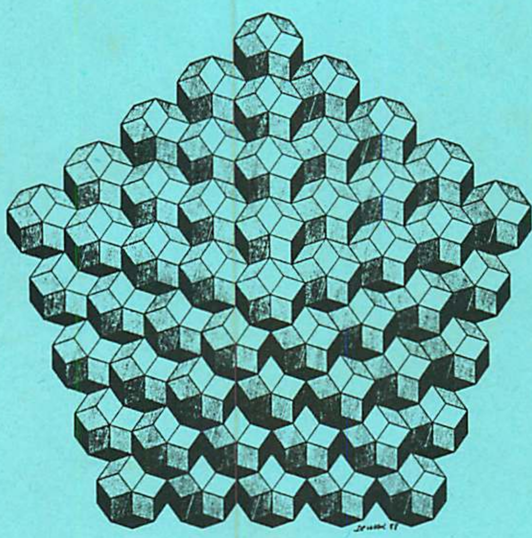
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



Edited by Gy. Darvas and D. Nagy

Buda
pest

August 13-19, 1989

hungary

STUDY OF DYNAMICS OF SYMMETRIC SYSTEMS

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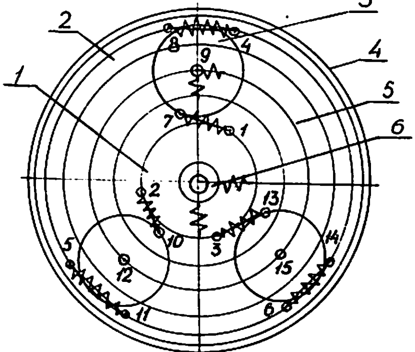
Most of unique properties of dynamics of a system are cau... by its symmetry. These include multiplicity of natural frequenc... number of independent forms of vibrations, effect of disturbing forces symmetry, etc.

If,however, the role of symmetry and its effects are to be studied, it is helpful then to use the theory of symmetry groups. This approach allows to characterize the symmetry of an object under study in terms of exact mathematical language, thus enabling a qualitative analysis of the systems. Such a theoretical-group approach is efficient in numerical calculations of complex systems since it allows to overcome the "misfortune" of large measures through decomposition of vector space.

When studying a system by symmetry group methods the dynamic model of a structure under study is represented as a lattice . However, full symmetry of a mechanism is defined not only by relative positions of similar elements,but also by symmetric properties of each of them.2.7

Therefore let us make the dynamic model of Fig. 1 more clear by describing each element's configuration with a finite number of points. On the one hand, the number of points and their arrangement must fully define element configuration ; on the other,they must represent,partly by intuition, the symmetry of the entire system as much as possible.



1 - sun gear "S"
2 - epicycle "E"
3 - satellite "P"
4 - body
5 - pinion carrier
6 - bearing

Fig. 1

Now,mark by encircling, the assumed equivalent points, characterizing the symmetry of separate elements; then groups of points will characterize symmetry between them.

Thus, three groups of points (7,8,9),(10,11,12),(13,14,15) define symmetry between elements "Pi" - symmetry group C3, while points (7,8),(10,11),(13,14) define symmetry of each element "Pi" - symmetry group C2. On the others hand, each group of points (1,2,3) and (4,5,6), when taken separetely, characterize symmetry of elements "S" and "E" - symmetry group C3; then taken on the whole, they give symmetry group C2. Under the circumstances shown the symmetry of the mechanism under consideration can be represented as a sum of product of groups $C2\otimes C3\oplus C3\otimes C2$, where $\otimes$ - product of characters of group irreducible representations,and $\oplus$ - their direct sum. Finally, the symmetry of the entire mechanism - C3h+C3h.

