Symmetry

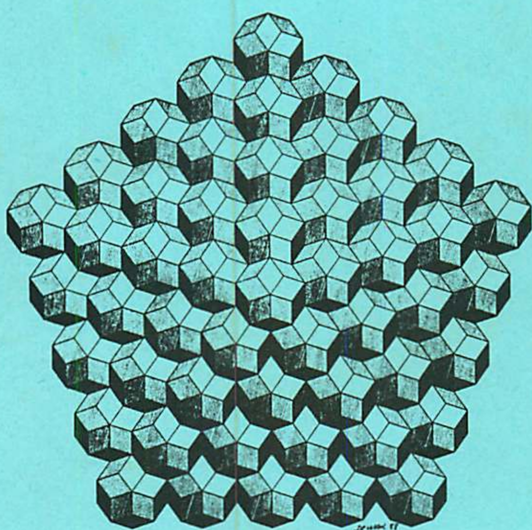
of

STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



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Mathematics and Textiles

This lecture proposal is written in three parts, an introduction followed by comments based on the three questions asked for the essay answers for "Symmetry in a Kaleidoscope."

Introduction

My field is mathematics education. By definition this draws on a blend of disciplines that involve at a minimum mathematical, psychological, social and philosophical aspects of how people, particularly children, learn mathematics. For the past ten years I have been working on a project that tries to encourage closer links between the sort of mathematics that goes on inside school and that which goes on outside it. My work is more concerned with realism for students than vocationalism for employers.

In the past I have been involved in research into the mathematics done in the workplace by "unskilled" school leavers and have come to reject the narrow definition of mathematics used in such research. As a way of counter-acting such narrowness I have researched and published school learning materials that exploit mathematically some of the practical problem-solving work that happens on factory floors but tends not to be recognised as such by employers.

Mathematics itself is very much a man's subject. Most histories of mathematics never mention a woman mathematician. Most discussions of the history of the development of mathematics within the social context of the time stress male-oriented activities - engineering, astronomy, architecture or ballistics. Until very recently school mathematics textbooks were written with boys in mind, often explicitly. Girls doing mathematics have always been treated as oddities or worse. As a corollory to this masculine interest, traditional girls' and women's activities tend to be judged as trivial and are normally perceived as non-mathematical and non-scientific.

When it comes to textiles, this attitude becomes more obviously irrational. Even clear geometric designs woven by a woman into a tribal rug (Fig 1) tend to be dismissed as being non-mathematical activity. My challenge to this attitude rarely produces rationally defensible responses, normally only a further defensive, narrowing of definition of mathematics.

Thus I work the union of several sets, some of which do not wish to join the union at all.

1. The methodological and heuristic role played by symmetry (and asymmetry) in your field of study and cultural circle.

In English primary school mathematical symmetry is studied as heuristic. The emphasis is on learning about shapes and their orientation in space and that the shape of a toy or other familair object is not changed though its position in space may be.

Children are encouraged to explore shapes with mirrors and repeat shapes to make patterns. Almost the whole mode of mathematics education in primary schools is practical and exploratory. (Fig 2)

In secondary schools the study of symmetry tends to be more abstract and formal. Children are introduced to the different symbol systems and formal methods of mathematics and practical work on symmetry tends to be done with symbols, diagrams and shapes that are already mathematical, for example in secondary school it is likely that a pupil's exploration of reflection and rotation will do so with a triangle or some other abstract asymmetric shape. (Fig 3) Symmetry in secondary schools tends to be taught as an end in itelf, a part of mathematics leading to more mathematics. A pupil going on to do mathematics at university would become aware of the great unifying power of symmetry in mathematics. A pupil leaving school at 16, if s/he remembers anything about formal symmetry at all, will be unlikely to see its relevance to the whole of life.

