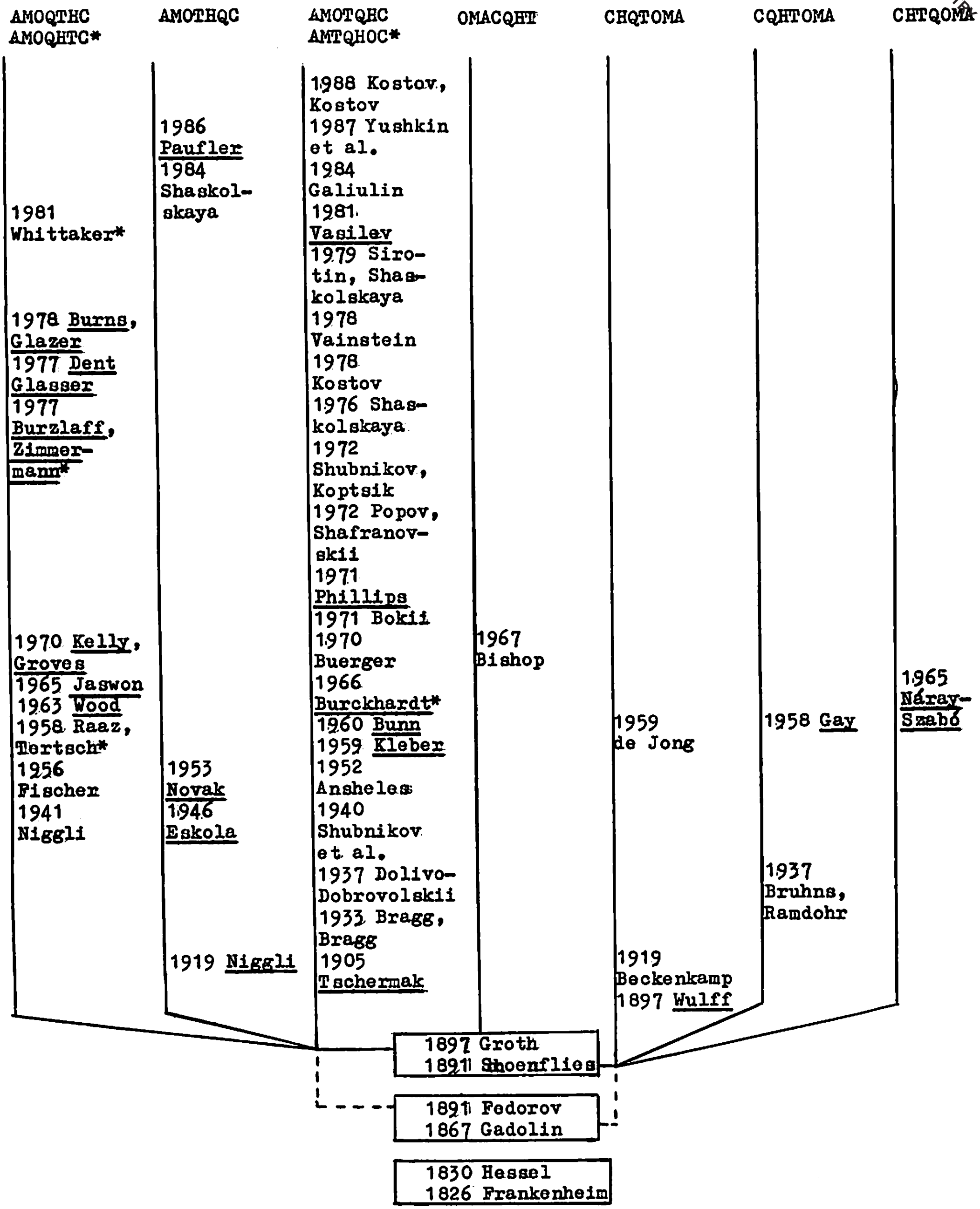

Fig. 1. Trends of the sequences according to crystal systems.