After mechanism symmetry has been defined one can pass over to analyzing its properties in terms of quality. For this: put down characters of irreducible representations of point symmetry group C3h into Table 1, compiled for group algebra in the field of complex numbers, where $\varrho = e^{2\pi i/3}$; $\varrho^* = e^{4\pi i/3}$

Table 1

Ch	E	C	C	C	S	S
χ_1	1	1	1	1	1	1
χ_2	1	1	1	-1	-1	-1
χ_3	1	ξ	ξ^*	1	ξ	ξ^*
χ_4	1	ξ^*	ξ	1	ξ^*	ξ
χ_5	1	ξ	ξ^*	-1	$-\xi$	$-\xi^*$
χ_6	1	ξ^*	ξ	-1	$-\xi^*$	$-\xi$
$n(R)$	15	0	0	3	0	0
$\chi(R)$	15	0	0	3	0	0

For better convenience some data is below under the Table. This data refers to the number of immovable points n(R) (for various operations) and to characters of Cartesian reducible representations [3]. The data allows to define number m ,showing how many times each of irreducible representations is contained in a Cartesian representation: $m_j = \frac{1}{h}\sum_{R\in G} \chi_\tau(R)\, \chi_j(R)$, where h - number of elements in set groups G; R -operation on group; χ_τ, χ_j - characters of reducible and j-th irreducible representations respectively. For symmetry group C3h m is expressed as

$$m_j = \frac{1}{6}\left[15*1*\chi_j(E) + 3*1*\chi(C2)\right] \qquad (1)$$

Substituting (R) values from Table 1 into (1) one will obtain m1 = 2; m2 = 2; m3 = 2; m4 =3; m5 =3; m6 =3. Hence the original vector space L (with duly accounted symmetry of the mechanism's elements) splits into a sum of orthogonal subspaces of similar vectors L = 15 = 2*1+2*1+2*1+3*1+3*1+3*1.

Whereas in a molecule type model, analyzed in the work , the equivalence of similar points is self-evident, the equivalence of points in the mechanical model has yet to be substantiated. When the equivalence conditions are not satisfied, attempts should be then made to construct symmetry. Ensure that equivalence of points does exict, using operator A,which interrelates the original, Cartesian system of coordinates "q" and the point system "u". A q = u (2)

Thus for elements "S" and "E" expressoin (2) is:

$$\begin{pmatrix} \alpha_1 & \beta_1 & r \\ \alpha_2 & \beta_2 & r \\ \alpha_3 & \beta_3 & r \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ \varphi \end{pmatrix} = \begin{pmatrix} u1 \\ u2 \\ u3 \end{pmatrix} \qquad \begin{matrix} \alpha_i = \cos(2\pi(i-1)/3) \\ \beta_i = \sin(2\pi(i-3)/3) \\ i = \overline{1,3} \end{matrix}$$

r - radius of gear pitch circle. Put down the expression of matrix of stiffness ($\tilde{C}$) and mass ($\tilde{M}$) in new coordinate system "$\tilde{u}$" in terms of their original expressions [4] $\tilde{C} = (A^{-1})'\, C\, A^{-1}$; $\tilde{M} = (A^{-1})'\, M\, A^{-1}$, where M and C - matrices of masses and stiffness in the original coordinate system "q", u_i^k - vector of displacement of the i-th point of the k-th element.

The expressions obtained for $\tilde{C}$ and $\tilde{M}$ allow one to see whether the points are equivalent. Verification proves that points (7,8), (10,11) and (13,14) are not equivalent; hence, the system symmetry is defined by a direct sum of groups C3h and C3. This explains lack of parity in the measures of subspaces, as in the case of molecule NaCl cited in work [3]. Such transformations can be used to construct symmetry. Thus, for a group of points 1,2,3 and 4,5,6 it is necessary to satisfy the conditions:

$$B_s'\, \tilde{M}_s\, B_s = B_e'\, \tilde{M}_e\, B_e, \qquad B_s'\, \tilde{C}_s\, B_s = B_e'\, \tilde{C}_e\, B_e \qquad (3)$$

where $B = A^{-1}$. In (3), for example, parity can be assured by proper choice of inertia moment of element "E",given that gears "S" and "E" have similar masses (Ms = Me) and coefficients of elastic links. If masses and stiffnesses of elements "S" and "E" are different, the point equivalence condition can directly be inferred from equality of partial frequencies of sybsystem "S" and "E", i.e. conditioned that $\tilde{H}s = \tilde{H}e$, $H = M^{-1}\ C$.

