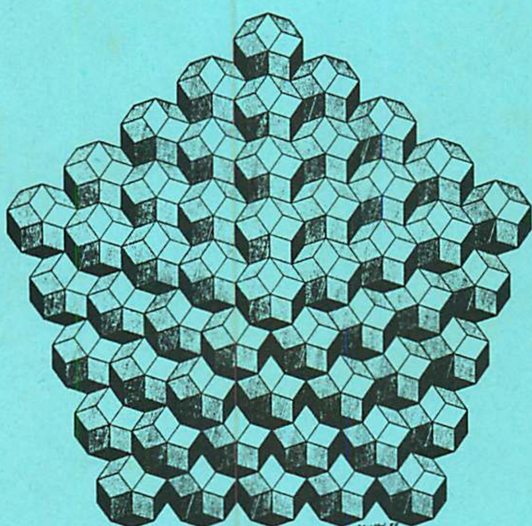
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



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SYMMETRY IN THE ANALYSIS OF STRUCTURES

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The existing symmetries influence the numerical analysis of structures remarkably. They reduce the mass of mathematical operations which should be done. Four different levels of this "facility" can be found in the analysis of structures.

1. Discrete Symmetries in the Body

The discrete symmetries of analysed body are the mirror symmetry, the cyclical symmetry and the discrete shifting symmetry (periodical skeleton construction).

In the case of mirror symmetry the order of algebraic problem which should be solved is divided into two (roughly) equal parts decomposing the unknown quantities into symmetrical and asymmetrical parts. In the case of existing of double or triple mirror symmetry the order of algebraic problem becomes a quarter the value or eighth one. Of course two, four or eighth (independent) algebraic problems must be solved accordingly to the number of symmetry.

In the case of cyclical symmetry or periodical skeleton construction the structure of matrix of the algebraic equations simplifies. In the case of skeleton construction this matrix is a symmetrical, uniformly continuant one (its elements themselvs are matrices too), while at the case of cyclical symmetry the system "closes", i.e. an element, standing in the neighbour to the principal diagonal line, gets into the left lower and rigth upper corner.

If the cyclical symmetry and/or periodical skeleton structure exists in two or three directions too, then the blocks of the matrix also have the structure described above.

The existing of discrete symmetry can give a help in the last step of the numerical analysis of structures, i.e. in the solving of the algebraic equations.

2. Continuous Symmetry in the Body and in the Mechanical State

In the Eucledian space there are two (independent) motions: the shift and the rotation. The first is the continuous case of the skeleton structure, while the second is the continuous case of the cyclical symmetry. The combination of them is the translation along a helical curve (heliodromic). Accordingly to these three operations three symmetry names are applied: shifting, rotational and helical symmetry. These three symmetries generate three body: prismatic one, body of rotation and the helio-symmetrical body.

The mathematical state of the body having continuous symmetry also can be fulfilled a continuous symmetry. Then the mechanical state - plane, cylinder-symmetrical problems and helio-symmetrical state - reduces to the twoo-variable problem.

Of course the double and triple continuous symmetry can be interpreted both in the case of bodies and mechanical states, the last ones are one-variable, or zero-variable (the real coefficiens are the unknown qualities) respectively.

Finally it must be mentioned, that the cylinder-symmetrical and plane problems can be divided into symmetrical and asymmetrical (which is the torsional one) problems.

The continuous symmetry reduces the number of variables from three to two (separetely to one or zero), that implies the reduction of the order of algebraic problem (which should be set), too.

3. The Symmetry Group of a System of Equations

The solution of the partial differential equation of the setting problem (independently of the exterior geometrical symmetry and symmetry in boundary conditions) has the symmetry property. Namely (usually) there is a symmetry group of the solutions of the equation.

The symmetry group plays an important role in the solution of partial differential equations. On the one hand the symmetry group indicates those coordinate systems in which the method of separation of variables can be used to solve the given differential equations, on the other hand the symmetry group makes possible to determine those special functions, which determine the function space of the solution for the separated one dimensional space. Usually these functions turn to be the characteristic (eigen-)functions using which the (initial and/or) boundary value problem can be solved "easely".

4. Symmetry in the System of Equations

The analysis of structure - solid, deformable bodies - is based upon the consequences of the continuum mechanics. In this system of equations the symmetry is observable at several points.

Firstly the symmetry of constitutive equations is mentioned. It is in need, because both the strain and the stress tensor are symmetrical. But the cause of this symmetry is deeper, it has a thermodynamic explanation.

Secondly let us see the symmetry of strain and stress tensors. Both are consequences of the fact that continuum mechanics is discussed in a Euclidean space, the metric tensor of which is symmetric. And both mechanical tensors are in close connaction with the metric tensor.

Finally the symmetry in a system of equations must be touched upon. This is also the consequence of the fact that the mathematical theory of the physical phenomenon is constructed in a Euclidean space, identifying the physical phenomenon with it.

If the (geometric) space in the mathematical theory is generalized or the continuous description is already irrelevant, then the above mentioned symmetry property "is lost" partially: asymmetrical tensor functions and differential expressions appear in the system beside the symmetrical ones.

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