

# Symmetry of STRUCTURE

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Abstracts

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# UNIFORM POLYHEDRA FOR BUILDING STRUCTURES

by

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## General introduction

Many building structures are in their overall shape or in their internal configuration based on the geometry of one or more kinds of the so-called 'uniform' polyhedra. The cube and the prisms are very popular in this respect, but these are in fact members of a large family of forms that have great potentials for application in building. It is therefore interesting not only but highly relevant also to undertake a systematic study after the properties of this group of figures in order to find out what their characteristics are, what their binding factors and what their differences.

## Definition of the uniform polyhedra [1]

1. They are composed of one or more kinds of plane, regular polygons with 3, 4, 5, 6, 8 or (maximally) 10 edges.
2. The polygons meet in pairs at a common edge.
3. The dihedral angle at such an edge is always convex (= less than  $180^\circ$ , if seen from the interior).
4. All vertices of a polyhedron lie on one circumscribed sphere.
5. All these vertices are identical, which means that around each vertex of a particular polyhedron the polygons are grouped in the same number, kind and order of sequence.

## Different kinds of polyhedra

It is easy to understand that within these limits the minimum total number of polygons around a vertex is 3, the maximum number 5 and it is also simple to prove, that not more than 5 totally regular polyhedra can exist. These regular solids are composed out of one kind of faces each.

Polyhedra are called semi-regular if more than one kind of polygons is used for their construction. Thus a group of 30 polyhedra in total, answers the definition of uniformity (see Fig. 1):

- 5 regular or Platonic solids, consisting of 8 or 20 triangles, of 6 squares or of 12 pentagons
- 15 semi-regular or Archimedean solids (including the enantiomorphic or left-handed versions of the snub cube and the snub dodecahedron)
- 5 prisms, having two parallel congruent regular polygon faces with squares as the interconnecting sides (the square prism is identical to the cube)
- 5 antiprisms with two twisted parallel polygons and with triangles as sides (the triangular antiprism is identical to the octahedron)

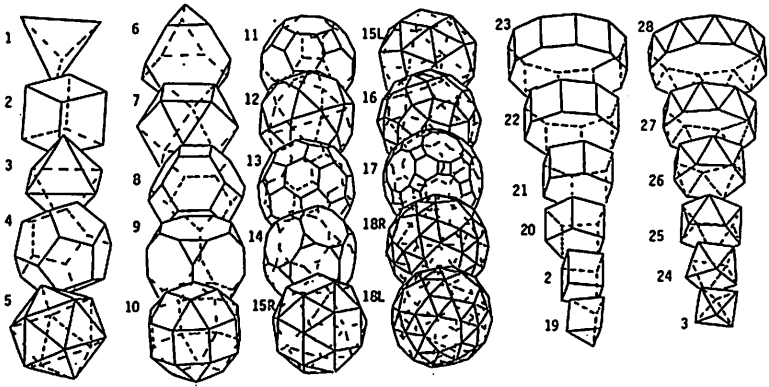


Fig. 1. Review of the regular and semi-regular uniform polyhedra:  
 A. Platonic: tetrahedron(1), cube(2), octahedron(3), dodecahedron(4), icosahedron(5)  
 B. Archimedean: truncated tetrahedron(6), cuboctahedron (7), truncated octahedron(8), truncated cube(9), rhombicuboctahedron(10), truncated cuboctahedron (11), icosidodecahedron(12), truncated icosahedron(13), truncated dodecahedron(14), righthanded snub cube(15R), lefthanded snub cube(15L), rhombicosidodecahedron(16), truncated icosidodecahedron(17), righthanded snub dodecahedron(18R), lefthanded snub dodecahedron(18L)  
 D. Prisms: trigonal(19), square(2), pentagonal(20), hexagonal(21), octagonal(22), decagonal(23)  
 E. Antiprisms: trigonal(3), square(24), pentagonal(25), hexagonal(26), octagonal(27), decagonal(28)

Computer technique for visual representation [3]

The coordinates and other geometric aspects of uniform polyhedra can be generated automatically with the help of computers by firstly forming each of the different kinds of polygons out of which the polyhedron consists and then reproducing it by combined translation and rotation in space as many times as it occurs in the considered polyhedron. Fig. 2. shows the principle of such a procedure as developed for study purposes on personal computers.