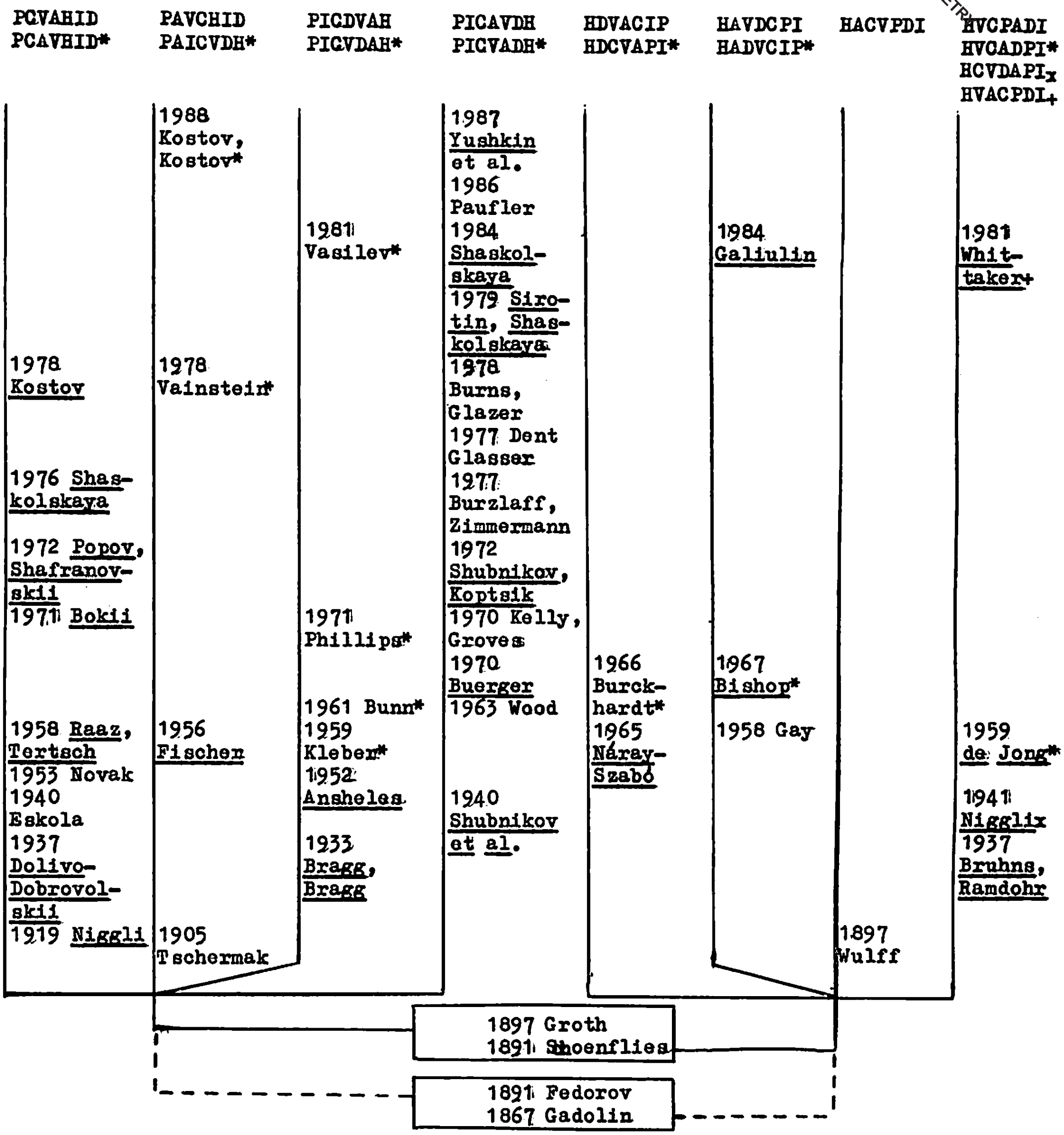

Fig. 2. Trends of the sequences according to type of symmetry.

