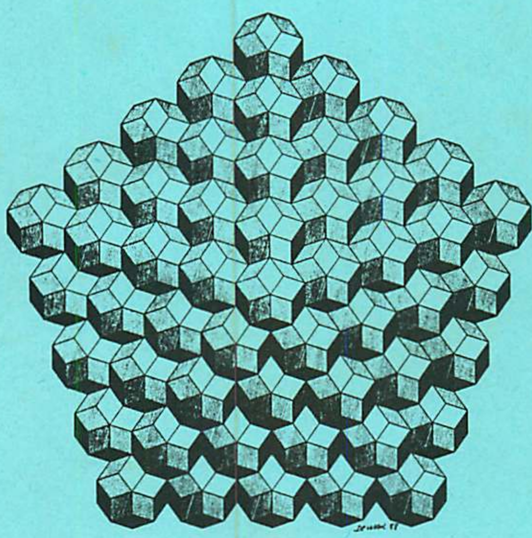
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



Edited by Gy. Darvas and D. Nagy

Buda Pest

August 13-19, 1989

hungary

THAT UNORDINARY MIRROR-ROTATION SYMMETRY

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The history of *mirror-rotation* (or *rotary-reflection*) symmetry begins in the 19th century with crystallographers; but manifestations in its prehistory can be traced to the advent of civilization, when man began to record observations of astronomical bodies.

The mystery of the solar eclipse surely awed man in distant times; and, for those who functioned beyond dismay, there were wonders to observe. Aristotle recorded what many had certainly noticed before him: the holes of a sieve cast round images of the sun, even if they were square or triangular. The leafy canopy of a woodland does the same; and, during an eclipse, a multitude of crescents appear. Strangely, these crescents are inverted in relation to the crescent of the partially eclipsed sun.

Those not located in the total eclipse's corridor will see that, during the event, the position of the sun's crescent will appear to rotate either clockwise or negative clockwise. Whichever way it seems to go, the images will seem to rotate the other way. An observant person would not only be puzzled by the inverted image, but by its perverted rotation. Up becomes down; clockwise becomes negative clockwise.

The sun's images are caught in rooms from holes in roofs. Dangerous to stare at with the naked eye, the eclipse can thus be safely observed. Other apparitions materialize from chinks in window shutters. Whole exterior scenes are projected onto interior walls: figures, colors, movements, all that is outside. The projections are, however, upside down; handedness is reversed.

Hearsay history credits da Vinci, Alberti, Roger Bacon, and other Renaissance notables with the invention of the camera obscura. In fact, they were making use of a device with a remote origin. The earliest known record of it comes from the tenth century Arabian scholar Alhazen, whose writings indicate that he was describing something already known.

A true mirror-rotation configuration is isometric. Though the sun's image can be set to the same size that the sun appears from earth, the sun and its image are a "similitude," not an isometry. Contained apparatuses, such as the boxes of Athanasius Kircher, can be rigged to precise isometric relationships. An image on one wall of a chamber is lighted by candles; a shutter with a hole is set at the center; the wall of the darkened chamber on the other side catches a perverted image.

For all his scientific formalities, Brother Kircher was careless. Embarrassing errors are immortalized in his tomes. The images in Kircher's illustrations of camera obscuras are all duly inverted. In one case, handedness is not reversed; in two others, though letters and words are reversed, the positions of the words are clumsily scrambled. Observation was inconclusive; bungled reversals of handedness evidences defective witness. Kircher, for one, cannot be credited with a faithful presentation of the mirror-rotation principle.

Lenses were introduced in more sophisticated camera obscuras at the shutters' apertures in order to effect clearer images. Those dealing with lenses also saw upside-down, reverse-handed worlds through the honed glasses. Aristophanes is the first to have written about a "burning glass"; and it is reported that Roman emperors viewed gladiatorial games though jewels. Yet, the great innovators of lensed instruments from Galileo on can no more than Kircher be credited with actually specifying the conditions of mirror-rotation symmetry; and modern text books on optics, alas, often present illustrations that repeat faults found in Kircher's works.

The necessity to derive the concept of mirror-rotation arose with the crystallographer. *Inversion*, rather than the "coupled" *mirror-rotation* operation, could stand as definition for two Crystal Classes which were later given the notations S_2 and S_2 by Schönflies. The elusive and critical four-fold mirror-rotation operation, S_4 could not be described as a straightforward inversion.

Frankenheim, using analytical geometry, is given credit for first identifying the 32 Classes (1826), including, of course, S_4, which spontaneously appeared among the permutations of his algebraic expressions. Hessel's confirmation, presented by a peculiar and tedious terminology, soon followed (1830). Gadolin's enduring contribution was his stereographic projections (1867) -- Frankenheim having produced no illustrative aids and Hessel, less than artful diagrams in a supplemental publication (1862).

Logic suggests that, even without Frankenheim or Hessel, S_4 would have shown up for Gadolin through the simple exercise of mapping all the combinatory possibilities that his projections allow. Schönflies, whose notations remain in use today, did, of course, handle S_2, S_6, and S_4 with group theory and applied the term *Drehspiegelung*.