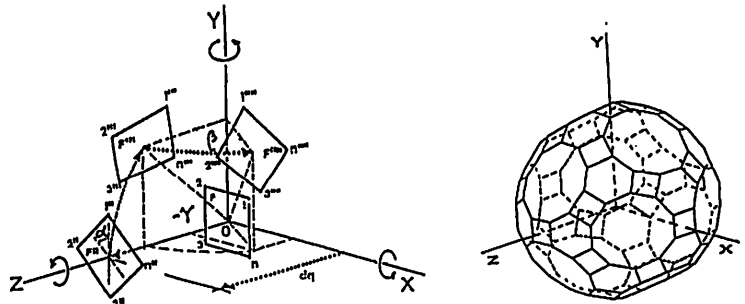


Fig. 2 Combined translation and rotation of polygons for the formation of polyhedra

The use of polyhedral forms in building [3]

The role of these polyhedra for the formgiving possibilities in building is very important, although this is not always recognized.

- all trivial uses of cubic and prismatic shapes (vertical or horizontal, pure or distorted versions)
- many solitary applications with macroforms, derived from one of the more complex polyhedra or of their reciprocal (dual) forms
- prismatic and antiprismatic folding structures
- close-packings of one or more kinds of solids in conglomerates or in space structures
- pyramidized or polar versions of solids in order to reduce the relative size of larger polygons
- basic geometry of geodesically subdivided dome structures - mainly on the basis of octahedron, icosahedron, or triacontahedron (reciprocal of icosidodecahedron)

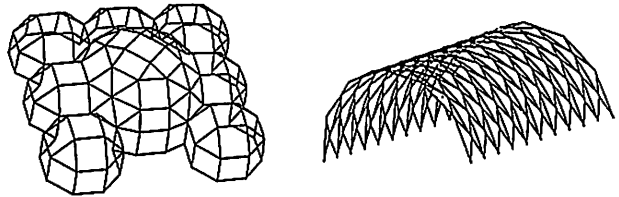


Fig. 3. Polyhedral building and antiprismatic structure

Tri-axial ellipsoids [2]

Polyhedra can be considered as subdivision patterns of spherical shapes. A higher break-down frequency provides a closer approximation to the sphere. If to such a dome structure different radii are given in X-, Y- and Z-direction of the coordinate system, then a tri-axial ellipsoid is obtained. A horizontal cross-section according the XY-plane (see Fig. 4) has the form of an ellipse. Any vertical cross-section through the Z-axis is an ellipse also, having its foot points on the horizontal ellipse. The equation of the ellipse is of the second power, but the main form of this ellipse can be influenced considerably by raising or lowering this power. (squared or flattened if respectively larger or smaller than 2, straight or hollow if respectively equal to or smaller than 1).

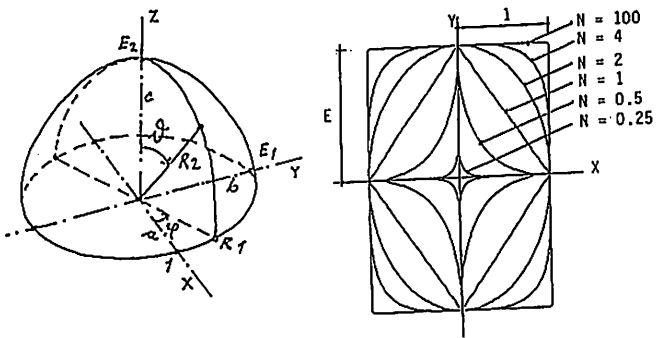


Fig. 4. Characteristics of tri-axial ellipsoids

them squarely on the road to progress. The revolutionary significance of this new organon lies in the fact that it has overstepped the original limits set for it. However, the idea of combining concepts and ideograms into a single coherent schema, basically corresponding to the structure of the world and of our knowledge of it, has been less successful. It is founded on perception of the unity of human knowledge and reality. Such a schema is a projected unity, established as a point of reference and correlation, but does not state whether anything is "actual" or "existing". The semantologic multiplicity and arbitrary nature of any language provide many possibilities for communication between conceptual and visual languages. The basic conviction that a feasible "canon" must exist for bridging them, has its roots in the fixation that a rational constructivistic relationship overlies the arbitrary level of a metalanguage.

The organon is an illuminating instrument of science, exposing unknown truths and establishing a common method which contributes to science, philosophy, the arts and architecture. All historical, cultural and scientific development might be viewed as harmonization of vital images, formal structures and diverse ideas. The result of this process of human knowledge and consciousness lies in the realm of essences, which find their formal expression in the "conceptual and ideographic organon". The fundamental principles of the organon impose order and pattern on nature and on human knowledge. If it successful it might yield possible principles of morphogenesis.