Organization of high symmetry grade in a system suggests that disturbance symmetry grade is as high. Only in this case will each subsystem be "engaged" with "its" disturbing force . These problems can be studied employing a theoretical-group approach and taking into consideration the symmetry of both the disturbance and the system per se . Let us exemplify the analysis technique by the model under consideration. Begin with determining the projective operator T^{6} ,which links basis vectors of original system,also with vectors $\tilde{u} \in G$ of symmetric coordinate system u =T u (4),where $T = \frac{f_j}{N_{REG}} \sum \chi_j\ (R)$, χ(R) - character of the J-th irreducible representation $\mathcal{T}$(R), f_j - measure of the J-th representation. Substituting (3) into (4), one will obtain $g = T^{-1} . A^{-1}\ u$ or $g = Q^{-1}\ u$ (5). The latter can be formulated in the form of Table 2, interrelating Cartesian coordinate system with symmetric one, after sorting subspaces Di. Using operator Q for transforming the original vector of disturbing forces F(t), one will obtain its written form over subspaces Di: $\tilde{F}(t)$ = Q F(t). Hence, each Di subspace will be expressed as $Di*\tilde{u} = Fi(t)$ (6)

Given as an example are disturbing forces from unbalance of bodies "S", "E" and "P". In this the unbalancing force on each satellite can act either in phase with others, $\alpha_1 = \alpha_2 = \alpha_3$; out of phase at an angle $\alpha_i = 2\pi(i-1)/3$; (i=1,3) or arbitrarily.

Having completing transformations, aided with operator Q, their structure can be accounted of in subspaces Di, table 3. Forming equations (6) on the results of table 2 and 3, one can argue that F(t) from elements "S" and "E" do not disturb vibrations in subspace D1. However, these forces, alike $\tilde{F}ic(t)$, "engage" with the rest of subspaces, destroying the system symmetry.

Table 2

Di	N coord.	Ui =F(g)
D1	1	u1 = $\varphi_s + \varphi_e$
	7	u7 = Xp
D2	2	u2 = Rs + Re
	10	u10 = Xp
D3	3	u3 = Rs +Re
	13	u13 = Xp
D4	4	u4 = $\varphi_s - \varphi_e$
	8	u8 = φ_p^{+}
	9	u9 = Yp^{+}
D5	5	u5 = Rs - Re
	11	u11 = φ_p^{+}
	12	u12 = Yp
D6	6	u6 = Rs - Re
	14	u14 = φ_p^{-}
	15	u15 = Yp

R = αX + βY ;
(+) - symmetric form;
(-) - oblique-symmetric form:

Table 3

Di	"S"	$\alpha_i = \alpha_j$	$\alpha_i \neq \alpha_j$	$\alpha_i = \frac{2\pi(i-1)}{3}$	"E"
D1	0	0	0	0	0
	0	1	0	1	0
D2	1	0	0	0	1
	0	1	1	0	0
D3	1	0	0	0	1
	0	1	1	0	0
D4	0	0	0	0	0
	0	1	0	1	0
	0	1	0	1	0
D5	1	0	0	0	1
	0	1	1	0	0
	0	1	1	0	0
D6	1	0	0	0	1
	0	1	1	0	0
	0	1	1	0	0

"S" -sun gear; "E" - epicycle;
"P" - satellite;

The theoretical-group approach can be applied to formulate the inverted problem as well, when a new structure F(t), disturbing only definite subspace is given and when it is necessary to derive the structure of original vector of disturbance that is desirable to obtain on the basis of inverted transformation $Q^{-1}\ \tilde{F}i\ (t) = Fi(t)$. Consequently, one can not only analyze mechanism qualitatively, but also sythesize symmetry with due account made to disturbing forces and their phasal relations. The described analysis technique allows to dispense with dynamics equations of a studied mechanism as it is based only on the properties of its symmetry.

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