Federov saw clearly the comprehensive nature of mirror-rotation. He submitted that, though S_2 can also be described as the inversion C^i and S_6 as C^i_3, inversion is an unnecessary operation in a succinct theory of symmetry:

> *I have repeatedly pointed out that the concept of the inversion center does not belong to symmetry theory at all, but to the theory of similitude, and is immeasurably older than symmetry theory.*

The mirror-rotation operation (Federov's "compound symmetry") covers the three Classes, S_2, S_4, and S_6, and an infinite array of members, S_8, S_{10}, S_{12} . . . S_{2n} (and S_{4n}), beyond the narrow range of crystallographic possibilities.

Failing an ability to read Russian, I take many clues from the Federov papers translated into English by David Harker. *Gadolin*, Federov declared, *did not yet know anything about compound symmetry and was connecting only one special case* (four-fold mirror-rotation) *with the word sphenoidal.*

Federov, as mentioned, was critical of Schönflies for retaining inversion. Interestingly, Federov's later papers indicate that he himself had not envisioned the comprehensive nature of a mirror-rotation operation at the time of his 1883 treatises: *I have been saying, beginning with 1889, not "sphenoidal symmetry," but "compound symmetry."*

Though four-fold mirror-rotation, rather than two-fold, created consternation (even some discredit for Bravais) and was the element that forced the rigor of a Federov upon the theory, except for Gadolin's often reproduced diagram, there were few early illustrations of it. Curie did provide an utterly straightforward one. Shubnikov and Koptsik resurrected an elegant example attributed to G. Wulff. In their 1956 *Symmetrie*, Wolf and Wolff, that unsymmetrical pair, not to be confused with the yet other Wulff, did not present a proper representation of four-fold mirror-rotation; they resorted to a regular tetrahedron with all its additional mirror planes.

The chemists were slow to adopt symmetry theory in their study of molecules. Of course, they had first to work out general concepts of how molecules are structured. Before the employment of X-ray, both the theories of crystallographic symmetry and of molecular structure were abstract exercises, albeit, compelling ones. Van 't Hoff and Le Bel made a major breakthrough with their brilliantly deduced "asymmetric carbon atom." An error was made, however, which again indicated unfamiliarity with mirror-rotation symmetry and its properties.

Van 't Hoff (and those who followed) depicted the meso-tartaric molecule as a mirror figure. It is now known that in solution there is "free rotation" of the meso-tartaric molecule's two tetrahedral components at the molecule's mid-point. Since like atoms repel one another and effect antipodal positions, the favored alignment, then, of the two components, out of all random possibilites, is not a mirror configuration but a mirror-rotation one.

Chemists, applying crystallographic symmetry theory, sought the hypothetical. While two-fold and six-fold mirror-rotation molecules were soon identified, it wasn't until 1955 that an actual four-fold mirror-rotation molecule was found.

At 1930's international conference in Zürich, Hermann and Mauguin proposed *rotary-inversion* to replace Federov's hard argued *rotary-reflection*. As early as 1897 Federov considered a system with the alternative "compound symmetry" of inversion-rotation; but he stayed with mirror-rotation.

If not from Federov, from where did Hermann and Mauguin's rotary-inversion, as a comprehensive operation, come? Only two years before the Zürich conference, Hermann wrote an article for the *Zeitschrift für Kristallographie* that cited *Drehspiegelung*, but not *Drehinversion*. Wykcoff's text of 1924/1931 presents both *rotary-reflection* and *rotary-inversion* as alternate ways to describe S_2 and S_6; it described S_4, however, only as rotary-reflection. Many agree with an esteemed scholar that rotary-inversion is "one of the worst concepts ever perpetrated on the scientific public."

Aside from the embodiment of pure isometric, two-fold mirror-rotation in certain optical instruments the only other familiar, utilitarian, man-made object with solitary mirror-rotation symmetry is the double-ended dental instrument. (A sphenoidal wedge with four-fold mirror-rotation may be used to stop a door, but it is not *pure*; it is encumbered with two additional mirror planes). The doubled-ended curette, having perfect balance, is easily twirled in the fingers, allowing the dentist, without changing instruments, to pick at both sides of the same tooth with right and left hooks.

Though instances of mirror-rotation are rare in natural and utilitarian objects, abstract objects with these properties can be made at will. Some sculptors have, perhaps knowingly, perhaps accidentally, produced pieces with this unordinary structure. There has been such a one, for example, before the Bahnhof in Zweibrücken.

Innumerable mirror-rotation objects have been made in my instructional design studio at various schools; small though they are, many evoke monumentality. Our first studies involved only two-fold mirror-rotation. Recently, four- and six-fold examples have been developed. Few, if any entities, larger than molecules, have solitary, four-fold mirror-rotation other than, perhaps, scaled-up models of molecules and idealized crystals. It is possible, then, that never in the whole of history have there been so many singular examples of four-fold mirror-rotation in one place as were in my classroom at the University of Hong Kong in 1986.