On the other hand according to the structure of the Canon, the organon possesses - as a central tool of human knowledge - the ability to analyse all information associated with acquisition or expression of knowledge. There is a duality between information or knowledge already possessed or acquired, and the senses or interpretations attached to it. Scientific and non-scientific inquiry is traditionally over time, despite revolutionary changes in the theoretical models and systems describing its subject matter. Analytical or dialectical use of the organon according to the different subjects of the canon means that its continuity and development are of hermeneutic nature. Such application of hermeneutical elements and methods means in turn that, through continuous reinterpretation of data, the real essences will gradually be exposed to human knowledge. The attendant identification and isolation of the same set of referents requires consistency with all possible changes in the theoretical models or systems. The importance of the hermeneutical approach lies in the ability to explain scientific observations, in finding links with alternative points of view and even in showing the way to new scientific and non-scientific perspectives.

In the attempt to construct our schema of concepts, we concluded that it could be expressed on a new formal basis. Each concept is assigned a visual object, a so-called "element house". The first element house used in this project refers to the concept of space, which - conceived as the foundation stone of human knowledge - has developed over the ages alongside its conceptual, symbolic and architectonic evaluation.

nondegenerate regular eight-member cycle  $P(2)$ , which is equivalent to the whole previous cyclic closing  $[P(1)]$ , but also two simplest possible additional degenerate two-member cycles  $p$  (the initial,  $p^i$ , and final,  $p^f$ , cycles):  $\iota(2) = S(2) = S[P(2)] = S_{p^i} + P(2) + p^f = S_{p^i} + S_{P(2)} + S_{p^f} = 2 + S[P(1)] + 2 = 2 + 8 + 2 = 12$ . The longitudinal skeletons of the blocks are of four types (allowing for the feature of the proline residual) and may be described by the proper spinal characteristic  $S$  with four pairwise-antisymmetric half-integer eigenvalues  $S(\beta) = \beta = \pm \frac{1}{2}, \pm \frac{3}{2}$ .

In psychology ( $g = 3$ ), the complete system of typical homo sapiens with their 13 characteristic inverse values of critical levels of intellectual potentialities  $1/x(i)$  or, in other words, with the appropriate successive whole numerical eigenvalues  $I(i) = 12/x(i) = i = 0, 1, \dots, L(3) - 1$  ( $L(3) = 13$ ) with the particularly singled-out common initial zero eigenvalue  $I(0) = D(0) = 0$ , which is inherent to the divinely-omnipotent genuises, at the center of symmetry of twelve remaining cyclicly-closed eigenvalues  $D(d)$  ( $d = 1, \dots, S(3) - 1$ ;  $S(3) = 13$ ) constituting the complete closed nondegenerate regular twelve-member cycle  $[P(3)] = P(3)$  which is characteristic of the proper mentality classification of typical homo sapiens and equivalent just to the previous cyclic closing  $[P(2)]$ , namely,  $\iota(3) - 1 = S(3) - 1 = S_{[P(3)]} = S_{P(3)} = S_{[P(2)]} = 12$ . The individuals with opposite traits (extraverts and intraverts) are of sign-opposite half-integer eigenvalues of the proper spinal characteristic  $S$ , namely,  $S(\beta) = \beta = \pm \frac{1}{2}$ .

The above mentined systems of the fundamental structural elements of matter at four successive basic levels of its natural self-organization are equivalent to the systems of uniformly-quantized eigenvalues of the respective universal characteristics  $I$ ,  $D$ , and  $S$  which are of the same type of symmetry and are defined deductively from an appropriate mathematical induction. In each of the cases ( $g = 0, 1, 2, 3$ ) the total number of so called standard elements ( $\Sigma^*$ ) coincides identically with the total number  $\Sigma = \iota + S + \sigma$  of all the possible eigenvalues  $I(i)$ ,  $D(d)$ , and  $S(\beta)$  realized by the standard elements:

$$\Sigma^*(g) = \Sigma(g) = \iota(g) + \delta(g) + \sigma(g) =$$

$$= \left\{ \begin{array}{lll} 7 + 7 + 2 = 16 & \text{at } g = 0, \\ 8 + 8 + 0 = 16 & \text{at } g = 1, \\ 12 + 12 + 4 = 28 & \text{at } g = 2, \\ 13 + 13 + 2 = 28 & \text{at } g = 3. \end{array} \right.$$

In principle, all these symmetrical periodic systems not only are determined by the proper dynamic laws of interaction, but also determine them themselves [1 - 8].

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