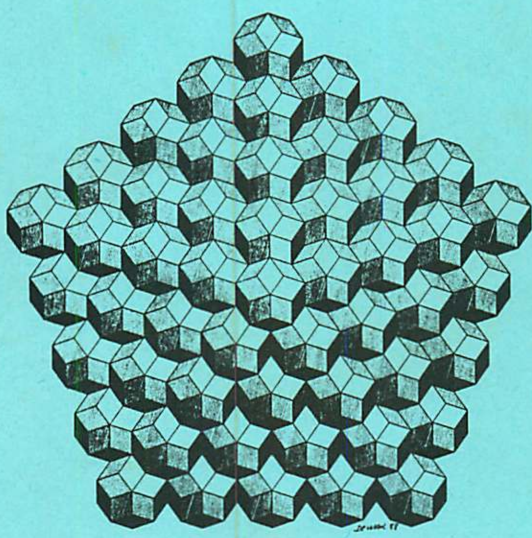
Symmetry of STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



Edited by Gy. Darvas and D. Nagy

Buda
pest

August 13-19, 1989

hungary

International Society for Interdisciplinary Studies of Symmetry

A B S T R A C T S

S Y M M E T R Y   O F   S T R U C T U R E

INTERDISCIPLINARY SYMMETRY SYMPOSIA, 1

Affiliated events: workshops, art exhibitions and performances

August 13-19, 1989, Budapest, Hungary
Timing of the Symposium is fitted to the 18th Congress of History of Science, August 1-9, Hamburg and Munich, and to the 12th European Crystallographic Meeting, August 20-29, Moscow

Organized by:

the Hungarian Academy of Sciences

Sponsored by:

International Union of Crystallography,
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Institute for General Technics, Eötvös L. University, Budapest
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Edited by
György Darvas and *Dénes Nagy*

Budapest
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COMMUNICATION WITH

--(1) The organizers of former conferences (with proceedings/abstract volume) associated with some aspects of our symposium

Fedorovskaya sessiya [Fedorov Session--regular sections on crystallographic groups], Leningrad, late May of each year (summaries in Zapiski Vsesoyuznogo mineralogicheskogo Obshchestva; sometimes formal proceedings, too); Simmetriya v prirode [Symmetry in Nature], Leningrad, May 25-27, 1971 (Leningrad: VSEGEI, 1971); Simmetriya struktur geologicheskikh tel [Symmetry of Structure of Geological Bodies], Leningrad, October 13-15, 1976 (Vols. 1-2, Moskva: VSEGEI, 1976)--I.I. Shafranovskii, V.A. Frank-Kamenetskii; V.I. Dragunov; L.M. Plotnikov

Symmetry and Function of Biological Systems at the Macromolecular Level [Nobel Symposium 11], Sdergarn [Sweden], August 26-29, 1968 (New York, N.Y.: Wiley/Stockhom: Amqvist & Wiksell, 1969)--A. Engstrm, B. Strandberg

La simmetria, Venice, April 1972 (Bologna: Il Mulino, 1973)--E. Agazzi

Patterns of Symmetry, Northampton, Mass., February 1973 (Amherst, Mass.: University of Massachusetts Press, 1977; Russian transl. Moskva: Mir, 1980)--M. Senechal, G. Fleck

Symmetrie, Vienna, April 22, 1978 (Wien: Springer, 1980)--A. Preisinger

Symmetries in Science [Einstein Centennial Symposium], Carbondale, Illin., February 23-March 2, 1979; No. 2, Carbondale, Illin., March 24-26, 1986; No. 3, Lochau [Austria], July 25-28, 1988 (New York, N.Y.: Plenum Press, 1980, 1986, 1988)--B. Gruber et al.

Kristallographische Gruppen, Bielefeld, September 2-16, 1979 (special issues of Match, Nos. 9, 10, October 1980, January 1981)--A. Dress, J. Neubser

Fearful Symmetry: Doubles and Doubling in Literature and Film, Tallahassee, Florida, January 24-26, 1980 (Tallahassee, Florida: University Presses of Florida, 1981)--E.J. Crook

Teoriya simmetrii i ee obobshcheniya [Theory of Symmetry and its Generalization], Kishinev [Moldavia, U.S.S.R.], 1980 (Kishinev: Kishinevskii Gosudarstvennyi Universitet, 1980)--A.M. Zamorzaev at al.

Symmetries in Physics 1600-1980, Sant Feliu [Spain], September 20-26, 1983 (Barcelona, 1984)--M.G. Doncel, A. Hermann, A. Lais

M.C. Escher: Art and Science [section on Escher and Symmetry], Rome, March 26-28, 1985 (Amsterdam: North-Holland, 1986)--M. Emmer

Symmetrie, Darmstadt, June 13-17, 1986 (Berlin: Springer, 1987; exhibition catalogues, Vols. 1-3, Darmstadt: Mathildenhhe, 1986)--R. Wille

Crystal Symmetries [Shubnikov Centennial Symposium], Moscow, April 7-9, 1987 (special issue of Computers & Mathematics with Applications, in print)--B.K. Vainshtein

Symmetry in a Cultural Context, Tempe, Ariz., May 1-2, 1987; No. 2, Tempe, Ariz., March 18-20, 1988 (Tempe, Ariz.: Arizona State University, 1987, 1988)--D. Nagy

--(2) The editors of comprehensive volumes

Module, Proportion, Symmetry, Rhythm (New York, N.Y.: Braziller, 1966)--G. Kepes

Printsip simmetrii [Principle of Symmetry] (Moskva: Nauka, 1978)--N.F. Ovchinnikov; V.S. Stepin (Inst. of Philosophy), V.P. Vizgin (Inst. of Hist. of Sci. and Tech.)

Sovremennaya kristallogrfiya, Tom 1, Simmetriya kristallov [Modern Crystallography, Vol. 1, Symmetry of Crystals], (Moskva: Nauka, 1979; English transl. Berlin: Springer, 1981); Metodologicheskie problemy kristallografii [Methodological Problems of Crystallography] (Moskva: Nauka, 1985)--B.K. Vainshtein; E.N. Eliseev

Symmetrie (Kln: Aulis, 1985)--H. Stork

Symmetry: Unifying Human Understanding (New York, N.Y.: Pergamon Press, 1986; also special issues of Computers & Mathematics with Applications, Vol. 12B, Nos. 1-2, 3-4, 1986)--I. Hargittai

Zakony simmetrii v mineralogii [Laws of Symmetry in Mineralogy] (Leningrad: Nauka, 1987)--N.P. Yushkin, I.I. Shafranovskii, K.N. Yanulov

Field of interest: Geometric-morphologic-architectonic aspects of symmetry (dissymmetry, asymmetry) in an interdisciplinary and/or intercultural context; emphasis on concrete science-art, nature-technology or man-machine relationships; East-West comparisons.

GENERAL DESCRIPTION OF THE TOPIC

Some geometrical-topological structures with certain symmetry (or dissymmetry) properties play important roles in many theoretical and practical fields, e.g., in sciences (intuitive geometry, geometric crystallography, crystalphysics, stereochemistry, geomorphology, biomorphology); in engineering and applied sciences (design and construction principles in structural mechanics, electronics, robotics, computing); in arts (composition principles in ornamental art, architecture and other visual or acoustic arts). The concept symmetry has also non-geometric generalizations: conservation, invariance, homeostasis, selforganization etc., which gained special importance in physics and biology.

The existence or lack of symmetry contributed -- in an intuitive or concrete form -- significantly to human thinking and generated thoughts in a variety of ways in science, art and technology. Therefore symmetry vs. dissymetry principles (also broken symmetry, left and right problem, incommensurate structures etc.) and their cultural roots in Western and Eastern (Indian, Chinese, Japanese, Middle Eastern etc.) traditions are of special interest.

Present efforts would provide a forum for interdisciplinary approach to the concept of symmetry, where specialists of different disciplines, arts and skills have opportunity to contribute to exchange and enrich their experiences in a holistic form (some possible "meeting-places" are e.g., symmetry groups in crystallography and ornamental art; conformal symmetry in biology and robotics; parallel processing in brain research and computing; binary representation in nature and art; organizing principles in modern physics and oriental philosophy).

PROGRAM

Aiming at interdisciplinarity the program consists of *plenary lectures* (no parallel sections) and *workshops* (some contributed lectures and informal discussions on concrete topics). In addition to the main scientific event workshops and *exhibitions* (Symmetry in Art/Architecture/Music) are organized partly at the conference center, partly in several galleries in Budapest. The Program Committee -- taking into consideration the suggestions of the International Scientific Advisory Committee and the Hungarian National Committee -- invited plenary lectures, exhibitions and formed workshops from the proposals. The philosophy of the symposium tries to avoid both extremities: too specialized disciplinary approaches and too general indirect surveys.

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A GROWTH MODEL OF PHYLLOTAXIS:
The Dynamics That Produce a Living Crystal

IRVING ADLER

North Bennington, Vermont 05257, U.S.A.

1. The Subject. Phyllotaxis is the study of the patterns formed by leaves, scales or florets on a plant. A phyllotactic pattern is like a living crystal.

2. Preliminary Facts and Concepts, Mathematical and Botanical.

A. A generalized Fibonacci sequence is determined by its first two terms a_1, and a_2 (which may be any two integers), and the recurrence relation $a_{n+1}=a_n+a_{n-1}$, $(n>1)$, which generates the rest. Sequences that play an important role in phyllotaxis are those whose first two terms are 1, t, with $t>0$, and t, 2t+1, with $t>1$. Examples of the first type are $(F_n)=1, 2, 3, 5, 8, 13, \ldots$ (usually called the Fibonacci sequence because it was the first one ever studied), and 1, 3, 4, 7, 11, 18, ... (known as the Lucas sequence). An example of the second type is 2, 5, 7, 12, 19, ...

B. The golden section g is the positive root of the equation $x^2=x+1$. $g=(1+\sqrt{5})/2$.

C. On a mature stem, if there is only one leaf at each node (level on the stem), they all lie on a helix known as the genetic spiral .Since the size of a stem does not appear to be relevant, we exclude it as a factor by picturing the stem as a normalized cylinder whose girth is taken as the unit of measure. Leaves are numbered in the order of their emergence, starting with 0. The internode distance between leaves i-1 and i is designated by r_i, and is called the rise. The angle of rotation around the axis of the cylinder between leaves i-1 and i expressed as a fraction of a turn is designated by d_i and is called the divergence.

(Fig. 1. Normalized cylindrical representation)

D. The picture looks different when the units (leaves, scales or florets) are crowded, as on the growing tip of a stem, or on a pineapple, pine cone, or sunflower head. What is observed then are two sets of conspicuous spirals called parastichies. One set goes up to the left, and the other set goes up to the right. If there are m left parastichies and n right parastichies, we say that the plant has (m,n) phyllotaxis. The conspicuous parastichies are determined by joining each unit to its nearest neighbor on the left and on the right. They are secondary spirals determined by the genetic spirals that are themselves not conspicuous. It can be shown that if the phyllotaxis is (m,n), the number of genetic spirals is the greatest common divisor of m and n. Consequently, in the case of a single genetic spiral m and n are relatively prime.

E. The phenomenon of phyllotaxis occurs on many different kinds of surface. We convert them all into normalized cylinders by means of appropriate conformal transformations.

F. A plastochrone is the length of time between the emergence of any unit i and its successor i+1. Time T is measured in plastochrones starting with the emergence of unit 0.

3. The Principal Facts of Phyllotaxis. a) As the number of units increases, there is a period during which the phyllotaxis (m,n) rises to higher and higher values, that is, with higher values of m and n. b) During this period of increasing phyllotaxis, (m,n) with m<n is succeeded by (m+n,n). c) d_i converges rapidly to a limiting value. In nearly all cases of phylllotaxis (m,n), m and n are consecutive Fibonacci numbers, and the limit to which d_i converges is g^{-2}.

4. The Central Problem. What is the explanation for the three facts cited in paragraph 3?

5. Assumptions for the Growth Model.
1) Successive units arise at equal intervals on the genetic spiral, so that d_i and r_i are independent of i, hence have values d and r that are functions only of time T. 2) r(T) is a monotonic decreasing function of T that approaches 0 as a limit as T increases to infinity. 3) Beginning with some instant T_c, d(T) is such that the minimum distance between units is a maximum relative to the values of the minimum distance that correspond to neighboring values of d.

6. The Phase Space. With these assumptions, the state of a system of phyllotaxis is given by two parameters, d and r. The phase space then is the (d,r) plane, and the development of the phyllotaxis of a plant as time passes may be pictured as a path in the plane.

7. The Problem Solved. On the basis of these assumptions we obtain an explanation for each of the facts cited in paragraph 3: a) Rising phyllotaxis is a consequence of decreasing r. b) The addition rule which governs rising phyllotaxis, that is, that (m,n) phyllotaxis with m<n is succeeded by (m+n,n) phyllotaxis, is a consequence of the maximization of the minimum distance between units. c) If maximization of the minimum distance begins early, that is, before $r<\sqrt{3}/38$, or while T<5, then it is inevitable that in succeeding values of (m,n) phyllotaxis the m and n be consecutive Fibonacci numbers and the limiting value d of the divergence be g^{-2}.

8. Consequences of the Assumptions (Adler, 1977). Suppose that at time T_c units m and n are the units that are nearest to unit 0. Then the phyllotaxis is (m,n). We assume without loss of generality that m<n. Maximization of the minimum distance between units implies that dist(0,m)=dist(0,n). This equation is the equation of a circle in the (d,r) plane. Consequently, as T increases, the point that represents the state of the system descends along an arc of this circle until dist(0,m+n)=dist(0,n)=dist(0,m). After that the point that represents the state of the system descends along an arc of the circle whose equation is dist(0,m+n)=dist(0,n), and the phyllotaxis changes from (m,n) to (m+n,n). With further decrease in r, the state of the system switches from an arc of the circle dist(0,m+n)=dist(0,n) to an arc of the circle dist(0,m+n)=dist(0,m+2n), then to an arc of the circle dist(0,m+2n)=dist(0,2m+3n), etc. These successive arcs form a

connected zig-zag path with narrower and narrower swings to the right and left. The projections of these arcs on the d-axis form a nest of intervals, and the divergence d converges to the value of the point that is inside the nest.

(Fig 2. The Phyllotaxis Path)
(if maximization of the minimum distance begins early)

9. <u>The Vortex Metaphor</u>. Fig. 3 is a series of graphs of the square of the minimum distance between units as a function of d, each drawn for some fixed values of r and T. The graphs show that for high r and low T there is at first only one maximum. Then, as T increases and r decreases, more and more maxima appear. If maximization first occurs at some time T, any one of these maxima may be the starting point of the kind of zig-zag path just

(Fig. 3. The Square of the Minimum Distance as a Function of d)

described. It is as though there are many vortices in the (d,r) plane, with more and more of them present in the regions closer to the d-axis. <u>When maximization of the minimum distance between units first occurs, the point (d,r) that represents the state of the system moves to the nearest vortex and then descends into it.</u>

(Fig. 4. Vortices in the (d,r) Plane)

10. <u>Weakening the Assumptions</u>. The model described above used the very strong assumptions that d_i and r_i are independent of i. Weakening these assumptions does not affect the conclusions reached for the following reasons: 1) The argument can still be carried through using the average of the d_i as the value of d, and the average of the r_i as the value of r. 2) It can be proved that maximization of the minimum distance between units compels an equalization of the values of the d_i.

11. <u>The Wave</u>. Using reasonable equations for $r_i(T)$ for units on a parabolic surface or on a disc shows that as more and more units emerge, the younger ones recapitulate the history of older ones, that is, the values of d_i and r_i, as they change with increasing T, recapitulate the values that d_1 and r_1 passed through at an earlier time. This means that the vibration of d travels as a wave from the older to the younger units.

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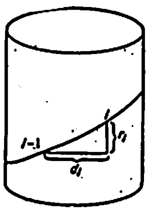

Fig. 1. Normalized Cylindrical Representation

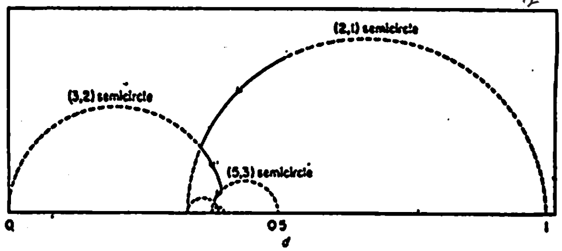

Fig. 2. The Phyllotaxis Path if maximization of the minimum distance begins early

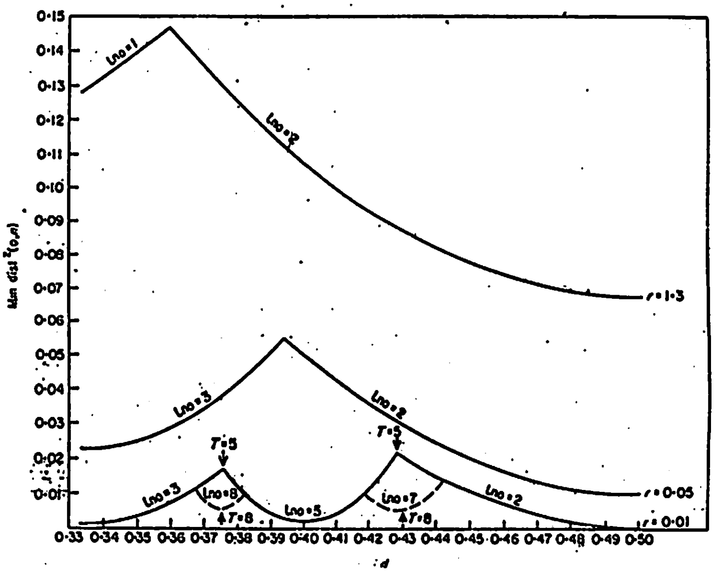

Fig. 3. min dist² (0, n) as a function of d when T = 5 and T = 8. (Lno = leaf nearest zero.)

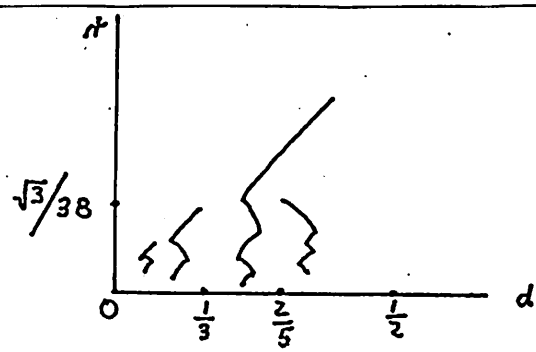

Fig. 4. Vortices in the (d,r) plane (Schematic diagram)

ERWIN BAUER'S THEORETICAL BIOLOGY AND THE PROBLEM OF ASYMMETRY

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1. If there is to be some hope today - in the age of the Babel's confounding of languages - for unification or synthesis of such branches of knowledge as natural and social sciences, philosophy and the art, then it has to be connected to the principle of symmetry (and of asymmetry, as will be demonstrated later).

2. The type of a physical regularity is based on symmetry A physical law equals symmetry i.e a regular reproduction of the same consequences under similar conditions. Moreover, it is possible to unite physical laws through symmetry, and thus symmetry is sometimes referred to as the law of physical laws.

3 Physical world is being described by splitting its structure into two incompatible poles: laws and initial conditions. Hence, irregularities, random events, asymmetrics are being put outside the physical theory from the very beginning: they are being placed into the area of initial, or boundary, conditions. As for the theories of physics, they are only dealing with the symmetry (preservation. invariance) or its distortions.

4. Presently, the concept of distorted symmetry (a spontaneous distortion of symmetry, the thermodynamic imbalance, the irreversibility of time, etc.) draws particular attention of physicists. However, the phenomenon, that truly deserves attention and study, is hidden under the name of symmetry. Thus, asymmetry is a more significant event that just a distortion of symmetry.

5. However, asymmetry, when thrown out of the door, i.e. out of the framework of the physical theories, returns through the window, i.e. it assumes the role of the most basic principle in the theories of fundamental particles, in the cosmological models of the development of the Universe, or in the global theory of evolution.

6. It became obvious in the past few decades that physics lacks a theory of evolution towards the formation and development of increasingly complicated structures. The existing theory of evolution, expressed by the law of the growth of enthropy, describes just only destruction of structures, degradation and death.

7. The existence of two theories of evolution - Darwin's concept of the origin of species and the second principle of thermodynamics. - which describe two directly opposing tendencies and which do not have any points in common - illustrates the gap between the studies of the living and non-living matter

8. If we are going to set up a physical theory of evolution in the sence of self-organisation of systems, we have to operate with a new concept of the irreversible time, as Ilya Prigogine says now. But the asymmetry of time introduces an obvious distortion into the symmetrical physics. hopelessly complicating it.

9. Asymmetry. which is met by physics at its frontiers. belongs to the foundations of biology as a cornerstone in the study of the living matter. Trying to discover the essence of life, scientists get stuck with the problem of asymmetry: both at the level of molecules, structure, functions and at the cognitive level. The problem remains unsolved; moreover, it rests beyond the frontiers of the symmetristic science.

10. Can there be, after all, a <u>scientific</u> structuring of

biology, or "does the science of life, i.e. biology, really exist, or is it just a branch of applied physics and chemistry ?" - this is how the question was put by Erwin S. Bauer, the author of "Theoretical biology", a book published in 1935.

11. Prior to explaining his own point of view, Bauer assesses mechanicism and vitalism which have existed since times immemorial. Then he formulates a principle vividly expressing the most specific feature of the phenomenon of life: "Where forces working against the equilibrium present themselves as a regular event, which is the case with the living systems, there we have to deal with new regularities. These cannot be described by amendments to the old concepts any more, since the distortions turn into new regularities."

12. Thus Bauer formulated his "principle of a persistent thermodynamic non-equilibrium", a law not yet known to the science of which physicists started to speak ten years later as if neither E. Bauer nor his "general principle of biology" had ever existed.

13. A distortion of the thermodynamic equilibrium or a deviation from symmetry persist in a living system as a self-preserving variable which may be expressed as a regularity. It can be interpreted as asymmetry being a general law of biology - a fact all the more overwhelming since Bauer could not yet correlate his law with the optical activity, which was discovered by Louis Pasteur who thus declared the molecular asymmetry to be "the only distinctive border line between the chemistry of living and non-living matter".

14. Bauer discovered and formulated the law which describes the drive of the living matter towards building all the more complicated structures, as opposed to the trend towards their distortion and decay, expressed by the second principle of thermodynamics. The new "source of life" is a peculiar principle

of <u>preservation</u> <u>of</u> <u>asymmetry</u> in the living nature, and it may be presented as a major principle of the science of life (then the principle of the growth of enthropy will take the second place here in terms of its significance). Bauer had conceived it as a law of science, and he had even tried to ascribe a quantitative measure to it.

15. One can argue that Bauer's concept is the most significant event in the theoretical biology after the works of Charles Darwin. However, it went unnoticed by the scientific community - probably because it is only now, over 50 years after Bauer's book was published, that asymmetry starts to be perceived as a principle shaping up structures; it begins to attract a particular attention of the physicists, though still and well it is being interpreted as the distortion of symmetry.

16. A different interpretation of the regularity of the living nature, offered by Erwin Schroedinger, was commonly accepted: as the negative enthropy, which is very unlikely from the point of view of thermodynamics. Also, it was not noticed, that the statement: "chaos taken with the negative sign is the measure of regularity" is inconsistent both in the common sence and in the scientific one, because conversion of sign of such an irreversible variable as the enthropy contradicts the second principle of thermodynamics which asserts that an irreversibly decaying system cannot return to the initial state. Thus, the science had chosen reductionism and, after closing the circle, came back to the problem as worded by E. Bauer.

17 Prior to solving this problem biology will most likely have to absorb most of the achievements of modern science and to acquire a new dimension of a dynamic synopsis of various fields of knowledge so hopelessly separated today.

MULTIDIMENSIONAL SYMMETRY AND ITS ADEQUATE GRAPHIC-ANALYTICAL REPRESENTATION IN THE SYSTEM "MAN-MACHINE-ENVIRONMENT"

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Every "man-mashine-environment" system has many degrees of freedom, is multidimensional and as a rule symmetrical one (Frolov, 1979). It is actual therefore to try to describe multidimensional adequate graphic-analytical information, its symmetry, "isomery" (this term means geometrical ambiquity and is taken from chemistry) etc. Such adequate representations are well known in 1- and 2-dimensional cases. So 1-dimensional representation

$$Y = f(X) = X_1 = i_1 X_1 = e^{i_1 \alpha_1}$$

corresponds to a point on the real axis (a circle means the importance of the Nature i.e. of a "mathematical dimension" of a number, for example: a number is reel, imaginary or other one). The birth of 2-dimensional adequate representation may be set at 1673 when John Wallis suggested the geometric representation of complex numbers by points in a plane:

$$Y = f(X) = X_1 + X_2 = i_1 X_1 + i_2 X_2 = 1 \cdot X_1 + \sqrt{-1} \cdot X_2 = e^{i_1 \alpha_1 + i_2 \alpha_2}$$

"Mathematician expected that the extension from the complex number with $n=2$ to $n=3$, would be childs play, but considerable time elapsed before they found that no such extension seemed to be possible without violating a rule of ordinary algebra" (Moon, 1986).

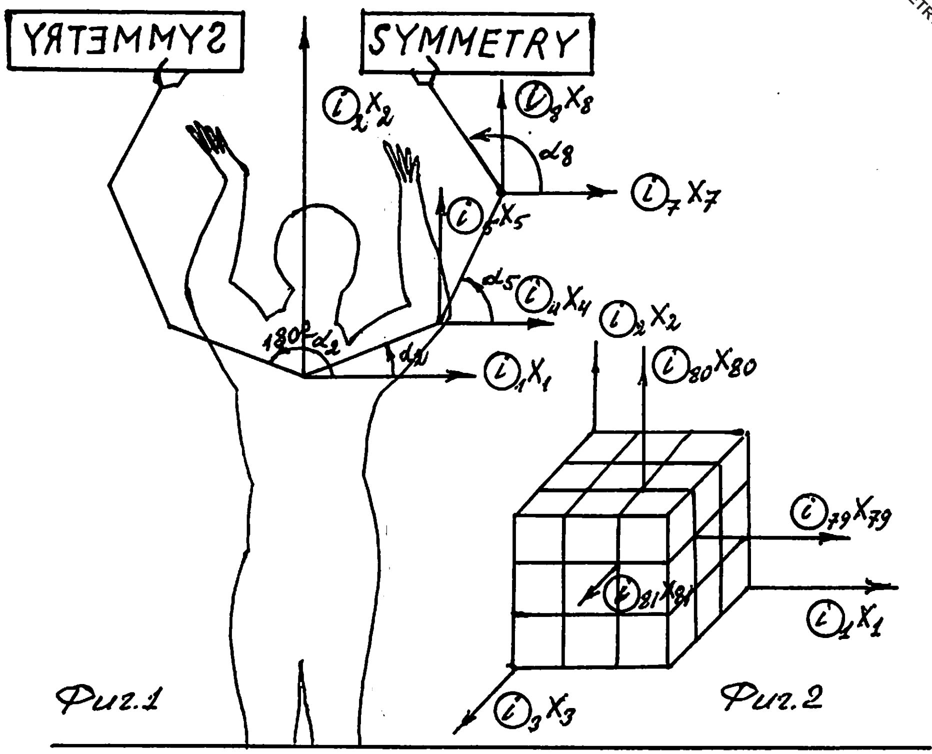
With the aim of simplicity we do not describe here the curvlinear coordinates and symmetries (Petukhov, 1981; Bunin, 1971, 1985) and describe only multidimensional Descartes coordinates (Bunin, 1971).

What is the cause for beauty and power of complex numbers? This cause is for different Nature of axes. The result numbers after algebraic operations may be again distributed on corresponding axes: reel part of result on reel axes, and imaginary one-on imaginary axes. But such a distribution is impossible if the Nature of numbers is the same. The base of n-dimensional representation here is the same. We also use unit numbers of different Nature

$$i_1 = 1\ , \quad i_2 = \sqrt{-1}\ , \quad i_3 = \sqrt{-1} \cdot {}^{\sqrt{-1}}\!\!\swarrow\!{-1} \qquad \text{etc.,}$$

where (Bunin, 1967, 1985) $i^{\,1,4,7,10\ldots} = +1$; $i^{\,2,3,5,6,8,9\ldots} = -1$.

The symbol $\swarrow$ denotes here "superroot" (Bunin, 1967) inverse to "superpower" $\nearrow$ (for instance ${}_2 2^2 = {}^3\!\nearrow 2 = 16$; ${}^3\!\swarrow 16 = 2$).



Let us consider a simple exsmple of multidimensional symmetry: symmetrical 9-dimensional arms of a robot Fig.1 (partly taked from the well known Weil's book "Symmetry"). n =9 independent values of coordinates corresponds to one point on Fig.1, for example to grab holding the word "SYMMETRY", as usually take place in multidimensional coordinates. Analitical exponential representation analogous to that of 1- and 2-dimensional one is

$$Y = f(X) = e^{i_1\alpha_1 + i_2\alpha_2 + \ldots + i_9\alpha_9} + e^{i_1\alpha_1 + i_2(180^\circ - \alpha_2) + i_3\alpha_3 + i_4\alpha_4 + i_5(180^\circ - \alpha_5) + i_6\alpha_6 + i_7\alpha_7 + i_8(180^\circ - \alpha_8) + i_9\alpha_9}$$

We put on the Fig.1 $\alpha_1 = \alpha_4 = \alpha_7$ (all lincs are equall); $\alpha_2 = 30^\circ$, $\alpha_5 = 60^\circ$, $\alpha_8 = 120^\circ$; and $\alpha_{3,6,9} = 0$ was taken to demonstrate how may be <u>convoluted</u> n-dimensional picture on the flat screen of display (we hawen't multidimensional screens now). Analogous method of representation was described in application to another objects: 3-dimensional spiral (Bunin, 1985), athom (Bunin, 1971) etc. It is interesting to applicate this method to analyse of a Rubic Cube rotations, which obviousely needs in 81-dimensional coordinates (Fig.2).

This quantity of coordinates may be decreased because of multidimensional symmetry of Cube, its parts and movings. It must be noted that our results are absolutely in no contradiction to so-called "Fundamental theorem of algebra", which ban going out "field of complex numbers" only by using of power polinomial, e.g. the operation of 3-d step (power, root, logarithm), and said nothing about possibility or impossibility of such going out by using more powerful operations, for instance, 4-th step ("superpower","superroot","superlogarithm"),etc. Let us consider an example of <u>convolution</u>. The result of an experiment have often the form of a system of n equation with unknowns $X_1, X_2, \dots, X_n$. If we write this system by units of dimensions $\ell_1, \dots, \ell_n$, we become:

$$q_1 = a_{11}\ell_1 X_1 \dots a_{1n}\ell_n X_n$$

......

$$q_n = a_{n1}\ell_1 X_1 \dots a_{nn}\ell_n X_n,$$ solution of which is

$$\ell_1 X_1 = \frac{\Delta \ell_1 X_1}{\Delta} \quad \dots \quad \ell_n X_n = \frac{\Delta \ell_n X_n}{\Delta}$$

Convolution of this solution on 3-dimensional screen comprise a set of three coordinates $\ell_1 X_1, \ell_2 X_2, \ell_3 X_3$ which definits the origin of other three coordinates $\ell_4 X_4, \ell_5 X_5, \ell_6 X_6$ etc. More dence convolution take place if we use 2-dimensional screen and a set of pairs of such coordinates. The graphical result on the screen of display may show klasters, symmetry, decomposition and help in operate in systems "man-machine-environment".

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VIBRATIONS OF SYMMETRIC MECHANICAL SYSTEMS

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Systems,that have geometrical symmetry,find wide application in many branches of mechanical engineering: they comprise various foundations, reduction systems, bladed rotors,etc.

The dynamics of such systems has certain features which make their use "a must".Typical of them, in particular, are: a)independence of various classes of motion (e.g.progressive and torsional);b) existence of a comparatively "quiet" zone - the symmetry centre,which is a node for all torsional vibrations.

Symmetric systems in mechanics have unique features, which strongly influence the work of such systems; they are:

1) technological scatter of parameters, resulting in asymmetry (quasi-symmetric systems);

2) hierarchy of subsystems, each having a symmetry of its own;

3) existence of extended solid bodies with 6 degrees of freedom and indefinite type of symmetry;

4) multidimensional displacements of characteristic points, defining the type of symmetry (generally having 6 degrees of freedom). Certain specific features are introduced due to the employment of finite element method (FEM).

All these features have necessitated generalisation of the existent approaches and creation of symmetry block operators. To this end block projective operators have been introduced, such operators comprising diagonal blocks which allow to account for a multimeasurable nature of system nodes; "equivalent points", chosen for extended solid bodies, are unique in that their displacements are concerted with group symmetry of an entire system.

Block operators of symmetry can be expressed as

$$P_\mu = \frac{f_\mu}{n} \sum \chi^\mu (g^*)\, g \qquad (1)$$

where n- order of group G; f_μ - measure of μ-th representation; χ^μ - diagonal matrix, comprised of characters of the μ-th irreducible representation; g - element of group G.

Transformation of coordinates

$$h = P^\mu y \qquad (2)$$

is equivalent to matrix transformation of the original dynamic matrix of stiffness

$$\bar{D} = (P^\mu)^T D P^\mu \qquad (3)$$

On the power of orthogonality of various irreducible representations, matrix D falls into independent blocks, each of which describes irreducible representation of its own

$$\bar{D} = K - \lambda M \begin{bmatrix} D_1 & & & \\ & D_2 & & \\ & & \ddots & \\ & & & Dn \end{bmatrix} \qquad (4)$$

For quasi-symmetric systems such falling is effected with accuracy down to tiny magnitudes of the order of ε , occurrable in out-diagonal elements of matrix D, which is a sign of weak interrelation between independent classes of motions.

Thus, representing the original matrix as (4), will mean decomposition of system in conformity with independent classes of motions, defined by (2,3); in this sense, (2) can be regarded as generalized forms of virations. Such generalized forms of vibrations are convenient when analyzing multimeasurable systems, since it is unreasonable to follow each form of vibrations separately. This is particularly important for forced vibrations because external forces are usually distributed in conformity with one of subspaces (2), so that only one block remains in (4) which describes this subspace.

To analyze forced vibrations on the basis of the group representation theory it is necessary to decompose external force vector by symmetry operators as

$$F = P^{\mu} F \quad .$$

The analysis of operators (2) permits the following tendencies in symmetric system vibrations be revealed without aid of computers: a) to establish undulating character of vibrations; b) to establish the number of independent classes of motions and their configurations, also the number of multiple frequencies; c) to find out how different classes of motions are interrelated depending on asymmetry distribution; d) to define optimum methods of distributing asymmetric elements under different applications of load; e) to account for symmetry in a hierarchy of subsystems; in this case the resultant operator will be the product of operators for each of the subsystems, i.e. the product of undulating motions for corresponding types of symmetry. In engineering practice such a hierarchy is, in fact, a routine procedure involved in improving design models, in the course of which they become progressively more detailed.

The approach has been used to analyze vibrations in symmetric and quasi-symmetric frames-foundations for power generating plants. Fig.1 shows a damped pentagonal frame; its symmetry group is C .Offered for observation is a finite element model with two intermediate nodes 6-15 on either side.

It is reasonable to choose the coordinate system for each vertex 1-5, due account made of the entire frame symmetry (Fig.1a).

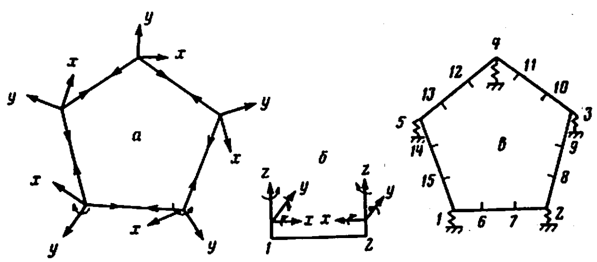

Fig. 1

Then the dynamic matrix of stiffness will assume a simple form, comprising blocks of two types

$$D = \begin{vmatrix} a_{11} & a_{12} & 0 & 0 & a_{21} \\ a_{21} & a_{11} & a_{12} & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{12} & 0 & 0 & a_{21} & a_{11} \end{vmatrix} \qquad \begin{aligned} a_{11} &= \theta_{-\varphi}^{T} k_{11} \theta_{\varphi} + \theta_{-\varphi}^{T} k_{11} \theta_{\varphi} \\ a_{12} &= a_{21}^{T} = \theta_{\varphi}^{T} k_{12} \theta_{-\varphi} \end{aligned} \qquad (5)$$

Character T means transposition, θ_{φ} - matrix of torsion through angle φ . Basis vectors of subspaces form blocks of projective operators P, so that blocks a are transformed similarly to scalars for unimeasurable nodes. The analysis of operators shows that the system has one unimeasurable and two two-measurable representations, i.e. there are 2n/5 two-fold roots, corresponding to representation U_2, 2n/5 two-fold roots, corresponding to U_3, and n/5 - one-fold roots. If additional nodes are present on the pentagon sides, representation (2) will decompose original matrix D as

$$D = \begin{bmatrix} D_{11} & & & & \\ & D_{22} & D_{23} & & \\ & D_{32} & D_{33} & & \\ & & & D_{44} & D_{45} \\ & & & D_{54} & D_{55} \end{bmatrix}$$

when classes of motions for either of subspaces U_2 and U_3 are interrelated. This is attributed to the fact that the forms of vibrations of 1-5 nodes and those of 6-14 and 7-15 nodes are linear combinations of $h_2 \in U_2$ and $h_3 \in U_3$ vectors; generally speaking, they are non-orthogonal to the former, thus explaining the emergence of terms Dij. This conclusion will obviously hold for any finite-element model.

Fig. 2 shows the results of calculation of low natural frequencies and forms of vibrations, whose analysis confirms theoretical conclusions about quantities of multiple frequencies and configuration of vibrational forms. The analysis of amplitudes of vibrations shows that displacement 1-5, 6-8-10-12-14 and 7-9-11-13-15 vary by representations, belonging to one subspace, which means correct choice of projective operators in a block-like form (2).

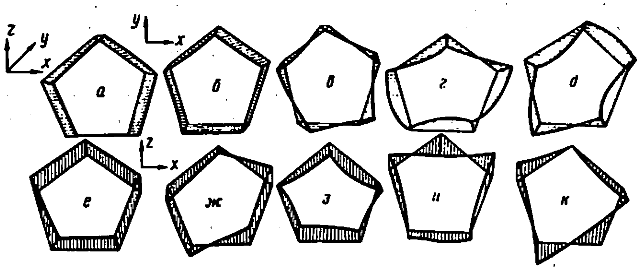

Fig. 2. a,б-21,7;в-24,3;г,д-29,8;e-30,9;ж,з-35,2;и,k-50,8 gz

Study of quasy-symmetric systems. For quasi-symmetric systems the dynamic matrix of stiffness can be presented, after having made group transformations, as

$$\bar{D} = \begin{bmatrix} \bar{D}_{11} & \varepsilon \bar{D}_{12} & \cdots & \varepsilon \bar{D}_{1n} \\ & \bar{D}_{22} & \cdots & \varepsilon \bar{D}_{2n} \\ & & \cdots & \cdots \\ \text{symmetric} & & & \bar{D}_{nn} \end{bmatrix}$$

which reflects weak interrelation between independent subspaces. A solution for natural and forced vibrations can be represented as converging series by exponents ε .

At forced vibrations the interrelation of various subspaces is defined both as distribution of external force and as occurrence of resonance states at $\omega = \omega_{ext}$ in some of a subsystem.

Fig.3 shows a damped square-shaped frame intended for power shaped frame intended for power generating equipment. As a rule it is practically impossible to make the frame perfectly symmetric: there is always a scatter of parameters leading to interrelation of vibrations and occurrence of beatings as a result of detuning of multiple frequencies. Let us consider the interrelation of vibrations caused by the scattering of dampers` stiffness parameters.

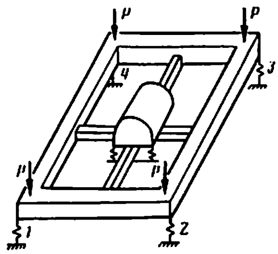

Fig. 3

The frame stiffness matrix after being expanded by symmetry operators

$$\begin{bmatrix} 4a_{11}+8a_{12}+\Sigma H_i & H_1-H_2-H_3-H_4 & H_1+H_2-H_3-H_4 & H_1-H_2-H_3+H_4 \\ & 4a_{11}+8a_{12}+\Sigma H_i & H_1-H_2+H_3-H_4 & H_1+H_2-H_3-H_4 \\ & & 4a_{11}+\Sigma H_i & H_1-H_2+H_3-H_4 \\ \text{symmetric} & & & 4a_{11}+\Sigma H_i \end{bmatrix} \quad (6)$$

where a_{11} and a_{12} are determined similarly (5).

When distributing asymmetry to type U_2, i.e. $H_1=-H_2=-H_3=H_4$ as evident from (6), interrelations arise between subspaces U_1 and U_2.

Hence:

- if the external force is distributed to type U_2, i.e. $F_1 = F_2 = -F_3 = -F_4$, then resonance states may arise on natural frequencies in subspaces U_1 and U_2 ; in this case there will be no torsional vibrations about axes x and y.

Similar conclusions can be drawn for other cases of asymmetry and external force distribution.

As obvious from the example cited above, frame asymmetry appreciable influences the dynamics of system as a whole: progressive and torsional vibrations are generally not decoupled; beatings arise as a result of detuning of natural multiple frequencies; placing the rotor into geometrical centre of symmetry does not help in dividing shapes of vibrations. However, knowing the scaterring of dampers characteristics and distribution of external force, one can, employing the approach offered, arrange the dampers in such a manner that their asymmetry will not induce any interrelation between certain classes of motions, ensuring good vibration resistance of the object. These qualitative conclusions can be obtained without computer-assisted design, by merely analyzing the projective operators.