The trends in Figure 2 show similar symmetrical branching along four lines each of the two sets according to the type of symmetry (P - polar, A - axial, I - inverse-primitive, C - central, V - planar, D - inverse-planar and H - planar-axial). The sequence in the tetragonal system has mainly been used. On the left hand side are the P → other transitions and on the right hand side the H → other transitions. The variety of arrangements in this case is greater, but most authors prefer the P → other transition type.

In both figures along each separate line underlined are the authors who use vertical and not horizontal manner of presentation. Asterixes mark small variations in the arrangement.

Crystallography is a fundamental science and the table of alignement of the crystal systems and symmetries like the Periodic Table of Mendeleev has its logical meaning and significance. Further development of this table including icosahedral, formal pentagonal and infinite groups of symmetry has also been made (Kostov, Kostov, 1988).

REFERENCES

Burckhardt, J.J. 1984. N. Jb. Miner. Mh., 11, 481-482.

Groth, P. 1897. Physicheskaya Kristallografiya (in Russian). 850 p.

Kostov, R.I., I.Kostov. 1988. Comp. Rend. Acad. Bulg. Sci., 4, 45-47.

Schoenflies, A. 1891. Krystallsysteme und Krystallstruktur. Leipzig, 638 S.

Shubnikov, A.V., E.E.Flint, G.B.Bokii. 1940. Osnovi Kristallografii (in Russian). Moscow-Leningrad, 487 p.

Yushkin, N.P., I.I.Shafranovskii, K.P.Yanulov. 1987. Zakoni Simmetrii v Mineralogii (in Russian). Leningrad, 335 p.

THE AESTHETICS OF STRUCTURAL SYMMETRY IN PAINTING, MUSIC AND POETRY: APPLICATION TO EDUCATION

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The lecture will develop according to the following guidelines:

Introduction: a. Legitimacy of Discussion
b. Definitions

Painting : a. Liniar symmetry
b. Transfer symmetry
c. All-round symmetry
d. Spatial symmetry

Music : a. Tonal symmetry (vertical and horizontal)
b. Dodecaphonic symmetry (vertical and horizontal)

Poetry : a. Echo symmetry
b. Symmetry of meaning
c. "Anti-symmetry"

Conclusion : a. Modern vs. modernistic
b. Enlargement of concept
c. Application to other Art-media
d. Application to Education
e. Learners' activities -- A model proposal (see fig. 4)

At the end of the lecture the following questions will be raised for discussion:

A.

In light of the ideas offered above, what patterns of symmetry -- reflection, rotation, translation and their combinations(Shubnikov, 1974; Senechal, 1977) -- can be discerned in the following works-of-art:

1. Paul Klee's "Running-away from Oneself" (see fig. 1).

2. Arnold Schoenberg's Dodecaphonic Series from String Quartet no. 4 and the opening melody based on it (see fig. 2).

3. Timm Ulrichs' Concrete Poem (see fig. 3).

In the above works-of-art, which are the aspects of transformation (change) and which of conservation (invariance) (Shubnikov, 1974) ?

Does the enquiry of symmetry operations in one art-medium -- e.g. the visual -- contribute to the understanding of symmetry operations in another art-medium -- e.g. the verbal?

Should the discussion on the Level-of-Form -- namely of structural symmetry --aspire to reveal truths on the Level-of-Content as well as on the Level-of-Meaning?

B.

Jerome Bruner introduced the concept of "The Structure of Knowledge" in relation to Curriculum Planning (Bruner, 1960). He advocated that instead of memorizing multitudes of details, the learner should become acquainted with the principles of a discipline. Harry Broudy proposed to apply Bruner's ideas to Art Education (Broudy, 1966). According to Weyl, "all a priori statements in physics have their origin in symmetry" (Weyl, 1952); is this true also of the art-media? If so, can symmetry be regarded as one of the depth-patterns at the basis of not only one art-discipline, but of The art-disciplines as a whole?