The symmetry of preference in architecture is, by no coincidence, bilateral symmetry -- its imprint lodged in our own bodies. Formal buildings are often given a second, transverse plane of symmetry. The facade of Palladio's Villa Rotunda proclaims the symmetry of the square, the combination of four mirror planes and a four-fold rotor. There are buildings in the shapes of hexagons, octagons, and circle, but the orders of symmetry of their essential bodies are often offset by openings that do not match to the same degrees.

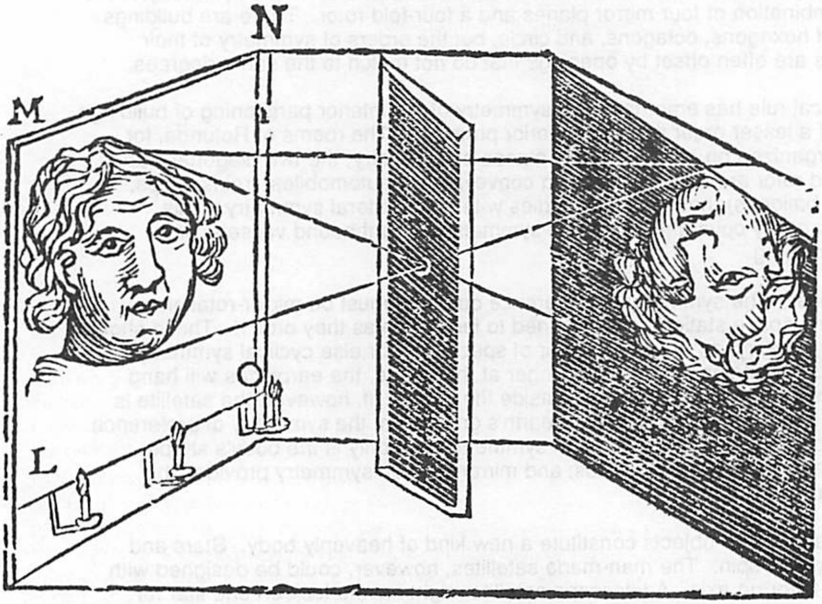
A sort of empirical rule has emerged: the symmetry of the interior partitioning of buildings is predictably of a lesser order than the exterior promises. The rooms of Rotunda, for example, are organized on only two cross planes of symmetry; the two diagonal planes and the four-fold rotor are missing. Moving conveyances, automobiles, trains, ships, aircraft (except balloons), have exterior bodies with only bilateral symmetry. This summary substantially covers the extent of symmetry for earthbound vessels, both stationary and moving.

But what of space? The symmetry of preference out there must be mirror-rotation symmetry. Some space stations are designed to face earth as they orbit it. These should have bilateral symmetry, as abides in a pair of spectacles (or else cyclical symmetry). When spectacles are balanced on one's finger at the bridge, the earpieces will hang downward, since the center of gravity is outside the bridge. If, however, the satellite is meant to rotate freely without the effect of earth's gravitation, the symmetry of preference would, without question, be mirror-rotation symmetry. Not only is the body's shape critical, but so is the distribution of mass; and mirror-rotation symmetry provides the blueprint for that distribution.

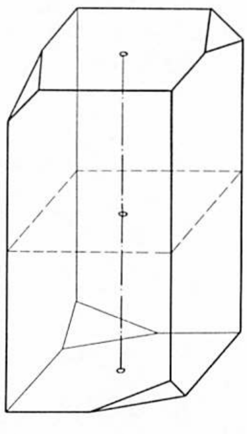
Certain man-made space objects constitute a new kind of heavenly body. Stars and planets have axes of spin. The man-made satellites, however, could be designed with good reason to favor no axis. A telescope satellite might have a lock on one star for a time and, then, by command, swing to a lock on another. This would be accomplished with ease, if the satellite has a center of symmetry. The sphere, cylinder, and icosahedron all have large arrays of mirror planes and multi-fold rotors. They also have centers of symmetry and, therefore, must be classified among mirror-rotation objects. Because of their high orders of symmetry, however, a consistent distribution of massing, in order to preserve equilibrium, would be highly demanding.

Less regular bodies, retaining a center of symmetry, would reduce the redundant matching of mass for mass. Two-fold and four-fold mirror-rotation are prime candidates for the new sedentary vessels of space. Such satellites could have spherical exteriors; and, taking a cue from architecture's mode of reduced interior symmetry, their interior arrangements could be stripped-down mirror-rotation symmetry. [An alternative body with centered equilibrium is one with three, mutually perpendicular, two-fold axes (*V*).]

Mirror-rotation, observable, yet not identified for over six millennia of man's past, has a future in the stars.



1.
camera obscura: Athanasius Kircher; *Ars Magna Lucis et Umbrae*; 1671



2.
prism, modified to exhibit characteristics of four-fold mirror-rotation, after Curie: Pierre Curie, *Œuvres*, 1908