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APPROXIMATE SYMMETRIES OF EQUATIONS WITH A SMALL PARAMETER

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Roughly, the investigation of different physical processes is made based on the simpliest model representations. They are chosen to truly describe a given process on the whole. When investigating such models and constructing their particular solutions, of great help are methods of group analysis (i.e. Lie and Lie-Backlund transformation groups). The consideration of subsequent approximations gives us a succession of models, each of them considering some additional factor. Though these factors are little, they can materially change the general picture of the process. As a rule taking into account the subsequent approximation narrows the allowed group of the simpliest model equation. It greatly limits the use of classical symmetries for the investigation of differential equations.

The introduction of approximate symmetries allows to overcome this problem and to construct the theory of stable relatively small perturbations of differential equations symmetries. The theory of approximate symmetries is based on a concept of the approximate transformation group, which is allowed (with a definite degree of accuracy) by an equation with a small parameter. Based on the analogue of Lie's theorem for approximate groups, an infinitesimal description of approximate single-parameter transformation groups is developed, and the determinative equations for the construction of approximate symmetries are isolated.

In general, the perturbated equation (the model taking into account the additional factors) admits not any symmetry of non-perturbated equation in the form of approximate symmetry. If such a succesion of symmetries is admitted, such symmetries will be called stable symmetries.

In the way of example we made a group classification of nonlinear wave equation (which for example describes iso-entropic motion of the lequid in a tube with a little dissipation in Lagrange coordinates). It is demonstrated, that not every symmetry of the wave equation is stable, i.e. not all of it's exact symmetries are succeded in the form of approximate symmetries of the wave equation, which takes into account the small dissipation. For convenience of comparison we calculated the exact symmetries of the wave equation with small parameter and isolated the cases when the use of approximate groups gives an additional expantion.

The whole group on non-perturbated equation is fully succeded by a perturbated equation (in the form of a group of approximate symmetries) only exceptionally. In case such a succession takes place with any degree of accuracy, one can introduce for consideration a new object - the formal symmetries. The change from approximate symmetries to formal ones allows us to get rid of the condition of parameter smallness and consider it as a calibrating ellement (i.e. the formal symmetries represent the formal power series). To stable symmetries belong the symmetries of transfer equation which are succeded by evolutionary equations with an arbitrary degree of accuracy. Among these one distiguishes such symmetries, which satisfy the condition of breaking the formal series. In case such a condition is fulfilled, we get known Lie-Backlund operators. The approach we propose gives a new method of constructing Lie-Backlund symmetries. Contrary to the usual method, here the process of calculating the coordinates of the canonic Lie-Backlund operator is directed not from the higher (by derivatives) numbes to lower, but vica versa. Such an approach allows also to find out how does it happen, that the groups, which are separately admitted by the equations of Burgers and Korteweg-de Vries, vanish (within the limits of the Lie-Backlund theory) when we consider the Burgers-Korteweg-de Vries equation. When we go over to the Burgers-Korteweg-de Vries equation it appeares, that the corresponding Lie-Backlund operators are transformed into the formal ones which don't satisfy the condition of breaking the formal series.

With the approximate (formal) symmetries closely connected are the approximate (formal) Backlund transformations. The stability of all symmetries of the transfer equation indicates the possibility of on approximate transformation (with any degree of accuracy) of an evolutionary equation into transfer equations. Such transformation is executed with the aid of the formal Backlund transformation. As far as the transfer equation is linearized by means of a point transformation, all evolutionary equations are formally linearized.

SPIRAL DIFFERENTIAL STRUCTURES AS A PRINCIPLE FOR HARMONIC ORGANIZATION OF ARCHITECTURAL SPACE AND FORM.

L.E.BELOVA,architect,USSR

There is a persistent tendency, without any historical connection, to use organic forms and rythms in architecture.

It's cause lies in deep correspondence of the laws of perception to the structure of organic matter. The structure of organic matter, its symmetry corresponds to one of stable stereotypical models of perception of the man to surround himself by natural environment.

The aim of this environment is normal functioning of the human organism in an adequate cycle. In an adequate cycle one could spend less energy than in other cycles with maximum efficiency, least weariness and the greatest reliability of function.(Achmerov U.,1987).An adequate cycle appears with theaesthetic way of perception and is fixed subjectively by theaesthetic feeling of pleasure.(Kant I.Aesthetics.).

That's why a formal-aesthetic problem of human perception of the environment is an ecolojical problem at the same time. Its solution guarantees maximum adaptation to the environment and as a result provides the maximum survival because the environment gives more chances for su vival because the greatest information of the environment easies the processof its comprehension.

The analysis of plants meristem structure of the point of growth ca ried out by Richards and his diagrams of growth and the arrangement oi sprouts reveals the process of differentiation in the spiral systems. But Richards considered this process as a visual phenomenon characteristic for big flat meristems.

Investigations,carried out by the author, show that the differentiati appears in all the spiral structures when the diametre of its system changes and it is not a paradox of perception, but a deep property of the spiral structure, which reveals later in the organic matter on it higher levels as a separation of elements of the organic matter (viens of leaves, branches of the tree etc.).

All these things give us the posibility to consider organic forms as a spiral differentiation structures of the growth and developement.

Spiral structures as well as differentiation processes can be describe by recurrent numbers (numbers of Fibonaccy) and in this way it is connected with proportions of famous so called Golden cross-section - the traditional instrument of architects in their creative searches for harmony. These structures have a developement and at the same time preserve the whole. The high combinations of structural elements provides for its plastic variety. All these properties of spiral structures create a lot of opportunities for their wide use in architectural practice as a principle of structural organization of the architectural space and form.

On the base of Richards's works and the works of the author there were made some flat spacial structures and developed some variants of their use in architecture on the town planning levels:

1) town planning sheme
2) space-volume design
3) design

Architecturally interpretted properties of spiral structures reveal themselves as organic harmony of proportions, opening of system for futher developement, formal variety and functional flexibility.

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Illustrations:

Fig.1. Flat Spiral Differentiation Structures on the Base of Similar and Standard Elements.

Fig.2. Spacial Spiral Differentiation Structure.

Fig.3. Children's Playground on the Base of Spiral Differentiation Structures. Authors: Belova L.,Jernakova N.

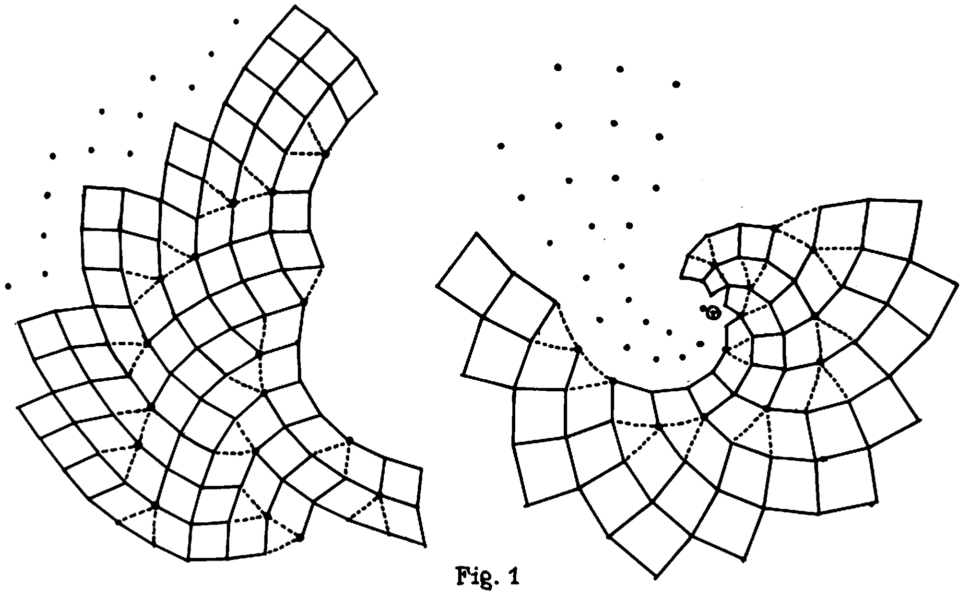

Fig. 1

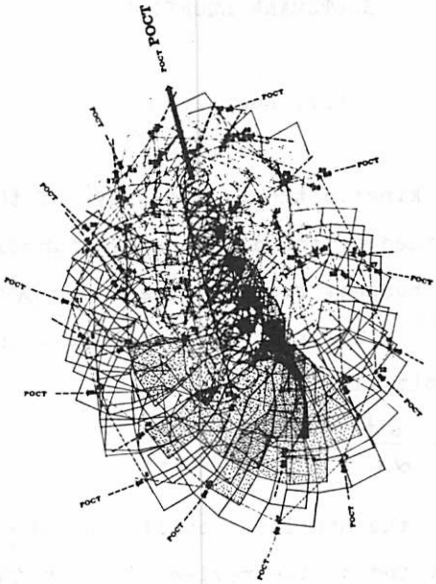

Fig. 2

Fig. 3

NONTRIVIAL SYMMETRY PROPERTIES OF THE NONLINEAR BOLTZMANN EQUATION

A.V. Bobylev

In the classical kinetic theory the state of the gas at the time t is characterized by the distribution function $f(x, v, t)$ of its molecules with respect to the coordinates $x \in \mathbb{R}^3$ and the velocities $v \in \mathbb{R}^3$. The time evolution of this function is described by the Boltzmann equation

$$\frac{\partial f}{\partial t} + v \cdot \frac{\partial f}{\partial x} = I(f, f),$$

where $I(f, f)$ is the nonlinear collision integral.

The report is devoted to the review of the mathematical results connected with the group properties of this equation. It is the integro-differential equation, that's why it is difficult to use the standart methods of the group analyses. Obviosly we can find the set of the symmetry transformations for the Boltzmann equation which are connected with the shifts, the rotations and the scale transformations of the independent variables and the function f. We call here these transformations (with the Galilei group) the trivial transformations because these properties are well-known and don't give us any new information about the Boltzmann equation and its solutions.

We have also at least two types of the nontrivial transformations for certain intermolecular potentials. Firstly it is the Lee-Backlund group for the potential $U(r) = \alpha / r^5$ (Maxwell gas) in the spatially uniform case [1]. Secondly it is the Lee group of the projective transformations for the potential

$U(r) = \alpha / r^2$ [2] in the spatially nonuniform case. We analyse in the report the consequences of these two classes of the transformations (the conservation laws, exact solutions) and their connection with the symmetry properties of the Euler and Navier--Stokes equations.

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SOME ASPECTS OF SYMMETRY IN SCHOOL EDUCATION, OPTIMAL CONTROL AND RELATIVITY

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In this report three aspects of symmetry are considered. These aspects are united together not only on account of interests of the author, but in consequence of their organic connection too.

1. The question on a part of symmetry in a school education is extremely actual now. The attention to this aspect of symmetry was attracted, first of all, by F. KLEIN and H.WEIL. Since the time of EUCLID the proofs in school teaching of geometry are based on consideration of "chains" of equal (more precisely, congruent) triangles. This method of praving makes reasonings to be logically strong. However this method represents a blind alley, which is not connected with the present and the future of the science and does not give ways into other branches of the science and into applications. A sharp criticism of "triangular" proofs in school geometry was given by a very known French mathematician J.DIEUDONNE. Nowhere, except school and entrance examinations to colleges, this method does not apply.

On the contrary, the ideas of symmetry are directed to future. They provide a deep connection between school geometry, other branches of mathematics and applications. The question is not only to apply the reflection, the central symmetry and other geometrical transformations in school, but in general to use a group approach for comprehending of geometrical facts. The set of all geometrical transformations, which keep distances between points and map a given figure onto itself, is named to be the symmetry group of the figure. A knowledge of symmetry group of a figure determined its geometrical properties. The symmetry group of a parallelogram contains a central symmetry, and all the properties of this figure are followed from this fact. The existence of two reflections in the symmetry group of a rhombus implies additional its properties. All the properties of regular polygons are determined by their symmetry groups etc. (Fig. 1).

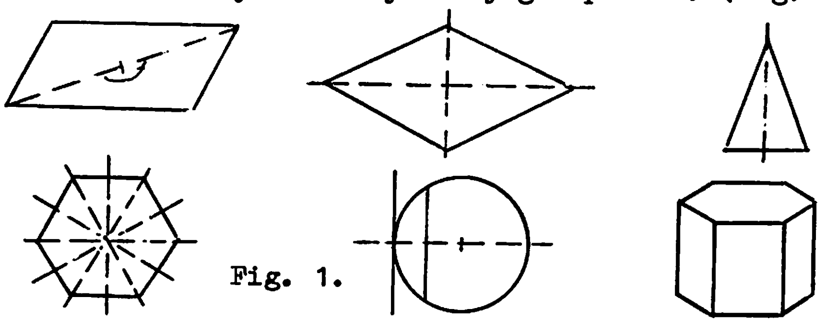

Fig. 1.

In essential, the whole variety of the facts of the school geometry is a manifestation of the Symmetry.

The group approach is very important for nuclear physics, partial relativity theory, crystallography and other sciences. That is why a rebuilding of school geometrical teaching on the foundation of symmetry aspects (instead of an archaic method of "triangular" proofs) is a very actual problem of modern school education.

2. The mathematical theory of optimal control is an important achievement of mathematics in XX century. The maximum principle in many its versions is the central result of this theory. It was advanced (as a hypothesis) by L.PONTRJAGIN. A proof of the maximum principle in linear case was given by R.GAMKRELIDZE and in general, nonlinear case it was given by the author of this report. A paramount role in this theory (both in statements and in proofs) belongs to ideas of symmetry. There is a duality of HAMILTON's type in the statement of the maximum principle, which is expressed by a symmetry of formulas relatively phase coordinates and auxiliary variables.

This symmetry has an essential importance for theoretical reasonings and numerical solutions of different optimal control problems. In the most frequent case an optimal control problem has a symmetric structure. It means that the control region, which describes a set of admissible contrôls, is centrally symmetric (relatively the origin) and the equations of motion of an object are symmetric relatively the origin of the phase space. As a consequence of the symmetric structure of an optimal control problem we obtain a symmetric picture of the BELLMAN's sphere.

The symmetry of the statement of the maximum principle and a symmetric character of the phase portrait of the system of optimal trajectories are a display of a deep symmetry, which is contained in a proof of the maximum principle. This proof is based on using of separation theory of convex cones, which as a matter of fact is deeply symmetric and generalizes symmetry principles of elementary geometry. Just the geometrical separation theory of convex cones was used by the author of report for developing of a "tent method". This method is now the most effective one of solution of optimisation tasks and other extremal problems. There are some ideas of symmetry in its basis too.

3. We have already mentioned, that the partial relativity theory is an original four-dimensional time-space geometry, which is based upon the LORENTZ group as the fundamental symmetry group. There are very interesting and rich in content aspects of symmetry in the general relativity theory too. These aspects are closely associated with the optimal control theory. An intention consists in using a postulate of "displacement" of a light sphere. This postulate allows to deduce the following

four-dimensional time-space metric:

$$ds^2=\left(c^2-\frac{2Gm}{r}\right)dx^0dx^0+\frac{2\sqrt{2Gm}}{r^{3/2}}x_p dx^p dx^0+h_{pq}dx^p dx^q. \qquad (ж)$$

From the metric (ж) it is possible by a very simple manner to obtain the SCHWARZSCHILD's metric, which is very known in relativity. This way twice uses ideas of symmetry. First, the gravitation field of a resting mass possesses a spherical symmetry, and this fact is used essentially for deducing of the metric (ж). Second, the passage from (ж) to SCHWARZSCHILD's metric is based upon a symmetrizing too. This symmetrizing consists in a passage to the "location" metric, for which the values of the light velocities in two opposite directions are equally. Even the fact, that the SCHWARZSCHILD's metric can be obtained by such a simple way, is unlikely an accidental coincidence. It is very likely, that the "displacement" postulate describes a mechanism of an interaction of a particle with gravitation field. Perhaps, this mechanism will allow to obtain physical foundations of the general relativity theory (which must differ from EINSTEIN's, pure geometrical one). Let us note, that the metric (ж) gives the same good coincidence with experiments as the SCHWARZSCHILD's one, since the equations of geodesic lines are in both the metrics the same.

Fig.2 shows the sense of the "displacement" postulate.

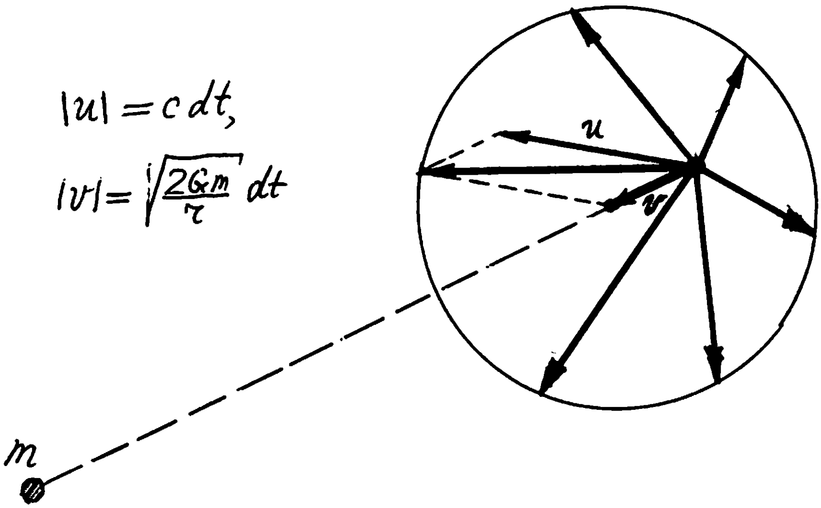

Fig. 2

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THE POWER IN ACKNOWLEDGING MUSIC'S SYMMETRY AS PHYSICAL MOTION WE PERCEIVE THROUGH OUR AURAL SENSE

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"The Symmetry of musical sound, is actually the symmetry of physical motion that we perceive through our aural sense rather than visual." This statement is an accurate and valid definition of music. When an individual accepts this definition, the stage is set for a major shift in that individual's perception of the academic information regarding music. This definition also presents all individuals with an opening for an exciting, personal, intimate relationship with music.

Such a shift in an individual's perception also puts that person in a new powerful position to deal with music as a communicative medium. Music's universal language is found in the symmetry of motion sensations. The production, transmission, and reception of melody, harmony, and rhythm, deals with this universal motion language. Musical aesthetics, either as first hand experience or by intellectual recall, is an acknowledgment of the normal, natural response to the human body's physical, sensual acknowledgment of motion's symmetry percieved as sound.

When a person accepts physical motion as music's communicative language they place themselves in position to personally work with all the symmetry of the musical processes. They can respond, rehearse, and perform, using their own bodies to communicate the physical sensations generated by the music. Using motion symbols they can compose and notate in the musical medium.

A person who has personally worked with the musical language is in a position to mentally become a participant rather than a spectator of musical performances. They can experience 'with' the performer, rather than sitting back and waiting for the performer to do something 'to' them.

The acknowledgement of music as a sensual symmetrical experience opens a new space for human beings. In this space, they personally can expand their musical enjoyment. They can be in personal relationship with music rather than having to rely on the "musical priesthood."

Music shows up as not one but two disciplines involving symmetry. One is scientific, the other artistic. Traditional breakdowns come out of the assumption that music be studied and experienced first as a science in order to appreciate it as an art. This assumption sets up an unspoken requirement that only those individuals fortunate enough to have spent years gathering scientific information, and/or who possess special talent can appreciate the finer artistic levels of music. However, when people use their own natural capacity to first experience the sensual symmetry of music, they establish a foundation and develop a goal for gathering information about music's scientific formulas.

STATEMENT

I. Music is artistic symmetrical physical motion perceived through the hearing sense rather than the visual

II. Motion symmetry is a universal language

III. Motion symmetry is an artistic language

IV. The human body is the primary musical instrument

V. Music is symmetry in two discplines

DEVELOPMENT

I. Sound IS the symmetry of physical motion
- A. Production
  1. Vibration
  2. Volume & Intensity
  3. Timbre
- B. Progressive
  1. Durational-one sound
  2. One sound to other sound
     - a. melody
     - b. harmony
     - c. rhythm

II. Motion qualities deal with universal symmetry
- A. Articulated-sustained
- B. Fast-slow
- C. High-low
- D. Suspension-release
- E. Others

III. Language of artistic communication is motion symmetry
- A. Formal performance descriptions
  1. Tempo
  2. Quality of forward motion
  3. Performance styles
- B. Informal performance descriptions
  1. Cooks
  2. Swings
  3. Moving experience

IV. The human body is the primary musical instrument
  A. Generates-composes
  B. Inputs-controls and manipules mechanical instruments
  C. Responds-hears, reacts, and evaluates

V. Two Disciplines
  A. Scientific
    1. Scientific approach leading to art
    2. Scientific perfection often accepted as artisitc
    3. Approach becomes the goal
  B Artistic
    1. Supposed purpose of music
    2. Sets the goal for scientific approach

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ROSE, WHEEL, RONDO AND MANDALA: ROTATIONAL SYMMETRY AS FOCUS OF RESEARCH PROCESSES FOR THE PLANNING OF AN INTEGRATIVE CURRICULUM IN POLYAESTHETIC EDUCATION (ABSTRACT)

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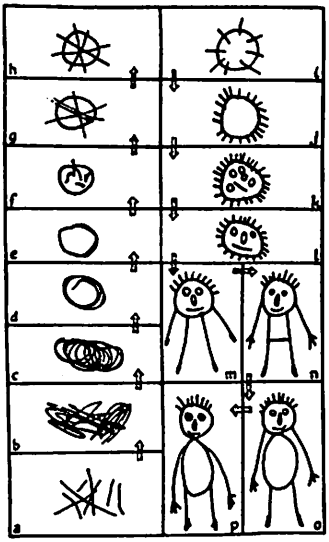
The paper attempts an integrative examination of several crucial educational questions, of some central issues of aesthetics and of a number of related fields. The starting point is one of the major problems of curriculum planning, i.e. the looking for the rational justification of the selection of particular objectives and learning experiences. For the construction of a comprehensive curriculum in aesthetic education a particular form, appealing strongly to the learner, was regarded as one of the possibilities of a connecting, integrative basis of the curriculum. The problem arose, what form should be chosen and how this choice could be rationally justified.

Comparisons with anatomic and cosmological phenomena revealed the central role of the circle as the basic form of rotational symmetry. The reviewing of research related to aesthetic activities of animals and a small child pointed again to the circle, as probably more appealing to a person than other forms. Rensch's Test of Aesthetic Preference (Morris 1968, 160) showed that all the four examined animals (crow, jackdaw, Guenon Monkey and Capuchin Monkey) preferred generally "regular patterns". The Capuchin Monkey preferred the regular pattern in all examples. Only in two examples all the four animals preferred the regular pattern: in the forms of circle and half circle.

Desmond Morris showed similarly in his experiments that forms of more or less imperfect circles constitute the optimal creative achievement of a chimpanzee (cf. Fig. 1).

Fig. 1: Circular Forms Drawn by a Chimpanzee (Morris, 1968, 133).



R. Kellogg points to the importance of circular forms symbolizing a face as a decisive stage in the gradual development of the drawing of a human figure by a child (cf. Fig. 2).

Fig. 2: Schematic Representation of Developmental Stages in the Drawing of a Human Figure by a Child (Morris, 1968, 124).

In the attempt to examine the question of form-preference in an empirical way it was presumed that the intensity of interest in a certain form or of the contact with it were positively correlated with the level of creative expression based on that form. An examination related to the Torrance-Test in Visual Creativity was carried out, in which the examinees had to complete various forms in a "creative" way (Torrance, 1966, cf. Fig. 3). The data were examined by means of Analysis of Variance for Repeated Measurements - (ANOVA) (Hays, 1973) and Tukey's Test of "Honestly Significant Difference (HSD)"(Runyon and Haber, 1971).

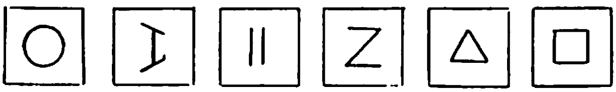


Fig. 3: Test of Visual Creativity (Torrance, 1966, 32).

The result showed that in all criteria the average achievements based on the circle were higher, in some of them significantly higher, than those based on other forms. Consequently the form of the circle (or rondo or mandala) was considered to be an appropriate integrative expression and a fitting basis for the planning of a polyaesthetic curriculum devoted to the fields of visual arts, literature, music, architecture and broadly including even areas outside the realm of aesthetics like certain related subjects in the behavioral sciences, and technology.

The circle is seen by scholars as one of the "symbols of perfection in its unlimited multitude of symmetries". Far beyond the fields of geometry and aesthetics it has great significance in the area of religion. It is furthermore used as "means of contemplation and meditation"(Wille, 1986, 457).

It should be important to examine the function of this basic form of rotational symmetry for education in general and for curriculum planning in particular.

Polyaesthetic education is defined for these purposes as the framework for a curriculum in which "aisthesis", activity of the senses in the broadest interpretation of the concept, should play a central role. In the suggested curriculum intermedial and interdisciplinary relationships are going to be emphasized and the three activities of evaluating apperception, reproduction and creativity will be of importance (cf. Fig. 4).

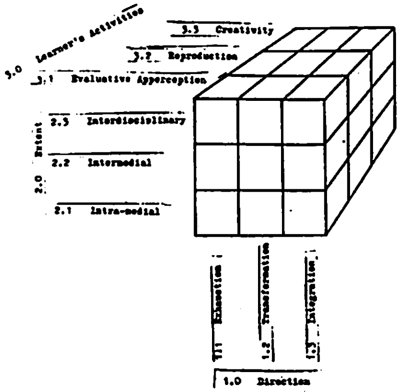


Fig. 4: Theoretical Model of Polyaesthetic Education

The polyaesthetic curriculum "Rose, Wheel, Rondo and Mandala" will be organized according to the following subject fields:

1) Forms of rotational symmetry in nature, in technology and in art: The sliding stone, the circular form in botanic and zoological phenomena, the wheel and the church-window (oculus, wheel-and rose-windows).

2) The circle as important form in plastic art in the course of time and in the diversity of cultures.

3) The circle in Jewish art (cf. Fig. 5).

4) The circle in literature and music.

5) The circle in architecture.

6) Circle, religion and myth: Sun-gods, zodiac and mandala.

7) The mandala in psychological research.

8) Forms of rotational symmetry and creative expression.

The last item will enable us to "close the circle" and to go back to the point of departure, to the scribbling of the ape and the small child, to the empirical experiment.

The circle occupies a central place in the brain and in the heart of man. He enjoys to perceive it, he is interested to use its particular form in order to express his important thoughts and his vital feelings.

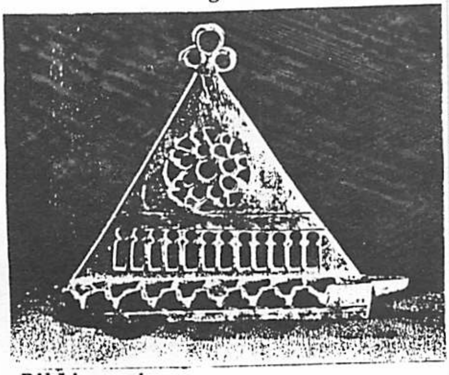


Fig. 5: Bronze Hanukkah Lamp, Lyons, 14th century (Wigoder, 1972, 63). Decoration reminiscent of rose-window of Gothic cathedral.

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THE SYMMETRY OF CRYSTALS

by J.J. Burckhardt

I. Introduction

I received my education at Zürich (Switzerland) and it was also there, where I have been teaching. The athmosphere of this city has formed me. It was also our great poet Gottfried Keller (1819-1890) who has lived there. That's why I beg you to allow me to start my statement by a citation of Keller's: In his autobiographical novel "Der Grüne Heinrich" Keller describes a conversation between himself and a philosopher:

> "Look at this flower", I said to the philosopher, "it is impossible that this symmetry with its counted points and jags, these white and red little lines, this little golden crown in the middle, not has been premeditated! And how delightful it is: a poem, a piece of art, a gay and fragrand joke! All those things could not possibly have made themselves." (Grüner Heinrich, vol. 2, chapter 9, ed. 1916, Cotta, Stuttgart-Berlin, pg. 324.).

These words of the poet show the same conception which lies at the base of my book "Die Symmetrie der Kristalle". I cannot but emphasize that the symmetry which we find in nature is not accidential but premeditated, because such a thing has been made on purpose. It could never have been formed by itself, it is premeditated. At its base lies therefore a mental frame, and we call it "Mathematics".

And now allow me to mention that the mathematitions who have influenced me at Zürich:

1/ I doctorated by Andreas Speiser whose book "Die Theorie der Gruppen von endlicher Ordnung" has influenced my work permanently. Speiser's way of thinking found a big echo at Zürich, I mention his connection with Heinrich Heesch and Wolfgang Gräser for instance.

2/ For a long time I have studied the fundamental work of Paul Niggli (Geometrische Kristallographie des Diskontinuums", which together with the book by Arthur Schoenflies "Krystallsysteme und Krystallstruktur" was the foundation of my own book "Die Bewegungsgruppen der Kristallographie" (1947, ^{2}1966).

3/ Lectures and exercises with Georg Pólya and especially his paper "Über die Analogie der Kristallsymmetrie in der Ebene" were extremely valuable to me.

4/ I was introduced into the forms of the different crystals by Leonhard Weber.

5/ Hermann Weyl showed his connection with our own themes only in later years in his book "Symmetry"; an unmeasurable influence in this direction came from his lectures.

6/ I learned to see the connection between physical processes and their mathematical formulation in the lectures and practica of Erwin Schrödinger.

7/ In the second half of my book I mention the work of my friends who have contributed to the problem of symmetry.

II. The disposition of the book

A. The Origin

1. The origin of the mathematically formulated conception of Symmetry goes back to A.M. Legendre.

2/ R.J. Haüy was the first one who formulated the law of symmetry of the crystals. This means that he not only noticed certain symmeteies - as did many other scholars - but that he noticed that those symmetries were guided by a generally valuable law which he called "Law of Symmetry", in German "Ebenmassgesetz".

3/ From Haüy to Laue. It was Christian Samuel Weiss - who was greatly influenced by Haüy - to whom we owe the nomination of the seven crystal systems. It was only in 1984 that it was discovered that - in 1826 already - Moritz Ludwig Frankenheim had distinguished the 32 crystal classes (J.J.Burckhardt, Die Entdeckung der 32 Kristallklassen durch M.L. Frankenheim im Jahre 1826. Neues Jahrb. Mineralogie Monatshefte 31, 1984. pg. 481 f.). Until 1984 Johann Friedrich Christian Hessel was believed to have discovered them in 1830 as first one. The method which Frankenheim uses is remarkable: He discusses the general linear equation of the surface of a crystal. The treatise was published in the now rare journal "ISIS", edited by Lorenz Oken, the first rector of the University of Zürich, and exponent of natural philosophy. The discoverer of the 32 crystal-classes end his treatise with the remark: "that only a very small part of the 32 classes have been observed". We admire the courage of the writer who published his work nevertheless, believing in the value of the mathematical deduction. Hessel was the first one who drew a graphic picture of the 32 classes. Later on, Gadolin could represent them with the help of the stereographic projection; you can see these pictures in many books. Frankenheim was not only the first who found the 32 crystal-classes, but in 1835 already, he mentions 14 lattices and the 5 networks of the plane. A. Bravais followed him in 1850. Leonhard Sohncke went into a new territory when he showed 14 of the 17 ornaments of the plane, and the

65 space groups which are named after his name. The geometrical part of crystallography is crowned, and at the same time finished, by the work of Fedorov and of Schoenflies. Erhard Scholz gives an overview of the results of these two scholars in a modern way.

B. Transition into modern times

Max von Laue, strongly connected to Zürich, was able to show in marvellous experiments, that the theory of the lattice-structure of crystals is true.

C. The School of Zürich

Paul Niggli saw that the enumeration of the 230 space groups by Fedorov and Schoenflies did not suffice to interprete Laue's X-Rays diagrams. That's why he wrote his work: "Geometrische Kristallographie des Diskontinuums (1919, ^{2}1973). Some of Niggli's collaborators worked in this direction: You will find a list of the works of Heesch, Laves and Nowacki in this book. The multi-coloured ornaments are largely treated: At Zürich, the 80 black and white ornaments of the double plane were simultaneously treated with different methods by L. Weber and H. Heesch, and illustrated by Weber. An ingenious method, using windet threads, is shown by Ingeborg Hund-Seynsche. It was Heesch who, as the first one, deducted certain double-coloured space-groups with the aid of geometrical considerations; Burckhardt used for this purpose arithmetical methods. Van der Waerden and Burckhardt gave a methodical instruction of how to findmulticoloured groups.

The author was able to show that the 230 space-groups are not only the result of combinations of geometrical symmetries, but that an algebraic-grouptheoretical structure lies at their base. You can find it if you modify the conception of the crystal-class which Frankenheim and all his successors have been using: The theory of equivalence of quadratic forms uses unimodular transformations with integer coefficients which you will be able to transfer to the crystallography, and which will bring you 13 arithmetical classes in the plane and 73 in the space. Simple laws of group theory enable you then to find to each of them the space-groups. We mention as examples the derivation of the plane groups, illustrated in a table.

BROKEN SYMMETRIES IN SCIENCE AND ART

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Scientific values and artistic values are both pursued by man, and they are not extraneous to each other. The aim of this report is to explore the nature of the relationship between art and science, and suggest arguments supporting the thesis which asserts that art and science have a common meeting ground in symmetries, and especially at the point where symmetry is broken (1).

Symmetry is invariance under a transformation. Symmetry is indiscernibility of the transformation. Symmetry transformations are associated to the concepts of invariance or conservation, and therefore to the permanent significance of structures. Although it exists, so to speak, outside time, symmetry is nevertheless essential to a description of the fundamental characteristics of a structure.

By definition, symmetry implies the impossibility of perceiving or measuring differences in systems at equilibrium without provoking perceptible changes in the structure. To perceive and measure, to remove uncertainty and create information and knowledge, it is necessary to make choices. And each choice involves a loss of equilibrium, that is, a breaking of symmetry. Paradoxically, symmetry can be understood providing it is broken.

We, as living beings, and as such removed from thermodynamic equilibrium, form self-reproducing information systems. Consequently, not only is symmetry agreeable to us - associated to ecstatic, detached and passive contempla ion - but so are broken symmetries - associated to an acquisition of information which is at the same time attentive, interested and active. As living beings we obey the laws of nature, and the mere following of these laws gives us satisfaction, or at least cause for serenity. Still, we find even greater gratification if we become attuned to nature to the point of making such laws our own, and being able therefore to understand and express them adequately. To understand and express the laws of nature, the artist-man and the scientist-man will each use his own instruments.

Perception is a fundamental instrument for the artist in his relationship with nature. It is the perceptive process which permits the artist to grasp the correlations which exist or may exist among forms, colors and sounds; once he has acquired these correlations, the artist filters them, subjectivizes them and transfers them in sculpture, on canvas or in music. His work will then be a stimulus for those who make use of it. And it will arouse emotions if this stimulus is created so as to resound on the observer's state of mind: a state of mind that has matured precisely because of a continuous exercising of the perceptive activity.

An essential instrument for the scientist in his relationship with nature is measurement. It is measurement which enables the scientist to grasp the correlations that govern the formation of natural structures, eliminating superfluous details and revealing the functional aspects: once these correlations have been acquired, the scientist elaborates his theory or his model of reality. His work will then be a stimulus for the person who utilizes it: it will inspire consensus if it has been created so as to pass all the experimental tests to which the theory or the model lend themselves.

The perceptive process and the measuring process are therefore at the basis of creativity in art and in science, respectively. These processes however have one characteristic in common: both are completed only after a critical stage has been overcome. In the critical stage the subject who perceives or who, with the aid of an instrument, measures, enters into symbiosis, so to speak, with the object perceived or measured, and in this way he gathers its aesthetic or gnoseological value. This critical stage is characterized by a breaking of symmetry: it appears during every crisis taken as a choice, or as a transition between a before and an after, between a state in which I was not aware and a state in which I became aware, between a state in which I didn't know and one in which I know.

To clarify in what sense the breaking of symmetry is to be found at the point where art and science converge, think, for example, of a baroque fugue (2). Like the vast majority of musical compositions, the fugue is divided into a succession of beats which, with translational symmetry, measure the time in equal steps. On these beats, all of the same length, the theme is developed and repeated, again with translational symmetry: a theme which, to follow itself, runs from itself along the score. And as it fades and reappears it evokes a rythmic and periodic movement to which, just as in flight, a momentum (or a velocity) is associated, the value of which remains, on the average, constant in time. On the other hand, it's the melodic line of the theme which breaks the translational symmetry, and with its informative content modulates the meaning of the structure, which otherwise would be purely rhythmic.

Passing from music to poetry, Montale comes to mind: "If I think of poetry as an object, I believe that it was born from the need to add a vocal sound (word) to the beating of early tribal music (....). Also in the first Nibelung sagas and later in the romance epics, the true nature of poetry is sound." (3). In poetry, it is sound that breaks the translational symmetry associated to the rhythm of the meter.

Other examples can be drawn from figurative art, especially from Greek vase painting.

But the example which most deserves our attention here is that reported in fig. 1, taken from Gestalt psychology. The reader who wishes to actively make use of this image may be led to discern an analogy (4) between the triggering of the "chemical" bond resonance at the breaking of symmetry provoked by the "charge transfer spectrum" during a spectroscopic measurement (5), and the perception of the "structural" ambiguity of the information bit: information that is released at precisely the critical stage of the perceptive process or the measuring process. Having reached the critical stage, the structure loses its equilibrium, the inversion symmetry is broken and the center of symmetry disappears (1).

In conclusion, with the passing of classical determinism and the advent of quantum physics,the physics of cooperative phenomena (6) and irreversible thermodynamics of systems away from equilibrium (7), the concept of symmetry breaking is assuming ever greater importance in increasingly wide-ranging fields of applicability. It offers a unifying interpretation for the understanding of complex structures and phenomena of the scientific and the humanistic cultures.

Caption for fig. 1

Graphic condensation (by Franco Grignani). The wall common to both cubic moduli of the condensed structure (in the lower portion of the figure) behaves , during the perceptive process, rather like an electron in an hydrogen molecule does during a spectroscopic measurement, under the action of an electromegnetic field whose frequency matches the resonance or exchange integral. Both the dynamic perception of the gestaltic structure and the measurement of the resonance frequency between bond-symmetric and antibond-antisymmetric levels of the biatomic molecule which this structure resembles , take place at the onset of the dynamical instability which marks the disappearance of the center of symmetry.

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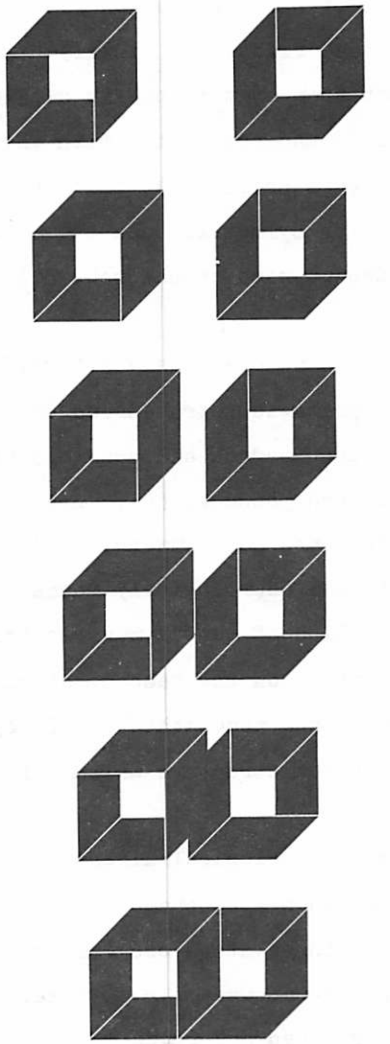

K.I.Chapigeni

DYNAMIC MODELS OF QUASISYMMETRICAL CRYSTALS
(Exhibition)

Classic mineralogical notions about the idealized forms of crystals are exchaneged by homologic ones. Homologic mineralogy is based not on 32 classes of symmetry but on 2I5 classes of homology (I) from which 32 classes of abstract (ideal) crystals are grown as private cases. Apart from symmetrical in quasisymmetrical crystals the axes of homology and conjugated with them elements are not the lines but Fedorov's primas (2). That is why not to mix quasisymmetric bodies with symmetric ones they are called prima-bodies: prima-tetrahedron, prima-octahedron, prima-cube , prima - rhombohedron etc (3).

Of unique importance is homogeneously deformed prima-tetrahedron quanting into four prima-subtetrahedrons and one prima-octahedron. From the prima-tetrahedron all possible variety of quasisymmetric crystal forms are produced (4). Crystals of minerals always, at least a little are not symmetric and they should be considered quasisymmetrical (homological) forms i.e. prima-bodies. The most precise description of quasisymmetric crystal formation (Earth minerals) is obtained on the base of homologic approaches. Prima-bodies as distinct from monolithic ideal crystals have strictly arranged overatomic block structure which defines real properties of crystals.

Connected with this in homological mineralogy under systematic approach they use not a two-stage scheme description (crystal - atomic structure) but a three-stage one (crystal - overatomic block structure , atomic structure). In this method overatomic block structures are described according to the levels of Hierarchy (from large to small). While describing block structures of great importance are compositions on the basis of golden cross-section and connected with it Fabonachi's numbers indicated by the row I,I, 2,3,5,8,I3,2I etc.(3,4,5). Blocks of different morphological types are inseparably linked with initial (starting) prima-tetrahedrons as they are deduced from them by the way of Fedorov's deformations proper (tension , compression, shears, twisting). From the mentioned above point of view the growth of each prima-tetrahedron (block) is described as its growing tension according to its corresponding axes of homology (vicinal growth). In this case the crystal growth is not superficial but a volumetrical process accompanied by plastic deformation of the whole volume and this brings about a correspon-ding distortion of its form. This means that growth deformations build up a real form of the crystal.