In textile work the study or use of symmetry can fill an heuristic or methodological role. For example it is not possible to design or make a garment without some notice being taken of the fact that the human body is symmetrical, that a properly fitting right sleeve or trouser leg is the mirror image of the left one. When it comes to the decoration of cloth or or garments, there is no limit to the explorations and imagination of weavers, knitters and dyers. (Fig 4 and Zazlavsky, Gerdes, Harris etc)

2. The interdisciplinary impact of symmetry - used in your field - on other scientific and/or cultural spheres.

In mathematics and mathematics education there is a tendency to study symmetry in isolation. Weyl, Coxeter and Budden are unusual. There is also a snobbery within mathematics itself that endows pure mathematics with a moral superiority over its practical application. The study of pure symmetry represents the humanistic side of mathematics, the use of symmetry in controlling the flow of water round the piers of a bridge represents it practical, slightly regrettable side. This snobbery filters down into school from the universities and adversely affects the school curriculum.

Within schools there are now initiatives to encourage the teaching of mathematics across the curriculum but there are not enough of them and they are chronically under-resourced. A possible advance could be made in the field of home economics. There is now an increasing amount of mathematics software for studying pattern generation and so on. One such program, called "Freize" being developed at Homerton College Cambridge encourages children to construct and analyse such patterns. The children can print out their designs either on paper or on a knitting machine. Thus there is some advance in looking at a topic, previously the preserve of home economics through mathematical eyes. The mathematicians however seem unwilling as yet to consult the home economists on what the constraints of their discipline are in this case. In other words, they enjoy printing out their designs on cloth but are not interested in adapting them to the use to which the cloth will be put. They do not respoind to the criteria of another discipline. The potential for cross disciplinary work remains nonetheless and it is only just beginning to be exploited to the benefit of both disciplines

3. The special meaning of symmetry influenced by your cultural background (external artifacts.)

England is now a very multicultural society but only recently has she begun to exploit her immigrant cultures for what they can offer instead of regarding them simply as defective in English culture. Most people of all cultures wear clothes. Most, though not by any means all of these clothes and the cloth from which they are made, are made by women. Many women are mothers - and it is safe to say that all mothers are women. The making of cloth and of clothes is thus a very familiar part of the background of all the children in our schools.

Teaching materials and an exhibition of textiles and mathematics produced by the Maths in Work Project both exploit the richness of traditional women's activities as a mathematics resource. A research question that it has not yet been possible to pursue is the extent to which women, exhausted after a hard day in the house or the fields turn their minds to the more intellectual pursuits or exploring symmetry when they do their household needlework. Traditional thinking views these activities as trivial and purely decorative. My work challenges this thinking and brings the designs of a Fair Isle sweater, of a Botswana basket, of printed Adinkra patterns from Ghana, of woven patterns from Bangladesh and of a Turkish rug into school as part of a pack of teaching materials for mathematics. Children are learning about more formal mathematical symmetry from their own garments and from textiles from their own cultures and from each others. Issues of gender and culture as problems in mathematics education have been reversed. The issue is now the narrow mindedness of school mathematics and how to reverse that.

