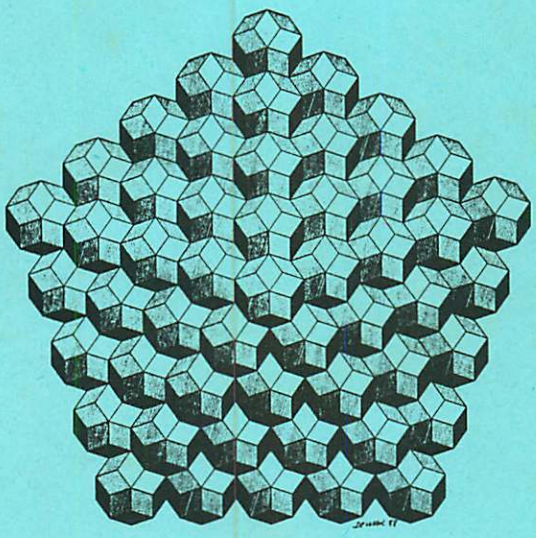
# Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



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# NON-LOCAL SYMMETRIES OF DIFFERENTIAL EQUATIONS

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The property of invariance of differential equations is a criterion, by which one can classify the mathematical models of the physical phenomena we consider. The knowledge of the admitted group of transformations may be used to construct the exact solutions and conservation laws, as well as to clear out the questions of the equivalence between the systems of differential equations, etc.

The theories of Lie and Lie-Backlund transformation groups give us a constructive methods to calculated the local transformations, i.e. the transformations which depend on independent and dependent variables and their derivatives (hereafter called local variables). The local groups are generated by the infinitesimal operators with the coordinates, which are the analytical functions of an arbitrary finite number of the local variables. At the same time in practice there exest the equations, admitting the operators with the coordinates dependent from local as well as non-local variables, or the integrals (i.e. non-local symmetries). As a rule one calculates them by means of special methods: applying the recurrence operators, applying transformations, reducing a given equation to some others. Generally it is impossible to suggest the same constructive method for calculating the non-local symmetries. It is caused by the fact, that the space of analytical functions of a finite number of local and non-local variables is unlocked with respect to the integration operation.

The constructive method to calculate the non-local symmetries is developped for the equations, admitting Backlund transformations (like differential substitutions). The application of Lie-Backlund groups theory to such equations allows us to calculate for them the non-local symmetries of a special type, which we call quasi-local. Corresponding non-local variables appear to be associated with the conservation laws (which are generally non-local). By means of suggested method we had calculated the new quasi-local symmetries and conservation laws for the non-linear heat diffusion

and gas dynamics equations.

When classifying the equations of non-linear heat diffusion type by quasi-local symmetries, we isolated a new broad class of equations with expanded set of symmetries. Thermal conductivity coefficients of these equations remain finite at an infinite growth of temperature. With the help of quasi-local symmetries the new invariant group solutions of heat diffusion equations were found.

The peculiarities of applying the algorythm for calculating the quasi-local symmetries for the systems of equations is illustrated by the equations of one-dimensional adiabatic gas dynamics.In I958 these equations were classified in Euler coordinates accoding to the group of point transformations they admit. Lately it was show, that in Lagrange coordinates these equations admit a more extended group of point transformations. To create common classification by the method of calculating quasi-local symmetries, one introduces an intermediate system, which is associated with the equations of gas dynamics in Euler and Lagrange coordinates via Backlund transformations of differential substitution type. Such common classification had led to thirteen main types of gas dynamics equations with extended set of symmetries. Within these types we can isolate for example such gas dynamics equations, which are invariant with respect to changover to the uniformly accelerated coordinate system. The approach we suggest allows to make visible the hidden symmetry of Chapligin's gas equations.