C.

"Quarendo Invenietis" -- Seek and Ye Shall Find -- wrote J.S.Bach on one of the three "puzzle canons" in his Musical Offering: the composer provided a theme and the task was to determine by what symmetry operations he intended the theme to be repeated (Senechal, 1977). Indeed, through symmetry we can discover a norm for many things, thus unifying large bodies of knowledge. Therefore we keep on Seeking and hope to Find. Yet, should not the warning implied in the following questions/statements be re-heeded: when does this search for symmetry -- in the creation of works-of-art, as well as in their interpretation -- "begin to tend toward excess, toward a need for proportion and repetition and regularity so acute that it becomes obsessive and life-denying? How do we know when symmetry-seeking is still human?" (G. Fayen, "Ambiguities in Symmetry-Seeking: Borges and Others", in Senechal, 1977). Where is the limit?

SHORT BIBLIOGRAPHY

Broudy, H.S. "The Structure of Knowledge in the arts", in R.A. Smith (ed.), Aesthetics and Criticism in Art Education, Chicago, Rand Macnall, 1966.

Bruner, J.S. The Process of Education. Cambridge, Mass., The University of Harvard Press, 1960.

Senechal, M. and G. Fleck (ed.). Patterns of Symmetry. Amherst, University of Massachusetts Press, 1977.

Shubnikov, A.V. and V.A. Koptsik. Symmetry in Science and Art. Translated from Russian by G.D. Archad. New York/London, Plenum Press, 1974.

Weyl, H. Symmetry. Princeton, Princeton University Press, 1952.

LIST OF ILLUSTRATIONS

Fig. 1: Paul Klee. "Running-away from Oneself" (Phase I), 1931; Paul Klee Foundation, inv. 778, K5(25).

Fig. 2: Arnold Schoenberg. The Series from String Quartet No. 4 and the opening melody based on it; in S. Sadie and A. Lathan (ed.), The Cambridge Music Guide, Cambridge University Press, 1985, p. 62, ex. III.2.

Fig. 3: Timm Ulrichs. A concrete poem; in P. Garnier, Spatialisme et poésie concrète, Gallimard, 1968, p. 99.

Fig. 4: "Symmetry in Art Education: A Three-Dimetional Model":

X-axis: Learners' Activities: (Xa)--Assimilation; (Xb)--Reproduction; (Xc)--Creation; (Xd)--Interpretation.

Y-axis: Modes of Symmetry: (Ya)--A-Symmetry; (Yb)--Symmetry; (Yc)--Anti-Symmetry; (Yd)--Re-Symmetry.The modes correspond to phases in the historical development of the use of symmetry in The Arts.

Z-axis: Art-Media: (Za)--Painting; (Zb)--Music; (Zc)--Poetry;

fig. 1

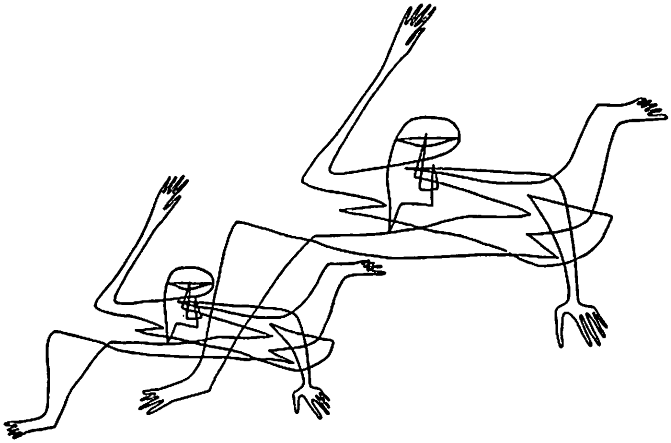
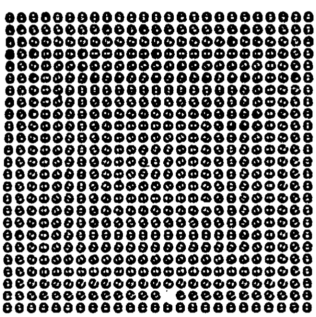
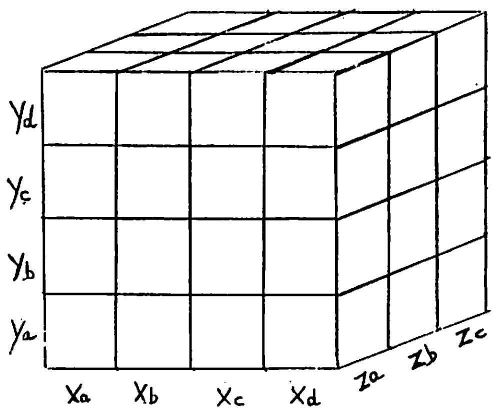