As the prima-bodies (forms of real crystals) are quasisymmetrical bodies with irrational (vicinal) facing , on the basis of packing such bodies it is impossible to fill in an endless space as it can be accomplished by packing, e.g. Platon's cube. That is why in every crystal (mineral) which attained a certain size big stresses arise as a result of lack of coincidence in prima-cubes (blocks). These stresses result in forming various deffects (waviness,patchiness, twisting, blocking, cracking). As a result of this, as Denesh Nad stated, symmetry is linked with density (6). Model constructions made as well coincide with the point of view of M. Seneshal who says that"rational structures are particular samples of more common type" (7). In our case for example the structure of Platon is considered as a particular sample of prima tetrahedron structure which in com-parison with Platon's tetrahedron is a more common form.

The procedure of describing the composition and characteristic features of the earth minerals in accordance with 2I5 classes of homology discovered by V.I. Mikheev requires the establishment of the stock of dynamic homologic crystal models able to describe homologic transformations connected with different deformatuins. On the basis of prima-tetrahedron the author has made more than 2000 homologic models of quasisymmetric crystals. In contrast to static models of 32 classes , the crystal models of 2I5 classes are dynamic : they can be subjected to homogeneous deformations of tension,shearing, compression and twisting.

Together with the models which are able to deform without breaking a group of models has been designed which is obtained by shearing an intitial model by its transformation (deformation) and by subsequent gluing, thus forming a new body. Among such models there is for example quasihexagonal prima-antiprism (more common analogue is the hexagonal antiprism of Archimed) transformed into prima-icosider which is more common analogue of Platon's icosider.

At the exhibition there are models on display which illustrate the common analogues of Platon's bodies: of prima-tetrahedron, prima-octahedron, prima-cube, prima-icosider, prima-pentagonal dodecahedron. The evolution of enumerated prima-body forms is investigated with the help of models in the process of their vicinal growth. After the example of diamond,quartz,silicon,gold,zircon,calcium and ather minerals here are demonstrated dynamic models strict geometrically describing (after E.S.Fedorov - crumbled) composition of quasisymmetric crystals produced with the help of golden cross section and Fabonachi numbers. On display there are models of crystals (minerals) of different morphological types including twins inosculated sperolytes, druses, splitted, thread-like, twisted, straight and turned out crystals as well as polygonally pipe-like crystals. At a quite new level dynamic models permit to know the block (regularly quanting) composition of crystals. Such models can be used not only while interpreting the composition natural synthetic crystals and paracrystals but also while investigating and interpreting the composition of biological quasisymmetric crystals (fags in their namber). These modeles can be used in architecture, robot-engineering and in other spheres of knowledge, where are applicable such notions as tension, compression, shearing, twisting and other transformational combinations described on the basis of Fedorov's primas.

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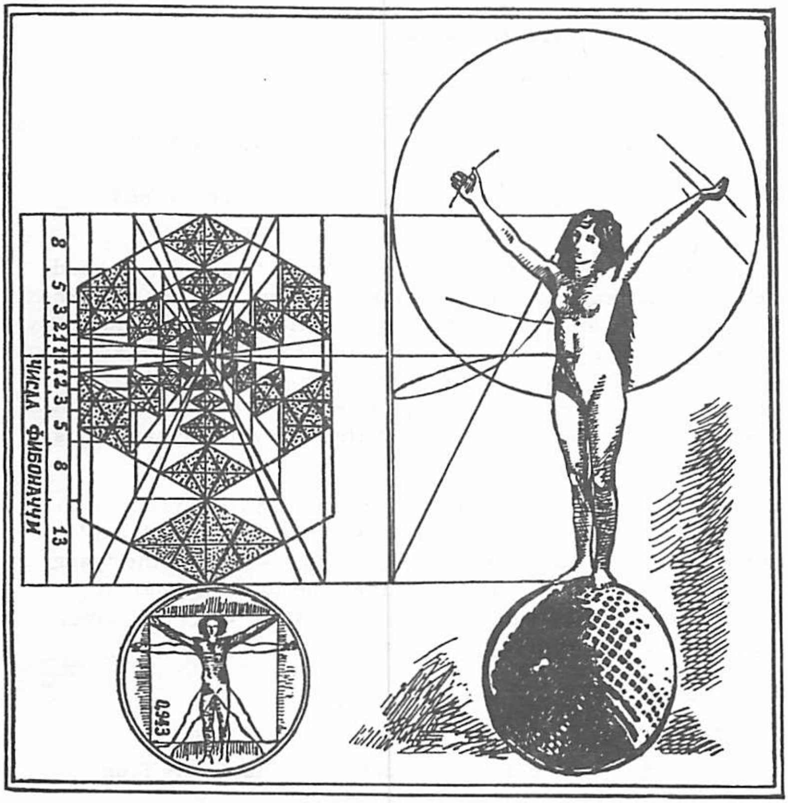

Рис.1 Золотое сечение является той общей платформой, на основе которой описываются объекты исследований гомологической минералогии и гомологической биоминералогии.

В правой части рисунка приведено построение, показывающее, что хорошо организованная фигура человека поясом делится в отношении золотого сечения (61,8% от пяток до пояса и 38,2% от пояса до верхней точки головы). Слева вверху показано золотое сечение природного скелетного кристалла кварца в плоскости 0001. Слева внизу - построение Леонардо да Винчи.

Binary and Cyclic Symmetries in Early Chinese Interpretation of Nature

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The abstraction of binary and cyclic patterns of nature occurred early in Chinese civilization.[1] In the course of time, binary patterns acquired the abstract significance of being associated with productivity and cyclic patterns acquired the abstract significance of being associated with continuity. Such abstractions formed the dynamic bases for the analyses of the processes of nature. The binary dynamics is governed by the 'yin-yang' 陰陽 principle while the cyclic dynamics is governed by 'i' 易, the nature of 'yin-yang' changes.

An early symbolic representation of a binary system is found in the system of 'kua' 卦 constructed in terms of the 'yin' and 'yang', two types of symbols known as the 'yin yao' 陰爻 – – and the 'yang yao' 陽爻 —— . Taking n 'yao' at a time, the total number of combinations, allowing all possible permutations, is 2^n. We have, for example, the system of eight trigrams,

☷ ☶ ☵ ☴ ☳ ☲ ☱ ☰

obtained by taking three 'yao' at a time.

The 't'ai-chi' 太極 model for productions of nature is found in the I Ching 《易經》 (Book of Changes)[2]:

是故易有太極，是生兩儀，
兩儀生四象，四象生八卦。

which in English reads[3]

> Therefore, in [the system of] 'i' 易, there is the 't'ai-chi' 太極 (Primordial Pole) which produces two entities (i.e., the 'yin' 陰 and 'yang' 陽). These two entities produce four symbols and the four symbols produce the eight 'kua' 卦 (i.e., the trigrams).

Matthew Ricci interpreted the 't'ai-chi' as the 'materia prima' of the Scholastics.[4] John Rodriguez, later in 1631, made the following comments[5]:

> According to the Occidental interpretation, the Great Ultimate (i.e., the 't'ai chi') is something material, is matter without intelligence and without consciousness. Unless there is the infinite, omnipotent, wise, and intelligent factor, how could it ever produce things?

The fact that the 't'ai-chi' model for productions of nature contains no elements of creationism was a source of difficulty for occidentals in this early period of scientific revolution. The 't'ai-chi' model views productions as natural processes governed by the binary dynamics of the 'yin-yang' principle which are spontaneous, requiring no infinite, omnipotent, wise or intelligent factor.

The essential feature of the 't'ai-chi' model lies in the symmetry of the 1, 2, 4, 8 binary pattern. In fact, the statement for the 't'ai-chi' model given above is an algebraic statement of the 2^n binary expansion for $n = 0$ to 3. This symmetry is best illustrated in terms of the 'yin yao' and 'yang yao' symbols (see Fig. 1).

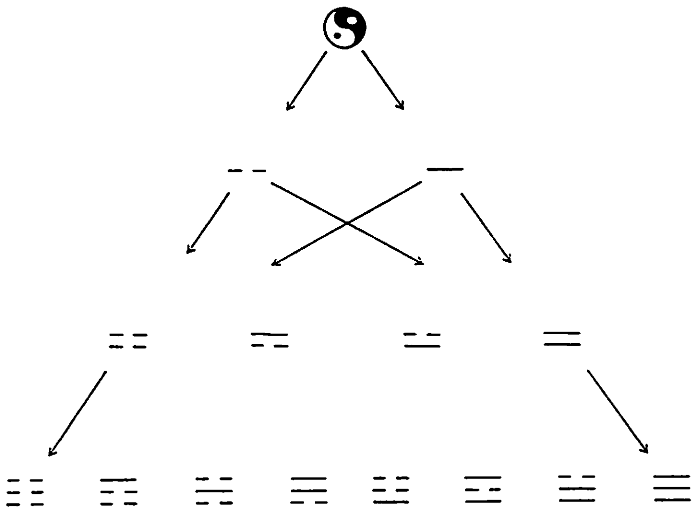

Fig. 1. The 't'ai-chi' model for productions of nature.

The algebraic nature of the binary expansion is evident since the elements in the expansion are open elements and may be variously substituted.

In favor of interpreting productions of nature as being ruled by chance, the early Chinese devised a divination procedure, known as the 'ta-yen' 大衍 procedure,[2] to consult with the 't'ai-chi' model. After the divinatory rhetorics are removed, the procedure may be stated as follows[6]:

(a) Divide the total 49 sticks into two portions arbitrarily and then remove one stick from one of the two portions.

(b) Count the sticks in each portion by fours, and then remove the remainders. (For the portion by fours evenly, take the remainder to be four.)

This simulation of the 2^n binary expansion certainly was intentional.

The question that one must address is whether the inventor of the 'ta-yen' procedure was aware of the mathematical content of his procedure. There is no question that the procedure was devised with the desire to simulate the binary symmetry of the 't'ai-chi' model. But this can be accomplished within the scheme of the procedure by simply imposing the appropriate conditions on the remainders resulting from counting the sticks by fours. The presentation of the procedure was, however, purposely made intriguing by aribtrarily allowing the division of the total number of sticks into two portions. To foresee the possibility of such a division necessitated an understanding of the mathematical content of the procedure. Despite the fact that the 't'ai-chi' model has long been superceded by other models, the use of symmetry in the model and the attempt to relate mathematics with the model at this early stage of our conceptualization of nature remains to be of great interest to the history of science.

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5. These comments are from a letter written by John Rodriguez in 1631. The letter is still preserved at the library of the University of Seoul and has been published in the Shigaku Zasshi 《史學雜誌》 (The Historical Journal of Japan), vol. XLIV. The translation in English was made by Rufus Suter and Matthew Sciascia, see Pasquale M. D'Elia, Galileo in China (Harvard University Press, Massachusetts, 1960), p. 43.

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(c) Take the remaining sticks (i.e., the total number of sticks minus those removed) to be the new total.

(d) Repeat the procedure two more times.

In current algebraic notations, the 'ta-yen' procedure may be written in terms of the following equations[6]

(1) $x = 4n_1 + R_1 + 1\ ,\qquad 4 \geq R_1 > 0$

(2) $y = 4n_2 + R_2\ ,\qquad 4 \geq R_2 > 0$

where n_1 and n_2 are positive integers, R_1 and R_2 are the remainders to be removed, and $x + y$ is the total number subject to the procedure.

Beginning with the number 49, we have $x + y = 49$. Eqs. (1) and (2) then reduce to

$$4n + R = 48\ ,\qquad 8 \geq R > 0$$

yielding two possible solutions

$$n = 10,\ R = 8\ ;$$
$$n = 11,\ R = 4\ .$$

This gives two possible paths, the path associated with the first solution with the number 40 and the path associated with the second solution with the number 44. The procedure is then repeated with one of the two numbers to generate the next possible paths. It is significant that the 'ta-yen' procedure allows a random generation of the paths and the paths so generated simulate the 2^n binary expansion (see Fig. 2).

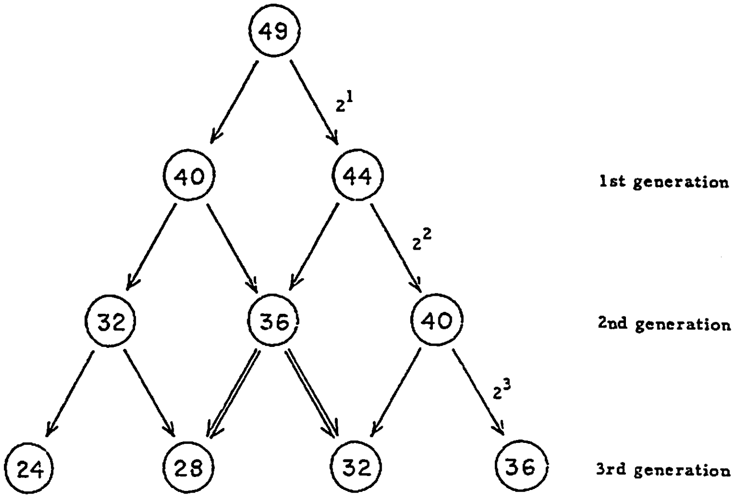

Fig. 2. The 2^n binary expansion for the paths generated by the 'ta-yen' procedure.

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MEYERHOLD'S BIOMECHANICS: HISTORY OF THE TERM AND ITS RELATIONS WITH CONTEMPORARY SCIENCE

"Man - machine" metaphor came into existence as early as in the I8th century in the epoch of French materialism. The metaphor was becoming a reality while machine constructors started imitating living organism employing the language of their contemporary technology. In I920ies in the Soviet Russia certain social conditions were such that "man - machine" formula came into use in the ordinary sense of the word. It was connected with the necessity of quick speeding up labour productivity, with the strive for the soonest scientific and technological progress following the social revolution.

Artistic culture of that period should be regarded in the context of scientific and technological thought. This opinion has been reaffirmed by the experience of studying theatrical art of I920ies, especially the works on biomechanics by the great theatre producer V.E.Meyerhold (I874 - I940).

In I922 Meyerhold was lecturing at the Higher State Theatrical Studios and called his pupils assistants as if they were employees of a scientific research institution. Meyerhold's experiments in stage direction and theory of theatrical education were akin to those of Central Labour Institute founded in I92I where the outstanding neuro- and psycho-physiologist N.A.Bernstein was working.

The genesis of the term "biomechanics" has not been completely cleared out in the Soviet science. The first Russian handbook on the subject was published in I926 (Bernstein N. Biomechanics for Instructors). The author defined biomechanics as a science dealing with the way "the living machine, i.e. every human being, is constructed, what the design of its moving parts is, how they work".

Meyerhold's biomechanics in everyday meaning of the term had come into being even earlier. It was a school for training actors and a practical system of their stage existence. He put forward an idea of "showing perfectly organized human mechanisms on the stage".

Another stage producer of theatrical avant-garde N.M.Foregger (I892 - I938) was developing - parallel to Meyerhold's biomechanics- his own system of theatrical and physical training. His demonstra - tion of perfectly organized movement resulted in the imitation of labour process, of various productive mechanisms operations and was called "machine dances". "Machine dances" were actors group formations with various types of symmetry. Looking-glass-like similarity of performers' groups was stressed by uniformity of their accentuated geometrical costumes. Actors were imitating the work of transmission, rotating shafts, the movement of driving-belts, transporters, pistons, wheels. Complex combinations of simultaneous turns, gestures and movements of arms, legs, trunks, heads were included in a geometrical stage volume and were carried out in accordance with the rules of rotary and translational symmetry. Coordinate axes in this movements system were stage dimensions. Performers' physical actions were constructed according to mathematical laws deduced from living nature observation, from biological systems by productive mechanisms inventors. In "machine dances" these laws were visually demonstrated by people.

Accurate and reasonable movements were, thus, subjected to the one and the same algorithm defining the essence of a device opera - tion, of a machine detail, of their dynamic peculiarities.

Biological symmetry laws and accentuated geometrical forms in the epoch of I920s constructivist art had their logical meaning, distinct esthetic ideals and social landmarks. From theatrical biomechanics point of view, these exercises were important for defining an actor's physical actions morphology, studying interconnection of movement elements and perfectioning performers' technique.

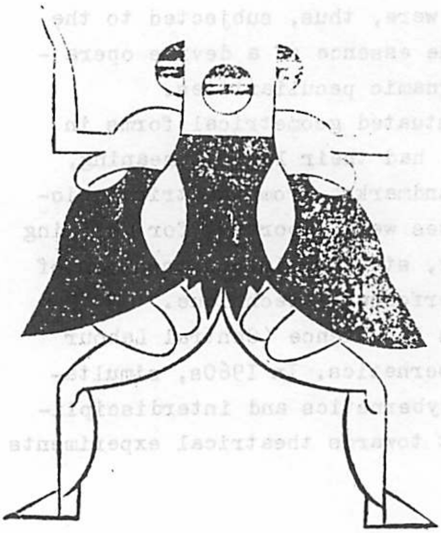
Parallel biomechanical experiments in science (Central Labour Institute) foretold the concept of cybernetics. In I960s, simultaneously with the exuberant growth of cybernetics and interdisciplinary branches of science, the interest towards theatrical experiments of I920s reappeared.

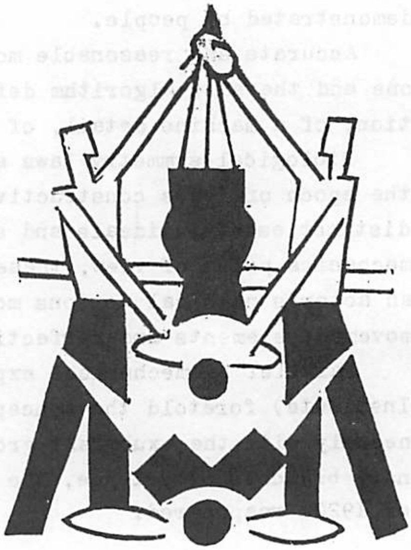
as well as some peculiarities of complex artistic thinking and scientific approach of modern prominent figures of stage towards the problem of creation.

From modern scientific point of view, Meyerhold's biomechanics and Foregger's "machine dances" have helped to define structural mechanisms of biological mobility. For them movements and structure symmetry was a device for studying and creating beauty, purposefulness and perfection in a given epoch. Movements algorithms were determined by dynamics of modern processes.

In present conditions of complex approach towards art and progressive theatrical figures' scientific thinking, artistic problems algorithms are becoming still more complicated. Complex symmetries concept helps modern artists to realize new esthetic principles of the late 20th century.

The main ideas of the present extended abstract have been presented by the author in the article "Theatre is a non-euclidian space" published in the magazine "Theatre" (I986, N II, p. I6I - I69).





Механические танцы Форегера.

SYMMETRY PRINCIPLES IN NONLINEAR CIRCUITS

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Various forms of symmetry can be identified in a large class of nonlinear circuits. This paper will present an in-depth analysis of the more subtle forms of symmetry principles and their circuit-theoretic implications. The properties of <u>reciprocity</u> and <u>anti-reciprocity</u> and their implications in terms of various stationary and variational principles in nonlinear circuits will be analysed. In particular, decomposition of an arbitrary vector field in terms of a reciprocal and solenoidal n-port will be presented. Finally, the symmetry principles will be used to relate nonlinear conservative circuits to Newtonian mechanics and thermodynamics.

KARL CLAUSBERG

SYMMETRIES IN ART HISTORY — 'ACTUAL GENESIS' AND THE 'AESTHETICS OF RECEPTION'.

An introductory abstract

In my lecture I shall focus upon an abandoned if not forgotten aspect of perception and artistic creativity and try to put it into a new frame of reference. I am speaking of the findings on *Aktualgenese*, which were initially presented by members of the new Leipzig School of psychology in the second quarter of our century. The new framework proposed is the grand and supposedly unifying theory of reception, wich has been recently adopted by art historians. My task will be a threefold one:
1) to summarize briefly the main characteristics of these two seemingly incompatible concepts in order to mark promising fields of interchange connecting psychology and art history;
2) to point out where interdisziplinary reasoning, running for decades on almost parallel tracks, was derailed and aborted - and where it might be taken up again using further evidence from the abundant fields of art history; and
3) last but not least to consider the role of symmetries as some sort of catalyst or fallout of likely feedbacks between 'actual genesis' and 'reception'.

'Actual Genesis'

The term *Aktualgenese* was first used by Friedrich Sander (1928), one of the junior stars of the 'new' Leipzig school of psychology. Due to ambition and/or conviction Sander turned into an open supporter of fascism. Thus, research into the phenomena of 'actual genesis' appears to be akin to and stained with his later political orientation. It is debatable, however, whether Sander himself was actually the first to notice the peculiar effects of *Aktualgenese*; perhaps the credit should go to his pupil Wohlfahrt (1925-/1932) instead. Anyway, Sander was a clever and quickwitted science manager, who instantly grasped the importance, put a catching label to it and instigated, supervised and edited a highly remarkable series of his pupils' studies during the thirties.

At that time in Germany, at least two major schools of psychology had been competing for dominance (A. Wellek, 1950). There was a young and vigorous band of Berliners, Köhler, Koffka and Wertheimer; they were to become the world-famous protagonists of *gestalt* psychology. The even 'newer'

Leipzig school was headed by the less well known Felix Krueger. The latter had been called a successor to the late pope of experimental psychology, Wilhelm Wundt. Against the bold but simple "theory" released from Berlin the Leipzig school set a much more complex and flexible *Ganzheitspsychologie* which never made it into the public vocabulary of science by slogans even though it had some stunning discoveries to offer.

The progenitors and adepts of *gestalt* psychology were and still are convinced to have reached the unalterable foundations of human perception and understanding. Their belief rests on a twofold set of well defined, terse, shapes (*prägnante Formen*) coexisting inside and outside the head, one governed by physical topology of the brain, the other by analog neural representation. Every time a match is attained between one of the preconceived pairs, the person experiencing this falling into place receives a cognitive kick ranging from simple satisfaction to higher revelation. Even though this notorious theory was based on experimentally justified fundamentals, its authors and followers advanced farflung consequences. These - often critizised - conjectures related certain outstanding properties of perception with invariants of visual thinking aloof to historicity.

Komplex- and *Ganzheitspsychologie*, as conceived by the Leipzig school (F. Sander / H. Volkelt, 1962), didn't place persons and their experiences - even under reduced experimental conditions - outside history. The sharp precision of geometric shapes said to be pushing along *gestalt* perception was, according to Leipzig scholars, only a minor subset of rather undefined complexes carrying in addition moods and feelings like affluent colour-tinting. Cognition and emotion were seen as inseparable states of overall attitudes which made elementary mechanisms of perception highly susceptible to the momentary environment, i.e. historical conditions. Fear or happy expectation can easily trigger, as we all know, quite different apparitions from identical sets of sensory data. This kind of commonsense wisdom was preserved and put into no uncertain scientific terms by members of the Leipzig school. Especially the 'actual genesis' of shapes and objects taking place on the doublesided projection screen of the eye bore testimony to the highly diversified strategies persons apply to handle and match incoming data to already preconceived or quickly contrived scenarios.

'Reception'

The 'aesthetics of reception' resemble a more or less coherent bunch of concepts specifically concerning the reader's part in literature. Rooted in and derived from ideas propounded by Prague structuralism, reception theory is frequently considered a domain if not invention of the school of Konstanz (H.R. Jauss 1970; Wolfgang Iser 1972). Beside structuralism proper and the pure dogma of semiotics the receptionist theory has clearly become the most pragmatic and promising of the systematic ventures to overcome modern solipsism, which had put the creators and receivers of artificial communications into separate 'black holes'. While literary receptionism has been debated extensively for more than two decades (R. Warning ed., 1975), translation into the fields of art history has only recently been advocated (W. Kemp ed., 1985).

The crucial postulate of receptionist aesthetics bestows upon any adressee of messages a vital part; it is comparable, though by no means similar, to the labour of the sender. The receiver has to fill in, to complete, to reenact the script or picture from his own point of view. The corollary of such distributed effort obviously can be stated as a question: To what degree can and will intention determine the empty elements reserved for possible participants in these artificially delayed – alas: asymmetric – communications? Some theoreticians (W. Kemp, 1985, p. 205) blandly state that all art is deliberately designed to accomodate or even to impersonate the receiver! On the other hand apparently the proverbial creativity of missunderstanding would aptly provide a third cornerstone for receptionistic concretion of artful subjectmatter. And from there we should turn all the way around and consider the possibilities of actual-genetic imagination within the framework of aesthetic production and reception.

Symmetry

Since one of the characteristics of actual-genetic evolution is the regularization of features, meaning in most cases an application of symmetries, it stands to reason to take symmetry as a standard of reference when the effects of fading memories or changing interests do their subconscious modelling of ideas and shapes. On the other hand perhaps the historical evidence preserved in the works of art may shed additional light upon shifts and mutations in the emergent or waning symmetries.

Evidence

Some of the most intriguing staging aereas of actual-genetic processes seem to be accessible in mediaeval miniature-paintings. Evidence ranges from obvious scientific representations recycled through repeated copying – for instance *Mappae Mundi* (Clausberg, 1988) – to the very realms of visionary enlightenment as preserved in some outstanding works by Hildegard from Bingen (Clausberg 1980). It can hardly surprise that science and hallucination not only coexisted, but depended upon each other where almost no external first hand observation was possible and methods of verification were quite different from our modern standards. Imaginations of the inner eye resembled and initiated recurrent loops of perception, memorizing and reproduction to be eventually put down on parchment and explained by written commentary. Looking backwards through art history we may perceive these visions materialized and stretched into iconographic chains of pictorial proliferation. Another, more recent case of selfcentered 'actual-genetic reception' may be found in the work of William Turner, who is famous for his regress to almost abstract patterns of light.

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DYNAMIC SYMMETRIES IN THE MOLECULE-LIKE STATES OF ATOMIC NUCLEI

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Inspite of its small size the atomic nucleus is a composite object, and its structure shows considerable variety. From the viewpoint of theoretical description the number of its constituent particles, called nucleons, is an important parameter. It ranges between 1 and 260, being too large for most of the nuclei to treat each nucleon individually, and too small to treat them statistically. Due to this situation nuclear models play a crucial role. They are simplified theories of the nuclear structure which make the problem tractable but, hopefully, do not neglect the most important features. There are quite a few of them, giving good approximation to different structures. A model is called microscopic if its basic picture is that of the many-nucleon sytem, and it is called macroscopic or phenomenological if the nucleus is treated as a whole.

During the last one and a half decades several models were introduced which make use of the mathematical tools of group theory. Among them we can find the interacting boson models. The best-established and most successful version of them describes the rotational-vibrational motion of nuclei having either ellipsoidal or spherical shapes, or their shape change dynamically between these limits (Iachello and Arima, 1987).

In this model the building blocks i.e. the bosons have dual interpretation. Phenomenologically they are excitations of the collective motion, and microscopically they represent nucleon pairs.

Another interacting boson model proved to be succesful for the description of the rotational vibrational motion of molecules (Iachello and Levine, 1982).

There are good reasons to belive that some nuclear states are similar to molecule-like configurations, having dumb-bell shapes rather than ellipsoidal ones. Based on these arguments suggestions were made to apply the interacting boson model of molecules to these nuclear states (Iachello, 1981; Daley and Iachello, 1986). This can be done after some simple modification of the model assumptions (Cseh and Lévai, 1988).

The group theoretical models are intimately related to the concept of symmetry. By symmetry we mean the invariance under a transformation. Since the atomic nucleus has a shape, there is a possibility for shape symmetry, and many of the nuclei show it.

There is, however, an additional possibility to display symmetry; and this is related to the Hamiltonian of the system. Just like for any other microphysical object, for the atomic nucleus the most characteristic quantity is the energy of the sysytem represented by an operator called Hamiltonian. The symmetries of the Hamiltonian are deeper symmetries of the structure, they are consequences of the special type of the interaction between the constituent particles, i.e. they appear due to the special form of the Hamiltonian. These symmetries are referred to as dynamic symmetries, and they correspond to the limiting cases of the algebraic model.

The Hamiltonians corresponding to dynamic symmetries provide us with energy spectra which can be compared to those of the experimental studies. It turns out that for many nuclei the model spectrum approximate the real one fairly well (Iachello and Arima, 1987). An example of this kind (Cseh, 1988) is shown in Fig.1.

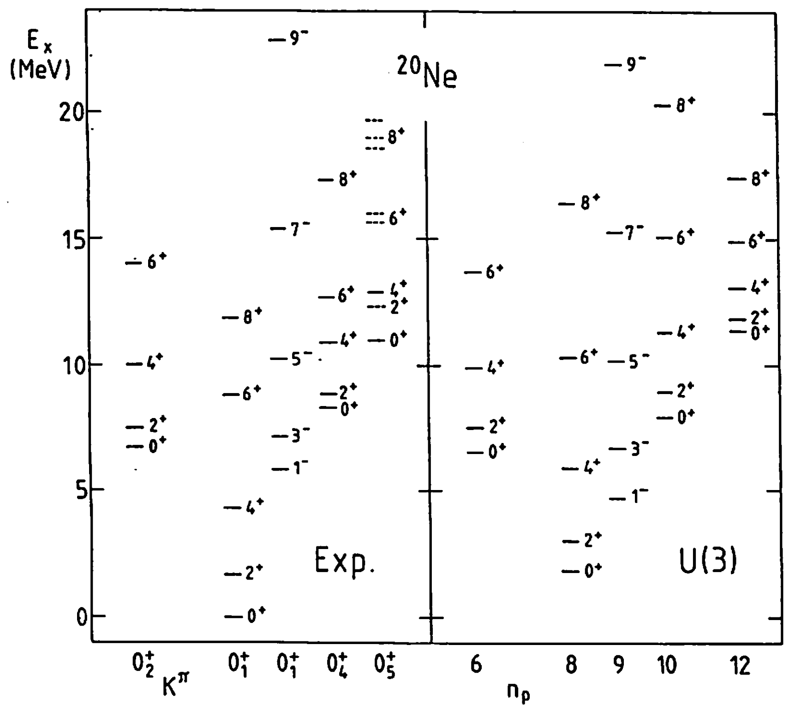

Fig.1. *Molecule-like states of the ^{20}Ne atomic nucleus in comparison with a model spectrum corresponding to a dynamic symmetry.*

To sum up: the dynamic symmetries of the nuclear and molecular physics are model symmetries, because the symmetry transformations act in a model space. They are approximate or broken symmetries rather than exact ones. The dynamic symmetries give the connection between the macroscopic or shape symmetries and the microscopic symmetries of the system . The reason why these nice features of the concept of dynamic symmetry are really valuable is that Nature shows evidence for the approximate realization of these symmetries.

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SOME ASPECTS OF THE PROBLEM OF SYMMETRY IN THE PYTHAGOREAN "MUSIC OF THE SPHERES"

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The disciples of Pythagorus conceived of the world as a manifestation of harmony "which comprises everything and exists in everything". But it is a kind of hidden harmony and it makes itself evident in the way the planets' movement is organized in the cosmos. According to this view the Cosmos in its variety and man as a microcosmos were considered commensurate, reflecting in their relationships "numbers" and "Numeric relations" as an invariant manifestation of "God's wisdom" (Sophia). They seemed to discover in numbers many similar characteristics with everything that exists and happenes - more than in fire, earth, water ... /Aristotle,1934/

Numbers and numeric relations were perceived as the beginning and becoming of everything that has structure as the basis for an ordered miscelaneous world, built on the principle of harmony and on a particular summetry which is to be found in the so called musical harmony.

The disciples of theorphic-pythagorean school claim that the appearance of number and numeric relations in the Universe in man and in human relations (ethical as wellas aestetic) contains in themselves one common invariant - harmoneous musical relations. As Aristotle rightfully suggested" ... thel saw in numbers qualities and relations characteristic of harmoneous structures..."/Aristotle, 1934/. So composition of the universe is seen as dependant on musical relations. This explains the so called "the music of Spheres" which finds its geometrical expression in the "golden proportion".

It should be noted that numbers and numeric relations were interpreted not only in terms of quantity but in terms of quality as well.

If we consider christianity - the most syncretic religion of all,we also find meaningful numbers. Satan's associates are seven and so are the realm of heaven.

One should note olso that the sum of the seven tones of the Diatonic (the european musical system) and the five tones of the pentatonic scale (the eastern asiatic musical system) comes to the twelve tonic classical chromatic scale, that is, the sum of the black and white keys of the piano.

If we consider the classical sonnet excluding the final couplet, we shall be left with the three basic four line couplets - 12 lines in all. Such is the number of the tribes in ancient Israel. Christ's apostles were twelve too. The year has 12 months in agreement with the signs of the sodiac.

Let us consider the situation in Greek mythology: the "argonauts" in the story of Jason and the golden fleece are twelve. They appear to be the greatest heroes in Illium. Could the dosen be considered as a casual invention by the people. The orbit of every planet comprises one "octave" which is divided into 12 parts. The movement of every planet around its orbit at every moment crosses these intervals. In this way different tones could be "heard" and so we have a cosmic music.

It should be noted that the monochord was invented by Pythagorus and it was this instrument that helped the great philosopher and scholar make his great discoveries.

He noticed that if a string is broken it raises the tone one octave. Acoustically this doubles the frequency. On the basis of this law the ancient philosopher invented his own order based on the qaintile pronciple.

Pythagorus has own hypothesis about the"music of the spheres". He assumed that each planet emits its own sound, which the human ear can not hear. Pythagorus looked for that which is common for the gravitational centres of the planets and musical tones.(Dankov - Nagy Denes,1984).

Understood in Pythagorean terms "harmony" includes all kinds of synchrony or conconance which are expressions of a particular kind of summetry. In music this harmony "based on the law of accoustics" with which the idea of "the music of spheres" is associated. There are three main funktional elements forming a triad. They are marked in the following way:

Tonic	- T
Subdominant	- S
Dominant	- D

They exist in a strict functional dependence and all musical compositions are based on them.

The pythagoreans found strict relationship between the order in musical functions and the order of planets in the solar system.Predominance is here given to the triad in the quality of an equivalent that finds expression in the uniform structure of the chords and theplanetary cycle.On this principle three "inner" planets stand distinct and can be seen with the naked eye. They are Mercury,Venus and the Earth. Next came three "outer" planets which can also be seen with the naked eye: Mars, Jupiter and Saturn. The remaining planets which had been discovered with the help of the telescope are also three: Uranus, Neptune and Pluto. There also as in the diatonic (which has three basic tones) exists a relationship of three elements to the centre of gravity: the Sun - planet - satelites - exactly as is the case with the chords.

The principle of summary in the musical acoustics was seen by pythagorean as a heuristic tool for the expression of certain laws in the order and the movement of the planets. This discovery was highly esteemed by Newton himself. He believed that the image of the Pythagorean lyre holds the sicret of the law of gravitation. The music that comes from the seven strings of the harp were seen by pythagoreans as an expression of the"music of the spheres". Newton was surethat sounds and tones are determined by the length of the string in the same way in which gravitation is determined by the thickness of the matter.Thus he rightfully believed that Pythagorus expressed in different terms what people before him had known. In connection with that we should note, that the four constant strings of the lyre stand in relation 6:8 = 9:12, or to put it in another way are in "golden proportion".

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Psychological Research on Perception of Symmetry

by Prof. Jan B. Deręgowski

The paper presents a brief review of psychological research on perception of symmetry and includes some recently obtained data.

It is argued that the perceptual effects of symmetry cannot be simply explained by considering symmetrical patterns as highly redundant (since one enantiomorph predicts fully the other), when compared with asymmetrical figures having the same number of cells. Symmetry it is claimed to bestow is a special perceptual attribute.

The role of this special attribute in perception of flat patterns and of patterns which although flat are seen to have depth, is examined. For the 'flat' patterns new empirically obtained data describing the relationship among the enantiomorphs of a pattern is adduced and their relevance to perception of symmetry of patterns and confusability of symmetrical patterns, such as letters b, d, p and q is considered.

The role played by symmetry of those patterns which although flat are seen as three-dimensional is analysed with special attention being paid to data obtained from pictorially relatively unsophisticated cultures.

ASYMMETRY AS THE NECESSARY CONDITION FOR MASSTRANSFERE BY TRAVELLING WAVES IN SOLIDS, LIQUIDS AND GASES

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Some theoretical and experimental investigations of travelling waves of deformation in solids, liquids and gases are described. The examples of the waves analysed are well known travelling waves on water surface, waves of deformation travelling along the bodies of moving caterpillar, earth-worm, terrestrial snake, fish. The peristaltic waves in digestive system of animals and men, the wave-like motion of elastic unit of wave (harmonic) mechanisms and machines and at last the global tidal waves generated in the Earth and planet bodies by gravitation forces are analysed as well.

The examples of wave deformation on elastic elongated bodies, solids and liquids are presented in Fig.1. The global tidal waves on the Earth crust generated by the Moon gravitation and the Earth rotation are shown in Fig.2.

The theoretical investigation of mentioned waves were carried out from physics, mechanics, hydromechanics points of view. The special devices for modelling and demonstration of masstransfere ability of travelling waves in solids, liquids and gases were designed and made.

The general one-dimensional model of all above mentioned waves is proposed. This is wave of linear density made by the body mass projection on the x-axis. Function of linear density of such projection is also wave-like. The properties of this model make it possible to find out following parameters of travelling waves : averaged step of particles, averaged velosity V_x in arbitrary cross section of canal, instanteous flow rate, masstransfere caused by running waves.

The main conclusion of our analysis is : only asymmetric relatively neutral condition wave can transfere the waved body mass. It is shown that masstransfere process is relay-like (substitutional). It means that transfered mass is continiously renewed during wave moving. The parameter of wave masscontent (Δm) equal to excess ($+\Delta m$) or mass deficit ($-\Delta m$) in wave crest or trough respectively is introduced (shadowed in Fig.1, c,d,g). It is shown that the travelling solitary waves with positive masscontent ($+\Delta m$) transfere the mass forward (Fig.1, b,c,g) and the solitary wave with negative masscontent ($-\Delta m$) transfere the mass backward (Fig.1, a,d).

The examples of symmetric (and therefore not mass-transferring) travelling waves are wind driven sea waves, waves on the water surface caused by fallen stone, acoustic waves, wind driven grassy field waves. The examples of asymmetric (and therefore mass-transferring) waves are the waves travelling along the body of moving caterpillar, earth-worm, peristaltic waves, waves in the elastic elements of wave mechanisms, tidal waves in atmosphere, hydrosphere, lithosphere and in internal part of the Earth and other planets.

Our theoretical and experimental analysis of asymmetric travelling waves led to some non-traditional approaches in different fields of science and technology :

- In biomechanics (Alexander, 1968). Contrary to the existing view (Gans, 1966; Trueman, 1974) our analysis shows that the locomotion of terrestrial snakes is based not on the "slope plane principle" but on the principle of travelling asymmetric waves of deformation (Dobrolyubov, 1986,b). It is also shown that peristaltic transport is based on the above mentioned ability of asymmetric longitudinal travelling waves to transfere the mass (Dobrolyubov, 1986,a).

- When applied to mechanism and machine theory the analysis of asymmetric waves travelling along the elongated elastic bodies (belt, rope, flexible band, chain etc.) permits to create new wave (harmonic) mechanisms and devices (Dobrolyubov, 1984).

- In geophysics :

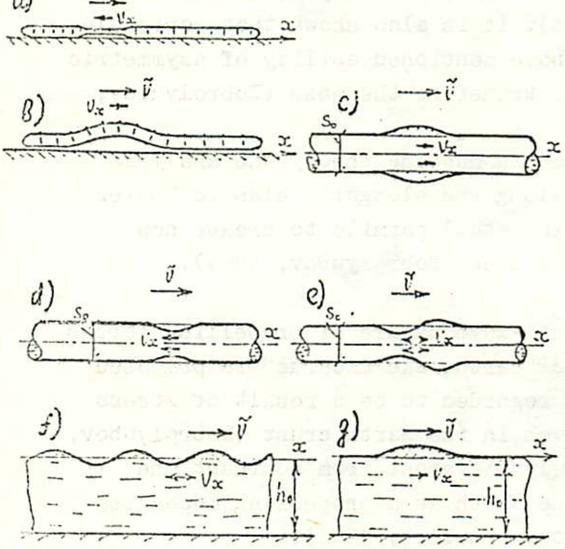
-- the hypothesis about tidal-wave nature of propelling forces of global tectonic processes and "earthquake machine" is proposed and substantiated. Earthquake is regarded to be a result of stress accumulation caused by tidal waves in the Earth crust (Dobrolyubov, 1987). This hypothesis is entirely different from dominant one about convective processes in the Earth as a propelling mechanism of the global tectonics (Turcotte, Schubert, 1982);

-- the hypothesis about transportation of ocean water from East to West by tidal waves generated by the Moon and the Sun gravitation is proposed and substantiated. It is shown that the largescale currents in the tropic oceans (Atlantic, Pacific, Indian) are the symmetric towards equator circulations which are a mirrow reflection of each other. This geographic argument says in favour of our hypothesis (Dobrolyubov, 1987);

--the conseption of masstransfere by asymmetric solitary waves is used for explanation of superrotation of the Earth and other planets atmosphere and solar corone. With the help of our hypothesis we gave new explanation of the Uranus atmosphere circulation picture delivered by Voyager-2 in January 1986 (Ingersoll, 1987);

--the hypothesis about wave peristaltic nature of driving mechanism of hydromagnetic dynamo which controls magnetic field of the Earth is proposed and substantiated. The peristaltic motion is in toroid ring canal in the Earth melted core (Dobrolyubov, 1988).

During the lecture the photographs of special devices demonstrating masstransfere ability of asymmetric travelling waves will be displayed.



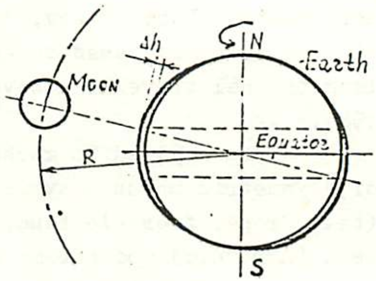


Fig.2 Scheme of generating global tidal travelling waves in the Earth crust.

Fig.1 Different kinds of travelling waves of deformation: a,b - longitudinal and transverse asymmetric waves on the bodies of caterpillar and earth-worm; c,d,e - peristaltic waves: asymmetric (c,d),symmetric (e); f,g - waves on the water surface: f - symmetric towards undisturbed (neutral) level, g - asymmetric.

