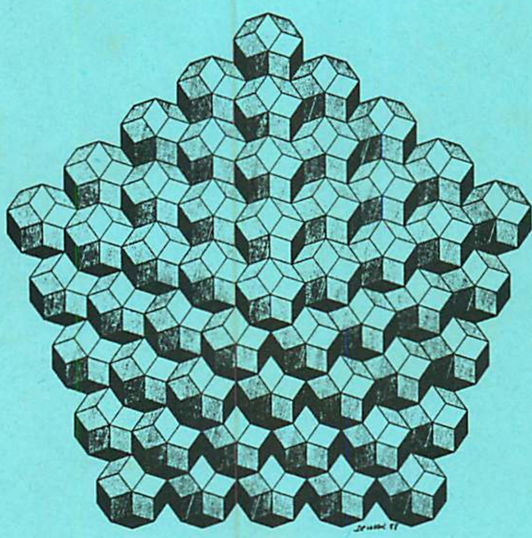
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



Edited by Gy. Darvas and D. Nagy

Buda pest

August 13-19, 1989

hungary

ON THE ASYMMETRY OF THE BIRCH LEAF ROLLER'S INCISIONS

GÁBOR HORVÁTH

Hungarian Academy of Sciences, Central Research Institute for Physics, Biophysical Group, P.O.B. 49, H-1525 Budapest, Hungary

Previously it has been stated that the leaf cone construction of the birch leaf roller (Deporaus betulae) is determined by the principle of optimal cost, which would result symmetrical incisions cut by the beetle. However the real incisions are asymmetrical. This contradiction and the reason of the asymmetry are explained.

Birch leaf roller rolls a regular, well-closed leaf cone for its offsprings (Fig. 1); it cuts two special S-shaped incisions into the leaf blade to make the leaf twist easier (Fig. 2). It can be seen in Fig. 2 that the birch leaf roller's incisions are asymmetrical.

A widely quoted general view is that the optimal shape of the incisions is determined by the principle of optimal cost: the work needed to roll the leaf halves is as minimal as possible at a given leaf mass rolled into the leaf cone [1,2]. If this variational principle was valid for the twists of both leaf halves, it would result symmetrical incisions. Therefore the question is raised: 'Why are asymmetrical the Deporaus betulae's incisions ?'.

The applicability of the above mentioned principle is refuted in [3], and a new bionical explanation and biomathematical description was presented in [3,4]. Let us sketch out this explanation.

After the cutting of the incisions the beetle begins to roll leaf half 1 into a leaf cone and incision 1 helps in this cone construction. Deporaus betulae first makes a small cone from the leaf lamina on the leaf border and then rolls the whole of leaf half 1 around this core; a regular, slender cone is formed. A suitable microclimate can be insured for the grubs if the peak of the cone is well closed: there may not be any gap on this peak. Figure 3(b) shows the situation of the leaf cone near its last stages, and Figure 3(a) shows the situation of leaf half 1 when it is uncoiled. The external layer of the leaf cone allows the internal core to rise out easily and to close the peak of the leaf cone only if leaf half 1 forms a slanting cone section in the uncoiled stage of Figure 3(a). Therefore Deporaus betulae cuts incision 1 so that after the twist of leaf half 1 the external leaf layer constitutes a slanting cone section. This can be realised if the uncoiled leaf half 1 forms a slanting cone section.

On the basis of these the theoretical incision 1 is the curve of a slanting cone section laid out in the plane. The expression of this curve can be determined [3,4]:

$$\rho(\delta) = y(x_0)\left[1+1/y'^2(x_0)\right]^{1/2} \times \frac{1 + \dfrac{\tan\beta \arctan y'(x_0) \cos\gamma}{\left\{4\pi^2 - \left[\arctan y'(x_0)\right]^2\right\}^{1/2}}}{1 + \dfrac{\tan\beta \arctan y'(x_0) \cos\left[\gamma + 2\pi\delta/\arctan y'(x_0)\right]}{\left\{4\pi^2 - \left[\arctan y'(x_0)\right]^2\right\}^{1/2}}} \qquad (1)$$

where the definition of the parameters $y(x_0)$, $y'(x_0)$, β and γ can be found in [3,4].

After the rolling of leaf half 1 into a leaf cone the beetle twists leaf half 2 around this cone. Incision 2 helps this rolling and prevents the uncoiling of the twisted leaf. The flexibility of the leaf lamina plays a primary role in the physics of the leaf twist, therefore consider the torque needed to roll a leaf blade around a cone with half aperture angle α. The thickness of the leaf lamina is a; the width of the rolled leaf blade along the generatrix of the cone is b. The nearer edge of the rolled leaf lamina is at distance x along the generatrix from the peak P of the cone. If E is the Young's modulus of the leaf blade, the torque needed to roll the leaf cone is

$$M = \frac{Ea^3}{12 \mathrm{tg}\, \alpha} \mathrm{Ln}(1+b/x) \qquad (2)$$

Cutting the midrib of the leaf causes the leaf tissue to wilt, that is, its cells lose their normal turgor. This is a crucial in the strategy of the birch leaf roller, because the mechanical properties of the flaccid, wilted leaf lamina are more advantageous for the leaf twist than those of the normal, turgid lamina. The Young's modulus E of a wilted leaf blade is much smaller than that of a turgid blade, so on the basis of (2) the torque M needed to flex a flaccid lamina is much smaller than that of a turgid one.