fig. 2

fig. 4

fig. 3





Quasiperiodic Patterns and Golden Sections.

Prof.Dr. Peter Kramer, Institut für Theoretische Physik der Universität Tübingen, FRG.

In this lecture we review the transition from periodic to quasiperiodic patterns, describe the new features of quasiperiodic structure and its links to the golden section, and illustrate the appearance of these patterns in the new physics of quasicrystals.

1. The appearance of periodicity:

The most familiar roots for the notion of periods is our experience of time. Time is experienced in terms of units like days, months, years. These units or periods follow each other in a sequential repetition pattern. With the expansion of experience and science, we have learned about the astronomical origin of these and other much longer periods, and we have also found extremely short periods of time on the atomic and subatomic scale. Among the arts, music is a prominent field where a pattern in time is created, very often with a background of periodic rhythmic units.

Periods in space were created, a long time before science, in the sequential order of band ornaments, and the additional dimensions of space allowed for the extension of periodic structures to ornaments in the plane. The inclusion of new dimensions opened the way not only to doubly periodic patterns, but also brought the new repetion patterns of rotational, mirror and color type. The richness of ornamentics is obtained from the variation of motives and from their combination. In the structure of crystals one finds up to triply periodic structures from the macroscopic down to the atomic level. The motives of this periodic structure are formed from atoms and have as their typical length scale multiples of an Angstrom.

The systematic description of these periodic patterns in science has a long history up to present-day crystallography.

2. The structure of periodic patterns:

A simple way of representing sequential periodic patterns is given by forming artificial words, that is, sequences from an alphabet of letters a,b,c,.... As an example consider the words cde, cdecde, cdecdecde,....

which are generated recursively from the motive cde. In the limit of an infinite number of repetitions we get a periodic sequence which is very similar to a periodic decimal fraction.

In mathematics we describe the periodic property of this infinite sequence by a shift map. This shift map moves each motive cde by three letters to the right, so that the unshifted and shifted patterns are

...cdecdecde...... ,

......cdecdecde... .

Since the points are meant to represent the infinite sequence, the unshifted and shifted sequences represent the same pattern, and so the shift map generates a symmetry map of the sequence. The elaboration of the possible repetition patterns for multiply periodic patterns in two- and three-space is the subject of crystallography.

The notion of various types of symmetries, of maps and motives and their interplay in art and science will be illustrated by selected examples.