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CRYSTALLOGRAPHIC STRUCTURES AND SPACE PARTITIONS: NEW TRENDS OF THE THEORY

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1. LOCAL THEORY OF REGULAR POINT SYSTEMS AND SPACE PARTITIONS.

What convex polytopes can tile Euclidean space in a regular way? What way can we enumerate all their combinatorial types ?

Answers to these principal questions are related very closely with important results obtained by collaboraters of Delone's geometrical school in the middle of 1970-s.

First of all let's remember the definition of the *regular system* (crystallographic monostructure) as a point set S in the d-dimensional Euclidean space $\mathbb{R}^d$ which fulfils next conditions:

(i) The point set S is Delone (r,R)-system. That's here r is a lower limit of distances between any two points of S and R is such a value that any ball of radius R contains at least one point of S.

(ii) The point set S "looks" the same if seen from every point of S.

The first condition is of a general kind and is fulfilled in a rather wide class of point sets. It's important for instance in the theory of amorphous maters. The second one characterises the class of regular point systems only. Indeed that condition predetermines the set S to be a point orbit with respect to some Fedorov space group.

The condition (ii) of a regularity has a "global" sense: *infinite* sets of *all* straight line segments, drawn from any point of system S to its remaining points, must be congruent. Therefore it was naturally trying to find some "local" substitution of that condition. In 1974 a problem of such a kind was supposed by Delone and Galiulin. Soon the grounding answer was found [1]:

Theorem 1. For a given dimension d there exists number c_d (it can be calculated effectively) such that if for every point of the (r,R)-system S its neighbourhood of radius $c_d R$ is the same, then the system S is regular one.

The main idea of a proof of the theorem (Dolbilin & Stogrin) consists in using of the fact that a sequence of finite groups of neighbourhoods of fixed point, when they are being extended, is stabilizing. As further investigations have shown the notion of the stable group had been introduced in [1] is very essential for that problem. Here we are thinking about the problem of a determination of regular systems by means of a comparison of point neighbourhoods of the discrete system with themselves. An underestimation of this fact reduces to an appearance of probable confirmations but for the time been unfounded ones, for instance that $C_3 \leqslant 6$.

It's clear that the value of c_d can't be taken too small. The main idea of a proof of the Theorem 1 allows to express it in the next more concrete form:

Theorem 1a. Let S be a (r,R)-system. Assume that for any points A, $A' \in S$ the sets of straight segments, drawn from points A and A' respectively, are congruent within a finite sphere of radius $(\nu + 2)2R$. Here ν is the number of prime factors in order of group H_0, where H_0 is the complete group of rotations around the point A (or point A' no importance) moving the point subset of the set S laying inside a sphere of radius $2R$ and with its center at the A (or A' respectively).

The final result in the case $d = 2$ belongs to Stogrin. He has proved that $c_2 = 4$ and the value can't be diminished. The analogous result but under more weak assumption as compared to the condition of congruency has been obtained by Dolbilin:

Let S be a (r,R)-system in the plane, and for $A \in S$ let $S_A(\rho) \subset S$ be a set of all points $A' \in S$ with $dist(A,A') \leqslant \rho$. Let also $N_A(\rho) = \{ dist(A',A''),\ A'A'' \in S_A(\rho)\}$, that is, the number set $N_A(\rho)$ is the set of all distances between each two points of $S_A(\rho)$.

Let's now assume that the (r,R)-system S is such that for each their two points A and $B \in S$ the sets of distances $N_A(\rho)$, $N_B(\rho)$ coincide:

$$N_A(\rho) \equiv N_B(\rho)$$

for any ρ with $0 \leqslant \rho \leqslant 2R$. Then the system S is regular.

This at the first vue very dificulte condition indeed is much weeker than the condition of congruency of neighbourhoods $S_A(4R)$ and $S_B(4R)$, which as it's enough easily to see implies the condition $N_A(\rho) \equiv N_B(\rho)$ for each $\rho \in [0,\ 4R]$. The controverse affirmation isn't trivial and its proof is based on a revise of monstrous number of combinations and we haven't full assurance in their completness.

Let $d = 3$. With the help of one Stogrin's lemma it's not dificulte to prove that we can take as a value of c_3 a number 14. It was more dificulte to prove that it has been proved that $c_3 \leqslant 10$. P.Engel [6] gave an example of the partition disproving the probable conjecture ($c_3 = 4$) and demonstrating $c_3 > 4$.

Recently **there was** announced the generalization of the local theorem for any crystallographic structure, that's, for multiregular systems but not only simple regular ones (Dolbilin & Stogrin [5]). The results have been received by authors it has to say more ten years ago.

On a base of the idea of a stabilization of finite symmetry groups and with the help of topological reasonnings similar to Poincare's ones used by him in his works on the theory of automorphic functions, there has been obtained a geometrical criterion of the convex polytope being a stereotope (Dolbilin [3]). Let's remind that a *stereotope* is called a convex polytope being a tile of some regular partition with transitively acting on it Fedorov group.

Let P be a convex polytope and $C_1, C_2, \dots, C_n$ be its coronas (if there exists they) consisted of congruent to P polytopes (for the definition of corona see, for instance [6]).

Let also $H_0 \supseteq H_1 \supseteq H_2 \supseteq \dots \supseteq H_n$ be a sequence of symmetry groups of the polytope P and of its following envelopes

$$\mathbf{E}_1 = P \cup C_1,\ \mathbf{E}_2 = P \cup C_1 \cup C_2, \dots,\ \mathbf{E}_n = P \cup C_1 \cup C_2 \cup \dots \cup C_n$$

respectively.

Theorem 2 (the criterion of a stereotope). Assume that for a given convex polytope P there exists such a sequence of its coronas $C_1, C_2, \dots, C_n$, which fulfils next condition:

in the corresponding sequence of symmetry groups the first equality occurs at the n-th step of the chain

$$H_0 \supset H_1 \supset \dots \supset H_{n-1} = H_n \quad n=1,2,\dots$$

and for each polytope $P_i \in C_1$ there exists such an isometry $\mathbf{g}_i$ that $P \xrightarrow{\mathbf{g}_i} P_i$ and the intersection $\mathbf{E}_n \cap \mathbf{g}_i\mathbf{E}_n$ of the complex $\mathbf{E}_n$ and its image $\mathbf{g}_i\mathbf{E}_n$ with respect to $\mathbf{g}_i$ represents a subcomplex of the complex $\mathbf{E}_n$. Then the polytope P is a stereotope.

It's evident that the conditions are necessary. The proof of their sufficiency isn't an easy problem. It's important to point out that an upper bound for the number n of coronas depends of the estimate for the coefficient c_d in the Theorems 1 and 1a. In particular for $d = 3$ we may set the next limit: $n < 5$. It's too probab that this limit can be diminished to 2, but at present this conjecture hasn't been settled yet. Also we note that the sence of the theorem is similar to Venkov's criterion of polytope, rediscovered recently by McMullen.

From this fundamental theorem with the help of the well-known Tarskii theorem it follows an existence (together with its description) of an algorithm listing for each given dimension d all combinatorial types of d-dimensional stereotopes (Dolbilin [4]). So far there was a similar algorithm for a listing only of types of Diriclet-Voronoi stereotopes (Delone & Sandakova [2]).

2. MONOTYPIC SPACE PARTITIONS.

Let's go over to a consideration of more general space partitions. We shall consider a face-to-face and normal partition with convex tiles on condition that all tiles of a given partition are isomorphic polytopes. We call such partitions *monotypic*.

Let's remind that a partition $\mathcal{T}$ is called *normal* if there exist two positive numbers ρ_1 and ρ_2 such that for each tile $T \in \mathcal{T}$ it holds that $B_1 \subset T \subset B_2$, where B_1 and B_2 are suitable balls of radius ρ_1 and ρ_2 respectively. For instance, tiles of normal partitions of the Euclidean plane can be only 3-, 4-, 5- and 6-gons.

An investigation of monotypic partitions is connected with the interesting conjecture of McMullen which says that a tile (a facet) of interesting conjecture of McMullen which says that a tile (a facet) of any monotypic d-dimensional sphere partitions has the same type as a tile of some monotypic d-dimensional space (euclidean) partition.

In the case $d = 2$ the McMullen conjecture is true evidently, for $d \geq 3$ it hasn't been settled yet. We point out that an extremely interesting cycle of works in this field belongs to E. Schulte.

In particular due to one simple idea which was expressed and used as far as we know for the first tyme by Danzer and Schulte, one can construct a wide class of monotypic partitions in the d-dimensional Euclidean space with the help of monotypic partitions of the d-dimensional sphere S^d of a special kind.

The matter is that if a monotypic partition of S^d contents at least one primitive vertex, that's $(d+1)$-valent one, then there exists a monotypic partition of R^d with tiles of the same combinatorial type. The construction of the mentioned partition is realized for two steps. At first with the help of a special projection almost all tiles of the sphere partition are projectered onto a d-dimensional simplex T^d yielding a face-to-face tiling of T^d. At the second step the simplex T^d together with its tiling is mapped affinely onto a simplex T' which is a fundamental simplex of a Fedorov group generated with reflections, that's some Coxeter space group. The Coxeter group extends the local tiling of T' through the space. Due to the reflections in faces of fundamental simplex T' the local tilings in adjoining images of T' are connected one with another one by a face-to-face fashion.

In due time there was obtained a rather rich class of regular 3-dimensional sphere partitions (particularly these results were exposed in Dolbilin's dissertation). Not having here a possibility to expose them we formulate just two thesis bearing relation to monotypic partitions:

1. A combinatorial type of 3-dimensional space monotypic tile can have very complicated structure, for instance, with any large number of faces. In particular n-gon prisme is a 3-dimensional tile for any n.

2. An infinite series of 3-dimensional tiles with infinitely increasing quantity of faces has been found. An interest in the series is determined by unnexpected its tie with such geometry number objects as Klein's and Newton's polygons in the 2-dimensional lattice. In particular, structures of these polygons determine the structures of the tiles of the mentioned series by a constructive and one-to-one way.

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SYMMETRY AS A MEASURE OF DEFECT CONTAINABILITY AND ENERGY OF CRYSTALLINE STRUCTURES

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Symmetry is characterized by the figure matchings whose number is expressed by what was qualified by E.S.Fedorov (1) a value of symmetry.

The value of symmetry (Σ) of a point class is equal to a number of faces (edges, apexes) of a general form (ν), but the same value can be deduced by multiplying a symmetry sub-group value (σ) of any face (edge, apex) by a totality of faces (edges, apexes) of a given form (2,3). The class m3m value of symmetry, for instance, is equal to 48 i.e. the maximum number of matchings as obtained through the elements of symmetry. The same value is deduced by performing the following operations:

6 cube faces (001) - 4 mm - 8.6 = 48
12 edges 110 - 2 mm - 4.12 = 48
8 apexes : 111 : - 3 m - 6.8 = 48.

Thus the value of symmetry of a point class (Σ) is a product resulting from multiplying a number of faces (edges, apexes) of general form (ν_n) by the value of symmetry proper to a facial (edge-type, apex-type) form i.e. sub-group describing the symmetry of a face, edge, apex (σ_n): $\Sigma = \nu_n \sigma_n$

The value of symmetry Σ reflects the level of energy bound within lattice. Values of Σ do not change monotonously and can be conformable with the well known set of whole rational numbers: 1,2,3,4,6, 8,12,16,24,48. It is easily understood that Σ is a discrete function having the aspect (Fig.1):

$$f(K) \begin{cases} 1,, & \text{if } K=1 \\ 2^{k/2}, & \text{if } k \text{ is an even number} \\ 3.2^{\frac{k-2}{2}}, & \text{if } k \text{ is an odd number.} \end{cases}$$

Mechanical, radiation, heat and other effects generate defects in the crystal structure thus disturbing the lattice site order. The critical number of defects that the structure may contain before it becomes amorphous is limited by a defect saturation level

(Dk). The level of defect containability saturation characteri free (excess) crystal energy.

Thus the cumulative lattice energy is made up with intrinsic (bound) energy proportional to the value of symmetry and therefore changing discretely, on the one side, and free (excess) energy which grows and relaxes monotonously, on the other side. From this it transpires that the properties of ideal (defectless) crystals change in compliance with "quantum numbers" of symmetry, while for actual crystals quantation is shaded by energies of a huge number of structural defects.

Quantized physical properties are also characteristic to polymorphous transformations accompanied by jumps of symmetry values. As this takes place, $\lesssim$ values may drop from high to low bypassing intermediate numbers (diamond, graphite) or else they follow a sequence of quantation degrees (cristobalite - tridymite - quartz).

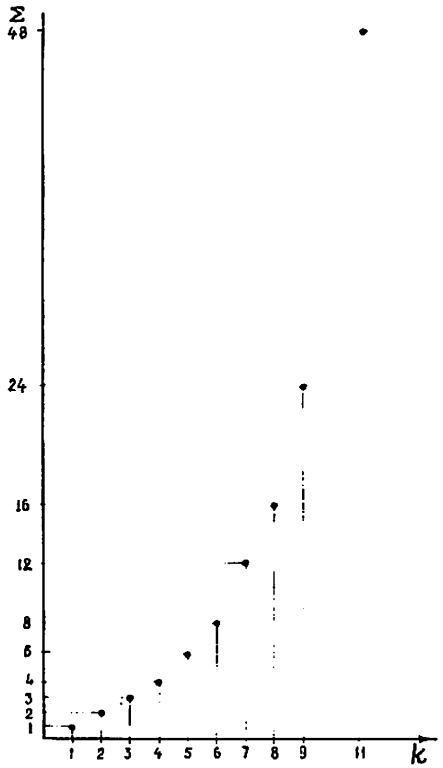
As for the ideal crystals, their quantized properties appear either at very low temperatures or with very fine individuals, such as fine metal particles, whiskers, particle fragments.

Polyform quartz species provide a classical example of how the physical properties are quantized depending on a descrete application of $\lesssim$ values.

At high temperatures in isotropic medium (under a normal pressure) forms β-cristobalite featuring the value of symmetry $\lesssim =48$. This is a manifestation P.Curie principle of symmetry superposition, namely, a highly symmetrical (and highly energetical) medium generates a highly symmetrical and highly energetical structure. Under the action of external factors (mechanical, radiational) the β-cristobalite becomes defect saturated to form α- cristobalite with $\lesssim =16$. Intrinsic lattice energy lowers, while the defect containability level grows. Under the further-on external impact the structures being formed are progressively lower symmetrical until they reach $\lesssim =4$ (coesite), the silica properties alter in a discrete manner, too - lattice parameters, refraction characteristics, spectral features (see the Table).

Twins are characteristic to the structure of quartz and its polyformous modifications. The readiness to twin formation should be considered as a manifested lattice energy quantation localized within a structural volume. The twin symmetry boundary is above (or beneath) that of monocrystals. Therefore "breaking" of a crystal into twin sub-individuals means a texturized distribution of bound energy $\mathcal{U}$ and free energy D.

Fig.1 Discrete function Σ(K)



SiO_2 properties quantizing as a function of value of Symmetry Σ

Polymorphous modification	Chemical formula	Molecular mass	Value of symmetry Σ	Parameters of elementary sell a_o	b_o	c_o	Volume of elementary sell, $Å^3$	Specific mass	Refraction parameters N_g	N_p	X cm^{-1}
I	2	3	4	5	6	7	8	9	I0	II	I2
β-cristobalite	β-SiO_2	60.084	48	7.II	-	-	360.94	2.I9	I.487	-	II98
											I096
											793.7
α-cristobalite	α-SiO_2	60.084	8	4.97	-	6.93	I7I.I8	2.32	I.486	I.484	-
β-tridymite	β-SiO_2	60.084	24	5.04	-	8.24	I8I.26	2.I8	I.474	I.469	II05
											787
											568
α-tridymite	α-SiO_2	60.084	8	9.90	I7.I	I6.3	22I3.84	2.3I	I.483	I.479	-
β-quartz	β-SiO_2	60.084	I2	4.999	-	5.457	II8.I0	2.53	I.539	I.53I	-
α-quartz	α-SiO_2	60.084	6	4.9I3	-	5.405	II2.07	65.	I.565	I.544	II59
											I093
											693
Stishovite	SiO_2	60.084	I6	4.I79	-	2.665	46.54	4.33	I.826	I.799	9.49
											769
											672
											885
											730
											623
Kitite	SiO_2	60.084	8	7.456	-	8.604	478.3I	2.50	I.522	I.5I3	-
Coesite	SiO_2	60.084	4	7.23	I2.52	7.23	566.76	3.03	I.597	I.594	I2I8
					β =I20^o						II52
											I07

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"THE SYMMETRY OF FINITE-DIFFERENCE EQUATIONS"

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Attempts of adaptation of the group analisis of differential equations to finite-difference equations were failured because of nonlocal nature of finite-difference operators. Therefore some authors applied preliminary localisation replacing the finite-difference equations by various differential approximations.

The first steps towards the group analisis of the finite-difference equations are made in the present work.

The formal transformations groups in the space of differential and mesh variables are considered. It is shown that the conservation of the difference derivatives sence tends nesessaraly to the Lie-Backlund groups. One of them - the Taylor group is used obtaining the formulas of transformations of mesh variables. The criterion of invariance and uniform conservation of the difference mesh is stated. The criterion of invariance for difference equations is applied for obtaining the finite-difference equations allouring the group isomorfic to the natural differential model group.

The group isomorfic to the Taylor group is constructed by means of formal Newton series. This group is applied for factorization of Lie-Backlund operators on uniform mesh. The discrete Noeter identity for some classes of group transformations is obtained, and the conservation criterion of invariance equations is settled.

THE BIRTH OF SYMMETRIES IN THEORETICAL PHYSICS : LAZARE CARNOT'S MECHANICS

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Lazare Carnot wrote a first book on mechanics in 1783[1] and a second book in 1803[2]. In both books the core of his theoretical thinking is the same and highly original with respect the other authors of his times, Lagrange included. From a theoretical point of view, his basic notion is that of geometric motion. The definition is the following: Any motion will be called geometric if the opposite motion is possible. He offered many examples of such motions. For us it is interesting the fact that the translatory motions and rigid rotatory motions of a system as a whole constitute geometric motions.

Geometric motions do not equate virtual motions, contrarely to what Gillispie seems to hold.[3] Rather, they play a crucial rôle in the development of L. Carnot's theory. They allow to obtain the laws of motion from a fundamental equation L. Carnot states (unfortunately in an obscure way) for the shock problem of a system of particles:

$$\sum m_i \vec{U}_i \cdot \vec{u}_i = 0$$

where m_i is the mass of the i-th particle, $\vec{U}_i$ the lost velocity, $\vec{u}_i$ the superimposed geometric motion, possibly in a specific way for any single particle.

Let us remark that since one may assign an infinte number of geometric motions, the $\vec{u}_i$'s velocities play the rôle of indeterminate quantities; then, any specification of $\vec{u}_i$'s provides a determinate equation which gives information about the system motion.

For example, when we assign to u_i's the same velovity , $u \neq 0$, we consider a translatory,uniform motion of the whole system; then, we obtain:

$$0 = \sum m_i \vec{U}_i \cdot \vec{u}_i = \vec{u} \sum m_i \vec{U}_i \ ,$$

being $\vec{u}$ an arbitrary quantity; it follows that

$$\sum m_i \vec{U}_i = 0$$

But $\vec{U}_i$, the lost velocity, is the same that $\vec{W}_i - \vec{V}_i$, i.e. the difference between starting velocity and final velocity; thus,

$$\sum m_i \vec{W}_i = \sum m_i \vec{V}_i$$

which is the law of conservation of momentum for an (isolated) system of material points.

Then, let us assign a new geometric motion which rotates rigidly the whole system about a fixed axis, with angular velocity $\vec{a}$. Then, $\vec{u}_i = \vec{a} \times \vec{r}_i$ and we obtain

$$0 = \sum m_i \vec{U}_i \cdot \vec{a} \times \vec{r}_i = \sum m_i \vec{a} \cdot \vec{U}_i \times \vec{r}_i = \vec{a} \sum m_i \vec{U}_i \times \vec{r}_i$$

Again, $\vec{a}$ is an arbitrary quantity, thus

$$\sum m_i \vec{U}_i \times \vec{r}_i = 0$$

or, what is the same

$$\sum m_i \vec{W}_i \times \vec{r}_i = \sum m_i \vec{V}_i \times \vec{r}_i$$

i.e. the conservation of the momentum of momentum.

Surprisingly enough, until 1970 L. Carnot's mechanics passed almost unnoticed , if not by some French authors[4]. The above derivations have been noticed by C.C.Gillispie in his analysis of L.Carnot's scientific work[5]; however he did not recognized in them the symmetry method of deriving laws.

Actually, L. Carnot was pride of having introduce the notion of geometric motion. He even foresaw an entirely new science, intermediate between geometry and mechanics[6]. The next development of science disregarded such design. However, it is well-known that Sadi Carnot produced almost the whole theory of thermodynamics by developing the main ideas of his father, Lazare. In particular its reversibility notion constitutes a filiation of the geometric motion[7]. Furthermore, H.Callen showed that thermodynamics may be reformulated as "a science of symmetry" since conserved and brocken symmetries offer its coordinates[8].

Therefore, in the historical development of science there exists a branch of theoretical physics (i.e. two basic theories) that presents an alternative mathematics to that of infinitesimal analysis. One may suppose that in past times the above branch has been under-evaluated because his mathematical techniques are not congruent with the dominant one, i.e. calculus.

To have identified the birth of symmetries in physics lead us to recognize the intellectual origins of such notion. Lazare Carnot had as main intellectual teachers D'Alembert and Leibniz. As an example, L. Carnot followed the Leibnizian principle of continuity till to adopt it as his main maxim for both scientific and political lifes(the"deplacement by insensible degrees")[9].

Now,it was a Leibniz' design to add to calculus a new mathematical technique , to be specific for geometry, in order to formulate a "characteristica universalis"[10]. After three centuries of development of theoretical science, there is no better candidate for such addition than symmetry mathematical techniques, that started by Lazare Carnot's theoretical mechanics.

When we ask for a representation in logical terms of such new way of reasoning, we find a possible answer in thermodynamics. S.Carnot did not believe in caloric theory as well as in mechanical theory of heat; as a consequence he did not stated what today is the statement of the first principle of thermodynamics. This one may be synthetized by saying that "heat and work are equivalent"; however an exact statement is "it is not true that heat is not work", provided that one neither can state "heat is work" nor "heat is not work". Since in classical logic a statement is logically equivalent to its double negation, the above statement of the first principle does not belong to classical logic. Really, it put a problem, i.e. to know when and in what terms heat is work.

In fact, Sadi Carnot's thermodynamic theory is aimed to this target. In other words such theory as a whole represents a logical cycle (which is far more important than the operation cycle S.Carnot introduced for representing the fonctioning of an heat engine); i.e. by starting from a doubly negated statement the theory introduces explications which lead us to an affirmative part of the starting statement. It is very interesting that one finds the same logical scheme in Lazare Carnot's books on calculus[11], geometry[12], mechanics[1].

In the last theory , the double negated statement is implicit in his version of the inertia principle, whose core we know to be the equivalence of both states (v=0) and (v=const.); or, more precisely, for the same reasons as above in the case of the first principle of thermodynamics, "it is not true that (v=0) is not equal to (v=const.)"(It is trivial to verify the falsity of the affirmative, corresponding statement as well as that of the simply negated statement). This one is not the version of the inertia principle by Lazare Carnot; however,he offered a new version of it, strictly opertive in nature, compatible with constructive mathematics and more adequate than the Mach's version;[13] furthermore the same notion of geometric motion is drawn from such equivalence principle, since one more definition L. Carnot gave of geometric motion is that of a motion which may be superimposed to a physical system without affecting its mechanical state.

As a consequence, symmetry technique in physics appears to have substantiated a new way of reasoning in science.

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ARTISTIC PATTERNS WITH HYPERBOLIC SYMMETRY

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Introduction

Probably the first repeating patterns of the hyperbolic plane were triangle tessellations (see Figure 1 below) which, though attractive, were not originally created for artistic purposes. Almost certainly the Dutch artist M. C. Escher was the first person to combine hyperbolic geometry and art in his four patterns *Circle Limit I*, *Circle Limit II*, *Circle Limit III*, and *Circle Limit IV* — see Catalog Numbers 429, 432, 434 (and p. 97), and 436 (and p. 98) of [Locher, 1982]. It is exacting and time-consuming to create such patterns by hand as Escher did. In the late 1970's, the power of computers was applied to the problem of creating such patterns. Since then, much progress has been made in this area which spans mathematics, art, and computer science [Dunham, 1986a], and [Dunham, 1986b].

We will begin with a review of hyperbolic geometry, repeating patterns and tessellations, symmetries of hyperbolic patterns, and color symmetry. Then the theory of repeating hyperbolic patterns will be related to that of Euclidean and spherical patterns. Finally, a computer-aided hyperbolic pattern-generation process will be described.

Hyperbolic Geometry

By definition, (plane) hyperbolic geometry satisfies the negation of the Euclidean parallel axiom together with all the other axioms of (plane) Euclidean geometry. Consequently, hyperbolic geometry satisfies the following parallel property: given a line ℓ and a point P not on that line, there is more than one line through P not meeting ℓ. Unlike the Euclidean plane and the sphere, the entire hyperbolic plane cannot be isometrically embedded in 3-dimensional Euclidean space. Therefore, any model of hyperbolic geometry in Euclidean 3-space must distort distance.

The *Poincaré circle model* of hyperbolic geometry has two properties that are useful for artistic purposes: it is conformal (i.e. the hyperbolic measure of an angle is equal to its Euclidean measure), and it lies within a bounded region of the Euclidean plane — allowing an entire hyperbolic pattern to be displayed. The "points" of this model are the interior points of a *bounding circle* in the Euclidean plane. The (hyperbolic) "lines" are interior circular arcs to the bounding circle, including diameters. The edges of the curved triangles in Figure 1 and the backbones of the fish in Figure 2 represent hyperbolic lines.

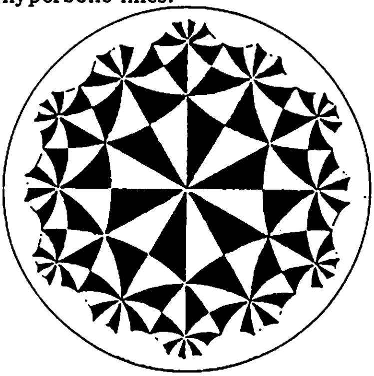

Figure 1. A pattern with symmetry group $[6,4]^+$.

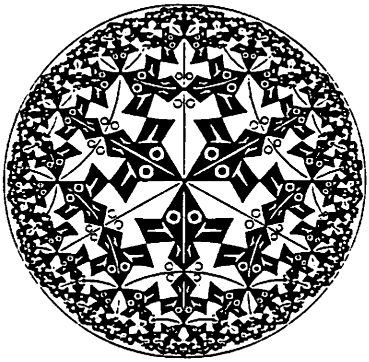

Figure 2. A computer generated rendition of M. C. Escher's *Circle Limit I* pattern.

Repeating Patterns, Tessellations, Symmetries

A *repeating pattern* of the hyperbolic plane is a pattern made up of hyperbolically congruent copies of a basic subpattern or *motif*. For instance, any adjacent black-white pair of triangles of Figure 1 forms a motif. Similarly, a black half-fish plus an adjacent white half-fish make up a motif for Figure 2.

An important kind of repeating pattern is the *regular tessellation*, $\{p,q\}$, of the hyperbolic plane by regular p-sided polygons, or *p-gons*, meeting q at a vertex. It is necessary that $(p-2)(q-2) > 4$ to obtain a hyperbolic tessellation. Figure 3 shows the tessellation {6,4} (solid lines) and its dual tessellation {4,6} (dotted lines).

A *symmetry operation* or simply a *symmetry* of a repeating pattern is an isometry (hyperbolic distance-preserving transformation) of the hyperbolic plane which transforms the pattern onto itself. For example, reflections across the backbones in Figure 2 and across any of the lines of Figure 3 are symmetries of those patterns (reflections across hyperbolic lines of the Poincaré circle model are inversions in the circular arcs representing those lines [or ordinary Euclidean reflections across diameters]). Other symmetries of Figure 2 include rotations by 180 degrees about the points where the trailing edges of fin-tips meet, and translations by four fish-lengths along backbone lines (in hyperbolic geometry, as in Euclidean geometry, a translation is the product of reflections across two lines having a common perpendicular, and the product of reflections across two intersecting lines produces a rotation about the intersection point by twice the angle of intersection).

The *symmetry group* of a pattern is the set of all symmetries of the pattern. The symmetry group of the tessellation $\{p,q\}$, denoted $[p,q]$, can be generated by reflections across the sides of a right triangle with acute angles of $180/p$, and $180/q$ degrees; i.e. all symmetries in the group $[p,q]$ may be obtained by successively applying a finite number of those three reflections. Thus, $[6,4]$ is the symmetry group of the tessellation $\{6,4\}$ formed by the solid lines of Figure 3 — in fact [6,4] is the symmetry group of the entire pattern of Figure 3. The orientation-preserving subgroup of $[p,q]$ consisting of symmetries made up of an even number of reflections is denoted $[p,q]^+$. The symmetry groups of Figures 1 and 4 are $[6,4]^+$ (it is just [6,4] if the color of the triangles is ignored), and $[5,5]^+$ respectively. For more about the groups $[p,q]$, see Sections 4.3 and 4.4 of [Coxeter and Moser, 1980].

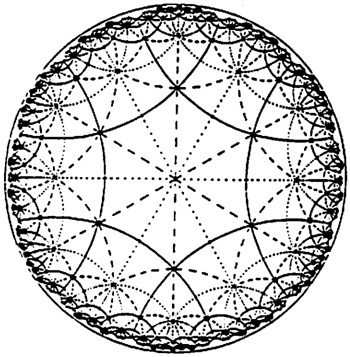

Figure 3. The tessellations {6,4} (solid lines) and {6,4} (dotted lines), and other lines (dashed) of reflective symmetry of the pattern.

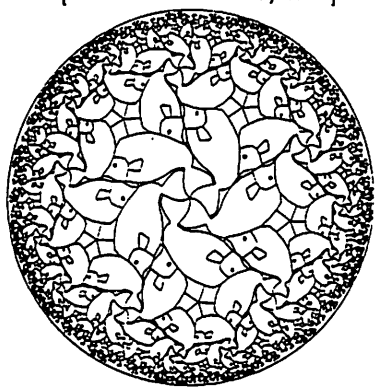

Figure 4. A pattern with symmetry group $[5,5]^+$.

Color Symmetry

A pattern is said to have *n-color symmetry* if each of its motifs is drawn with one of n colors and each symmetry of the pattern maps all motifs of one color onto motifs of another (possibly the same) color; i.e. each symmetry permutes the n colors. The pattern of Figure 1 has 2-color symmetry (as does the Euclidean checkerboard pattern): reflection of the pattern across the side of any triangle interchanges black and white; rotation about a triangle vertex through twice its angle produces the identity permutation — black triangles go to black triangles and white triangles go to white triangles. For more on color symmetry, see [Senechal, 1983], and [Shubnikov and Koptsik, 1974].

Relation to Euclidean and Spherical Patterns

If the ">" in the relation $(p-2)(q-2) > 4$ is replaced by "=" or "<", one obtains tessellations of the Euclidean plane and the sphere respectively. In the Euclidean case, the corresponding symmetry groups of these tessellations are $[4,4] = p4m$ and $[3,6] = p6m$ which contain all 17 of the plane crystallographic groups as subgroups (see Section 4.6 and Table 4 of [Coxeter and Moser, 1980]). In the spherical case, the groups $[2,q]$, $[3,3]$, $[3,4]$, and $[3,5]$, contain all the discrete spherical groups as subgroups. The notion of hyperbolic color symmetry also specializes to the usual notions of Euclidean and spherical color symmetry. If a pattern has symmetry group $[p,q]$ (disregarding color), we have found that there are 5, 2, and 16 possible kinds of 2-, 3-, and 4-color symmetry respectively for that pattern. Some of these kinds of color symmetry require that certain divisibility conditions hold for p and q (e.g. p must be even). Consequently, when the divisibility conditions are not met, these kinds of color symmetry cannot appear in Euclidean or spherical patterns with symmetry groups of the form $[p,q]$. However, in the case of hyperbolic patterns with symmetry groups of the form $[p,q]$, there are infinitely many values of p and q satisfying *all* the divisibility conditions for 2-, 3-, or 4-color symmetry.

The Hyperbolic Pattern-Creation Process

The present version of the computer program allows for the design of repeating patterns with color symmetry whose symmetry group is a subgroup of $[p,q]$ and whose motif lies within a p-gon of the corresponding tessellation $\{p,q\}$. The pattern-creation process consists of two parts: (1) design of the motif, and (2) replication of the whole pattern from the motif. The design of the motif is done most easily with a computer graphics input device such as a data tablet or mouse — the motif is outlined by a sequence of points entered by the input device and connected by line segments.

To replicate a pattern from a motif, first note that it is easy to replicate that part of a pattern within a p-gon of $\{p,q\}$ if that p-gon already has a copy of the motif within it — the copy of the motif is simply rotated about the center or reflected across lines through the center of the p-gon. The algorithm for replicating the whole pattern depends on the fact that the p-gons of $\{p,q\}$ form "layers": the first layer is a p-gon centered in the bounding circle, and each subsequent layer is defined inductively as the set of p-gons having a common vertex (only) or edge with a p-gon from the previous layer. Then it is merely a matter of moving a copy of the motif from one p-gon to another (either from one layer to the next or within a layer), using appropriate elements of the symmetry group. For more details on the pattern-creation process, see [Dunham, 1986a].

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DYNAMIC SYMMETRY: FILMS

by Michele Emmer

«......A succession of gradually changing figures can result in the creation of a story in pictures. In a similar way the artists of the Middle Ages depicted the lives of Saints in a series of static tableaux.....The observer was espected to view each stage in sequence. The series of static representations acquired a *dynamic* character by reason of the space of time needed to follow the whole story. Cinematic projection provides a contrast with this. Images appear, one after the other, on a still screen and the eye of the observer remains fixed and unmoving. Both in the medieval pictorial story and in the developing of a regular division of the plane the images are side by side and the time factor is shifted to the movement the observer's eye makes in following the sequence from picture to picture. It is possible to look at a film strip in the same way when it is held in the hand.»

These are words from Escher's book "The Regular Division of the Plane" [1]. It is well-known the interest of the Dutch graphic artist in the creation of figures that tassellate the entire plane and his attraction for mathematical forms. In the same book, a sort of theory on his own way of working, he claims [2]:

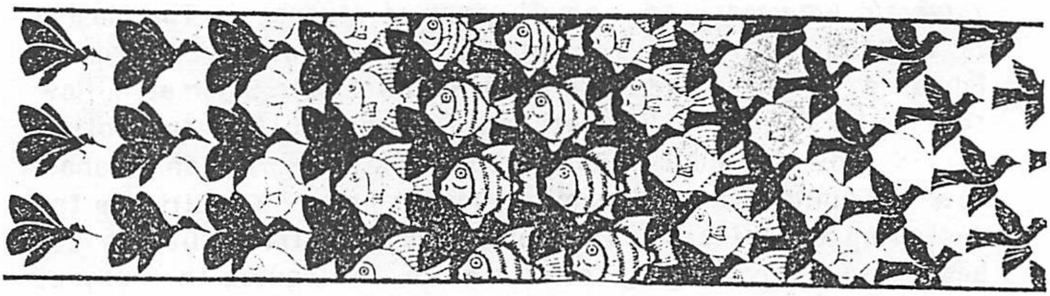
« In this book it is the images and not the words that come first.....For me it remains an open question whether the play of white and black figures as shown in the six woodcuts of this book pertains to the realm of mathematics or that of art.»

All the various aspects of the art of Escher were discussed at the Congress held at the University of Rome in 1985 [3]. In my introduction to the special issue dedicated to Escher by the journal *Structural Topology* I wrote that [4] : « the congress and the volume of the Proceedings cannot be considered in any case a final event but more correctly a starting point.»

But let go back to the Escher's words. In the creation of his symmetry figures a very important element was what Escher called « the dynamic equilibrium between the motifs» . In his book he asks the crucial question : « Is it possible to create a picture of recognizable figures without a background ?» and the answer is no .« Composition do not have a visual static balance but a dynamic one;.....a static balance is possible only if the whole figures seen as a pattern, separate from the representation of birds and fish.» As he pointed out, the fish and birds motifs were among the most useful for resolving the problem of creating interlocking patterns with easily recognizable objects.

The idea of create a dynamic symmetry patterns is a very old one.[5] As an example I can recall the following: « In the autumn of 1917, Jay Hambidge was giving a series of talks on *Dynamic Symmetry* to a small group of students.» The phrase is written in the foreword of a little volume by Edwards B. Edwards "Patterns and the design with Dynamic Symmetry: How to create Art Deco Geometrical Designs" [6]. In the introduction Edwards adds : « This book has been written to show how the principles of dynamic symmetry may be applied to the designing of pattern. The name Dynamarhythmic Design has been coined advisely by the writer to distinguish the subject from the term Dynamic symmetry, used by Mr. Hambidge..... In Dynamarhythmic design, not only are the forms related harmoniously, but the areas are related as well. Within these areas the artists may use such forms, natural or otherwise, as his caprice or fancy may dictate, providing that he keeps in mind the proportional relationships.... or, in other words, the parts consistently to the whole. »

Coming back to Escher, he wrote as a description of his work "Metamorphosis II": «First the black insect silhouettes join; at the moment when they touch, their white background has become the shape of a fish. Then figures and background change places and white fish can be seen swimming against a black background. A succession of figures with a number of metamorphoses acquires a dynamic character. Above I pointed out the difference between a series of cinematographic images projected on a screen and the series of figures in the regular division of plane. Although in the latter the figures are shown all at once, side by side, in both cases *the time factor* plays a part.»



Escher, before his death in 1972, partecipate to the making of a short movie in which drawings animation technique was used for two of his works. After his death his works on symmetrically repeating figures have been a source of inspiration for computer graphics animation.[7] As I pointed out elsewhere, in the making of the film series "Art and Mathematics" [8] I was attracted by the cinematographic quality of most of the Escher's works. If we use Escher's images in regard to the figure-background problem and the cinema technique, especially animation, it is possible to SEE the instant in which the instability is created, the passing from one form to another. And this is only an example of the various possibilities in using a camera with the symmetry drawings of Escher [9]. But of course in making a film, you always reach something new: the images flow quickly by, they are in real movement, not just side by side. In a sense we have teaken Escher's suggestions and extended them: a world of symmetry according to Escher, which moves and changes in the three dimensional space.

I have used the camera not only for the symmetry drawings of Escher but also to show various dynamic aspects of symmetry in three of the films in the series, including the famous mosaics of the Alhambra in Granada. With Roger Penrose we made an animation of his non periodic tiling of the plane with two birds.[10]

Looking at all these really dynamic symmetry images it is possible to quote from Escher's phrase: In these films, it is the images and not the words that come first.

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SYMMETRICAL STABLE SIMPLEX
INTRODUCTION TO THE RESISTANCE OF FORMS.

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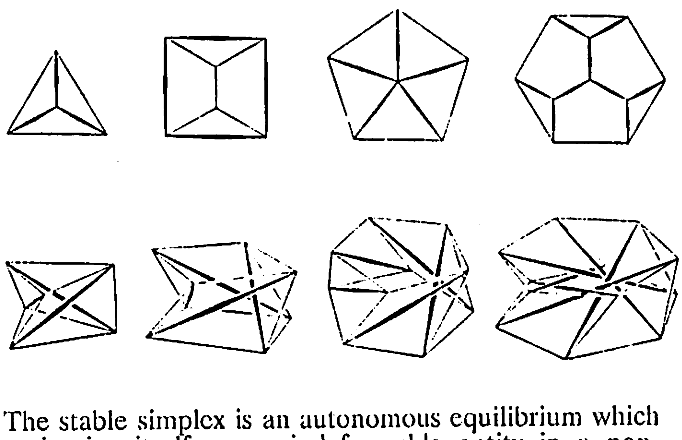
Symmetry is the generator of equilibrium, omnipresent forever in architecture, it also defines the great monumental orders as the elementary structural forms - lintels, arches, vaults - the stable simplex, which repeat themselves rythmically, being the characteristic motifs of the building.

In an industrialised space structure,based on the standardisation of elements and on the regularity of their assemblage, the symmetry becomes even more important : the employed simplex thus being polygons and polyhedra. The number of arbitrarily chosen forms being infinite, the invention and the systematic inventory of these new simplex stables, which are multidirectional and even multidimensional, would be impossible, as would be the study of their rigidity and the calculation of their resistance, without the introduction of the criterion of regularities - that is to say, symmetry.



The stable simplex is an autonomous equilibrium which maintains itself as an indeformable entity in a non-monolithic system, thus composed essentially of compression or tension bars, assembled by movable joints.

In ancient construction one can discern certain elementary configurations recognised and used as stabilising or bracing elements. These are the triangular brackets of the wooden frame, the St.Andrews cross in skeletons, the tensioned crossbracing in a metal framework. These stable simplex are nearly always planar, they have been very few in the number of types, and the art of building has been satisfied with them until now.

At present with the knowledge that construction is becoming lighter and more spatial, there is place to re-examine then enlarge the list of stable simplex which can enter into the composition of rigid bearing systems.

In an old article, appearing under the title "Principes Edifiants" in the "Techniques et Architecture" no.2-1966, I have already had opportunity to criticise established ideas and at the same time the validity of equations currently used by engineers to verify the indeformability of a configuration. Notably the formula A=3S-6, defining the relationship between the number of edges (A) and the number of articulated vertex (S), which is only valid for the convex compositions with continuous triangulations, thus without polygonal hiatus.

It is therefore suitable to replace the "Euler" formula with the "Ky-Fan Frechet" formula taking into consideration these hiatus. This gives a general controlling formula usable in the case of continuous triangulation : A=3S-6K, K being the number of connections of the enveloping surface of the configuration, even if the surface is torus-like or spongieuse, thus with one or several holes.