Furthermore it can be observed that the wilted leaf blade cutted by Deporaus betulae is a little twisted in ourself, and this makes also easier the leaf twist. It is important for the suitable microclimate of the grubs that the leaf tissue of the leaf cigar does not dry totally after the leaf cone is constructed, but the tissue does not regain its normal turgor after the cone construction. The upper part of the rolled leaf remains sound and turgid; and a smaller flow of the tissue fluid is possible through the gnawed midrib.

We can see from (2) that Deporaus betulae must cut incision 2 in such a way that during the roll the distance $\overline{PB_2}$ is not too small, because the torque needed to roll leaf half 2 would then be very great, or too large, because then little leaf mass would roll into the leaf cone. The beetle must choose a small distance $\overline{PB_2}$, then it cuts incision 2 so that the edge of the leaf moves away quickly from the point P during the twist, so x increases rapidly, M decreases rapidly, and the part of leaf half 2 near the point P can be rolled.

When the beetle is ready with the twist of the leaf halves, it fastens the leaf layers of the cone together with its proboscis; thus the leaf cone cannot uncoil. The last leaf layer must be tongue-shaped so that it can be fastened easily by the beetle, i.e., the torque M must be small. We see from (2) that M is small if b/x is small. Consequently the last, tongue-shaped layer must be narrow and its edges must be distant from the point P. Therefore Deporaus betulae cuts incision 2 in the leaf lamina so that the last leaf layer is a relatively narrow, long tongue far from the peak point P of the leaf cone.

Since the leaf cone nourishes the grubs, it is very important to have enough leaf mass in it; the beetle must roll as much leaf mass as possible into the leaf cigar.

Consider the angles between the borders of the lower part of leaf half 2 rolled and the generatrix of the cone. If these angles differ very much from each other during the twist, then the last, leaf layer will be suddenly very wide or narrow; either one would contravene the reqiurement of the narrow, long, tongue-shaped last leaf layer far from the peak of the leaf cone. Therefore incision 2 must be cut in such a way that the angles between the edges of the lower part of leaf half 2 rolled and the generatrix are equal.

Refering to Figure 2, assign polar coordinates with origin at P, and angle measured from the midrib AQ of the leaf, using the function $R(\varphi)$ of the leaf border the theoretical curve of incision 2 according to the above twist method is the following [3,4]:

$$r(\varphi) = \frac{\overline{PQ}\cdot\overline{PB_2}}{R(\varphi)} \qquad (3)$$

Expressions (1) and (3) describe well the shape of the Deporaus betulae's incisions [3,4]. The shape of the birch leaf roller's incisions cannot be explained by an only variational principle, this is the reason of their asymmetry.

REFERENCES

1 Daanje,A.: Verh.Konink.Ned.Akad.Wetensch., Afd.Nat., Tweede reeks, Deel 56, 1-215. (1964)
2 Rozen,J.G.: Natural History 71/4, 58-61. (1962)
3 Horváth,G.: Mathematical Biosciences 91, 131-150. (1988)
4 Horváth,G.: Bull. Math. Biology (in the press)

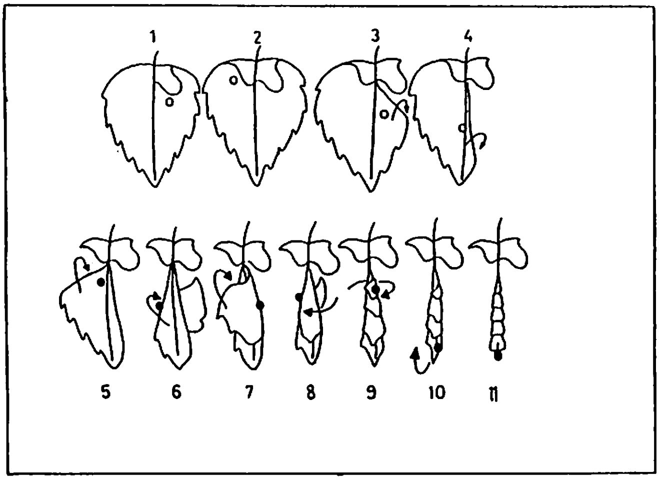


Fig. 1

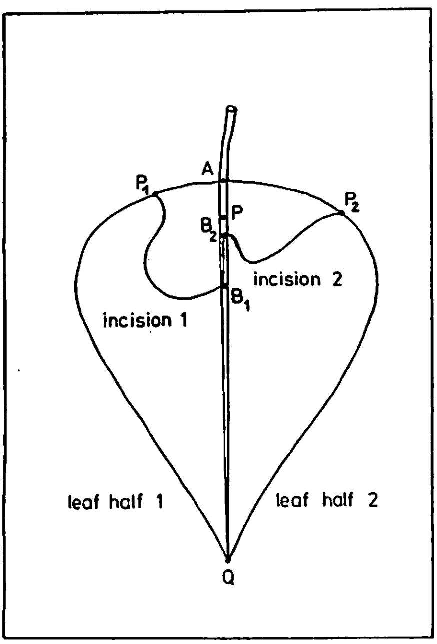


Fig. 2

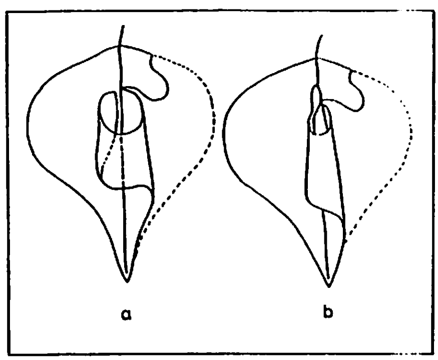


Fig. 3