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Symmetry of STRUCTURE

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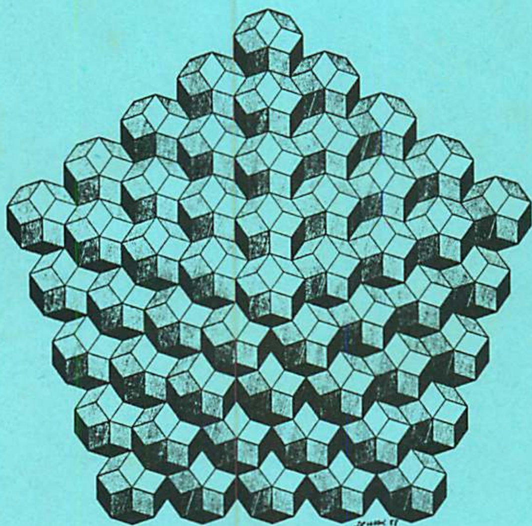
Abstracts

I.



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THE PECULIAR STABILITY BEHAVIOUR OF NON-SYMMETRICALLY LOADED SYMMETRICAL STRUCTURES

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Let us consider the simple symmetrical plane structure shown in Fig. 1a which consists of rigid bars and elastic springs, connected by hinges. The degree of freedom of movements is

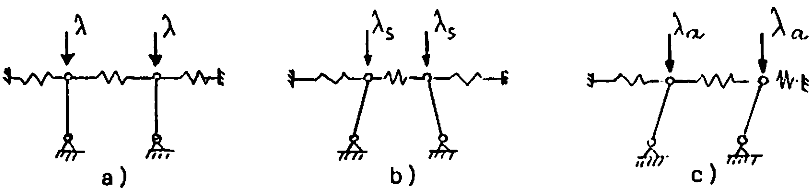

Fig.1

two. The symmetrical load can cause symmetrical (Fig. 1b) or antisymmetrical (Fig. 1c) buckling. The critical loads which belong to these buckling modes are denoted by λ_s and λ_α respectively.

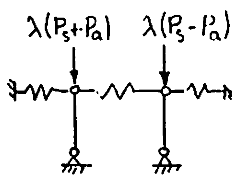

Fig. 2

Assuming different loads acting on the left and right bar (Fig.2), it can be shown that the critical loads (λ_{cr}) are the roots of the following equation of the second degree:

$$\lambda_{cr}^2 (P_s^2 - P_\alpha^2) - \lambda_{cr} P_s (\lambda_s + \lambda_\alpha) + \lambda_s \lambda_\alpha = 0 . \qquad (1)$$

Consequently, in the case of special non-symmetrical load arrangements shown in Fig.3, the critical load can be calculated from the geometric, arithmetic and harmonic means

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of λ_s and λ_a. By investigating some similar structures we have obtain the same result, namely that the critical load

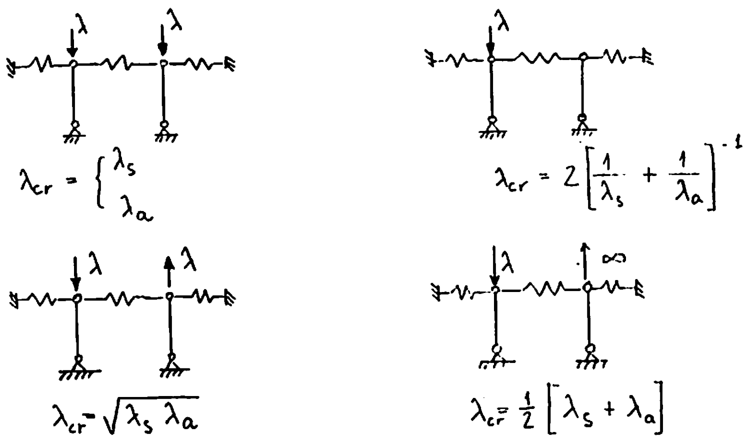

Fig.3

of a non-symmetrically loaded symmetrical structure can be obtained from the critical loads belonging to the symmetrical and antisymmetrical buckling modes of the symmetrically loaded structure on the basis of (1). Now the question arises whether this strange behaviour is a general feature of "symmetric" eigenvalue problems. The answer is no, but we can give the conditions with the aid of which it can be decided when this behaviour occurs.

Let us introduce the following bilinear functional:

$$\Pi = \frac{1}{2}\langle A_1\varphi_s, A_2\varphi_s\rangle + \frac{1}{2}\langle B_1\varphi_a, B_2\varphi_a\rangle + \frac{1}{2}\langle D_1\hat{\varphi}_s, D_2\hat{\varphi}_s\rangle + \frac{1}{2}\langle E_1\hat{\varphi}_a, E_2\hat{\varphi}_a\rangle + \frac{1}{2}\langle F_1\hat{\varphi}_s, F_2\varphi_s\rangle +$$
$$+ \frac{1}{2}\langle G_1\hat{\varphi}_a, G_2\varphi_a\rangle - \frac{1}{2}\lambda\left\{P_s\left[\langle C_1\varphi_s, C_2\varphi_s\rangle + \langle C_1\varphi_a, C_2\varphi_a\rangle\right] + 2P_a\langle C_1\varphi_s, C_2\varphi_a\rangle\right\}$$

where φ_s, φ_a, $\hat{\varphi}_s$, $\hat{\varphi}_a$ are the elements of the real inner product (pre-Hilbert) space $\mathcal{H}$, which form the vector :

$$\underline{\varphi} = \begin{bmatrix} \varphi_s \\ \hat{\varphi}_s \\ \varphi_a \\ \hat{\varphi}_a \end{bmatrix}$$

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and A_1 , B_1, ...,A_2, B_2 etc. are linear operators of $\mathcal{H}$. We deal with the

$$\Pi = \text{stationary} \quad !$$

eigenvalue problem. Assuming symmetrical loading, i.e. $P_s=1$ and $P_a=0$, we obtain the eigenvalue parameters λ_s and λ_a which belong to the eigenfunctions

$$\underline{\Phi}_s = \begin{bmatrix} \phi_s \\ \hat{\phi}_s \\ 0 \\ 0 \end{bmatrix} \qquad \underline{\Phi}_a = \begin{bmatrix} 0 \\ 0 \\ \phi_a \\ \hat{\phi}_a \end{bmatrix}$$

respectively. In the case of non-symmetrical load, the critical load can be calculated on the basis of (1) only if

$$C_1^* \; C_2 \; \phi_s = \alpha \; C_1^* \; C_2 \; \phi_a \quad ,$$

where α is a scalar, and C_1^* is the adjoint operator of C_1, see [Hegedűs and Kollár, 1989a] .

The results can be used for the investigation of discrete and continouos problems as well , among others for the flexural wrinkling of sandwich panels [Hegedűs and Kollár, 1989a,b] .

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