Otherwise, beyond this amelioration, there exists some stable compositions which are neither triangulated nor constructed as a continuum of rigid elements. And however, analytic studies are prolifering with heavy algebraic symbolism, it is not by such an abstract and non geometrical way that one can best approach the problem of rigidity of complex systems. The rigidity is not quantifiable as easily as those continuous and monolithic elements which constitute the study of the resistance of materials, one should now speak of the resistance of forms.

This resistance of forms - which is not concerning the opposition to the efforts because it depends on the intrinsic ownership of some geometric figures to resist mechanical deformations - brought to light in reality from the domain of kinetics, or more exactly from the field of anti- or non- kinetic, or better still, from all that is just non-kinetic or isostatic.

It is in fact easier to take the graphs one after another, beginning with the triangle which is altogether the most simple constructive subset, thus the most primitive simplex, and examine the polygons and the polyhedra having a more and more rich articulation towards the most complex spatial configurations ; the aim being to transform them by the addition of supplementary and a strictly necessary number of elements to obtain rigid or "instantaneously rigid" isostatic systems or following certain experts : critical or surcritical systems. Remarking that the addition of overabundant or redundant elements would produce hypocritical structures.

Instead of being content with the creation of a terminology which only classifies very different configurations under the same etiquette, one proposes thus a combinatory method which enumerates them systematically. This method is at the same time an experimental one, the figures can be easily modelised avoiding heavy and pedantic algebraic codes which makes the actual studies on the rigidity rather stern for the people engaged in structural research and even more for the practising constructors.

One thus takes a graph, preferably a symmetrical one, having a shape originally with a variable geometry with a more or less regulated liberty of movement, then by the addition of new elements and by the manipulation of the nature of the members, one transforms it by provoking any kind of conflicts in the relative displacement of the members until blocking his mechanism ; somehow like putting a stick, or a string, in the wheel-work.

The open figures - segments, stellations, aborescences- cannot be associated in a stable structure without being rigidly fitted together. They are thus composed of consoles or cantilevers, where a monolithism is established by the continuous embodding of their members, which excludes such structures from these studies, related uniquely to articulated ones.

Our objective is therefore limited to the study of closed figures like a loop, having a polygonal perimeter or composed of polygons. The membrures themselves can be struts or rigid bars working as well in traction as under compression, or slings or tensional members working only in traction.

The regular polyhedra, the solids, represent for three dimensional space the same elementary repetitive components as the regular polygons do for two dimensional space. Naturally, as in the space packing patterns, then in the planar regular tessellations one uses symmetrical basic geometrical figures. Otherwise, the polyhedra, whose edges are materialised by rigid bars and with articulated nodes, are not all solid.

After the theorem of Cauchy (1812) only the polyhedra with indeformable faces are indeformable. Thus among the articulated systems only the triangulated volumes are stable : among the regular polyhedra - the tetra, octa, and icosahedra - and among the semiregulars only the snub cube and the snub dodecahedron having continuous triangulated rings ; and all deltohedra generally as bipyramids, antiprims, etc...

Following this theorem other polyhedra can be rigidified theoretically by an indeformabilisation of the polygonal faces by triangulation by the use of one or another planar simplex. But practically, this method is not always successfull, the planar simplex being subject to warping most of the time. In principal, those with internal prestressed configuration having no cleavage-line or hinge are not warping ; thus, though planar, they remain stable even in space. Some others keeping their rigidity in an assembly of uneven order around a vertice ; disparallelity helps too, etc... But instead of making spatial items with planar ones, it would be more logical to follow again the morphological approach starting from the most simple volumes progressing systematically to the more complex ones and at the same time looking for their stabilisation, if possible, by stereometrical means.

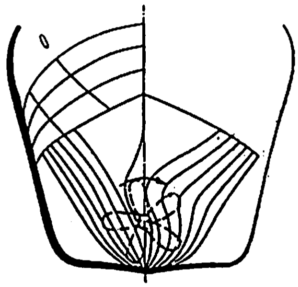
The result is astonishing enough : over the volumes entirely triangulated or stabilised by planar simplex faces, there exists elementery solids with articulated membrures, having not only rigid bars, but even flexible tensional membrures. These bodies are the stable spatial simplex which are at the same time practically empty and thus light volumes, ready to be used as building blocks, as was in the past the stone block in the hand of the mason.

These configurations, having generally screwed, left or right rotating, symmetry, present themselves as very surprising stable compositions, where the rigid bars - each one isolated in a continuous tensional network - appear to be floating. These self stressed solids are the most remarkable among the stable space simplex because of their extreme simplicity and their ability to enter in any kind of structural pattern - linear, planar, polyhedric, cristallographic - by their combinatorial facilities, their shape keeping a high degree of symmetry.

Black-and-white (antisymmetric) tiling and textile patterns

Endrei, Walter

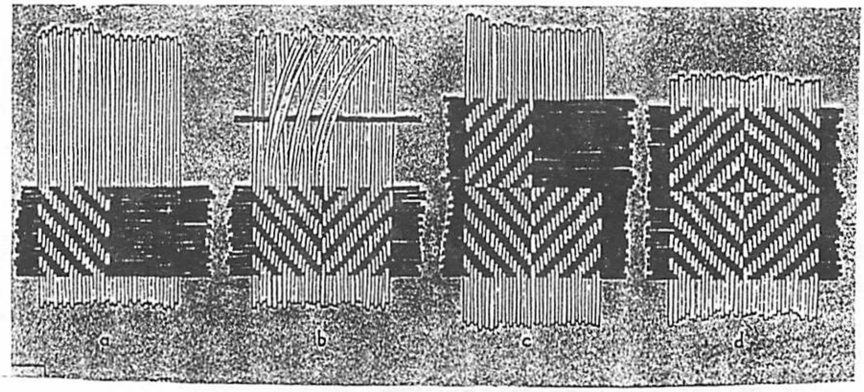
Common-sense will have it, that in the domain of technology - either procedure or material - symmetric structures ought to be preferred. A short survey undeceives us: often asymmetric designs are favoured, sometimes they offer the only solution. Reasons for this solution are obvious and manifold: biological facts like dexterity of man, as well as the important role of rotation in mechanics or molecular properties of certain crystal lattices.



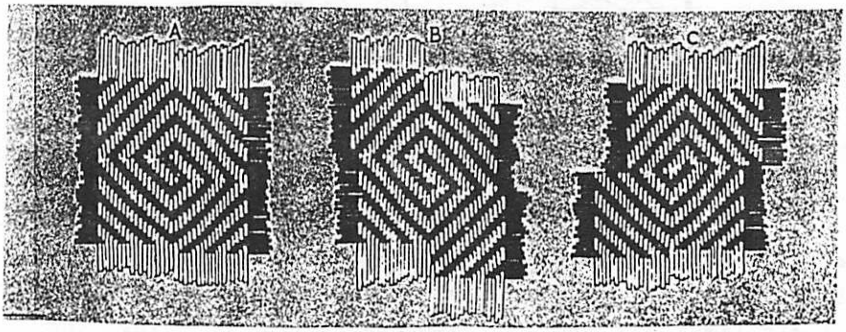
The latter ones - containing impurities - may serve us as a starting point into a technique of manufacture which, since prehistoric times has had an immense influence over the patterns of ceramics, architecture, leather-work etc. This is the plaiting and later on the weaving of patterns.

In order to explain our argument it is necessary to acquaint you with the notation of weaving-design. The two system of yarns - warp and weft are represented by different colours (usually black/white) and the gap between them is neglected. Consequently the sketch transferred to a cartridge-paper may serve as a more or less true image of the fabric, provided that warp and weft have different (but otherwise uniform) colour.

Reverting to our starting point, - structural faults, - a small example will elucidate our idea. If we take a simple twill and rotate it in the quarters of the pattern - as it is quite ordinary in basket - weaving - we shall get a diamond-twill.

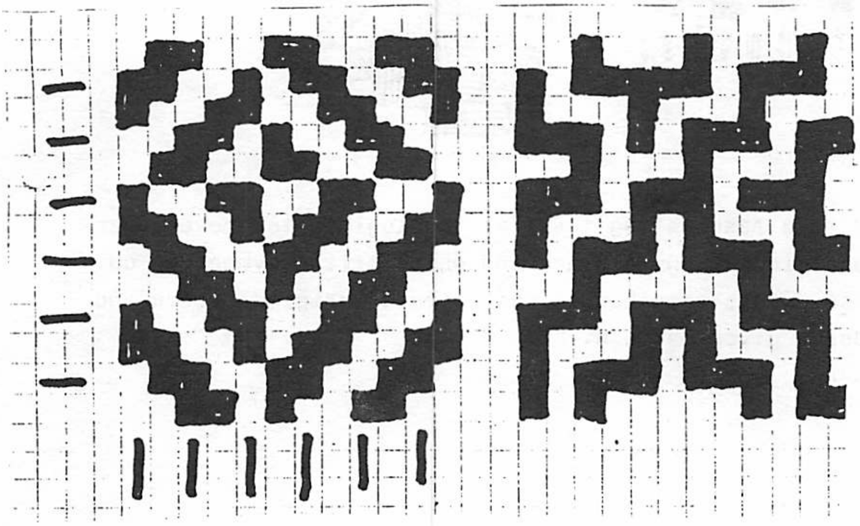


If the weaver misses the order of succession by two yarns only, he will get a meander, even in several varieties.

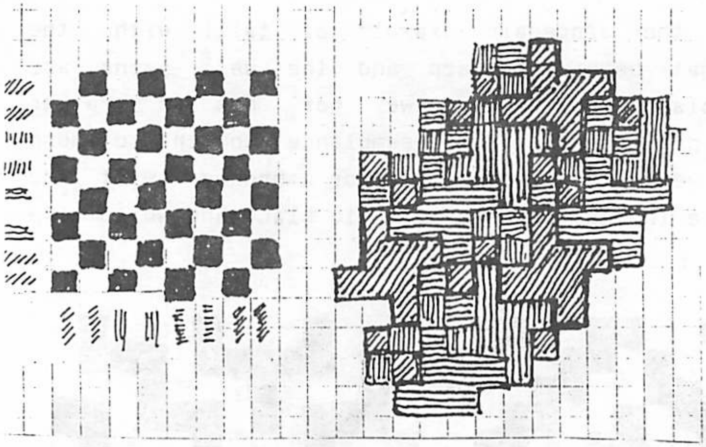


A special case of symmetry is that of black and white tiling, which is characterized by two identical surfaces being complementary to one another. They are omnipresent in human culture, we find them on leather-intarsia of Altai-people of the 5th cent. B.C., in Roman mozaic/work and recent Mexican coloured-woven fabricsl likewise. Tracing back their origin, we can show, that most of them are derivable from matting- or plaiting work and weaving.

Let us take the aforesaid example of twill with the difference, that both the warp and the weft yarns are alternately black and white: we get now a rather sophisticated pattern with no resemblance to the diamond twill. But if we took a simple twill, or a broken twill the result would be in most cases a splendid black and white tile pattern.



Varying them, we may get as good as all occurring variants of this symmetry-type; especially twill bindings are prone to produce them. Even three-colour combinations can offer such patterns



It would be a fascinating task for topologists to prove under which conditions turns a black and white tiling symmetry up. There also exists a method to reconstruct original weave and yarn order a given pattern.

Symmetry conservation and symmetry breaking in atomic structures

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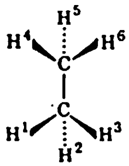
Symmetry will be used in its restricted sense as a repetition of a motif to compose a whole. The laws of repetition which we are considering are rotations, reflections and translations. These are called symmetry operations. The symmetry group of an object is the set of all symmetry operations which map the object onto itself. It is a group in the mathematical sense.

Atomic structures frequently exhibit symmetry. In crystals symmetry is realized in its highest perfection. We define a crystal to be an infinite, three-dimensional periodic object. In the first part of this lecture I will show that symmetry is imposed by some kind of "external forces". In the second part of this lecture I will show how symmetry breaks if the constraints imposed by the external forces are relaxed.

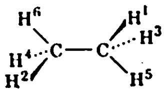
1. Conservation of symmetry

Symmetry can result from various sources. We will consider some cases:

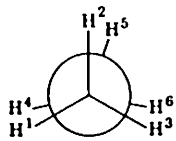
i) Symmetry can result from a low energy equilibrium state between attractive and repulsive forces among the atoms, e.g. in the ethane molecule there exist two equilibrium states, the eclipsed and the staggered conformation. The staggered conformations is lower in energy thus the resulting symmetry group is D_{3d}. The eclipsed conformation has symmetry D_{3h} while intermediate states have D_3 symmetry.



eclipsed

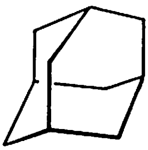
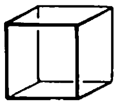
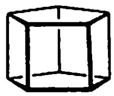
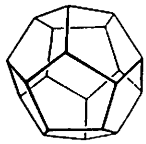


staggered

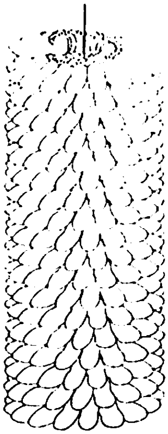


intermediate

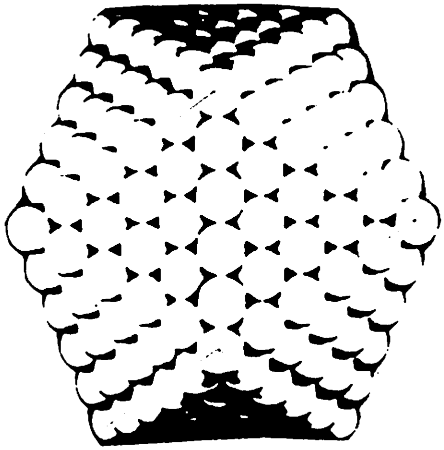
ii) Symmetry can result from the designed synthesis of a molecule. Recent examples are the successful syntheses of, among others, adamantane, cubane, pentaprismane, and dodecahedrane. All of them are highly symmetric molecules with symmetry group T_d, O_h, D_{5h}, and I_h respectively.

iii) In biological systems symmetry often results from economizing genetic information. In viruses the protein coats are built from identical subunits. Examples of this are the tobacco mosaic virus having a helical structure and the southern bean mosaic virus having icosahedral symmetry.

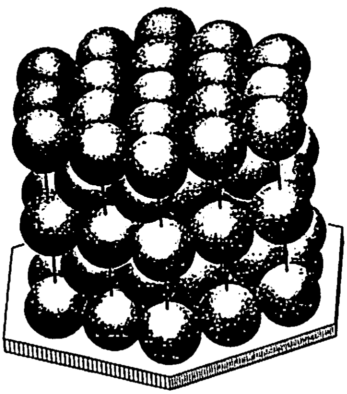


Model of the tabacco mosaic virus

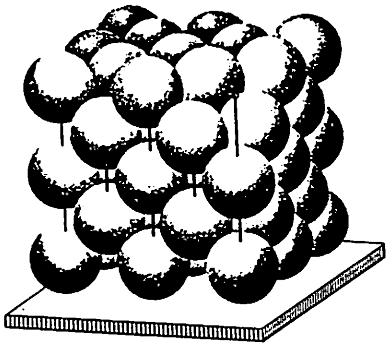


Model of the Adenovirus

iv) Symmetry can result from dense packing of equal atoms of spherical shape. Many elementary metals crystallize in cubic or hexagonal close packing with density 0.74. The corresponding crystals show symmetry groups $Fm\bar{3}m$ for cubic and $P6_3/mmc$ for hexagonal close packing.



Hexagonal close packing

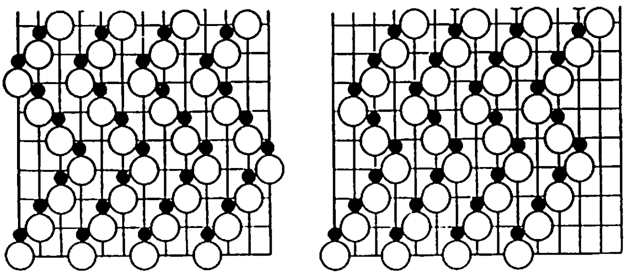


cubic close packing

We consider crystals as built up from small atomic aggregates. These building blocks, for simplicity, we take to be the content of a fundamental domain of a crystal. In a crystal each building block is surrounded in exactly the same way by all the neighbouring building blocks. Crystallization is considered as a higher level aggregation of identical building blocks. A fundamental law of crystallography states, that such an aggregate is crystalline if every building block is surrounded in exactly the same way within the first two neighbourhoods. Remarkably, in most cases it is sufficient to require a congruent first neighbourhood in order to establish periodicity. Thus symmetry results from the local requirement that every building block is surrounded in a congruent way within the first neighbourhood.

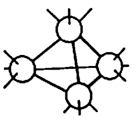
2. Breaking of symmetry

Of particular interest are the few exceptional cases where a congruent surrounding in the second neighbourhood is absolutely required to establish periodicity. If the constraint of congruent surroundings is partly relaxed then a break in symmetry could result. Examples are the frequent occurrence of polytypes which show a one-dimensional disorder.

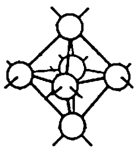


Polytypes of SiC

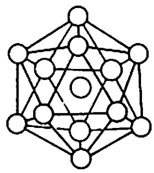
Thus, symmetry is not an intrinsic property of nature. If the constraint imposed by some external forces is relaxed then, in general, symmetry breaks. Particular interesting cases of symmetry breaking occur when non-spherical building blocks are densely packed. In metals, for example, we often encounter building blocks of icosahedral symmetry as a result of locally dense packing of atoms. The maximum number of atoms that can be placed in contact with a central atom is 12. At equilibrium the resulting aggregate shows icosahedral symmetry. I_h. For example such aggregates can be observed as gold clusters.



M_4



M_6

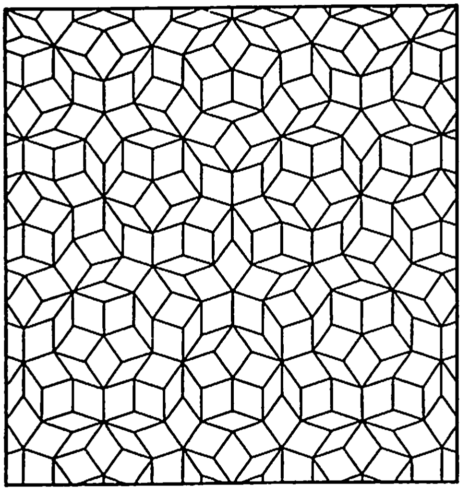


M_{13}

We may place another 20 atoms at the largest interstitial sites which occur above the triangular faces of the icosahedron to obtain a locally dense packing of atoms. Further shells cannot be packed as dense as that. An infinite non-periodic packing with density 0.734 has been described.

A classical result of crystallography states, that five-fold rotation is not consistent with periodicity in three dimensions. Since the icosahedron has five-fold rotation axes it is not possible to pack icosahedra densely in a crystal. Hence, in order to obtain a denser packing of equal atoms, at some embryonal state of crystal growth a rearrangement into a cubic or hexagonal closed packing has to occur. This means that the local symmetry is broken in order to obtain a higher global symmetry.

If the central atom of such an icosahedral aggregate is slightly smaller and has a relative radius of 0.902 then the twelve surrounding atoms are in contact with its neighbours. This situation may occur in binary alloys having two kinds of atoms of different radii. More stable icosahedral aggregates result which do not rearrange so easily Trying to pack such building blocks densely could result in aperiodic quasi-crystals. In quasi-crystals the local symmetry is retained at the expense of global symmetry. Examples are the quasi-crystals of Al/Mn/Si alloys.



Penrose tiling as a model for quasicrystals

THE SYMMETRY OF MUSIC

ESCOT, Pozzi

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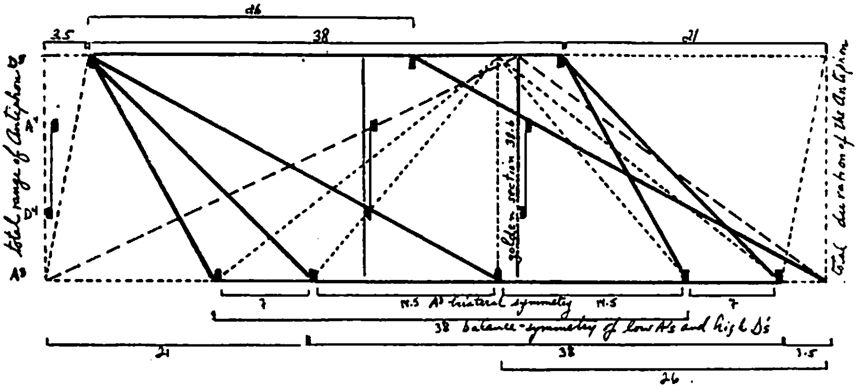
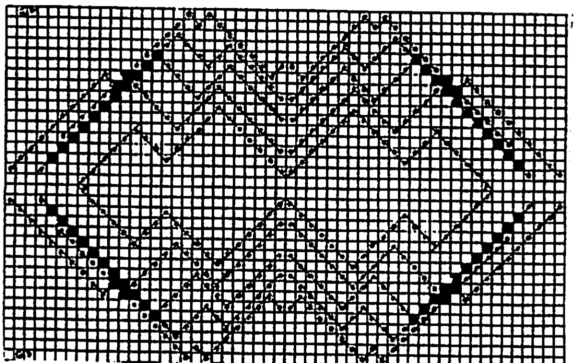
Bronowski writes that discoveries of science and the works of art are both explorations and explosions of a hidden likeness. It is this likeness that is often equated with symmetry. Pagers, in The Cosmic Code, says "If we could go back to the beginning of time, to the primordial fireball of quarks, leptons, and gluons, when the gauge symmetries were as yet unbroken, instead of 'Fiat Lux' we might have heard 'Let There Be Symmetry'". Physics searches for symmetry laws as it pushes to the boundaries of the material world -- fundamental laws of symmetry as that space is the same in all directions and places. In music the hidden likeness is a projection of mathematical models which often show stupendous spatial and time deployments. A myriad factors intervene or are combined to structure a sonic design where dynamic symmetries play the only role. Weyl, the philosopher of mathematics, writes that nature's surface beauty conveys no more than a hint of the loveliness hidden within; the mathematics is not in its skin, the symmetry must be uncovered. In music, a first audition does not discern the internal architecture; it is analysis of the written score which discloses the schema.

To Plato symmetry in nature is governed by mathematical laws and these are intuitively realized in the creative mind. For St. Augustine music is number and progressive divisibility affected the entire work down to the smallest details of its dimension: a hierarchy of symmetries. Bronowski again cites how Da Vinci was occupied with the logic of the processes he saw in people and machines; he looked for the hidden structure because it expressed that logic, proportion and symmetry. This essay presents that exploration and explosion of a hidden likeness as it uncovers the symmetries of musics from medieval Hildegard von Bingen to the contemporary Gyorgy Ligeti. Two brief examples follow.

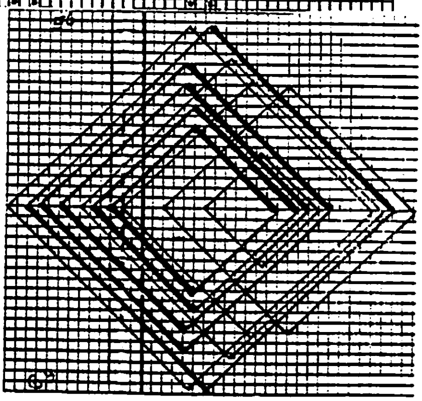
Using a complex plane (two-dimensional time-space figure) which translates accurately the temporal-linguistic measurements, we may see in Antiphon 61 of Bingen that the five same lowest and three same highest sounds are proportioned and symmetrical. The work Harmonies by Ligeti outlines a multiple symmetrical perspective. One order of this multi-faceted geometry shows the ten voices spanning 231 units of one total attack point each moving with parallel finger-exchange and in exact cumulative addition. To overview this space we may obtain through a one-to-one reduction (cancelling the cumulative addition) the geometry that pictures the symmetries of the parallel voices. Completing the lined squares of which Ligeti used only $\frac{1}{2}$, the dual bilateral symmetry is revealed.

Being responsive to the pattern which connects means being responsive to the critical aspect and esthetic experience, writes the biologist Bateson. Symmetry in biology results from the facts of growth. In music symmetry offers balance and correspondence between parts, parts and whole. These musics are strata of connective patterns involving remarkable invention. It is this discovery which conveys one other understanding of explorations and explosions of a hidden likeness.


26
3.5
38
21
total range of Antiphon
golden section 38.6
total duration of the Antiphon
7
11.5 A bilateral symmetry 11.5
7
38 balance-symmetry of low A's and high D's
21
38
3.5
26



Its mirror



STUDY OF DYNAMICS OF SYMMETRIC SYSTEMS

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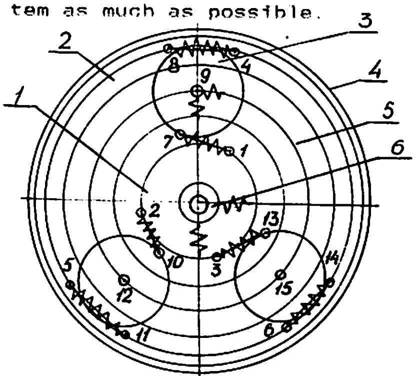
Most of unique properties of dynamics of a system are cau... by its symmetry. These include multiplicity of natural frequenc... number of independent forms of vibrations, effect of disturbing forces symmetry, etc.

If, however, the role of symmetry and its effects are to be studied, it is helpful then to use the theory of symmetry groups. This approach allows to characterize the symmetry of an object under study in terms of exact mathematical language, thus enabling a qualitative analysis of the systems. Such a theoretical-group approach is efficient in numerical calculations of complex systems since it allows to overcome the "misfortune" of large measures through decomposition of vector space.

When studying a system by symmetry group methods the dynamic model of a structure under study is represented as a lattice . However, full symmetry of a mechanism is defined not only by relative positions of similar elements, but also by symmetric properties of each of them. 2.7

Therefore let us make the dynamic model of Fig. 1 more clear by describing each element's configuration with a finite number of points. On the one hand, the number of points and their arrangement must fully define element configuration ; on the other, they must represent, partly by intuition, the symmetry of the entire system as much as possible.



1 – sun gear "S"
2 – epicycle "E"
3 – satellite "P"
4 – body
5 – pinion carrier
6 – bearing

Fig. 1

Now, mark by encircling, the assumed equivalent points, characterizing the symmetry of separate elements; then groups of points will characterize symmetry between them.

Thus, three groups of points (7,8,9),(10,11,12),(13,14,15) define symmetry between elements "Pi" – symmetry group C3, while points (7,8),(10,11),(13,14) define symmetry of each element "Pi" – symmetry group C2. On the others hand, each group of points (1,2,3) and (4,5,6), when taken separately, characterize symmetry of elements "S" and "E" – symmetry group C3; then taken on the whole, they give symmetry group C2. Under the circumstances shown the symmetry of the mechanism under consideration can be represented as a sum of product of groups $C2 \otimes C3 \oplus C3 \otimes C2$, where $\otimes$ – product of characters of group irreducible representations, and $\oplus$ – their direct sum. Finally, the symmetry of the entire mechanism – C3h+C3h.

After mechanism symmetry has been defined one can pass over to analyzing its properties in terms of quality. For this: put down characters of irreducible representations of point symmetry group C3h into Table 1, compiled for group algebra in the field of complex numbers, where $\varrho = e^{2\pi i/3}$; $\varrho^* = e^{4\pi i/3}$

Table 1

Ch	E	C	C	C	S	S
χ_1	1	1	1	1	1	1
χ_2	1	1	1	-1	-1	-1
χ_3	1	ξ	ξ^*	1	ξ	ξ^*
χ_4	1	ξ^*	ξ	1	ξ^*	ξ
χ_5	1	ξ	ξ^*	-1	$-\xi$	$-\xi^*$
χ_6	1	ξ^*	ξ	-1	$-\xi^*$	$-\xi$
n(R)	15	0	0	3	0	0
χ(R)	15	0	0	3	0	0

For better convenience some data is below under the Table. This data refers to the number of immovable points n(R) (for various operations) and to characters of Cartesian reducible representations [3]. The data allows to define number m ,showing how many times each of irreducible representations is contained in a Cartesian representation: $m_j = \frac{1}{h}\sum_{R\in G} \chi_r(R)\, \chi_j(R)$, where h - number of elements in set groups G; R -operation on group; χ_r, χ_j - characters of reducible and j-th irreducible representations respectively. For symmetry group C3h m is expressed as

$$m_j = \frac{1}{6}\left[15*1*\chi_j(E) + 3*1*\chi(C2)\right] \qquad (1)$$

Substituting (R) values from Table 1 into (1) one will obtain m1 = 2; m2 = 2; m3 = 2; m4 =3; m5 =3; m6 =3. Hence the original vector space L (with duly accounted symmetry of the mechanism's elements) splits into a sum of orthogonal subspaces of similar vectors L = 15 = 2*1+2*1+2*1+3*1+3*1+3*1.

Whereas in a molecule type model, analyzed in the work , the equivalence of similar points is self-evident, the equivalence of points in the mechanical model has yet to be substantiated. When the equivalence conditions are not satisfied, attempts should be then made to construct symmetry. Ensure that equivalence of points does exict, using operator A,which interrelates the original, Cartesian system of coordinates "q" and the point system "u". A q = u (2)

Thus for elements "S" and "E" expressoin (2) is:

$$\begin{pmatrix} \alpha_1 & \beta_1 & r \\ \alpha_2 & \beta_2 & r \\ \alpha_3 & \beta_3 & r \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ \varphi \end{pmatrix} = \begin{pmatrix} u1 \\ u2 \\ u3 \end{pmatrix} \qquad \begin{aligned} \alpha_i &= \cos(2\pi(i-1)/3) \\ \beta_i &= \sin(2\pi(i-3)/3) \\ i &= \overline{1,3} \end{aligned}$$

r - radius of gear pitch circle. Put down the expression of matrix of stiffness ($\tilde{C}$) and mass ($\tilde{M}$) in new coordinate system "$\tilde{u}$" in terms of their original expressions [4] $\tilde{C} = (A^{-1})'\, C\, A^{-1}$; $\tilde{M} = (A^{-1})'\, M\, A^{-1}$, where M and C - matrices of masses and stiffness in the original coordinate system "q", u_i^k - vector of displacement of the i-th point of the k-th element.

The expressions obtained for $\tilde{C}$ and $\tilde{M}$ allow one to see whether the points are equivalent. Verification proves that points (7,8), (10,11) and (13,14) are not equivalent; hence, the system symmetry is defined by a direct sum of groups C3h and C3. This explains lack of parity in the measures of subspaces, as in the case of molecule NaCl cited in work [3]. Such transformations can be used to construct symmetry. Thus, for a group of points 1,2,3 and 4,5,6 it is necessary to satisfy the conditions:

$$B_s' \tilde{M}_s B_s = B_e' \tilde{M}_e B_e, \quad B_s' \tilde{C}_s B_s = B_e' \tilde{C}_e B_e \qquad (3)$$

where $B = A^{-1}$. In (3), for example, parity can be assured by proper choice of inertia moment of element "E",given that gears "S" and "E" have similar masses (Ms = Me) and coefficients of elastic links. If masses and stiffnesses of elements "S" and "E" are different, the point equivalence condition can directly be inferred from equality of partial frequencies of sybsystem "S" and "E", i.e. conditioned that $\tilde{H}s = \tilde{H}e$, $H = M^{-1}\ C$.

Organization of high symmetry grade in a system suggests that disturbance symmetry grade is as high. Only in this case will each subsystem be "engaged" with "its" disturbing force . These problems can be studied employing a theoretical-group approach and taking into consideration the symmetry of both the disturbance and the system per se . Let us exemplify the analysis technique by the model under consideration. Begin with determining the projective operator T^6 ,which links basis vectors of original system,also with vectors $\tilde{u} \in G$ of symmetric coordinate system u =T u (4),where $T = \frac{f_j}{N}\sum_{R \in G} \chi_j$ (R), χ(R) - character of the J-th irreducible representation $\mathcal{T}$(R), f_j - measure of the J-th representation. Substituting (3) into (4), one will obtain $g = T^{-1} . A^{-1}$ u or $g = Q^{-1}$ u (5). The latter can be formulated in the form of Table 2, interrelating Cartesian coordinate system with symmetric one, after sorting subspaces Di. Using operator Q for transforming the original vector of disturbing forces F(t), one will obtain its written form over subspaces Di: F(t) = Q F(t). Hence, each Di subspace will be expressed as $Di*\tilde{u} = Fi(t)$ (6)

Given as an example are disturbing forces from unbalance of bodies "S", "E" and "P". In this the unbalancing force on each satellite can act either in phase with others, $\alpha_1 = \alpha_2 = \alpha_3$; out of phase at an angle $\alpha_i = 2\pi(i-1)/3$; (i=1,3) or arbitrarily.

Having completing transformations, aided with operator Q, their structure can be accounted of in subspaces Di, table 3. Forming equations (6) on the results of table 2 and 3, one can argue that F(t) from elements "S" and "E" do not disturb vibrations in subspace D1. However, these forces, alike $\tilde{F}ic(t)$, "engage" with the rest of subspaces, destroying the system symmetry.

Table 2

Di	N coord.	Ui =F(g)
D1	1	$u1 = \varphi_s + \varphi_e$
	7	u7 = Xp
D2	2	u2 = Rs + Re
	10	u10 = Xp
D3	3	u3 = Rs +Re
	13	u13 = Xp
D4	4	$u4 = \varphi_s - \varphi_e$
	8	$u8 = \varphi_p^+$
	9	$u9 = Yp^+$
D5	5	u5 = Rs - Re
	11	$u11 = \varphi_p^+$
	12	u12 = Yp
D6	6	u6 = Rs - Re
	14	$u14 = \varphi_p^-$
	15	u15 = Yp

$R = \alpha X + \beta Y$;
(+) - symmetric form;
(-) - oblique-symmetric form:

Table 3

Di	"S"	$\alpha_i = \alpha_j$	$\alpha_i \neq \alpha_j$	$\alpha_i = \frac{2\pi(i-1)}{3}$	"E"
D1	0	0	0	0	0
	0	1	0	1	0
D2	1	0	0	0	1
	0	1	1	0	0
D3	1	0	0	0	1
	0	1	1	0	0
D4	0	0	0	0	0
	0	1	0	1	0
	0	1	0	1	0
D5	1	0	0	0	1
	0	1	1	0	0
	0	1	1	0	0
D6	1	0	0	0	1
	0	1	1	0	0
	0	1	1	0	0

"S" -sun gear; "E" - epicycle;
"P" - satellite;

The theoretical-group approach can be applied to formulate the inverted problem as well, when a new structure F(t), disturbing only definite subspace is given and when it is necessary to derive the structure of original vector of disturbance that is desirable to obtain on the basis of inverted transformation $Q^{-1} \tilde{F}i(t) = Fi(t)$. Consequently, one can not only analyze mechanism qualitatively, but also sythesize symmetry with due account made to disturbing forces and their phasal relations. The described analysis technique allows to dispense with dynamics equations of a studied mechanism as it is based only on the properties of its symmetry.

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Symmetry as a Cultural Expression in Some Brazilian Indian Tribes

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Introduction

Brazil counts nowadays with about 200.000 indians devided in 150 different tribes. Each tribe with its own culture and many of them with no contact among themselves. They are very rich in handicraft and corporal paintingss, that is where symmetry appears with strong characteristics as part of the esthetic sense.

I chose three of these tribes to show this symmetry: Tapirapé, Kaxinawá and Kadiwéu.

The handicraft is done normally of straw braid wood, feather, painting, pottery, cotton weaving, etc. The corporal painting is basically and almost always done with two colors: the "urucum" red and the green "genipapo" black. The drawings are well elaborated and done in the body with a bamboo stiletto. This job is always done by the oldest women.

I am going to use an international notation to characterize the symmetry: translation, rotation, reflection and "glisso" reflection.[3,4]

1. Tapirapé

The Tapirapé tribe, in our days, has got only one village with 270 indians and is localized in central Brazil, alongside the Tapirapé river, affluent of the Araguaia, in Mato Grosso state. They are former indians, that is, not warriors, and have their survival based on agriculture and fishing. Their culture is rich in feather, strawbraid and brazilwood handicraft. All the time symmetry is found in these manifestations, but only the reflection. The importance of the duality makes reflection to be a strong reason of esthetic. We can see that in the corporal paintings[5].

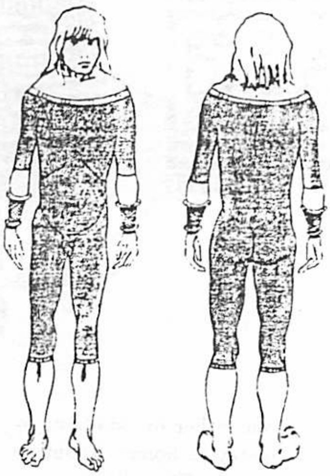


Fig. 2 — Pintura de corpo masculino *ruangkani*.

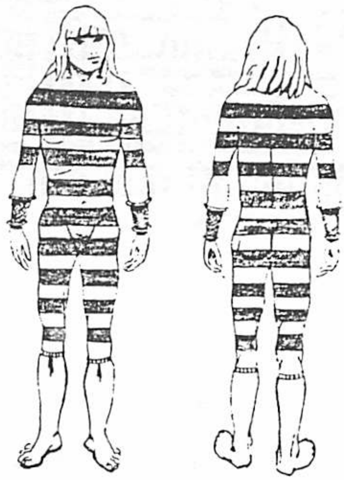


Fig. 3 — Pintura do corpo masculino *ipikaudua*.

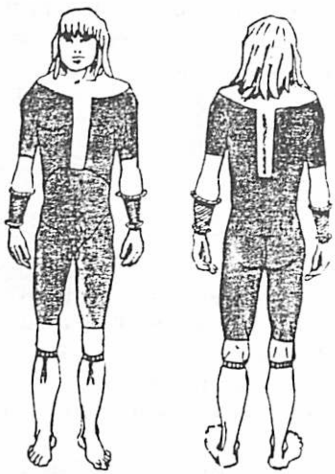


Fig. 5 — Pintura de corpo masculino *kananchakó*.

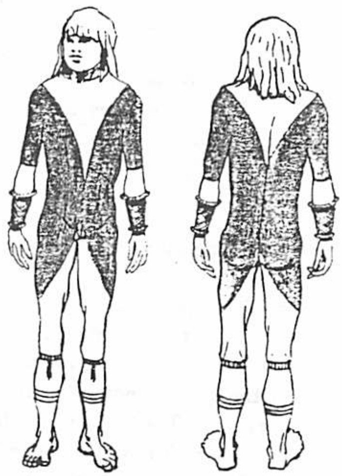
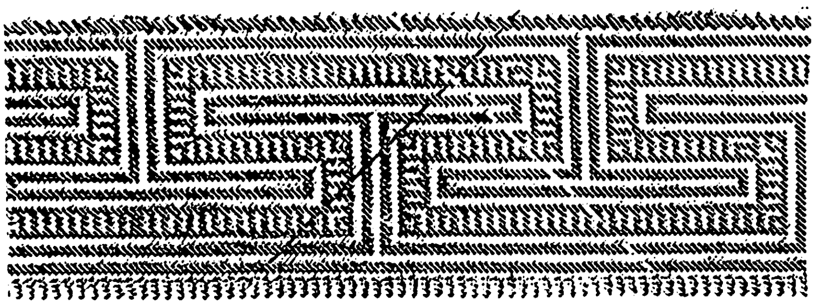


Fig. 4 — Pintura do corpo masculino *chankoaka*.

reflection

That are done to some commemorations as man and woman initiation, birth of a children the parents paint themselves), death, etc. The duality is also found in basket manufacturing



translation, reflection and rotation - band

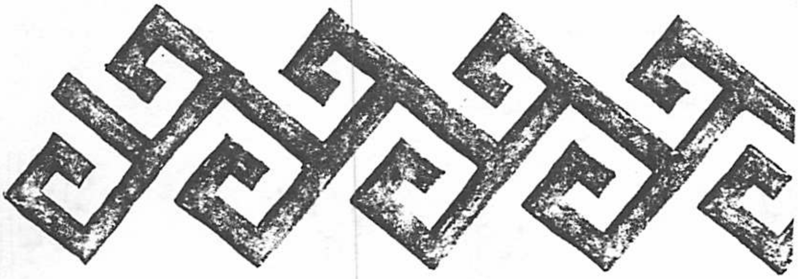
and also in games as the string games. This game is done either by adults or by children mainly in rainy days, when they are obliged to stay at home. About go different figures where registered, all symmetric by reflection[1]. The reflection axis can be vertical or horizontal. When braiding the string, the person, knowing. The kind of axis the figure has to have, repeats with one hand what was done by the other in the case of vertical axis, and in the case of horizontal axis what was done by the two back fingers must be repeated by the two front fingers.

The string game appears in may Brazilian tribes that supposely had never had any contact.

2. Kaxinawá

The Kaxinawá indians live in Acre State and Amazonas State by the peruvian border. Their population is of 2.500 indians, devided in 11 villages. Their economy is based in rubber extraction and Brazil nuts collection. This economy is the result of the contact with the whites, who led them to develop this economy for their survival. Their most significant handicraft is the cotton weaving with strings colored with roots, "urucum" or "genipapo".