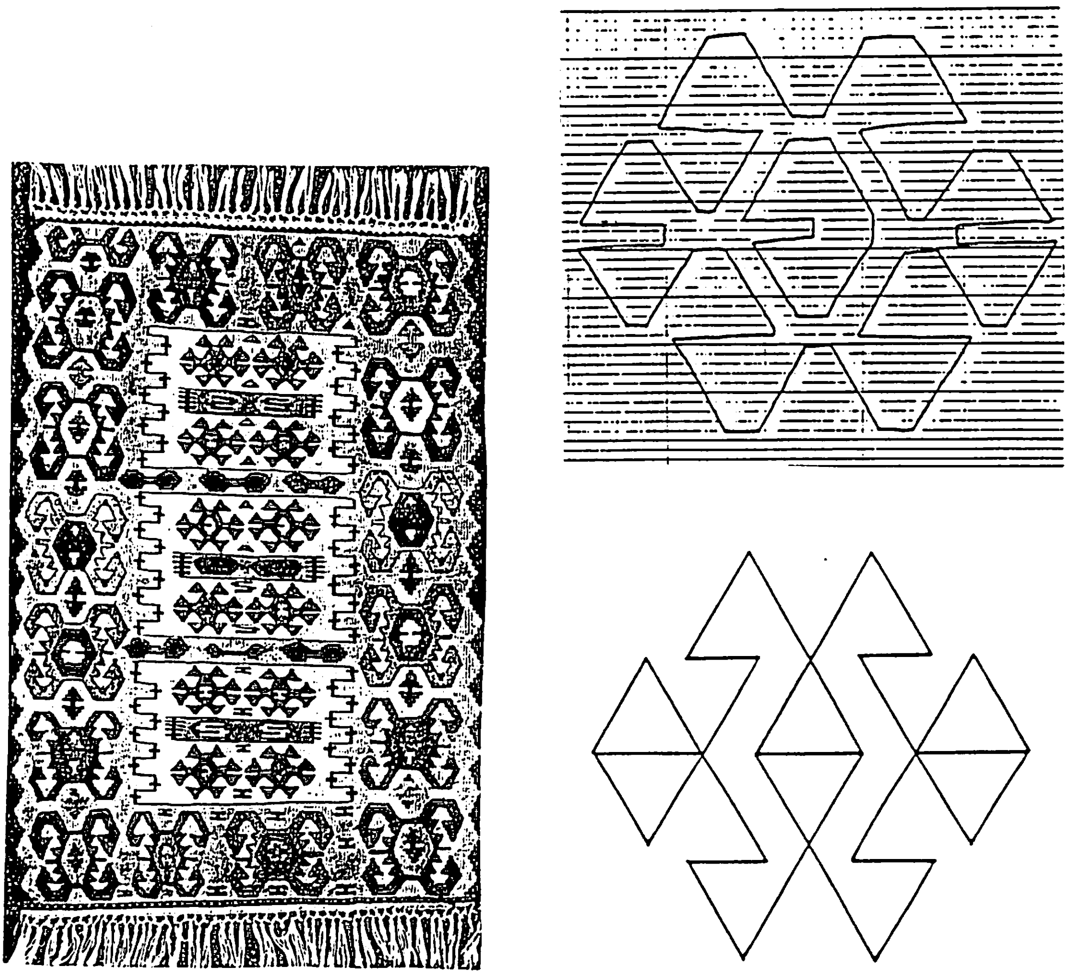

Fig.1

iii) Using a folded piece of squared paper, half of a given shape is copied. The squares can be counted to give accuracy. The other half is obtained by pricking through. This method brings out clearly that in a symmetrical shape each point on one side is matched by a point on the other side which is the same distance from the fold (Figure 5:31).

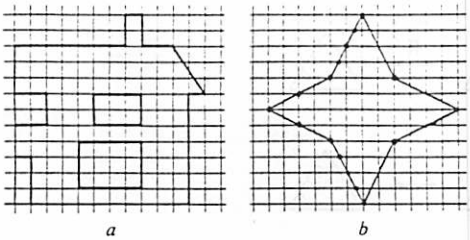

Figure 5:31

Up to the age of about seven children do not distinguish left from right with any certainty. This is understandable because it depends on the point of view of the observer which of two points is seen as left and which as right (Figure 5:32).

Figure 5:32

Making symmetrical shapes helps children to know left from right. Shoes and gloves and double swing doors have this symmetrical relationship and one of the pair must be distinguished as left. Turning a pair of shoes upside down makes it difficult for a child to identify which is his left one.

Folding a sheet into quarters introduces symmetry about two axes, and children can make interesting patterns by tearing or cutting round the edge and cutting out holes.