3. The structure of quasiperiodic patterns:
To introduce the concept of quasiperiodic patterns, we take as an example the Fibonacci words formed recursively from two letters a, b:
a, b, ba, bab, babba, babbabab, ...
To continue this sequence recursively, one simply links to the right of the last word its precessor. The infinite sequence obtained in this fashion is called the Fibonacci sequence. If one counts the frequency of the letters a and b in the sequence, one finds subsequent pairs of the well known Fibonacci numbers
1, 1, 2, 3, 5, ...
From the relation of these numbers to the golden section it is easy to demonstrate that the Fibonacci sequence cannot be a periodic sequence based on a finite motive formed from the letters a and b. So this sequence is an example of a so-called quasiperiodic object generated by a strict recursive rule. Patterns formed in a similar fashion by words are studied in the new field of mathematics called Combinatorics on Words, compare Lothaire (1983).
Now let us replace the letters a,b by two general motives to obtain the prototype of a quasiperiodic structure. Quasiperiodic patterns are not restricted to one-dimensional sequences, and richer patterns are obtained in two and three dimensions. Moreover, these quasiperiodic patterns may display rotational symmetries which are incompatible with all the well-known periodic structures. Patterns in the plane of this type with fascinating properties were invented by Penrose (1974). In three dimensions, similar patterns were proposed by Mackay (1982) and analyzed by Kramer and Neri (1984).
Properties and principles for the generation of these and similar patterns will be discussed and illustrated. The new interpretation of quasiperiodic patterns as sections through periodic patterns in hyperspace will be elaborated. Recently it has been realized that Kepler (1611) anticipated some important concepts of quasiperiodic structure. The significance of Keplers work was recognized already by Kowalewski (1938).

4. The golden section:
From the numerous ramifications of the golden section in the arts, in mathematics and science, as described for example by Huntley (1970) and by Richter and Scholz (1987), we select and discuss its relation to geometry and in particular to pentagonal and icosahedral symmetry. These symmetries necessarily are in conflict with periodic symmetry and hence were excluded from traditional crystallography, as will be illustrated by examples. The unexpected new result is that the golden section can be linked to quasiperiodic patterns rather than to periodic ones.

The connection between the golden section and quasiperiodic symmetry will be discussed in detail, starting from the Fibonacci sequence which is described above. As a result, quasiperiodic patterns with pentagonal and icosahedral symmetry will be interpreted as golden sections through periodic patterns in hyperspace. Periodic patterns formed from hypercubes as described by Haase et al. (198) will serve as a main example. Sections through these hypercubes generate families of cells in three-dimensional space known in part as zonohedra. These cells fit together in a subtle way to form quasiperiodic fillings of three-space.

5. Quasicrystals:

Experimental evidence for the existence of quasiperiodic structures formed by metal alloys was given first by Shechtman et al.(1984). Not only the study of these materials by X-ray and electron diffraction, but also the outer morphology of these quasicrystals given by Dubost et al. (1986) shows that icosahedral symmetry and hence the golden section is manifest in these alloys. Only part of the physics of quasicrystals has been explored and understood up to now. A non-technical survey of the present state of this field will be given, with emphasis on the evidence for a quasiperiodic cell structure.

For a long time, the golden section has played its main role in the non-scientific fields of culture. With the new role of the golden section played in quasicrystals, there is a much better chance of fruitful explorations from science and other fields of culture into the fascinating new landscape of quasiperiodicity.

References:

Dubost, B., Lang, J.M., Tanaka, M., Sainfort, P. and Audier, M. Nature 324 (1986) 48

Haase, R.W., Kramer, P., Kramer, L. and Lalvani, H. Acta Cryst. A 43 (1987) 574

Huntley, H.E., The Divine Proportion, Dover, New York 1970

Kepler, J., Strena seu de Nive Sexangula (1611)

Kowalewski, G., Der Keplersche Körper und andere Bauspiele, Köhlers Antiquarium, Leipzig 1938

Kramer, P. and Neri, R. Acta Cryst. A 40 (1984) 580

Lothaire, Combinatorics on Words, Addison-Wesley, London 1983

Mackay, A.L., Physica 114 A (1982) 609

Penrose, R., Bull. Inst. Math. Appl. 10 (1974) 266

Shechtman, D., Blech, I., Gratias, D. and Cahn, J.W., Phys.Rev.Lett. 53 (1984) 1951

Richter, P.H. and Scholz, H.-J., Der Goldene Schnitt in der Natur-Harmonische Prportionen und die Evolution, p.175 in: Ordnung aus dem Chaos, Ed. B.O. Küppers, Piper, München 1987