The handicraft pictured is from the Kaxinawá village of the Jordão River which holds 800 indians. The looms are done of more than two-meter high pieces of wood. It is an exclusively feminine job, taking a long time to make each piece. As symmetry, only the reflection appeas, but there is also a duble reflection. Because of the proximity with Peru we find in their drawings a very large *Inca* influence:



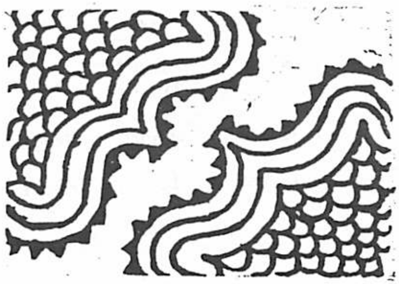
reflection and rotation - band

3. Kadiwéu

(In colaboration with Prof. Rodyney C. Bassanezi)

The Kadinawéu are indians who in habit one only village in Mato Grosso do Sul State by the Paraguaian border. They have got a population of 1.100 indians in an extense territory delimited by the government as a reserve. They were warriors, but nowadays they have a reaveful contact with the white population. What calls attentioı is their very elaborated corporal painting, that has appeared in ethnological books since 1.540. The contrast between their painting, also done with Green "genipapo" and the ones of other indians is that the Kadiwév prefer curved lines to straight lines.

reflection with perpendicular axle - Pmm

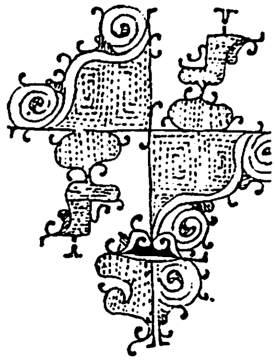


band - tranlation and "glisso" reflexion

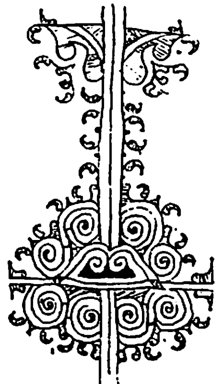
The symmetry is more elaborated, appearing all the kinds. and the corporal drawing has a pure esthetical finallity. They are done by the oldest women and do not have a figurative representaion. The fact that only the women are responsable for the corporal paintings appear very early as the anthropologist Darcy Ribeiro[2] proved in a set of drawings done by children of both sexes which shows that the boys make figurative drawings and the girls non figuratuve ones. The richest drawings are the ones of the face where normally appears a reflection on the mouth

are and 180° rotation on the nose region

rotation



rotation



reflection and rotation

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ASYMMETRIC EDUCATION: PROSPECTS AND DANGERS

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Humam beings have an external symmetrical appearance, however, their internal structure is asymmetric. For example, the heart is in the left side of the body. Moreover, the two cerebral hemispheres of the human brain are somewhat functionally asymmetric.

Several dichotomies were suggested to characterize this functional asymetry. One of these is the analytic processing of data by the left hemisphere one datum after another temporally versus synthetic data processing by the right hemisphere which creates a new whole out of several data. This dichotomy was suggested by Levi-Agresti and Sperry (1968). Ben-Dov and Carmon (1976) suggested that all the other hemispheric dichotomies can be obtained from this single dichotomy. The discovery of these two cerebral data processing mechanisms became possible by their asymetric locations in the brain, but the degree of lateralization is different in individual persons.

The existence of the two hemispheric mechanisms is already applied in the treatment of dislexia. Most of the cases of dislexia are due to a non-efficient left hemisphere which does not enable the children to process the letters one after another while reading. This difficulty can be avoided by teaching these children to read in the global method, namely, to read each word as a single picture. On the other hand children with a non-efficient right hemisphere fail to read in the global method, but they can learn to read one letter after another.

It was suggested in Fidelman (1984) that a dualic method of teaching related to the hemispheric mechanisms can be applied also in the teaching of arithmetic. It was found there that the concept of ordinal number, namely, number as a property of a single element, is related to the left hemisphere; on the other hand the concept of cardinal number, namely, number as a property of a whole set, is related to the right hemisphere. Therefore children with a more efficient left hemisphere can learn arithmetic more easily through ordinal concepts, while children with a more efficient right hemisphere can learn arithmetic more easily through cardinal concepts. The technique of computation may be the same in both methods.

It was suggested in Fidelman (1987) that learning calculus in the standard approach of potential infinity is related to the left hemispheric mechanism, while learning calculus in the non-standard (Robinson's) approach of actual infinity is related to the right hemispheric mechanism.

The hemispheric duality in human cognition extends over the whole of cognition and even of culture, see Bogen (1965). It was found in Fidelman (1989) that the dualic conception of physical phenomena as discrete particles and as continuous force-fields or waves may also be related to the hemispheric mechanisms, and this relation may possibly be applied in education.

It is possible to classsify children before entering school according to the efficiency ot their hemispheric mechanisms and to teach them both reading and arithmetic according to the method which suits their brain's functioning. A similar classification can be performed at highschools and universities in order to present physics and physical chemistry to students with a dominant left hemisphere through corpuscular presentation at the first stage of their studies, and to present these subjects to students with a dominant right hemisphere through continuous presentation at the first stage of their studies. Similarly, students can be classified in order to teach calculus more efficiently, the computation method in both the standard and the non-standard methods of teaching are the same, only the concepts and the proofs differ.

Each student should learn both systems of concepts, the left and right hemispheric related, of each subject. One system which suits his aptitude should serve each student in his own creative thought and be applied as "a native scientific language". The other system of concepts should serve as "a foreign scientific language" in order communicate with persons whose aptitude is different.

The advantage of this dualistic education is that it will increase the learning level of the whole population. It will prevent "slow learning" due to a method of teaching which does not suit the brain of the student. This is most important in the future society, in which automatization will replace the manual work and the importance of education will increase for both individuals and society.

The danger in this asymmetric educational method is that it may create two parallel cultures which do not communicate. However, the present situation is already the same. A controversy exists between ordinal and cardinal approaches to the foundations of mathematics. As a famous mathematitian, Lebesgue, stated about this conflict, no discussion between the two parties had been possible because they had no common logic so they could do no better than to insult each other.

A similar situation occurred in physics regarding the duality of phenomena. Einstein is the most outstanding representative of the wholistic, spatial and continuous approach to the perception of physical world. This approach may be characterized as right hemispheric. Einstein could not accept the consequences of the discrete quantum mechanics, though he himself discovered the photons.

We may conclude that mathematics and physics developed as two extreme hemispheric-related approaches. Asymmetric education may, indeeed, increase this extremity, but doing so it may accelerate the great discoveries of the future. Persons with equally efficient hemispheric mechanisms can operate as mediators between the extreme approaches and synthesize a united culture from the two extreme sub-cultures.

If each individual person will learn not only the "scientific native language" but also the "scientific foreign language" and will learn to communicate with members of the other hemispheric-related sub-culture and respect it (instead of insulting each other), than the asymmetric education may lead to a symmetric and harmonious united civilization.

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ON 3-PERIODIC MINIMAL SURFACES. II. TOPOLOGICAL PROPERTIES.

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Genus and Euler characteristic

A non-periodic surface in R^3 is said to be of **genus g** if it may topologically be deformed to a sphere with g handles. For 3-periodic minimal surfaces a modified definition must be used (cf. Schoen, 1970) counting only the number of handles per primitive unit cell. In other terms, the 3-periodic surface is embedded in a (flat) 3-torus T^3 to get rid of all translations, and then the conventional definition of the genus may be applied. This procedure corresponds to identifying the opposite faces of a primitive unit cell.

The genus of a 3-periodic minimal surface may be calculated in different ways, two of which will be discussed in the following:

(1) Labyrinth graphs: Each 3-periodic minimal surface without self-intersection subdivides R^3 into two infinite regions, called labyrinths, which are connected but not simply connected. Schoen (1970) proposed to represent the labyrinths by graphs in the following way: Each labyrinth graph is entirely located within its labyrinth; each branch of a labyrinth contains an edge of its graph; each circuit of one labyrinth graph encircles at least one edge of the other graph.

Any of the two labyrinth graphs may be used to represent topological properties of the surface. As each circuit of the graph corresponds to a handle of the surface the number of circuits per primitive unit cell may be counted to get the genus of the surface. In case of a **minimal balance surface** (intersection-free 3-periodic minimal surface that subdivides R^3 in two **congruent** labyrinths; cf. Fischer & Koch, 1987) the symmetry is best described by a group-subgroup pair G-H of space groups with index 2, and the genus has to refer to a primitive unit cell of the subgroup H. There exist two different possibilities to derive the genus with the aid of labyrinth graphs:

(a) In modification of a procedure proposed by Hyde (1989), a connected subgraph containing no translationally equivalent vertices may be separated from a labyrinth graph. Then the genus of the surface may be calculated as

$$g=\frac{p}{2}+q,$$

where p is the number of edges connecting the finite subgraph to the rest of the infinite labyrinth graph, and q is the number of edges that has to be omitted to make the subgraph simply connected. As p equals at least 6 the genus of a 3-periodic minimal surface without self-intersection is at least 3.

(b) Keeping in mind the embedding of the minimal surface in the torus T^3 a more crystallographic formula for the genus may be derived. g equals the difference between the number e of edges in the embedded labyrinth graph and the number e_s of edges in any simply connected subgraph with the same number v of vertices. With

$$e=\sum_i m_i e_i/2 \qquad \text{and} \qquad e_s=v-1=\sum_i m_i-1$$

it follows:

$$g=e-e_s=1+\sum_i m_i(e_i/2-1).$$

Here m_i means the multiplicity of the i-th kind of vertices referred to a primitive unit cell of H, and e_i is the number of edges meeting in this vertex. The summation runs over all kinds of symmetrically equivalent vertices of the labyrinth graph.

Details on the labyrinth graphs and the genera of the known minimal balance surfaces are tabulated by Fischer & Koch (1989c).

(2) Euler characteristics: An intersection-free surface in R^3 may also be characterized by a number χ, its Euler characteristic. χ is related to g by

$$g=1-\chi/2.$$

The Euler characteristic of an intersection-free surface may be derived in a simple way by defining a tiling on the surface, i.e. by subdividing the surface into tiles (disk-like surface patches). For such an arbitrary tiling the equation

$$\chi=f-e+v$$

holds, where f, e and v are the numbers of tiles (faces), edges and vertices, respectively, in the tiling. For a 3-periodic surface the tiling must be compatible with the translations of the surface, and the tiles, edges and vertices have to be counted per primitive unit cell of H (cf. Fischer & Koch, 1989c).

For a minimal balance surface generated from **disk-like-surface patches** spanned by skew polygons of 2-fold axes (cf. Fischer & Koch, 1987; Koch & Fischer, 1988) these surface patches may be used as tiles. Then χ may be calculated as

$$\chi=f(1-e_p/2)+\sum_i v_i,$$

where e_p is the number of edges of such a skew polygon, f is the number of skew polygons and v_i the multiplicity for the i-th kind of symmetrically equivalent vertices. f and v_i are both referred to a primitive unit cell of H.

If a minimal balance surface consists of **catenoid-like surface patches** spanned between parallel plane nets of 2-fold axes (Koch & Fischer, 1988) its Euler characteristic is given by

$$\chi=v_N-e_N,$$

where v_N and e_N refer to the plane nets of 2-fold axes. v_N means the number of vertices, e_N the number of edges counted for all nets of 2-fold axes and per primitive unit cell of H. Making use of the relation $f_N - e_N + v_N = 0$ for nets, the genus may be calculated from χ as

$$g=k+1$$

where $k=f_N/2$ gives the number of catenoids per primitive unit cell of H. The same formulae hold for minimal balance surfaces generated from **infinite strips** spanned between plane nets of 2-fold axes (Fischer & Koch, 1989b). Then k must be understood as the number of original catenoids (per primitive unit cell of H) that have been united to infinite rows.

Similar formulae have been derived (Fischer & Koch, 1989c) for three other kinds of minimal balance surfaces spanned also between parallel plane nets of 2-fold axes. Here k means the number of surface patches per primitive unit cell of H.

If a minimal balance surface is made up from **multiple catenoids** (Koch & Fischer, 1989a; Karcher, 1988)

$$g=km+1$$

holds. m gives the number of catenoids that must be united to form one multiple catenoid.

The genus of a minimal balance surface built up from **branched catenoids** (Fischer & Koch, 1989a) is given by

$$g=\frac{k(1+b)}{2}+1.$$

b is the number of branches at one of the ends of a branched catenoid.

If a minimal balance surface may be generated from **catenoids with s spouts attached** (Koch & Fischer, 1989b), its genus may be calculated as

$$g=ks+1.$$

Flat points

For each point of a minimal surface the defining condition

$$K_1+K_2=0$$

must be fulfilled, where K_1 and K_2 are the main curvatures in that point. Normally $K_1=-K_2\neq 0$ holds, i.e. the point is a saddle point. For exceptional points, however,

$$K_1=K_2=0$$

may be fulfilled. Such points are called **flat points** of the surface. In contrast to normal saddle points, the surrounding of a flat point shows n valleys separated by n ridges ($n\geq 3$). The simplest example with n=3 is the "monkey saddle".

For any point on an intersection-free minimal surface its degree of flatness may be characterized by an integer number ß, called its **order**. The order of a point P_o with normal vector $\vec{n}_o$ can be derived as follows: A second point P with normal vector $\vec{n}$ is moved on the surface around P_o. If P_o is a normal point, $\vec{n}$ rotates once around $\vec{n}_o$ during one revolution of P. If, however, P_o is a flat point, $\vec{n}$ rotates more than once (e.g. p times) around $\vec{n}_o$ per revolution of P. Then the **order ß** of P_o is defined as

$$ß=p-1.$$

Accordingly, a normal point has order $\beta=0$, and the order of a flat point may equal any positive integer. For 3-periodic minimal surfaces flat-point orders up to $\beta=4$ have been observed so far. The number n of valleys (or ridges) in the surrounding of a flat point is

$$n=\beta+2.$$

Each order of a (flat) point corresponds to a maximal site symmetry of such a point. This symmetry is $\bar{4}m2$ for $\beta=0$, $\bar{3}m$ for $\beta=1$, $\bar{8}m2$ for $\beta=2$, $\bar{5}m$ for $\beta=3$ and $\overline{12}m2$ for $\beta=4$. Therefore, most site symmetries of points on a minimal surface enforce the existence of a flat point. Conversely, only points with site symmetry $\bar{4}$, 222, 2mm, 2, m, or 1 can be non-flat points of a minimal surface.

There exists a relation between the genus of an intersection-free minimal surface and the order of its flat points (cf. Hyde, 1989; Hopf, 1983).

$$g=1+\frac{1}{4}\sum_i \beta_i.$$

The sum runs over all flat points within a primitive unit cell of H. This formula can be used in different ways:

(1) If all flat points with their orders are known the genus of a minimal surface may be calculated.

(2) If the symmetry of a minimal surface and its genus are known the relation between flat-point symmetry and flat-point order in combination with the above formula may be used to derive a complete list of flat points (cf. Koch & Fischer, 1989c).

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Sator Arepo Tenet Opera Rotas
or else:
Symmetrical Constructions in Webern's Rows ("Reihe")

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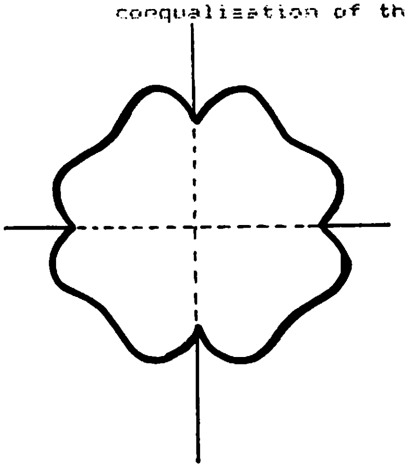
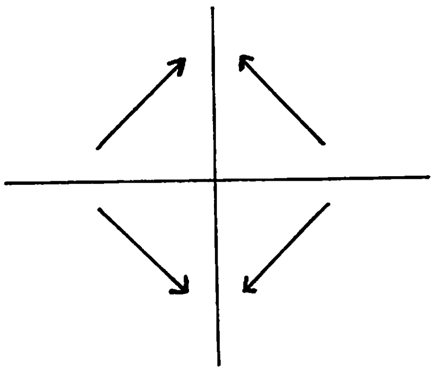
S A T O R
A R E P O
T E N E T
O P E R A
R O T A S

In case of stereoscopic compositions--be the object of survey natural or artistic composition--time is available to recognize the formal structure. (Let's think it over that a building can be walked around several times and from various directions.) And conversely, the formal inquiry at so called "temporal" compositions--such as musical ones--is more difficult: the composition itself "moves forward" in time and at the first hearing--without any help--only our vanishing acoustic remembrance can give a basis.

Thus the repetition can be considered as a primary form-creative factor. This helps the most the orientation. The larger form-unit is repeated, the easier recognition of the structure is: usually the "A-B-A", "da capo" structure can be perceived at first hearing.

Extremely difficult is the identification of the temporal progression if it is turned backwards. Perhaps Machault's short composition is the first "bravos-piece" of that sort: "Ma fin est mon commencement--et mon commencement est ma fin" (My end is my beginning--and my beginning is my end). It was followed by many enigma-canons of Flamish authors and the complicated polyphonic pieces of baroque music. In certain respect Webern wants to belong to this many-centuries-lasting musical progress, this is proved by his lectures in which he asserted that the birth of the "new Vienna school" was inevitable and this development was regular by the usage of the 12 chromatic degrees and by the conqualisation of the 12 tones.





The row, namely "Reihe" is a high level of musical organisation. Webern has pointed out that the constraint is useful in the interest of creating organic connections and at the same time it is exactly the constraints which can set free the thematic invention. He made clear the double-root of composing, based on the rules of the row, drawing attention to the fact that the new Vienna school was led by the same intention when used the 12 degrees of chromaticism in equal value, which sort of intentions occurred earlier in connection with the more various usage of individual scale-degrees.

op.31.

This rule corresponds to the law of the great melodic styles. It's enough to think of Palestrina's art, in which the prohibition of overusing of certain notes played determinant role. The same tendency is valid in the "harmonic" styles, too, because in the classicism the three functions didn't do other than exhausted fully the potentialities of 7 note diatonic scale. The diatonic row wasn't "found out" but it was discovered; similarly, the composition with the 12 chromatic notes is the result of a "natural order".

It can be easily believed that the method with the row binds in the componist, because--according to the row--a predestination would be established and, for example in a work with text it becomes incidental that for which sillable of the text which notes is associated. This question must be cleared to a certain extent.

Webern's method to create comprehensive connections rich to find abundant the possibilities of the row. Moreover, there are plenty of examples that he willingly narrowed the possibilities in order to crate rigourous relationships.

OP. 24.

We have to emphasize: although Webern explained the composition method of the new Vienna school, he didn't do it because of his technicist bias but he wanted to prove the justification of his new style.

Ot the one hand, the content played the primary role, on the other hand, he realised that the content can be transmitted only through a form which adequately for it.

This consciousness allows me to examine Webern's rows from formal and structural standpoints.

On the world of forms of Wilhelm Ostwald

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1. Who was Wilhelm Ostwald?

From his curriculum vitae.

1853	September 2, born in Riga
1872-1875	Student of chemistry
1878	Doctor promotion at the age of 25 years
1881-1887	Full professor of chemistry at the Polytechnicum in Riga
1887-1906	Full professor of physical chemistry at the university of Leipzig
1906-1932	free working in Großbothen
1909	Nobel prize for chemistry for his investigation in catalysis

He wrote several widespreaded books. Some are devoted to his color theory. Beside his color theory he developed a theory of forms which has not been taken into account. We will analyse his world of forms.

2. The symmetry concept in Ostwald's approach to ornamentics (Ostwald pattern)

Still as a student he was interested in antique ornaments. In 1922 he published a procedure to get the most symmetrically plane filling pattern. The starting point is the point lattice of the trigonal or the quadratic or the hexagonal mosaic. The order of the point lattice dependes of the size of the edges.
Two types of Ostwald pattern can be built up for each of the 3 point lattices: reflection type and rotation type. We explain the procedure in modern mathematical language. Consider the symmetry group S of the fundamental pattern of order n or the rotation group R of the pattern.
Now Ostwald takes an arbitrary segment s joining any two chosen vertices. The Ostwald pattern composed by this "line of theme" s will be the orbit

$\mathrm{orb}_S(s)$ resp. $\mathrm{orb}_R(s)$ in the plane.

3. Ostwald pattern from a cristallographic point of view.
Each Ostwald pattern has its symmetry group. This is one of the 17 Fedorov's Groups or wall paper groups. For the reflection type pattern there occur only the groups W_6^1 or W_4^1. (In the notation of the groups we follow [5]). For the rotation type pattern can be the groups W_6 or W_6^1 and W_4 or W_4^1.
4. Ostwald pattern generated by computer. The groups S and R are finitely generated. Therefore the Ostwald pattern can be given by a computer program.
5. How many Ostwald pattern there exist? Application of Polya's counting theory tells us the number of Ostwald pattern.
6. About Ostwald colored pattern. What are the symmetry groups? Many colored pattern could be found in the unpublished work of Ostwald. We will report on the answer to the mentioned question.

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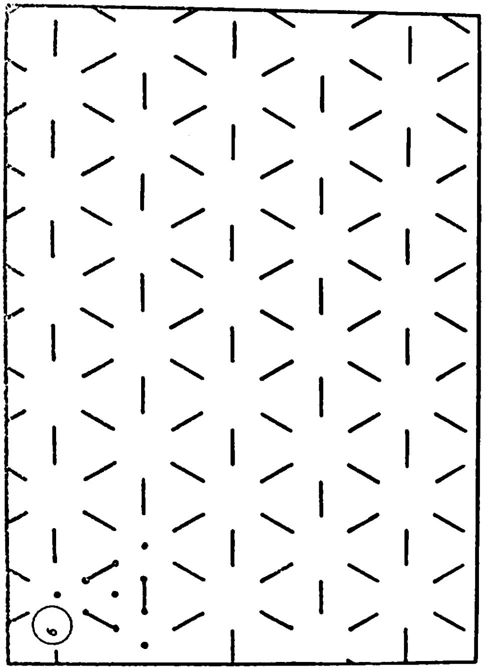

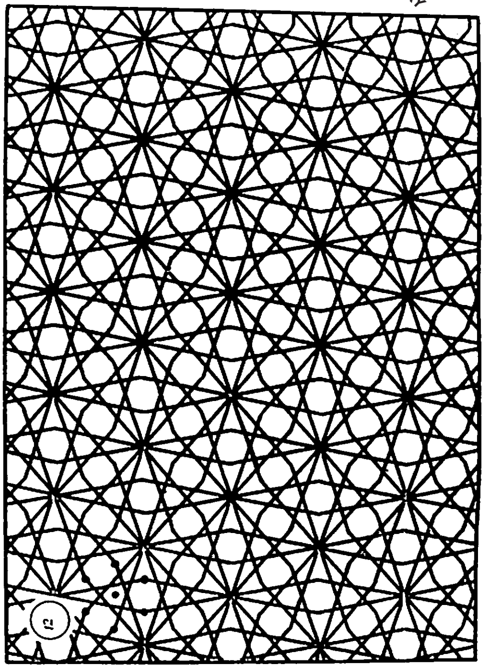
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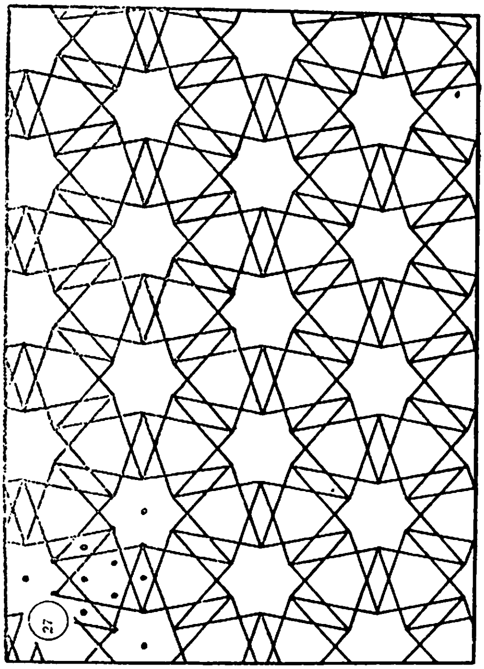
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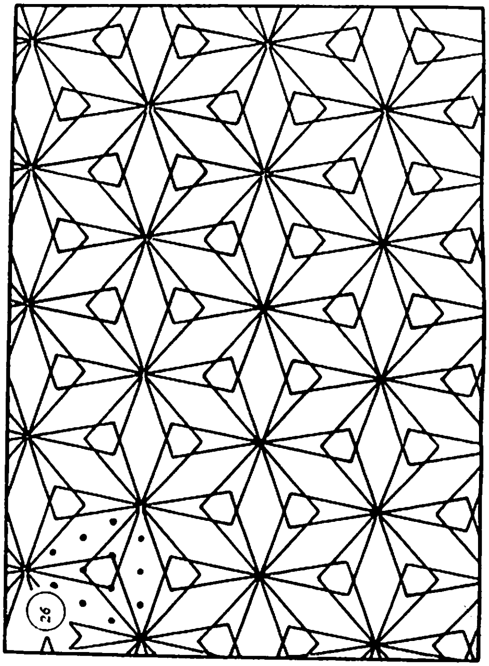
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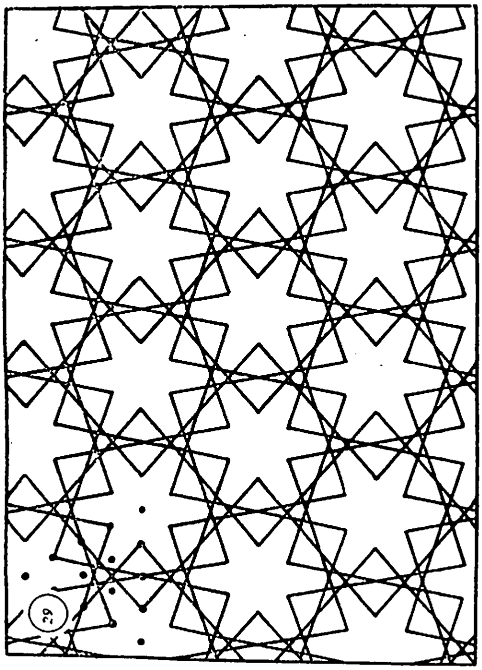
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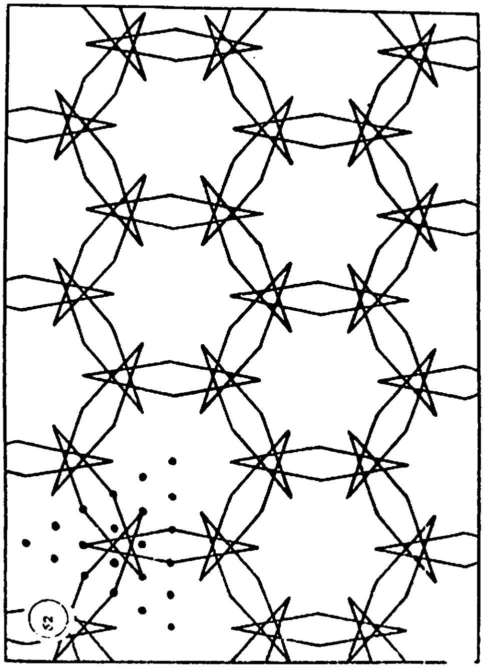
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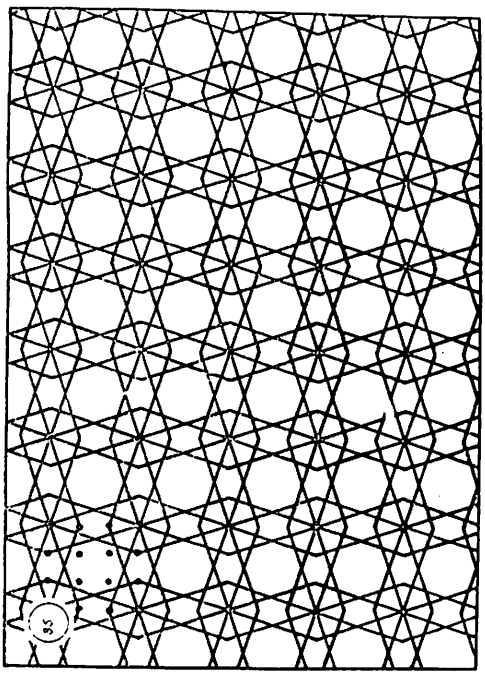

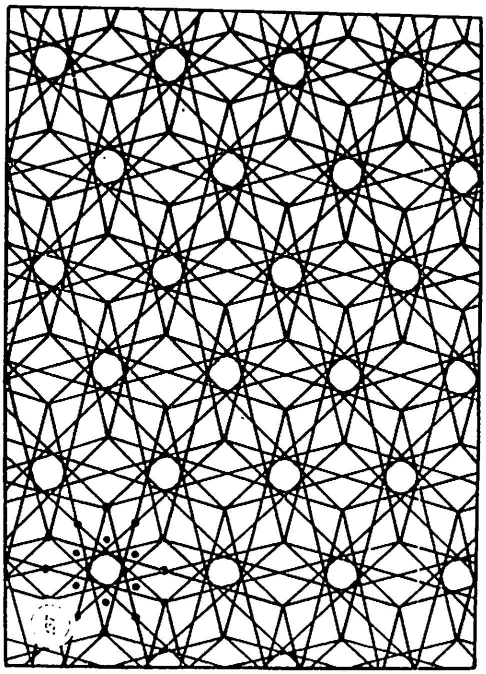

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DOME WITH CYLINDER VAULTS

Authors deal with a double-layers grid structure. A double layer grid structure has an external layer which constitutes a triangular grid and an external layer which is a triangular grid as well but with a four times greater number of triangles. The both layers are connected with stiffenig rods in such a manner that six rods converge in every vertex of the internal grid. These rods are connected, in turn, with six vertices of the hexagon external mother grid. The mother grid /fig.2/ is created from the triangular grid reduced to triangles and hexagons, in such a way, that every side of a triangle constitutes a common side with a hexagon, but neither triangles nor hexagons of this grid have common sides. The external grid is made from the icosahedron closed with twenty equilateral triangles in such position that the axis connecting the polyhedron opposite vertices is vertical. Let us draw an equilateral triangle on each face of the icosahedron, by connecting the sides centres. Those céntres are the apices of a semi-regular 32-hedron closed with 20 equilateral triangles and 12 regular pentagons. Drawing on each pentagon a regular pyramid with apex lying on a sphere described on a 32-hedron, we obtain an 80-hedron /fig.1/.

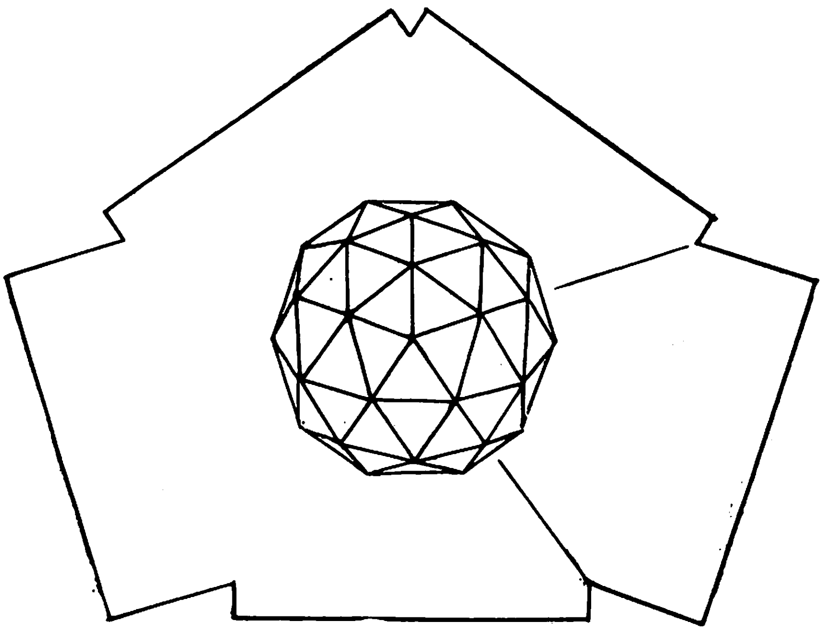

Fig.1

Let us consider a pyramid with a vertical height and pentagonal base, its five lateral faces constitute 1/16 part of an 80-hedron. The side of pyramid base measured in the angle between the radii of the 80-hedron sphere is 36°. The pyramid lateral side, measured in the same way, is equal to one half of the angle corresponding to icosahedron side i.e. $31^{\circ}43'03''$. So the dome will be formed from five isosceles spherical triangles with bases equal to 36° and sides $31^{\circ}43'03''$.

Let us now divide this spherical triangle into 400 spherical triangles. To do so we divide the triangle sides into 20 equal parts and we draw 19 big spherical circles, by corresponding points of division and by centre of the sphere. We divide these circles into 19, 18, 17, 16,... equal parts. So described grid apices are symmetrical collections of points with regards to the spherical triangle height. Thus, it is enough to take into consideration only one half of the spherical triangle. In this way we can fix position not only of the external spherical grid apices but also of the internal grid because every second apex will correspond to the internal grid apex and in this spherical triangle we obtain a four times smaller number of the internal grid triangles

At first let us designate angular coordinates of this collection of points. To calculate them we must find bases of the spherical grid triangles and their heights. Horizontal and vertical angular coordinates of the spherical grid apices are calculated on the respective tables. Having already angular coordinates, cartesian coordinates are calculated in tables for external and internal

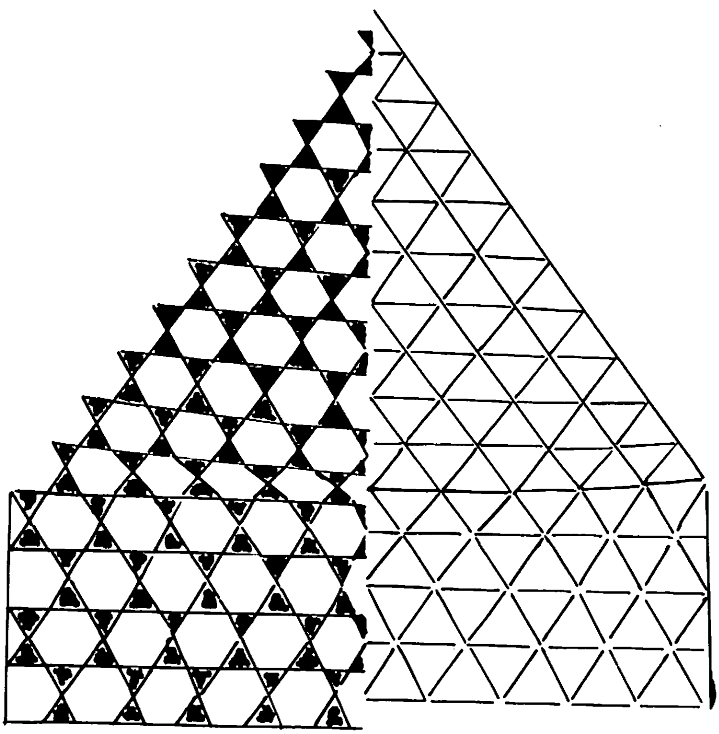

Fig.2

layer respectively. Thus we receive data to calculate length of the external and internal grid rods as well as rods connecting these two grids. These lengths are calculated on respective tables and are inscribed into the schemes of the 1/10 part of the dome. Grids on the cylinder vaults are designated in the following way. Let us consider one of the five arcs of the lowest circumference of the internal grid namely an isosceles triangle designated by points of support and the highest point of this arc. This triangle is leaned towards the level creating an acute angle. The base of this triangle is now the base of an isosceles vertical triangle with an apex having its ordinate equal to ordinate of the preceding triangle. Circumference circumscribed on this triangle is a cross section of the internal vault. The grid made of equilateral triangles is designed on the internal vault, taking as an example division of the lowest circumference of the spherical grid. Circumference having the same centre as the previous one but with radius longer respectively, being a cross section of the external vault depends also on the height of the highest apex of the external spherical grid lowest arc.
Analogically like in the equilateral grid, introducing additional meridians to the external vault we get an external isosceles grid with a four times greater number of triangles having a common axis with the internal grid. Then we calculate rectangular coordinates and lenghts of rods on the respective tables. Lenghts of rods are presented on schemes of 1/10 part of the structure.

"FUNCTIONAL ASYMMETRY" IN THE SYSTEM OF ARTS

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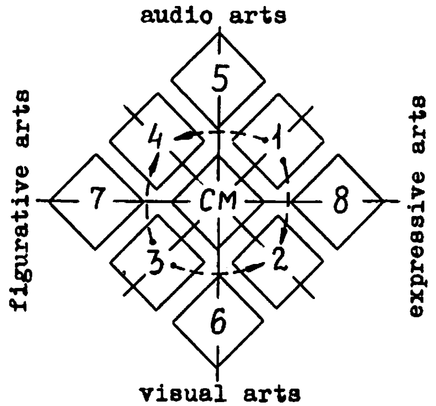
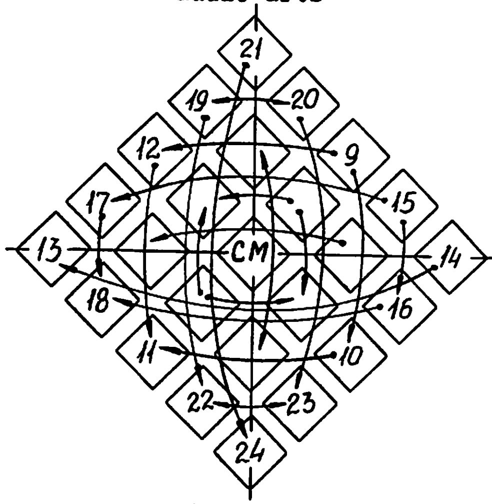
Aesthetics has been striking for a long time to find out if some sort of order exists in the family of Muses. The author has also made his own contribution suggesting a kind of a "periodic system" of arts, not so strict as that of D.I.Mendeleyev in chemistry, but possessing sufficient motivation and richness of content (1). We should like to specially point out that schematic execution of the suggested classification of arts has a symmetrical structure and along both axes at that (Fig.1).



What is this graphic-structural method of analysis that we have developed based on? Here we employ the method of spatial "splitting" of the essential powers of the integral man - conditionally he is placed in the centre as "the measure of all things" (in this scheme it is a "creative man"). "Bifurcation of the unity and the cognition of its contradictory parts" form, according to V.I.Lenin, "the essence of dialectics" (2,p.316). In keeping with this principle, this scheme enables to successively discover "the coordinate pair" of our interest of the related binary oppositions which characterizes a definite planar section of the multidimensional field of man's essential powers ("subjective abilities", according to K.Marx). This specifically provides for the differentiation of arts into "figurative" - "expressive (non-figurative)" and "audio" - "visual". Thus, such representation of oppositions is justified if only it offers a vivid illustration of the interdependence of opposites characterizing the system as an integral whole.

So the initial result of the artistic activities of "creative man" (CM) if we consider the genesis of art, is known, to be syncretic art (it is projected in the centre of the scheme). In the process of the division of labour and the specialization of the artistic creation objectivation takes place of certain essential powers of man assuming the form of independent arts: music (1); architecture, ornament (2); painting, sculpture (3); art of word (4). Although basically progressive acquisition of the kinds independence is fraught with deep-laid contradictions. It proves to be dialectically linked with the constant aspiration to preserve that complexity of reality reflection which was characteristic of syncretic art when perception was just as integral as it is in the case of the direct contact with the world and aesthetical apprehension of nature. This aspiration fascilitated the preservation of the bisensory character in the dramatic (7) and choreographic (8) art forms, and the "reflection-expression" unity - in the song-folclore (5) and the applied (6) arts. Moreover, at the proper level of social and material development the autonomous arts which have assumed independent status may be intentionally involved in various synthetic combinations as well (3 and 4 - book illustrations; 2 and 3 - monumental art; 1 and 4 - vocal music etc.). Besides, along with this obvious system-type relationschip as the "synthesis" there exist between the system components some sort of interactions "at a distance", characterizing the process of mutual influence, mutual imitation among arts - it is denominated "synaesthesia of arts" if it occours between visual and aural arts (there exist similar associative interactions between figurative and expressive arts). The most vivid examples are "pictorial" music by the expressionist composers, "musical" painting by V.V.Kandinsky, M.K.Ciurlionis etc. Hence, the procésses of differentiation and interaction of arts not only supplement each other but are interconditional as well. The analysis makes it possible to speak of the presence in the system of arts, besides centrifugal forces, of some sort of centripetal forces whose dialectical unity determines the possibility of the integrity preservation of the expanding system of arts which is consistently and substantionally mastering new aspects of its developing subject (reality, the world).

These conclusions are made on the basis of the analysis of traditional arts. But the technical stores of arts are being constantly renewed. This become particularly evident in the last century. The possibilities offered by the new technology have been immediately put to use to satisfy growing artistic requirements. And so with traditional arts have been preserved, "by side" with them (as shown by the system scheme of arts) new art forms begin to emerge not copying the old kinds but proceeding with the development of their possibilities owing to the potential of a new means of audio-visual communication. Taken as a whole they form the new outer layer of the system of arts (Fig.2). If we apply to this outer layer the conclusions drawn from the analysis of the system of traditional arts there appears a possibility to find out their viable forms which emerge due to the above system-type relationsohip.



To set it forth very briefly: 9 - sound-recording, electronic music; 10 - kinetic art (luminodynamics, video art, abstract cinema); 11 - fotography, cinematograph, holography; 12 - radio-theatre; 13 - television; 14 -concert music-kinetic art; 15,16,17,18 -sound and light scenography of the musical and, respectively, the drama theatre (revealing in them such specific synthetic-entertainment forms as "space music" - 15, "labirinth" - 16, "Son et Lumière" performances - 17, "Làterna Màgica" - 18). Bifunctional applied forms are arranged, respectively, in cells 19,20 - aesthetization of the sound medium, and 22,23 - aesthetization of the light medium. It is primarily exhibition-and-decorative art, discoteques, light architecture, sound and light performances in the open air. In cells 21 and 24 the light and the sound design is located.