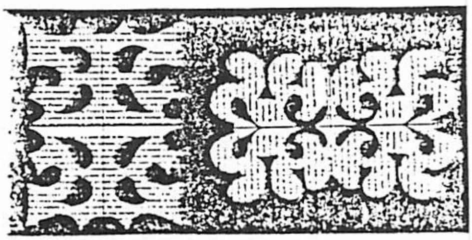

Figure 5:33

It can be seen that Figure 5:31(*b*) also has symmetry about two other axes, and this pattern could have been made by folding the paper into eighths. As children's dexterity grows they will invent ways of making very attractive paper mats using four-fold symmetry.

It is now possible to look again at squares, rectangles, circles, etc. and see which of them can be folded to make two halves which fit (*see page 50*). Any shape can be folded but the two parts are not, of course, usually the same shape. But even the most irregular flat shape, a leaf or a jagged piece of paper, can be folded flat so that the fold is a straight line. Stiff paper folded in this way makes a good edge for ruling a straight line. A further fold, keeping the parts of the first fold together, will make a square corner or right angle. Notice the irregularity of the paper used in Figure 5:34; this makes the right angle stand out clearly. A circle folded in this way is a useful alternative to the jagged paper. The intersection of the folds gives the centre; the folds divide the circle into quarters.

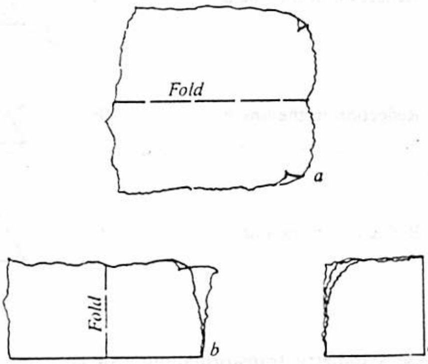

Figure 5:34

If the paper is unfolded the creases show the fitting together of four right angles (Figure 5:35). A right angle made in this way will serve as a home-made set square, and a child can use it to check right angles which occur in the classroom, and in particular to discover the way in

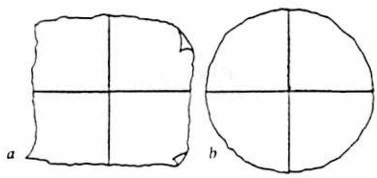

Figure 5:35

(*b*) Transformations which map a shape onto itself are called symmetry transformations. The effects of the six symmetry transformations of an equilateral triangle are shown in Figure 4.

The identity transformation	I
Rotation through 120°	R
Rotation through 240°	S
Reflection in the line *a*	A
Reflection in the line *b*	B
Reflection in the line *c*	C

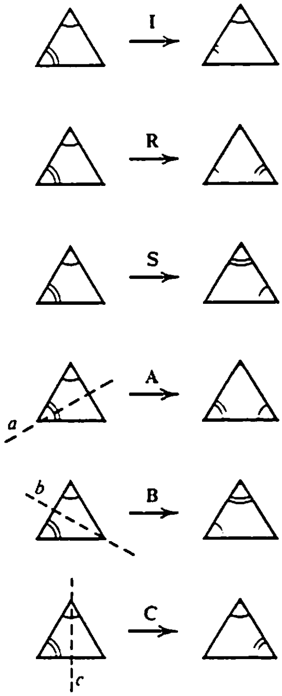

Fig. 4

These symmetry transformations can be combined in the usual way. For example, Figure 5 shows that SC has the same effect as A and we can write

$$SC = A.$$

Remember that SC is the combined transformation 'first C and then S'.

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Fig. 3

Figures and References.

Figures

Fig 1. A Turkish kelim in the possession of the author. One motif has been taken out for analysis and comparison with a more formal mathematical exercise of building a symmetrical design based on isometric transformations of an isosceles triangle.

Fig 2. A page from "Primary Mathematics Today" Williams E and Shuard H. Longman 1982 (third edition.)

Fig 3. A page from the "Schools Mathematics Project" Book Z, Cambridge University Press 1976.

Fig 4 Photographs of Aran and Fair Isle sweaters from the "Common Threads" exhibition. (see also the Common Threads Catalogue)

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