The analysis of the current practice shows that parallel with the process of alienation of the artificial means of audio-visual communication used in the outer cells, the scheme of art kinds under investigation can preserve its unity only at the expense of the maximum increase in the tension of the field interkind interaction forces in it. Not being autonomous kinds of art, the majority of artistic activity's kind singled out in the outer cells become viable only when they enter in the synthetic formations characterized by the action of the intrasystem ties revealed above (marked by arrows). Let us note that these arrows in the scheme are not always two-directional, they point to the preference of links by their direction.This is where specific character of audio communication manifests itself in human intercourse: if audio arts (radio, electronic music etc.) may lay claim to autonomy, their visual analogues (the movies, video art etc.) have to involve the sound.

The above phenomenon of the "functional asymmetry" is even more pronounced in relation to the other, vertical axis i.e.when comparing the left (figurative) and the right (expressive) half of the scheme. The difference in the rate and intensity of their development is evident at first sight with the left half outstripping the right in which even the names of new kinds are still uncertain. How can one account for this paradoxical fact? Maybe such is their eternal destiny? It is enough to recall, for example, much suffering lightmusic which claimed its rights as early as the 18-th century but has up to the present preserved the status of an experimental kind. The point is that if the equipment and technology of cinema and television are subject to unification and serial production (that, by the way, provides for their profitableness), any light instrument is, as a matter of fact, unique and cannot be wholly reproduced (provided we do not intend to repeat the work itself). The initial functions of photo- tele-film-technique are to store, reproduce and convey some given information. The functions of the light instrument are different being purely productive: to produce in the screen something, exists only in the mind of the artist. And if in photo-tele-film-technique the functions of the engineer and the artist are separated, in music-kinetic art they are inseparably linked so that creating a light instrument is in the point of fact the primary artistic act. Is the general public ready to cultivate on a grand scale this unwonted situation (uniqueness of the instrument, the work of art and the artist)? To all appearances not until social conditions have been created for the upbringing of the all-round development of individual, until all bureaucratic, interdepartmental barriers have been destroyed...

In conclusion I'd like to point out that the graphic-structural method used here may be employed in other research situation as well. Thus it has proved its worth in our analysis of audio-visual communication means, in studying of synaesthesia (the system of perceptable reflection). When discussing our method at various symposia there were even suggestions about the possibility of systematizing philosophical categories in accord with this scheme (placing "being" or "consciousness" in the centre of the scheme). This is highly problematic of course but in any situation when the human being in some function could be placed in the centre of the scheme (the system) this structural scheme is to all appearances valid. A consistent combination of these systems into a multidimensional metasystem of all essential powers of the integral man will be able, in our opinion, to fascilitate in a way the development of this single science of a man the possibility of which was predicted already by K.Marx. And, of course, in every cross-section of this metasystem - owing to the fact that the structural multiplicity of human qualities is heterogeneous - the "functional asymmetry" will manifest itself, and in each case specifically, depending on the content of these heterogeneous qualities.

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NON-LOCAL SYMMETRIES OF DIFFERENTIAL EQUATIONS

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The property of invariance of differential equations is a criterion, by which one can classify the mathematical models of the physical phenomena we consider. The knowledge of the admitted group of transformations may be used to construct the exact solutions and conservation laws, as well as to clear out the questions of the equivalence between the systems of differential equations, etc.

The theories of Lie and Lie-Backlund transformation groups give us a constructive methods to calculated the local transformations, i.e. the transformations which depend on independent and dependent variables and their derivatives (hereafter called local variables). The local groups are generated by the infinitesimal operators with the coordinates, which are the analytical functions of an arbitrary finite number of the local variables. At the same time in practice there exest the equations, admitting the operators with the coordinates dependent from local as well as non-local variables, or the integrals (i.e. non-local symmetries). As a rule one calculates them by means of special methods: applying the recurrence operators, applying transformations, reducing a given equation to some others. Generally it is impossible to suggest the same constructive method for calculating the non-local symmetries. It is caused by the fact, that the space of analytical functions of a finite number of local and non-local variables is unlocked with respect to the integration operation.

The constructive method to calculate the non-local symmetries is developped for the equations, admitting Backlund transformations (like differential substitutions). The application of Lie-Backlund groups theory to such equations allows us to calculate for them the non-local symmetries of a special type, which we call quasi-local. Corresponding non-local variables appear to be associated with the conservation laws (which are generally non-local). By means of suggested method we had calculated the new quasi-local symmetries and conservation laws for the non-linear heat diffusion and gas dynamics equations.

When classifying the equations of non-linear heat diffusion type by quasi-local symmetries, we isolated a new broad class of equations with expanded set of symmetries. Thermal conductivity coefficients of these equations remain finite at an infinite growth of temperature. With the help of quasi-local symmetries the new invariant group solutions of heat diffusion equations were found.

The peculiarities of applying the algorythm for calculating the quasi-local symmetries for the systems of equations is illustrated by the equations of one-dimensional adiabatic gas dynamics.In 1958 these equations were classified in Euler coordinates accoding to the group of point transformations they admit. Lately it was show, that in Lagrange coordinates these equations admit a more extended group of point transformations. To create common classification by the method of calculating quasi-local symmetries, one introduces an intermediate system, which is associated with the equations of gas dynamics in Euler and Lagrange coordinates via Backlund transformations of differential substitution type. Such common classification had led to thirteen main types of gas dynamics equations with extended set of symmetries. Within these types we can isolate for example such gas dynamics equations, which are invariant with respect to changover to the uniformly accelerated coordinate system. The approach we suggest allows to make visible the hidden symmetry of Chapligin's gas equations.

THE PROBLEM OF SYMMETRY OF "THE PRIMARY ELEMENTS" IN THE THRACIAN ORPHISM AND PYTHAGORISM

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Differenciation in the Orphic mythological consciousness results in the formation of definite concepts about the role of symmetry in the formation of the world. The cosmic view of life in Ancient Greece demanded a rational explanation of the Earth's and cosmos's organization as well as their invariant interrelationships. Water, fire, earth, air and ether were world forming elements in the Thracian Orphism and Pythagorism /Foll, 1986, Dankov 1988/. If philosophers of the Milet school accepted one of them as the world's forming element, Pythagoreans thought of all these together in their interaction and interdependence. Five regular geometrical figures correspond to these elements: a cube to the Earth, a tetrahedron to fire, an octahedron to air, an icosahedron to water and a pentagondodecahedron to ether.

From the point of view of symmetry the five elements can be represented as convex polyhedrons, Oiler's theorem being valid for all of them. The representation of the primary elements is also possible with the help of the so called metric theory of convex polyhedrons which is different from the combinatory ones in using such concepts like length, angle and volume.

When represented as polyhedrons primary elements represent a convex surface of a non-void closed set. Each polyhedron $P = \text{con. } V \{x_1,\dots,x_p\}$ can be called a k-polyhedron if $p = k$. This means that some subfamily of the family $/x_1,\dots,x_n/$, consisting of k+1 point is independent affinely and every subfamily, consisting of k+2 points is dependent affinely. The k-simplex is to be understood as a k-polyhedron, appearing as a simplex. Such a simplex appears as a k-simplex only and only if it has k 1 apexes. Thus 1-simplex is a segment, 2-simplex is a triangle, 3-simplex is a tetrahedron and 4-simplex is a pentogram. The latter is known as a world model.

The idea of the geometrical aspect of the material organisation of the structure forming elements is used successfully by de Beaumont /1852/ for the explanation of the architectonic aspect ofthe global organisation of the Earth. Some definitely perceivable parities between the structure of the earth's crust and the mentioned elements of symmetry gave him cause to hold the view that 20 hedrons of the icosahedron, one of Plato's figures, correspond to the firmest areas on the earth's crust, that 30 of its edges correspond to the most important mountain ridges /less firm areas on the earth's crust/. In connection with this Lapparan/1882/ considered the tetrahedron a form "which holds the earth's crust in case of deformation"/Shafranovskii, 1975/. Similarly Lalleman holds the view that the four apexes of that tetrahedron, together with the corresponding edges, represent the Alps, the Himalayas, the Rocky Mountains and the South Pole and considered the tetrahedron structural organisation of the Earth a form which determines a minimum reductions in the earth's crust/Shafranovskii, 1975/. The octahedron whose symmetry is

$3L_4 4L_3 6L_2 9PC$/besides its axis of symmetry, planes and centre/ is characterized with 12 one-dimentional edges, 8 two-dimentional hedrons and 6 0-dimentional apexes. The cube, having the same crystallographic symmetry as that of the octahedron, is characterized with 12 one-dimentional edges, 6 two-dimentional hedrons and 8 0-dimentional apexes. These numbers, according to Pythagoras, are in a relation of "a golden proportion".

The dodecahedron/$6L_5 10L_3 15L_2 15PC$/ is characterized with 30 one-dimentional edges, 12 two-dimentional hedrons and 20 0-dimentional apexes; the icosahedron, whose structure has the symmetry of a dodecahedron, is characterized with 30 one-dimentional edges, 20 two-dimentional hedrons and 12 0-dimentional apexes.

The importance of the symmetrical organization of the earth is seen from the fact that certain symmetrical elements correspond to places of natural resources, i.e. in the distribution of diamond formations. Unanium deposits have a similar symmetrical location. "The uranium-235 deposits in Gabon, considered to be an extinguished natural reactor, astonishingly correspond to one of the apexes of a polyhedron"/Kanev, 1975/. According to him that polyhedron has the symmetry of a dodecahedron. Its orientation is such that "if the dodecahedron coincides with the earth's axis, two of its edges coincide with the Atlantic submarine ridge. A number of well known submarine and continental anomalies in the earth's crust also coincide with other hedrons of the dodecahedron"/Kanev, 1975/. The ridges in the middle of the ocean appear to be the constructive boundaries of a plate that is a source of the lithosphere.

In that way the symmetrical relations in the earth's organization carry information in which the dynamics of exogenous and endogenous processes is reflected. That is why "tectonics represents a carcass, a skeleton, a coordinate system around which data of the distribution of natural resources gather and which allows the prognosis of new locations"/Kosigin, 1974/. Kanev holds a similar point of view when asserting that as a result of the presence of an analogical tectonic ordering in the structure of the Earth "different blocks of different structure and tectonic activity exist in the earth's crust. This circumstance gives grounds to believe that initiallyour planet represented a gigantic polyhedron crystal-like body. Gradually that polyhedron body turned into a geoid"/Kanev, 1975, p.192/. Analysing modern achievments of geology and first of all geophysics the author concludes:"deop into the earth a crystal-like surface exists or to be more precise a frame of force whose apexes and edges represent the most sensitive places in the earth's crystal"/Kanev, 1975, p. 193/. He writes further: "This frame creates and changes the tectonic field of force of the Earth and represents the main force in the movement of the earth's crust in relation to its development"/Kanev, 1975, p.197/.

At the same time the earth's crust is in an antisymmetrical relation to the ocean symmetry/as a result of which their surface tends to levelling/, i.e. to an equal number of hedrons which in fact represents an antiequation. Parallel to this oceans and continents are also in an antipodal relation /Lichkov, 1965/. This relation is genetically conditioned and is represented symmetrically with the help of two antisymmetrical tetrahedrons. According to Shafranovskii and Plotnikov /1975/ "the ideal model of the distribution of land and water on the earth's sphere can be that of the black-white "octahedron" /a combination of two tetrehedrans/ with the following antisymmetry - $3L'_4(3L_i4)4L_3 6L'_2 3P_6 C'$ - $m\dot{3}m$.

Actually if we take the white hedrons to represent the mainland and the black ones to represent the oceans and if the orientation of the octahedron is such that the "black" hedron is situated at the bottom and the "white" one at the top then the triple crystallographic axis of the symmetry will be perpendicular to these hedrons and will coincide with the axis of rotation of the earth. In this case the top "black" hedron represents the North icy ocean while the "white" hedron represents the Antarctic. The structure of this octahedron belongs to the crystallographic system with the following symmetry - $3L_4 4L_3 9PC$. According to it there are 3 oblique "white" triangles around the "black" triangle: "These are the continents - North and South America, Euroafrica and Asia"/Shafranovskii, 1975/. The rest of the "black" triangles correspond to definite oceans. The combinatory "octahedron", consisting of two antisymmetrical "tetrahedrons", separates visually "the basic regularities in the distribution of land and water on the earth's surface"/Shafranovskii, 1975/. Modern "global tectonics" does not take into full account these regularities, which is more than necessary since "modern plate tectonics...springs from the conception of a previously existing mainland /"Pangeo"/, consisting of a number of plates. These plates were separated from one another with boundaries along which each of the plates was subjected during the Mesozoic to spreading, going up or going down in relation to one another" /Gerasimov, 1976, p.6/.

The antisymmetry of these tetrahedrons differs not only in the sign of their enantiomorphism but also in their characteristics which also points to the dialectical character ofthese two antisymmetrical opposites. From the point of view of dialectics this antisymmetri cal organisation of the earth and its elements manifests its /antisymmetrical/ being only in relation to a certain structural dimension determining the dialectical unity of motion and nonmotion of the architectonics of the earth. The absolutisation of either of them has always caused discussions while the objective course of development of geology demands that these opposites be regarded from a new angle. Spilhaus's conception is an important step into this direction.

The formation of the isomorphic and antisymmetric relationships in the process of the development of the earth represent its selfregulation, selfpresentation and selfdevelopment. At the same time the influence of structural genetic links in the organisation and development of the earth helps the formation of certain isomorphic genetic relationships. All the structural variety in the process of the earth's development is based on these relationships. For example each of the 6 main plates which represent the base of today's continental mainland has a triectory of movement corresponding to the crystal structure of the tectonic field of the earth. The plates represent a continuation in the existence of elements of "Pangeo" in today's earth's structure. That is a definite quantitative transition from one structure into another /from a simpler into a more complex one/ that appeared in the history of the earth. The transition in the system of the geological movement is one of its selfrearranging conditions at a given stage of its development, e.g. "Pangeo" and it is the result of structural activity, which helps its symmetrical and harmoneous ordering of its inner elements. In that sense the system of the geological movement of matter in the state of "Pangeo" took on such a symmetrical ordering, which helped the "disruption" of "Pangeo", a necessary transition for the next stage of development of the earth ahd the formation of the ancient platforms. The "disruption" was conditioned by a definite symmetry of the tectonic field and in such a way was of a necessary, regular and ordering character, corresponding to the carcass symmetry of the tectonic field of the then existing tectonosphere.

Such regularities enabled Spilhaus to reconstruct the primary position of the platforms and to create an epistemologico-geological model, revealing the informational regularities in the symmetry and selfordering of the earth. The global two-dimentional model of tectonic structure, put forward by the author and obtained with the help of an equidistant projection, happens to be a derivative of the simplex structure of the earth/Dankov,1977/. Giving Plato his due, Spilhaus came at the end to Pythagoras's idea and to be more precise to the Pythagorean model of the dynamic harmony of the world /Dankov, 1976/. This amazing link in the global equidistant projection of the icosahedron was most probably the basis of an integrated theory of the development of the earth and in particular the basis for revealing the symmetry of the acting tectonic regularities in the history of the earth. All this is very important for the reconstruction of some invariant regularities representing the "logic" in the development of inorganic geological processes and the transition from geogenisis /crystay genisis/ to biogenisis and noogenisis in the history of the earth. Undoubtedly a further analysis of the Pythagoras's idea of the geometrisation of the world forming elements is necessary for the reconstruction of some crucial ideas in the history of geology.

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SYMMETRIES AND THEIR BREAKING

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Abstract of a talk to be presented at
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Symmetry of objects in space as defined by Hermann Weyl is closely related to unobservability. The concept is extended to the laws of nature. Absolute values of variables that are modified by symmetry transformations of these laws are unobservable. Symmetries imply conservation laws. Symmetric laws of nature are not inconsistent with asymmetric phenomena since in addition to the laws initial conditions are required to fix the phenomena. Symmetry in the mean as well as explicit and spontaneous symmetry breaking are described. Unified theories such as Newton's mechanics, Maxwell's electrodynamics and the standard model of elementary particle physics have a larger domain of applicability and thus a higher symmetry than the individual laws they unite in a modified form. As an extension of the rotation symmetry of their predecessors, the Maxwell equations are symmetric under velocity changes according to the rules of special relativity. They furthermore possess a certain local symmetry. Both symmetries, which are of utmost importance in present-day elementary particle physics, appeared in Maxwell's equations for the first time. Their validity is tested by these equations themselves and their quantum-mechanical version, quantum electrodynamics, with overwhelming success.

The laws of Maxwell and the standard model may furthermore almost uniquely be derived from the symmetries they possess. According to E. P. Wigner, the relation of the symmetries of the laws of nature to these laws is the same as the relation of the laws to the phenomena. Known and supposed symmetries therefore may help in the search for more general laws of nature than are presently known. The symmetry group of these laws presumably combines internal and space-time symmetries in a non-trivial way. The prominent role played by the requirement of local symmetry for the derivation of interactions is explained with the help of examples.

Outside of physics, symmetries are mainly used for the classification of objects. The laws of chemistry and biology leading to (or allowing) the breaking of mirror symmetry observed in living systems, serve as an example for the spontaneous breaking of symmetries.

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ADAPTIVE DISPARITY OF DISSYMMETRICAL FORMS OF PINUS SILVESTRIS L. AND PICEA ABIES L. KARST. IN THE NORTH-WEST OF RUSSIAN SFSR

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Population selection and genetics of woody plants lack now knowledge of easily identifiable and informative characters of trees which could facilitate insight into regularities of changes in the intrapopulational structure of the species in response to internal (genetic) and external (environmental) factors. Of particular theoretical and practical interest in this respect is dissymetry of plants which is essential for studies of the natural differentiation of the species and highly important for understanding evolution processes in natural populations (Urmantsev, 1974; Khokhrin, 1984; Nikulin, 1987).

Variability and natural selection have long been established to be the moving force of evolution of species. Intraspecific selection, in its turn, is based on adaptive disparity of individuals, which is determined by numerous factors whose joint influence affects the frequency of occurrence and the growth of different genotypes in a population.

All the characters that affect directly or indirectly the survivability or the competetivenes are generally agreed to be selective. Among such characters of the common pine (Pinus silvestris L.) and the European fir (Picea abies L. Karst.) is the phyllotaxis dissymmetry of the epicormic shoot.

The pine and the fir are the principal and most valueable timber species in the forests of North-Western Russia (Leningrad, Novgorod and Pskov Districts). With respect to the direction of the leaf arrangement spiral on the epicormic shoot, two forms are distinguished, viz. the laeotropic (L) and dextrotropic (D) forms.

Investigations of the intraspecific dissymmetric variability of the pine and the fir were carried out for many years both in natural populations and in plantation cenoses of this region, making it possible to establish a biological and ecological disparity of the L and D forms of these species (Golikov, 1981, 1985;

Golikov, Kartsev, 1987).

At present there is much straightforward evidence obtained from natural populations which indicates with certainty that the L forms of the species in question occur more frequently (by 5 to 26%) and grow better (by 8 to 21 % in diameter and height) on dry and fresh soils, while the D forms, on the contrary, are more frequent (by 7 to 26 %) on moist and wet soils. The analysis of the dissymmetric forms in terms of selection categories and habitat conditions shows that sample and growing-stock trees make up the bulk (67 to 84 %) of the L forms on drained soils. These categories are considerably less frequent among the D forms (58 to 65 %). The frequency of occurrence and the growth of the L and D forms were reported to depend noticeably on the forest type in pine forests of Karelia as well (Bakshaeva, 1975).

It was established by studies of the cone crop capability of the dissymmetrical pine and fir forms that on moist soils the D forms yield 13.8 to 18.2% more seeds than the L forms; seed yields of the L forms proved to be 12.9 to 16.8 higher on dry and fresh soils. These differences are statistically certain ($F_{act.} > F_{0.01}$). All this proves a reproductive disparity of the dissymmetric forms under different ecological conditions.

In order to estimate the adaptability and the growth of the L and D forms under different ecological conditions, studies were made of 5 to 8 year old test crops descending from different populations. The growth data for these plantations follow a pattern similar to that revealed for natural populations, thereby confirming that there is a substantial adaptability disparity between the L and D forms and that each of the forms needs specific environmental conditions for its fast growth and high yield.

The frequency of occurrence of the forms in question in the progeny of individual trees and populations was studied on one-year plants grown under identical conditions. It was found that the habitat conditions of the mother trees and populations affect considerably the numerical proportions of these forms in the first generation. The progeny of a majority of trees and populations growing on dry and fresh soils show a predominance of the L form, while seeds of humid habitat trees give mostly the D forms. The occurence difference for the two forms ranges from

3 to 18 %, with a high degree of certainty ($\chi^2_1 > \chi^2_{0.01}$). The reason appears to be related to the genetic mechanism, in accordance with the hypothesis by Khokhrin (1977) on numerically non-equivalent primary (zygote level) ratio of the L and D forms. Therefore the frequency of occurence of these forms is governed not by natural selection alone; of significant importance are also genetic factors. Thus a low adaptive value of this or that dissymmetric form, coupled to a negative correlation of the seed-yielding capacity and the growth rate, results in a lower frequency of occurrence of the form in new generations thereby disturbing the genetic equilibrium of the population.

An essential growth-affecting factor for the forms under study is the stand density. The experimental finding for 10 year crops with stand densities varying from 1 to 11 thousand per hectare indicate that at lower densities of crop, under identical soil conditions, the L form trees show a better growth. The largest reliable difference between the forms in favour of the left-hand one is recorded at sparse plantations (1 to 2 thou. ha), the differences un the diameter and the height being 13-16 % and 5-12%, respectively. In the high-density (11 thou. ha) crops the D forms demonstrated a better growth of 15 % in the diameter and 8 % in the height. The comparatively rigid intraspecific competition of the L and D forms develops with the planting density, showing the D forms to be more competitive under more severe conditions. Judging by the fact of a higher frequency of occurrence of the D form pines under the forest canopy (Khokhrin, 1984), the different response of the two forms to the crop density, appears to be related primarily to the illumination level. The light competition, influenced by tree shape parameters and stand density and apparently aggravated by the root competition, affects substantially the growth rates of the L and D forms of the fir.

The fir forms respond differently to herbicides (propaesin and glyphosate), the D forms in 5-10 year crops and in the nursery showing a better herbicide resistance. The difference materialized in numerical ratios of the forms in the nurseries and in their growth in the crops.

The above-quoted results of investigations of dissymmetric forms of the pine and the fir in North-Western Russia point with consistency to the fact that the L and D forms are adaptively non-equivalent and differ with certainty in such vital biological

GENERIC KNOWLEDGE REPRESENTATION AND FUSION USING SYMMETRY IN METASTRUCTURES

H. T Goranson

New insights into morphological properties of higher spaces (principally symmetry) are employed to create a knowledge representation environment of substantial capability. This environment implicitly provides an underlying methodology for indexing, data structuring, and generic conceptual modeling. An explicit common grammar, expressible in Ada, has been devised to support cooperative, integrated application systems. A general model for universal data and knowledge modeling and expression results. This general model can form the basis for a global enterprise framework for information systems in support of interdisciplinary design, manufacturing and support of complex systems.

BACKGROUND: This research relates to, and combines innovations from three technical areas which occasionally overlap: morphology (the science of form and symmetry as a basis for structured relationships); knowledge representation (the science of understanding and recording information for directed, automated manipulation); and the areas of conventional computer science which deal with data/ information modeling, and associated specialized languages.

The primary application of the research is to allow the creation of languages and modeling systems for the communication of information. Advanced morphology can be employed to enable robust, generic modeling, processing and communication. The results of the research are the subject of both patent activity and policy considerations by the US Defense Advanced Research Projects Agency.

The common unifying element which underlies the research is the concept of grammatic principles which directly result from morphological primitives in symmetry spaces. This forms the basis of a conceptual modeling approach for knowledge representation which can be expressed in a large class of ordered special purpose application languages, primarily using Ada. These models and languages are related by use of the grammatical foundations and common abstract data type vocabularies.

DISCUSSION OF THE BACKGROUND: A commonly discussed problem in computer science concerns the need for knowledge representation technology which is at once applicable to more generic application domains and advanced reasoning techniques, while simultaneously integrating the increasingly diverse and sophisticated data, process, product and index modeling technologies coming into use.

More specifically, the general field of artificial intelligence and its collected sciences of "conceptual modeling" have been identified in technical literature as the single greatest limit to progress for advanced computer applications. Consequently, a great deal of attention has been given the problem in the past decade, largely under government funding for research primarily performed by universities. Two broad approaches have resulted.

A first group of investigators has produced a large number of interim, special purpose approaches to the problem. These intermediate approaches each have applicability in specialized domains or applications, and are optimized for specific processing by special purpose programs which have formed the basis of a viable market in artificial intelligence. In general, these approaches combine logical calculii, semantic association models and networks, with specific chunking rules to produce "frames", "cases", and "scripts". Elements of these special approaches have been patented or otherwise protected, but are widely used in applications.

A second community of investigators has been studying the nature of the limits of these interim approaches which, by themselves, do not appear to be extensible to more general, broadly applicable uses. At the same time, other parameters as the growing capacity, the seed yield capability, and the competitive ability. The regularities established by these studies have significaht importance for population biology and forest selection.

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stronger, competitive trends in specific computer applications are increasing the distance among fundamental approaches of the artificial intelligence, programming language, and database communities. This branch of investigation has, however, resulted in an understanding of the nature of the requirements for high level unifying principles. The literature now includes a generalized statement of need for the underlying principles, and a vocabulary of terms to describe the open issues.

In general, the problems can be described in terms of underlying, common grammars which provide the generating foundation for characterizing and relating knowledge elements, and accessing and manipulating these elements and their structured combinations. Currently, it appears that if the grammar is sufficiently complex to describe special cases and qualifiers common to everyday knowledge, they become computationally large, ambiguous and inefficient. On the other hand, compact, ordered special purpose grammars are not sufficiently fundamental for broad applicability.

One useful technique for relating special purpose grammars has been the creation of meta-grammars, or grammars which abstract information from lower level, more application specific modeling and representation approaches. But as the levels of abstraction increase, a similarly cumbersome superstructure of qualifiers and singularities increases which chokes the process.

Compelling mathematical arguments have been made to suggest that the complexity of the representation information grows in "size" faster than the information being represented, if existing approaches to fundamental grammars are employed. The general problem is analogous to the search for a grand unified theory in physics, which can be used to express or predict any physical phenomenon. In fact, the understanding of what such a "metaphysical" approach would necessarily entail for a grand unified theory is similar to the projected requirements for a grand unifying grammar for generic knowledge representation.

In both cases, there is a predicted reliance on the most fundamental operations of the phenomenological world, expressed best by simple symmetries, and other primitive relationships which are found in morphological studies. A fundamental grammar for advanced conceptual modeling devised from morphological research must be orthogonal, that is complete within the target universe while each of the other primitive elements is independent. The grammar must itself be simple, but capable of directly expressing complex relationships. This principle allows the representation information to grow less fast in "size" than the information being represented. It also must be applicable, without modification, at arbitrary levels of abstraction. The grammar must be capable of modeling itself an unlimited number of times using the same methodology and tools as for external modeling.

Finally, as a practical concern, the grammar must be easily and unambiguously expressed using abstract data types which can be effectively manipulated by software engineering methodologies (such as those related to Ada) and practices coming into widespread use. An example is the "object oriented" methodology.

The need exists for a novel approach to understanding the underlying morphological principles at work (ie "metamorphology"), extracting the appropriate grammatical primitives to form a complete, succinct, orthogonal modeling environment, and providing a broad approach to applying this in common computer and communication environments, as a basis for "MetaSystems".

FIRST AREA OF INNOVATION: MORPHOLOGY (This section describes the morphological principles involved and the innovative primary morphological concepts claimed, both in pure morphology, and in the knowledge representation context. The contribution of H. Lalvani is emphasized. Structural Fundamentals of N-Dimensional Metasystems; Fundamental Regions as Lower Dimensional Structural Primitives; Polygonal and Polyhedral Generation to Cluster Primitives; Affine, Polyhedral, and Nonperiodic Pattern Generation; Periodic/Nonperiodic Space Fillings as Lattices; Infinite Polyhedral Lattice Symmetries; and Intersymmetry Saddle Lattices/Labyrinths; are briefly viewed as foundations for the

MetaStructure of symmetry.)

KNOWLEDGE SYMMETRY PRIMITIVES: The existence of symmetry operators which are both comprehensive and fundamentally atomic is of profound significance for conceptual and modeling grammars. Recent mappings of cognitive grammars to morphological primitives provides an enabling basis for conceptual modeling when given the results of advanced morphological discoveries. In this context, semantic networks of great conceptual density can be sustained. These networks are of substantially greater complexity and generic applicability than those networks generatable by conventional LISP, or similar, unstructured facilities.

A grammar results from this application of morphological primitives to describe network or framework relationships among existing knowledge representation schemes, allowing a number of innovative developments in metastructures, the subject of the present research. Specific results dealing with underlying morphological principles of MetaSystems, and which are to be discussed in the lecture include:

The MetaSystems concept which assigns morphological structures to elements of a metastructure as an ordered way of describing and exploring classification and generation of structures; The application of the MetaSystems concept to relate components of differing symmetry class, intermediate symmetries, and "trans"-symmetries; the application of the MetaSystems concept in extending to higher dimensions and projecting into lower dimensions; and the codification, in the MetaSystems context, of the decomposed elements of the structures, fundamental cells, regions and primitives.

Specific results dealing with utilization in the context of conceptual modeling include the following: The specification of a set of grammatical primitives which are at once orthogonal in the large application domain and applicable to efficient execution on a wide range of computer architectures as basis for an instruction set architecture; the mapping of this grammar to generalized semantic networks, metanetworks, and the operations and manipulations of those networks in the knowledge representation environment; an annotation of Ada, to "extend" the language to allow direct manipulation of the network/metanetworks in the programming environment; The combination of these elements as a conceptual modeling methodology, covering traditional bounds of complexity found in the artificial intelligence and data engineering domains; and a detailed mapping of the elements of the conceptual modeling methodology (via the grammar) to the continuing discoveries in abstract metamorphological research, which allows for discovery of additional classification schemes and representational innovation.

CANDIDATE SCHEMA: The candidate lattice schema as a basis for advanced conceptual modeling and very high level language grammars can employ a lattice as complex as the 3 hyper-schwarz types (infinite polyhedral labyrinth) described previously, because of the well developed integration of symmetries among the lattice and their lower dimensional articulations, minimal surfaces. The minimal surface operators of the hyper-schwarz surfaces themselves closely follow established computational trends in abstraction methods. Reflexive algorithms which reflect the ambidimensional transpolyhedra symmetry are a distinct possibility. Provisions for the representation of "doubt" by fractal resolution of the multiscale lattice can follow.

The resulting could appropriately be termed an Ada/Lattice Integrated Conceptual Environment (A/LICE). Features of A/LICE include: The relational function set is extended into an operator language of high level primitives, the language having a base logic (of some undetermined and flexible order). A transportable, underlying abstraction grammar which has a direct mapping to the network lattice transforms exists, and can be supported using the same facilities as the "application" features. These transforms have topological equivalences, a unique and interesting feature which allows the rules of operation among the fabric of abstraction layers to be ambiguous when mapping "up", but precise when mapping "down", or out of abstracted spaces. The grammar must be conceived with sensitivity to a number of related disciplines, described by Goranson in research and planning reports for the Defense Advanced Research Projects Agency.

The morphological equivalence across the various views also allows resolution at the machine level into a few simple operations (including as a subset, analog photonic operators). This is of interest for high speed, concurrent (artificially intelligent) processing in Ada on traditional and advanced parallel and optical environments in mixed mode. Having established a metalanguage with a corresponding metanet formal methodology for morphologically rigorous models and languages, the software engineering tool community will be faced with exciting possibilities. Because of the generic nature of the technology, knowledge of any kind can be "fused" and managed by simple procedures if the source calculus is known. These techniques can be used to support national distributed knowledge bases of heterogeneous origin, medium or domain, providing a flexible unifier for information modeling and a semantic unifier for abstraction grammars.

A PRIMITIVE GRAMMAR AND INSTRUCTION SET ARCHITECTURE: (This section describes the specification of a set of grammatical primitives which are at once orthogonal in the large application space and applicable to efficient execution on a wide range of computer architectures as an instruction set architecture.)

SEMANTIC NETWORK INSTANTIATIONS, LATTICES: (This section describes the mapping of the grammar to generalized semantic networks, metanetworks, and the operations and manipulations of those networks in the knowledge representation environment.)

THE ADA/LATTICE ANNOTATION: (This section describes an annotation of Ada, to "extend" the language to allow direct manipulation of the network/ metanetworks within the programming environment. Issues revolve around duplicating the special intimacy the LISP language has with LISP environments. A facility for "tagging" objects within validatable Ada is described.)

THE ADA/LATTICE INTEGRATED CONCEPTUAL ENVIRONMENT (A/LICE): (This section describes the prototype combination of all of the elements as a fully instantiated conceptual modeling methodology, covering a level of complexity beyond that normally found in the artificial intelligence and data engineering domains.)

IMPLEMENTATION COMPONENTS OF THE TECHNOLOGY: (This section describes the components in an implemented environment. Components in Data Modeling; Index Modeling; Product Modeling; Process Modeling; Horizontal Abstraction Facility; Vertical Abstraction Facility; Constraint Management Facility; and Administrative History Management Facility are discussed.)

IMPLEMENTATION COMPONENTS RELATED TO INTERNATIONAL STANDARDS: (This section decsribes the components of an environment as they are represented in international standards of interest. Components in User/Display Standards; Physical Data Standards; Programming Language Standards; Communication Standards; Operating System Standards; Database/Query Standards; File/Transfer Standards; and Information Modeling Approaches are discussed.)

OTHER APPLICATIONS: (This section describes some of the related research currently sponsored in the United States, which are to be affected by the national framework reference model. Efforts in Neural Nets; Aero and Hydrodynamic Design; Software Engineering Tools; Optical Computer Instruction Sets; and Architectural Design and Engineering are discussed.)

BROKEN SYMMETRY OF STRUCTURE OF GAS-DISCHARGE LICHTENBERG FIGURE WHEN BYOLOGICAL LIQUID IS PLACED INTO ELECTRIC FIELD

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The method of gas-discharge vizualization (GDV) which allows to receive the information about surface and volume properties of objects is well-known [1].

The essence of the method lies in the study of low-current gas-discharge, which develops near the surface of investigated object under the influence of high electric field ($E=10^5-10^6$ V/sm). The sliding gas discharge, which develops in the electrode system "spike-plane" is more suitable for the study of byological liquids[2]. This kind of gas-discharge leads to the formation of gas-discharge images (GDI) known as Lichtenberg figures. GDI represents the traces of ionization channels (strymmers) registrated on the photometerial. In general the length and branchihg extent of channels depend on the magnitude, shape and polarity of tension, configuration and structure of electrodes and also on gas properties such as composition, pressure, temperature etc.

If one of electrodes is the radial symmetry metall spike, Lichtenberg figure looks like halo with its shape, close to the circle and the uniform distribution of branches (Fig.1).

When byological liquid stands for the spike discharge figure looks like halo with non-uniform density of spoiling. Halo shape and dimension change depending on the phvsiological state of byological liquid (Fig.2). In the last case the parameters of external medium such as gas composition, humidity, temperatnre and properties of tension source being constant. The yeast crop of Candida kind was chosen as a model of byological liquid.

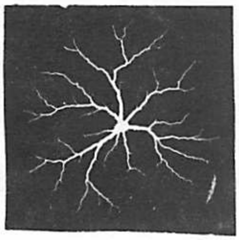
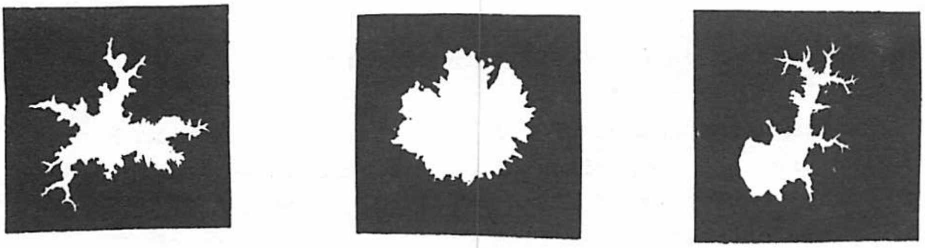


Fig.1. Lichtenberg figure for the radial symmetry metall spike.

Fig.2. Gas-discharge images for Candida yeast crop depending on time of cultivation a-t=24h., b-t=48h., c-t=72h.



a b c

The state dynamics of microorganisms during their development is described by the rise curve M(t) (the dependence of byomass increase on the time of cultivation) and by time-dependence of specific velocity of byomass increase $\mu(t)=\frac{I}{M(t)}\frac{dM}{dt}$ (Fig.3).

Developing in the periodical regime of cultivation the yeast crop passes a number of physiological states (rise phases) which are distinguished both by the presence of metabolits and ferments and by the velocity of byomass increase.

During the cultivation samples of yeast suspension were taken for the invfstigation after each I2 hours [3]. The magnitude byomass, the level of gas-exchange and pH were defined independently. A number of geometrical parameters which describe the broken symmetry of Lichtenberg figure was used for the quantitative estimation of the gas-discharge images. This broken symmetry becomes apparent in the deviation of the GDI shape from the circle and in the deviation of distribution of channels of ionization from the uniform one [4]. Such parameters as shape coefficient $K=\frac{p^2}{4\pi s}$, where p- the out-line perimeter of halo, s- halo area, N- number of discharge branches outgoing from the center of spoilihg spot, d- diameter of spoiling spot are belong to the mentioned parameters.

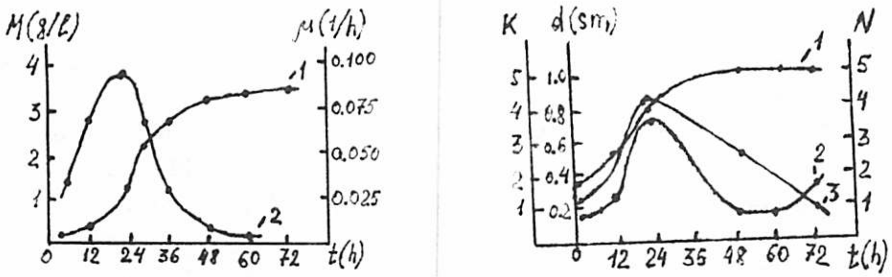


Fig.3.Rise curve of byomass (I) and time-dependence of the spe-

cific velocity of byomass increase (2).

Fig.4. Time-dependences of GDI parameters:
I- d, 2- K, 3- N.

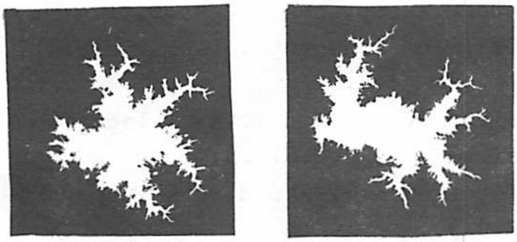
The time-dependences of GDI parameters are shown in Fig.4. It's seen from the comparison of dependences shown in Fig.3,4 that change regularity of K and μ , d and M coincide correspondently. This coincidence enables to use the totality of GDI parameters for the diagnostics of the physiological state of byological liquid.

The comparison of GDI appearance (Fig.2) with the mentioned curves (Fig.3,4) shows that the most characteristic phases in the development of crop (24,48,72 hours of cultivation) are distinguished by both GDI appearance and parameters tendency of change. In the phase of more active physiological state when specific velocity of byomass increase takes its greatest value, the asymmetry extent of GDI shape is also the greatest, and it corresponds to the maximum value of K. The number of discharge branches N also takes its maximum value and their distribution on the area of GDI is even (Fig.2,a). In the phase of steady equilibrium, which means the vital activity stopping of microorganisms (μ=0) GDI shape has a symmetry appearance, its out-line closes to the circle and internal ctructure is partially obligerated (Fig.2,b). The shape coefficient K takes its minimum value (K= =I) and diameter of central spot is in maximum. The phase of eventual states of yeast crop is characterized by partial or full destruction of cells. In this case shape asymmetry of GDI as a rule cnnects with existing of one discharge branche outgoing from the central spot (Fig.2,c). GDI parameter K increases a little but its value significantly smaller than in a maximum. Spot diameter doesn't change practically.

Human blood was chosen in the capasity of another byological liquid for the investigation. It is considered to be one of the most sensitive organism medium and it is a good indicator of organism state.

Additional to the described parameter was introduced for the estimation of the state of man in accordance with GDI appearance. It defines the prolation of the GDI shape ξ =L/1, where L and 1 represent the lengths of maximum and minimum mutually perpendicular chords passing through the central spot of spoiling.

Blood samples of patients were taken during the first three days after their entering into the department of infarction re-animation. It's seen from the Fig.5 that GDI of the blood of healthy (donor) and of patient being in condition of acute pathology differ considerably in asymmetry extent of shape and structure.



a b

Fig.5. Gas-discharge images of donor's blood (a) and infarctional patient (b).

The values of GDI parameters of the donor's blood were taken as standard ones (see the Ist line in table).

The more the deviation of the values of GDI parameters of the patient blood taken in total from the standard ones, the more serious cmplications accompany the illness.

During the investigation by blind way three characteristic group of patients were revwaled in accordance with the extent of heaviness of their illnesses (table):

number group	shape coefficient K	spot diameter d, sm	branche's number N	shape prolation ξ	prediction of illness's flow
donor	3.4 ± 0.5	0.20±0.05	5.2±0.5	I.2±0.2	
I	2.2 ± 0.4	0.66±0.08	3.9±0.4	I.2±0.2	without complications
2	I.8 ± 0.4	I.I ± 0.2	2.I±0.5	I.4±0.I	without fatal complications
3	I.3 ± 0.I	0.72±0.08	I.2±0.2	I.9±0.I	with fatal complications

The results of the prediction of the acute infarction miocardus flow which are received with the help of GDV method for a period of IO days after the investigation and confirmed during medical inspection of patients are presented in last column of the table.

Thus, the choice of the more informational GDI parameters on the base of the shape and structure symmetry enables reliably to distinguigh the physiological state of byological liquids of any nature.

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MOLECULAR STEREO-ISOMERISM OF THE LIVING MAT...

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The first half of the 19th century was marked by a very important event: Louis Pasteur discovered the dissymmetric spatial configuration of the molecules in the living matter. The discovery gave rise to the theory of molecular dissymmetry, which helped explain the optical activity of substances as a function of asymmetry in the molecular structures (L. Pasteur, J. Van't Hoff, J.-A. Le Bel). It also marked the moment when studies of the functioning of the living nature at the molecular level were started, thus creating a new approach to the investigation into the origin of life.

Studies of the significance of asymmetry in the spatial configuration of monomers and biopolymers for the processes of life brought into existence two directions of research: the "stereo-physiological" and the "stereo-biochemical" one. They study the role of steric factors in the realization of the stereo-specific complementarity, which is a basic condition for the biochemical reactions, of which the physiological process actually consists.

Two questions had to be answered within the said research directions. The first is <u>how</u>, by what molecular-structural factors is the complementarity of the interaction of substances in biochemical reactions brought about. The second is <u>what for</u> is the asymmetry (chirality) of the fundamental building stones of the living matter needed; what is the biological essence of L-aminoacids in the proteins, of D-ribose in the DNA and of D-oxyribose in the RNA. These problems are being investigated into both experimentally and by way of mathematical modelling and computation.

The sources of an answer to the first question can be found in the works of E.Fischer who offered the concept of "key and the lock" in 1894, which was based on the idea that "molecular geometry" plays a decisive role in the interaction of substances. Later on the problem of structural and physico-chemical mechanisms of complementarity was studied more in detail by P. Ritchie (1932), within the concept of the conformational intermolecular interaction by W. Astburie, L. Poling and R.Cori (1930ies), by the multipletic theory of cathalysis of A.A. Balandin (1963), and also in the course of research of co-operative systems of the intra- and inter-molecular forces assisting in the discrimination and structural matching of chiralic and achiralic molecules and biopolymers (D. Rein, P. Schipper and S. Mason, 1960ies through 1980ies).

Along with the study of the structural causality of the stereo-specific complementarity, a lot of attention was paid to the deciphering of the physiological reason for the chirality of the biomolecules. The foundation for it was laid by L.Pasteur (1840ies through 1850ies). Two aspects of the problem became noticeable in the course of research. One pertains to the study of the origin of the biomolecular chirality per se, and the other - to the question why is the molecular foundation of life represented by the L-aminoacids and the D-sugars, and not by their antipodes. Initial research was done by W. Kuhn (1920ies) who discovered that rate and efficiency of biosynthetic processes goes down if the spatial configuration of the enzyme's molecule is distorted. At the same time it was discovered that enzyme remains inactive or racemates are being synthesized in a medium consisting of a mixture of D- and L-isomers. W. Astburie made an important conclusion based on these data: an identical spatial configuration of monomers is the necessary prerequisite for their polymerization.

Study of the role of the chiralic purity of the biomolecules in the polymerization and of the significance of chirality for the methabolic reactions became the essence of the stereophysiology and stereobiochemistry since the beginning of the 1970ies. As the result of this research it was found out that the enzyme-substrate complex functions cooperatively. This means that discrimination of chirality is effected by a complex influence of various forces including molecular conformational restructurings.

After L.Orgel put forward (1968) the idea that along with the genetic code living beings also bear the chiralic code, of which the purpose is to encode both the chirality of the monomers and the regularity of the biopolymeric structures built from them, an extensive modelling and a thorough experimental investigation into the functions of the chiralic code was started.

Beginning with mid-1970ies the problem came under the scrutiny of L.A.Morozov and of a number of Soviet researchers under the guidance by V.I.Goldansky. Composition of a monomeric medium (purely chiralic or racemic) was varied, and a polynucleotidic matrix was inserted into it. Then it was proven that matrix assemblage of the complementary chain is only possible on a chiralically pure matrix and in presence of chiralic monomers.

The study of the significance of molecular-spatial factors for the activity of organisms yielded (in 1970-1980ies) some important data relevant for the theoretical and practical physiology, medicine and pharmacology. It turned out, for instance, that the absolute stereo-selection does not remain constant, if biologically active substances are introduced into the body. It was discovered that chiralic purity of monomers is required in order to maintain the chiralic purity of the methabolism. Presently, the mechanism of ageing of living beings, of immunic reactions, of various distortions in the methabolism and of the influence of pharmacological preparations on the organism are being studied within the framework of these ideas.

With the discovery of the molecular stereo-isomerism in the living matter a new dimension was introduced into the studies of the origin of life: attempts to understand the origin of an inherent property of the living matter which is the asymmetry of its molecular structures. The concept of L. Pasteur about the role of the "dissymmetrizin forces" was augmented by certain new ideas: the idea about the instability of the racemic equilibrium of stereo-isomers at the stage of the molecular-chemical evolution and the idea of spontaneous distortions of the mirror symmetry in the course of physico-chemical processes in the pre-biosphere. At present, the problem of the abiogeni origin of the molecular stereo-isomerism at the level of self-replicating molecular systems has become one of the issues of synergetics. Such is its contribution to the model of global evolutionism.

Symmetry and Tradition of Javanese Batik Patterns

by

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Batik is an ancient technique of decorating textiles by a resist dyeing procedure. It was wide spread over Asia, Africa and Europe; such relics were found along the so-called "silk route" from China to Minor Asia.

Today it is difficult to decide, where this technique really came from, because textiles are not resistant against climatic influences. However, it may be stated that on Java, the main island of the Republic of Indonesia, batik was developed to a higher standard than anywhere else. From there batik was spread over the world at the turn of this century, when modern dyestuffs already allowed easier processings. At that time European artists discovered the technique of creating new patterns for furnishing fabrics but they rarely obtained the perfection of the Javanese prototype.

The old vegetable dyestuffs such as "indigo", "alizarin"-varieties and bark solutions from native trees have to undergo protracted dyeing and fixation procedures to combine with the fibres. It is not possible to paint or print with those dyes on fabrics directly, because they will penetrate the material without a boundary, never giving a sharp line; pigment paints will impair the flexibility of the fabrics, because they are only glued on to the surface. Thus, in the past the resist method was the only possible way to get distinct patterns .

For batik, which is a Javanese word for "drawing with wax", hot mixtures of wax and resin are used to reserve parts of the fabric from the dye solutions. An original batik is the result of the sequences of various waxing and dyeing operations.

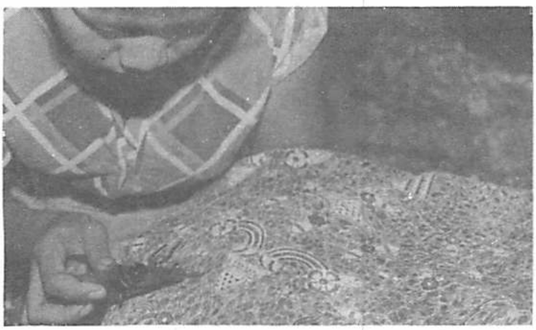
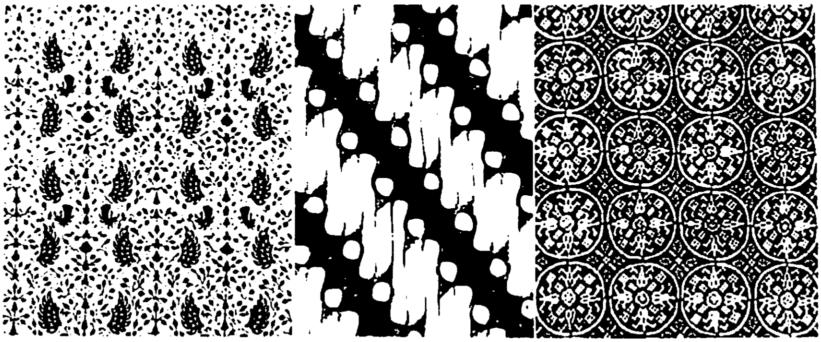


Fig. 1
Applying the wax resist

In old times the waxing was exclusively done by hand using a waxfilled pen only. For a good batik the resist has to be applied on both sides of the fabric. This time-consuming work is generally done by women. It may take weeks or even months to finish a single "kain", which means a cloth of about 1 m x 2.5 m cotton weaving, and which is part of the formal dress for men and women of contemporary Indonesia. Before the next dyeing more resist is applied to save the newly dyed parts of the pattern. After a final boiling to remove the resists a traditional Javanese batik appears in the following colours: darkblue/brown/white and black; the latter as a product of blue and brown. Resist drawings of the different waxing steps can give the impression of a many-coloured cloth.

The mixtures of resists for the different steps were kept secret by the families as well as particular ingredients and circumstances of the dyeing operations.

Batik is a real popular art in Indonesia, but the most interesting patterns are found among the traditional cloths with high symbolic content from Central Java. The symbols and/or their situation to each other were regarded as protection against evil influences.



a) b) c)

Fig. 2: Representative Javanese batik patterns
a) "semen", type b in fig. 4; b) "parang rusak";
c) "jelamprang"-variety

Most symbols in batik patterns are of pre-Islamic origin. Colonial settlers from India had brought Hinduism to Java since 100 AD. Several powerful Hindu-Javanese empires had been established in the Indonesian archipelago until the 16th century, when the last principality of Java had been converted to Islam. Contrary to other countries the Islamization of Indonesia went on gradually without open confrontation. This fact may explain why so many pre-Islamic (and even pre-Hindu) remains could survive and mix with the imported cultural value of Islam. (compare Wagner (1959), p. 146).

A comparative inspection of many old and new batiks from various Javanese provinces lead to the finding that there are relations between symmetry elements (see Fig. 3) and symbolism as well as tradition. A few types of the 17 plane symmetry groups (see Hahn, (1987), are preferably present in batik patterns, while others are missed completely.

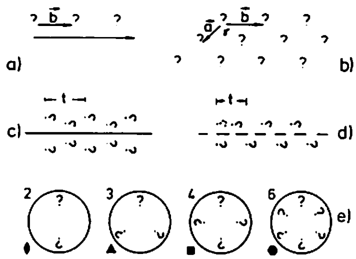


Fig. 3
Symmetry elements in infinite plane patterns
a) translation in one direction;
b) translation in two directions;
c) mirror line;
d) glide mirror line;
e) 2-, 3-, 4-, 6-fold rotational points

Due to the method of making the high symmetry of the infinite patterns is remarkable. Regarding handdrawn patterns containing a tiny elementary cell one may ask for the reason to do the boring work of hundred- or thousandfold repetition. It is proved that the concentration on the neat multiplication of a single motif had a meditational effect. On the other hand there was the wish to increase the magic power of the symbols by multiplying them on the same cloth.

Since about 150 years ago sometimes the resist is applied with a block, but the tool was created after the symmetric patterns were already existing. However, this tool is interesting in another symmetry aspect. Each block without a mirror line needs an identic counterpart for the reverse side of the cloth.

Certain symmetry elements in batik patterns represent the old Asian philosopic principles "mancapat" (the Javanese version of "mandala", the compass-card) and "dualism".

"Dualism" means roughly "coexistence of opposites". In batik patterns it is expressed as dark/light, left/right and symbols for upper/lower world and good/evil in general.
Thus, the symmetry elements mirror line and two-fold rotation can represent dualism. The mirror line is an important symmetry element in "semen"-patterns, which contain Hindu symbols symmetrically arranged on a floral background. Some motifs with an own mirror line are placed along a center line, pairs of others group on both sides of this line. "Columns" of those groupings can alternate with similar ones (Fig. 4b), are just repeated themselves (Fig. 4a) or repeated after shifting within the columns (Fig. 4c). There are also a few examples of which the columns alternate in direction of "head and foot" (Fig. 4d).

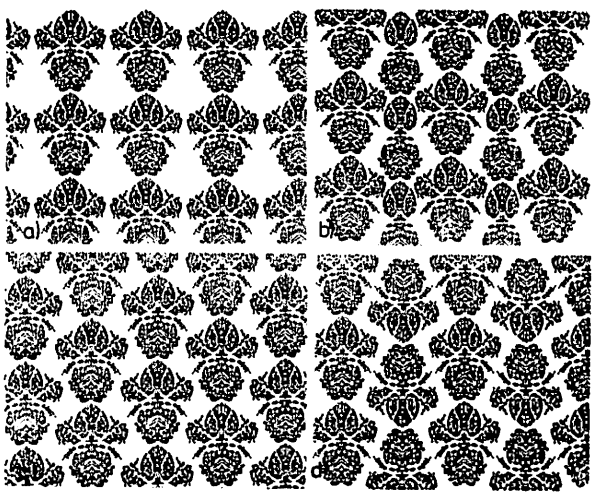


Fig. 4: Symmetry types of "semen" patterns; (schematic)

Due to International Tables, Vol. A (see Hahn (1987)) the symbols for a) and b) = pm (they only differ in size of the unit cell), for c) = cm and for d) = p2mg. In c) a glide mirror parallel to a mirror results in a centered unit cell, while in d) a glide mirror perpendicular to a mirror creates a two-fold rotational point.

Twofold rotation may be another example for the presence of the dualistic idea in batik patterns. In an old "semen" pattern this symmetry element p2mg is part of the name: "pisang bali(k)", which means "turned banana". The most respected pattern containing two-fold rotational points is the "parang rusak" (= destroying dagger; fig. 2b), which was in some versions reserved for the ruler only. As this ruler (after Javanese belief) is regarded as an incarnation of Batara Guru (Shiva), who signifies the union of upper and lower world, a connexion of rotational symmetry and dualism seems to be conceivable (compare Hardjonagoro (1980), p. 231).

"Mancapat" and "mancalima" (Jav. "manca" = outer, "pat" = four, "lima" = five), the Javanese versions of the cosmic model "mandala" are described in a compass-card. The four or eight directions, respectively, and the centre are dedicated to deities, colours, daytimes and all kinds of natural tendencies, of which the centre takes the highest rank always. This cosmic model played an important role for daily life during the Hindu period, but it can be regarded as pre-Hindu (see A. Veldhuisen-Djajasoebrata (1980), p. 204-205). "Jelamprang" is one example for "mancapat". The interpretation ranges from "cakra", Wishnu's wheel shaped wapon to "nine wali" (nine saints, who propagated Islam in Java) according to religious context.

Countless batik patterns are based on "mancapat", which shows the symmetry of plane group p4m (see Hahn (1987); Haake (1984), pp. 39-44 and (1989), but there are hardly any three- and six-fold rotational symmetries in traditional batik patterns. As these symmetries are quite common in Islamic art, the only reason for the lack might be the still existing and tolerated cultural relics of Hindu-Javanism.

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INTRODUCTION TO HARMONICAL FUNDAMENTAL RESEARCH

Sources and History

Fourteen years ago a new branch of science called "Harmonikale Grundlagenforschung (Harmonical Fundamental Research)" came into existence. At present it is only represented by a professorship and Research Institute at the "Hochschule für Musik und darstellende Kunst" in Vienna, and yet our publications have aroused world-wide interest. What is the purpose of this science? What, in fact, is "Harmonik"? The idea, that the universe originated in sounds or consists of them was widely believed in ancient civilizations. This was described in myths and symbols. When the ancient Greeks evolved from mythology to philosophy, this concept was also accepted and found its reflection in Pythagoreanism. The thinking was - from then on - rational, in proportions and harmonies. The Pythagoreans also incorporated analogies into their ways of thinking, and the representation of a universe consisting of sounds could thus have been formulated as follows : There are analogies between nature, Man and music. Meant were the musical intervals. that is, the numerical laws ruling our musical intervals, and with this it was claimed, that these are natural laws and correspond to psychic and intellectual dispositions of Man.

This theory did not emerge at once, since the school founded by Pythagoras in the sixth century b.C. exercised a very strict secrecy and the ancient Pythagoreans did not leave any written documents. Only about one hundred years later did fragmentary writings come to light, and closer connections could be evidenced even only by Plato, who codified most of the testimonies, some of which have not been deciphered completely. With the last Pythagoreans this strict secrecy probably became looser, since over five hundred years later, in the first centuries of Christianity,Pythagorean doctrines appeared again,which have been proved to be authentic,so that a hidden tradition must be assumed. Nicomachus of Gerasa and Theon of Smyrna were the main representatives of this new Pythagoreanism and from them some isolated ideas reached into the Middle Ages.

Apart from the theory of music,there is no question of a consistent tradition in other branches,and it is to the humanists that we owe its re-emergence in the Renaissance period.To the scholars,well into the Baroque epoch,the concept of a world harmony with musical laws was familiar ; they wrote on the subject quite matter-of-factly without producing any evidence or proof of the underlying theory.Only one of the great thinkers on the threshold of modern age made it his life's mission to demonstrate this world harmony.This scientist was Johannes Kepler,the famous astronomer and mathematician,who demonstrated in his "Five Books of the World Harmony" (Harmonices mundi libri quinque), that the orbits of the planets follow certain laws,namely simple intervals,predominantly consonant ones.His work is valid even today, although in his time it was misunderstood and later even derided.Thus at the end of the Baroque epoch,when the triumphal march of the natural sciences just was beginning,the Pythagorean theory of the world-harmony sank into oblivion.

It was only about one hundred years ago,that this situation changed when a scientist named Albert von Thimus picked up the old ideas again and rediscovered the ancient buried sources.He wrote a two-volume work entitled "Die harmonikale Symbolik des Alterthums" and brought to light many interesting ideas ; he mixed them,however,with his own philosophy, inspired by ancient symbolism.His life-work would hardly have received a noteworthy success,had it not been for the German-Swiss private scholar Hans Kayser (1891 - 1964),who took up his investigations and correlated them with other scientific achievements,including those of Kepler.

This so-called "Kaysersche Harmonik" was a great synthesis of the Pythagorean tradition,overlaid with highly speculative and peculiar metaphysics.The modern Harmonical Fundamental Research arose from the above,setting clear limits,however,on philosophical speculation and with its main emphasis,instead,on inductive,empirical methods.

<u>Harmonical Laws in Nature</u>

The antique harmonical tradition holds that the most important basis of music,the intervals,also appear as natural laws and are firmly rooted in the hearing disposition of Man.What are these laws ? We know them from school,where we have been shown - maybe with the help of a monochord - that our intervals inseparably correlate with simple numerical proportions.The octave has the proportion 1:2 , the fifth 2:3 , the fourth 3:4 and so on.These proportions are at the same time general natural laws,as it has been demonstrated by modern harmonical research, above all in a c o u s t i c s , especially in the overtone-series,which sound automatically with every tone produced.

Johannes Kepler had already demonstrated the existence of such interval-proportions in the planetary orbits,and,since then, a s t r o n o m y has discovered others,namely in those planets,which were still unknown in Kepler's time.At the beginning of this century c r y s t a l l o g r a p h y came up as an additional science,as a result of the observations of the crystallographer Victor Goldschmidt of Heidelberg,who discovered important proportional laws in the crystals' structure and identified their musical characteristics.But in p h y s i c s and c h e m i s t r y ,too, there are important laws of proportions,which are to be interpreted harmonically.Max Planck was completely convinced of the fact that his discoveries of the Quantum Theory were an anology of harmonics since only complete multiples of the Planck's constant (h) can occur - just as the harmonics are full multiples of the frequency of the fundamental tone.These same laws are also the basis of the Periodic Table of the elements,in relation to nuclear charge and number of electrons. Applying these laws to a monochord,the sounds produced are harmonics !

In b o t a n y and z o o l o g y there are harmonical laws,too.It is not obvious that birds' singing has the same basis as human music,since the birds' singing is not an imitation of it - which has been demonstrated - but it develops independently.The majority of the laws of proportion is found,however,in Man himself.Externally the human body is harmonioualy proportioned,which has already been recognized in the ancient theory of art,and at present it can be confirmed by a n t h r o p o m e t r y . The fact is essential,above all,that the physiological rhythms of Man are based on simplest proportions,which can be represented by the numbers 1 to 4 , so that without exceptions they are all consonant, as it has been demonstrated widely by Gunther Hildebrandt,M.D. : heart beat and breathing,for example,are in a proportion of 4:1 ; the many rhythms of the human body are similarly coordinated - especially during sleep - so that they don't run at random and in confusion.

Summing up these discoveries,one can see that the same proportional laws or very similar ones appear in all these sciences ; and in no case incidentally : they can rather be described as functions.Thus Harmonical Research indicates far-reaching analogies between different sectors,lateral connections that cannot be noticed from the individual sciences,but they evidently play an important role in nature.Besides, there are laws disposed in our hearing,about which we must talk in more detail.

Musical Disposition of Hearing

Already in Greek antiquity it was asserted that the human soul was "tuned" to musical intervals.Johannes Kepler professed a similar opinion that the origin of these intervals was to be found in the human soul.Hans Kayser,referring to this,spoke of prototypes,based on the theory of the archetypes of Carl Gustav Jung.For Harmonical Research it was hence an important subject to follow scientifically the question of such a disposition.

In doing so,relations were established to later empiric investigations of hearing by Heinrich Husmann,who verified that the anatomy of the ear adds new tones and intervals to those,which reach the eardrum ; these new sounds are harmonics again - called subjective harmonics - and combination tones,including some of higher order.In that way, complicated interferences which result differently for each interval, develop within the ear,so that characteristic differences are built up for each interval.The mathematic description of the results shows that in this way those intervals which are based an proportions of integers are preferred by the ear.It is,thereby,understandable that their use throughout the ages since the time of antiquity is no a coincidence.In addition an ability to distinguish consonance and dissonance results from this.By further evaluation of Husmann's experiments,one comes to the conclusion that the diatonic system, the chromatic and Major scales can also be explained by the physiological characteristics of the ear.The ultrashort memory should also be considered,as well as the psychic disposition of hearing.In the psychic sector of hearing,the intervals are disposed too,of course not as numerical proportions,because in this sector the numbers shift to sensual qualities or psychic sensations.On the psychic level it is important that here wider hearing ranges can be noticed for individual interval perceptions which can even amount to 80% of the distance between two half-tones.This explains why deviations from the proportions which can nevertheless be associated with the correct intervals,are possible.The tempered tuning is only possible for this reason.

This very complicated psycho-physical hearing disposition confirms the Greek theorists to a great extent,although they presented this human hearing disposition in a much simpler way,namely purely psychic.Their ideas,however,were right to a great extent ; and the Pythagorean theory of the identity of musical intervals with natural laws and hearing disposition had a true basis,as has been scientifically demonstrated today.

Furthermore,it has been demonstrated that the most important basis of our Occidental music are preferred by the ear,so that our music didn't develop accidentally in the way known to us,but teleologically,that is,subordinated to the human hearing.Composers of the past followed a reliable instinct and did not simply establish convention as it is often affirmed nowadays.Our Major scale is preferred by our hearing, as it links the five best consonances to a common fundamental tone. This same interval sequence reappears in the most important scale of Indian music,the Sa-Grama,and reversed,that is,from the higher to the lower note,it is identical to the old Greek Doric scale,which was the central scale ; this scale also appears in the primitive races as the first step in the development of tonality.The hearing disposition has,thus,decisively influenced the musical basis of the different cultures,since. such a striking coincidence cannot be explained in any other way.What music developed on these bases is a completely different question.It can be very different from ours and even sound strange to us - just as in painting,where the basis at all times was the same,namely colours ; what was produced from them was varied and manifold.

The Application of Harmonical Fundaments

The philosophical theory of the conformity of musical and natural laws and human disposition led,already in Greek antiquity,to the application of these laws - that is,the proportions of the intervals - to other sectors where they do not appear naturally.We call this application "applied harmonics",and the most important sector of this was a r c h i t e c t u r e .The Roman writer Vitruvius Pollio,who lived in the first century b.C. reports of this - and Hans Kayser has confirmed it through the analysis of the ruins of the Paestum Temples - that the measurements can be interpreted as intervals and represented as notes.

It may be mentioned,incidentally,that in other cultures there were also constructions which followed musical laws,f.i. in China and in India.In Europe the proportions of the intervals were again taken into consideration for buildings in the Gothic,but the true revival of this ancient tradition took place in the Renaissance.But it is not only the description of Vitruvius that attracted interest,as seen in the various translations and commentaries of the time,but before the beginning of the Vitruvius-Renaissance the famous theorist of art and architect Leon Battista Alberti calls attention to the importance of musical proportions.During the whole Renaissance period and up until Palladio,one of the last great architects of this period, construction . based on the intervals is widely known.

During the Neo-Classical period,which is also a retrospective to antiquity,this type of construction experiences a revival,and especially nowadays there are some architects in different countries who have taken up the ancient principles and build according to the proportions of intervals.One of the best known is the Swiss architect André Studer,in whose house Hans Kayser lectured to young architects about twenty yeras ago.The houses built by Studer were presented to the public by television in German-speaking countries.

Apart from architecture,harmonics are also applied to l i t e r a t u r e also already in antiquity,where the entire metric of language, especially that of poetry was measured according to these proportions with respect to shortness and length.Since then,proportions and other musical laws have been used in manifold ways,apart from the fact that the contents of literature,as well,have been exposed to the harmonical ideas,particularly during Romanticism : even in the 2oth century,in the novel "Glasperlenspiel" of Hermann Hesse,harmonical influences are to be found.
Of all sciences it is m e d i c i n e , that has benefitted most from harmonics.The investigations of the human physiological rhythms of Gunther Hildebrandt,which we have already mentioned,were stimulated by the Harmonical Research of Kayser.In addition to that,the German physician Hans Weiers successfully experimented in the field of high frequency irradiations and built a so-called "Bioscillator",which applies irradiation treatment with two different frequencies,which form a fifth.He also used intervals in hydrotherapy and was able to apply his method in veterinary medicine.Harmonical principles, however,are most important in musico-therapy,which has been extended considerably in the last decades with more and more success.The fact

that human physiology functions to a great extent harmonically and that the hearing is tuned on the harmonical basis provides for a very logical explanation of the therapeutical effect of music. It is surely not by accident that the Pythagorean tradition reports on the great significance of music for the harmonization and cure of the human psyche.

The Significance of Harmonical Knowledge

Harmonical Research verifies the theories of antiquity, particularly the Pythagorean doctrines on the extensive validity of musical laws in the universe. It draws up a theory of life, which is not only accessible to the intellect but whose laws can also be perceived psychically with human hearing, and that is related to the arts, especially music. It is of especial importance that this harmonical conception of the world does not set the human being in opposition to nature as is often the case in the natural sciences, but the human being is a part of it in its own very special way. The natural harmonical laws are known as consonances, intervals and even the Major scale appears in nature. These, however, are criteria which appear not to be permitted, since everyone knows that specific sensitivity to the intervals or to the consonances occurs only in the human being, whereas, in nature, only the corresponding numbers exist. In the same way it is always affirmed that colours exist only in the human being ; in nature on the contrary, they are only electromagnetic waves or other similarly quantities. This separation cannot be maintained because all these quantities can be discovered only through measurements carried out by Man and only he is conscious of them. Why then should colours, tones and other qualities be separate from the possibilities of cognition ? The well-known Swiss physicist Walter Heitler therefore demands that all that is related to Man be integrated to his cognition and not only that which can be measured and understood intellectually as it has been maintained since the time of Descartes.

If the human sensual capacities were to be omitted, then with regard to the harmonics in nature only numeral pairs would remain. The integration of hearing clearly indicates that there are certain proportions as intervals, consonances, etc., which brings one to the assertion that without human hearing it would not be possible. The integration of Man brings cognizance of nature a step forward, and crowns it.

Through the categories of hearing, the conformity among the sciences, the analogies within nature, hearing and music are visible - or rather, audible - and relationships are established,which could not have come to light from individual sciences.If we want symbolically compare nature with weaving,the natural sciences would be the longitudinal threads along which causal investigations of the different sectors are carried out ; Harmonical Research would be represented by the transversal threads,indicating analogies,linking the individual sectors intellectually.Both directions together offer an integral view of nature ; natural sciences give us a partial image only.

Because the theory of life of the natural sciences is one-sided - something that is being recognized more and more by leading scientists - the philosophical conclusions of this concept are also one-sided.This we refer,above all,to the materialism and the atheism which emerged in relation to the natural-scientific thinking and became more and more dominant.The harmonical perspective of the world leads one,however,to other results.This perspective admits the recognition that behind these natural laws there obviously exists, a great plan,which consists of simple numerical laws and which links sectors analogically all.In this plan,the human being is much more integrated than in the universal concept of the sciences,namely with psychical sensations and the basis of the works of art created by him.Johannes Kepler firmly believed in this plan and in the Creator who conceived it.On account of this belief he was the first to confirm some of the harmonical natural laws.We should follow his example and submit the abundant numerical material offered by the disciplines of the individual natural sciences to the harmonical perspectives.In this way we find the wayback to a harmonicous and coherent concept of the universe.

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THE TRADITION OF VITRUVIUS' CONCEPT OF SYMMETRY IN THE RENAISSANCE ARCHITECTURAL THEORY

by HAJNOCZI Gábor

One of the most important sources of the Renaissance symmetry and proportion theory is De architectura by Vitruvius (Ist century A.D.). Influenced by Vitruvius' concept Book III, Chapter 1) the Renaissance theoreticians of architecture

- treated symmetry and proportions as aesthetical values and looked upon them as parts of the definite connection between parts and the whole;

- discussed symmetry and proportions in connection with the church, since a church meant the top in the value hierarchy of buildings;

- accepted the principle of anthropomorphism, according to which the human body was regarded as the model of symmetry and architectural proportions

The views can be observed in L.B.Alberti's treatise De re àedificatória (1452) and Filarete's Trattato di architettura (1461-64), where they became the theoretical fundamentals of the central plan churches. In Francesco di Giorgio's treatise who worked at the end of the century the emphasis was put on the basilical and cruciform plan and its medieval symbology has survived. So beside the humanist principles we can observe that the medieval exegetical tradition had lived on: on the one hand the church signified Christ's mystical body (Corpus Christi misticum), while on the other hand it represented the proportions of Noah's Ark. The arguments of the exegetical tradition turned out to be compatible with Vitruvius' principles: with anthropomorphism and concrete proportions (the ratio of 1/6 of width and length). The philosophical basis of proportion principles is formed by Plato's ideas. The thought of imitating nature was based on Aristotle's imitation principle, while the principle of the connection between the divine macrocosm and the human microcosm had got a Platonic interpretation (although it had already been present in the medieval exegetical and mystical tradition). The teaching of the relation between visible and audible (i.e. musical) harmonies is a Platonic-Pythagorean one as well. In architectural theory this neoplatonic proportion doctrine is present in the most explicit way in Francesco Giorgi's Promemoria (1535) which was written in Venice.

The theory of the circular plan had developed at the end of the 15th century which was based mainly on the Vitruvian proportion principles and the neoplatonic cosmology. The circle was regarded as the most perfect form because it signified God's unity and infinity. At the same time the basilical and cruciform's symbology of medieval origin remained in parallel existence. This dichotomy can be observed in Palladio's treatise Quattro Libri (1570).

After the Council of Trent the Jesuit architectural concept had preserved some elements of the Vitruvian proportion doctrine, first of all the anthropomorphism. Since the idea of the circular church had been driven out by the basilical and cruciform, the elements of the medieval tradition strengthened which were supplemented by hermetic elements. This tendency is peculiarly depicted by Villalpando's work In Esechielem Explanationes (vol.II, 1604).

E.M.Hakimov

SYMMETRY, HIERARCHY AND EVOLUTION OF STRUCTURES

The role of symmetry in natural science as the fundamental principle of preservation and change is broadly investigated in scientific knowledge. But the role of symmetry as an element of the process of the development of natural systems' structure was less exposed. That is conditioned by specially scientific expression of symmetry based on complicated mathematic apparatus and also by the lack of investigations of correlation of the notion of symmetry with general scientific and universal categories. The solution of the problem appeared to be connected with revealing of general principles of the development of hierarchy systems. For the purpose of solution of the problem mentioned the author created a hypothetic - deductive conception, the notional apparatus of which is represented by the pithy expressed axioms /non-formalized owing to the common character of assertions/ and by partly and sufficiently formalized corollaries, deduced from the former. The object of analysis of the conception comes to be a multilevel structure of an abstract level of organization, based on the assertions of axioms and corollaries, regarded as a model of the development of hierarchy systems. The basis of formalization of the process of development is a possibility of mathematic description of the spatial structure of abstract Level of organization/ALO/. The system of notions of the conception of hierarchy, being a hierarchy system by itself, looks like the following:

I. Non-formalized axioms, expressing fundamental tenets of hierarchy as a teaching of the development of multilevel structu-

2. Partially formalized corollaries - notional apparatus of the conception.
3. Sufficiently formalized corollaries, describing topologically expressed structure of ALO on the ground of the algorithm, worked out by the author, connected with coefficients of Binomial theorem and the numbers of Fibonatchy.

Proceeding from the material stated, symmetry may be considered as a notion of different levels of community:

I. Non-formalized notion, corresponding to the philosophical category of identity.
2. Partially formalized notion of correlation of identity and difference in the structure of ALO. We understand formalization as arithmetical rules of unification /addition/ and division /substraction/, bringing to correspondence different number of levels to definite relations of identity and difference /stability and unsteadiness/.
3. Sufficiently formalized notions of symmetry, including mathematical apparatus of high level.

In the same aspect one considers a different degree of community of violation of symmetry /asimmetry/, which also appears to be a generalized notion.

The main conclusions of the conception of hierarchy are the following: The process of the development of hierarchy systems in connected with acceleration of the development by itself on the levels of higher organization. The mechanism of this phenomenon, which is an antithesis to the second law of thermodynamics, is considered to be connected with growing unsteadiness. The latter in its turn appears to be conditioned by the growth of violation of symmetry in the structure of ALO. In the conception of hierarchy, to overcome this contradiction, there is postulated an idea of compensation of the growing unsteadiness /asimmetry/ by means of acceleration of the process of development in evolutioning systems. That process is also accompanied by the development of homogeneous /identical by main parametres/ subsystems and by sharp spatial expantion of the structure of a newly arising system.

Corroboration of a number of thesises of the conception of hierarchy is connected with empiric foudation of its hypothetic-deductive thesises. The content of axioms is confirmed on the basis of empiric substantiation of sufficiently formalised corollaries, which are deduced from partially formalized corollaries. The assertions of the latter in their turn are deduced from non-formalized assertions of axioms of the conception, which expose the dependence of completeness of description of a system on the level of its organisation /number of levels of system considered/.

As a corroboration of the main assertions of the conception there comes out the revealing of expressed in numbers constants of breach of symmetry in the structure of objects, different by their nature.

As constants of breach of symmetry in the structure of ALO there considered invariable for different levels numerical ratios 1, 1/2, 1/3, 2/3 and inversed ratios - 1, 2, 3, 3/2, underlying in the basis of the algorithm describing the evolution of multilevel systems' structures.

Even at present stage of investigations it appeared to be possible to express, on the grounds of the algorithm suggested by the author, the elements of symmetry of crystals, the structure of the Platonic bodies, of DNA, of a certain number of amino acids, coordinative numbers arising while isomorphism in minerals and so on.

On the grounds of investigation of objects of physics, chemistry, chrystallography, and biology it was shown that:

1. The main thesises of hierarchy find their expression in the structures of real and ideal systems of inanimate and animate nature.
2. Notional-categoric apparatus of conception, including formal methods as well, may be used for the exposure of the regularities of systematic - symmetrical evolution of objects of nature and nature itself.

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Neurobiology of brain asymmetry in man

Although an indication of anatomical asymmetry of the brain has been demonstrated already in some animals (apes, monkeys, and particularly in birds), it is the human brain which exhibits morphological as well as functional asymmetries between the two hemispheres in a gross scale. This asymmetry is partly due to the fact that in most cases, the left temporal planum - which surrounds the speech center - is larger than the right one. With computerized tomography, it has been also shown that this asymmetry is present even in the human fetus, suggesting that an inherent anatomical asymmetry may initially favor the left hemisphere (in 95 % of all humans) for the development of language functions. Interestingly, however, the right ("mute", "minor") hemisphere as a whole, is larger than the left. - In addition to being asymmetrical, the two hemispheres differ in their capabilities. This functional asymmetry has been studied in detail in patients undergoing to neurosurgical ("split-brain") procedures. (In certain populaion of epileptic patients, a life-saving operation is to transsect the main connection between the two hemispheres, the corpus callosum, and the anterior commissure: studies of these patients show that each separate hemisphere is capable of functioning independently.) The sodium amytal test - by which one's left or right hemisphere can be unesthetized, while the other is awoken - is another possibility to study lateralized functions, as speech and, partially, also mood. - Results from several indirect, noninvasive methods were particularly useful in obtaining a detailed distribution "map" of different cognitive and other, "unconscious" functions in the two hemispheres. In one test, where tachistoscope is used, very brief visual stimuli are presented to the right or left visual hemifield of the eye. The nature of the visual pathways is such that the image of a visual stimulus that is restricted to one visual field is projected first to the opposite hemisphere. Another technique is applying dichotic auditory task, in which lateralization is assessed by simultaneously presenting different auditory stimuli to both ears and determining which ear (and, because of the mostly crossed auditory pathways which contralateral hemisphere) is better at recognizing the auditory inputs.

As a result of the combined studies, we may think of our brains (in a greatly oversimplified, but didactically useful way), as consisting of a left hemisphere (LH) that excels in intellectual, rational, verbal and analytical thinking, and a right hemisphere (RH) that is better in emotional, non-verbal and intuitive thinking, and is better in visuo-spatial processing. The LH appears to do best at tasks involving declarative memory, while the RH is more specialized for tasks involving reflexive memory. LH-processing is characterized as sequential, serial, temporal and analytic, RH-processing as parallel, gestalt or holistic. LH is superior in judgmenets of temporal order and the production of temporal sequences, which ties neetly with LH superiority for language skills. It is still unclear, however, whether these language skills depend on temporal processing or vice versa. In contrast, the minor hemisphere, although it is able to comprehend both written and spoken words to some extent, cannot express itself verbally. At the same time, RH is superior to the "major" LH in tasks involving spatial performances - it is clearly better in geometry, whereas LH is superior in mathematics (algebra).

Finally, it has to be emphasized that although the "emotional brain", the ancient limbic system is both structurally and physiologically symmetric, the expression and cortical realization of emotion becomes asymmetric, with a greater involvement of RH. As a consequence, it is only the RH which possesses sense of humor (if developed at all), and, importantly, deals with new, so far unknown and incomparable information. The logical LH is processing and stores only "familiar" information, most of them taken over from the RH. This shows an important aspect of the unified brain: a close cooperation of the two, functionally different hemispheres. This cooperation - through the corpus callosum, made up of 200 million interconnecting nerve fibers - together with the existing competition between LH and RH (for which examples will be also given) is the main factor in the establishment of the human personality.

DYNAMIC GEOMETRY

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Since my first book was published in 1975 *(The Measuring Numbers Systems)*, I had been expecting some feedback. Strangely enough, nothing happened. The book was distributed to major universities all over the country, and a few thank you notes were received for the book, but nothing more. Because of the length of time (14 years ago) I have a suspicion that in reality, nobody really knew what to do with the book's content.

It is a rather strange and very unusual experience to read this book. It does not fit into the accepted line of mathematics. I suspect that the learned minds of present day mathematics were totally confused and shocked to see such a publication appear for distribution. There is no reference in any of the world libraries where one could find an indication that the kind of development like the *Measuring Numbers System* is possible.

The *Measuring Numbers System* represents a totally new world of mathematics which falls outside of present-day mathematics systems. It is a very drastic departure from our present-day understanding of what mathematics really is, what its structure is and what the building blocks of the system are. I am convinced that we really did not pay enough attention to these facts because they seemed childishly simple and self-evident. I find them quite the opposite.

My investigation into mathematics would never even have been started, if I had not taken a very critical view of the basic ideas of the building blocks of mathematics. At this point, I came up with surprising discoveries of the nature of the foundations of mathematics and of science in general.

I am convinced that my present book will help to overcome these misunderstandings about my activity in this field and will help to start a concentrated effort to further develop the already known foundations of Dynamic Geometry.

I will tell you the story of what was on my mind when I worked on and developed Dynamic Geometry.

In 1958, I arrived at the time to take account of my past and decide what to do for the rest of my life. During the past several years, I had visited the local library to get some interesting books to read. The subjects were confined to different disciplines of science, such as physics, mechanics, mathematics, astronomy, philosophy, chemistry and others. I always enjoyed the freedom of the library's policy to freely browse among hundreds of books on the shelves. I was never short of interesting books to take home to read.

During those years, I had developed many new ideas about various subjects, and I felt that the time was right to make some serious decisions about which way I should direct my attention and settle down. So, in 1958, I made summaries about major scientific principles, such as physics, chemistry and mathematics. These three disciplines were the focus of my attention because I thought that in any of these three fields I would be able to do some serious work if I had to start anew.

Finally, I settled with mathematics because I felt I could accomplish the most in this field. First of all, I would not need to invest much money, only time. And the other surprising major discovery I made with my summary was the realization that with the proper work, and lots of luck, I might be able to make a major contribution to mathematics.

The discovery that I made was so overwhelming to me that I decided I would dedicate ten years of my life to try to develop the discovery I made. What was this unusual discovery I made with my summary?

My thinking started with the following questions:

> Why is mathematics so important to our everyday life?
>
> Why is mathematics so important to the sciences?

The only conclusion I could come up with was that mathematics is the only science which can give precise answers to the unknown, provided that the subject can be expressed with numbers. Mathematics has great logic behind it and a tremendous "order," an order which was developed through the millenia. These are the two great characteristics of mathematics: logic and tremendous order.

In very early times, the philosophers and other thinkers developed the idea of the Greek Democritus, that virtually everything here on earth, just as all over the universe is made of atoms. This atomic structure of the world idea was supported with the experiments of modern scientists. Besides, "nature" also produces animals, vegetation, atoms, molecules, particles, pieces and individuals in general. So, the existence of the atom is accepted as the true building block of everything around us. This atomic structure came to life in mathematics as "point structure."

In my study, mathematics is discussed in three major sections: GEOMETRY, NUMBERS SYSTEMS, and TOOLS.

In GEOMETRY, the building blocks are point and line. The point is dimensionless and has only one location in the field. The line has its length, and for this reason it is one-dimensional; however, it is made of an infinite number of points (dimensionless points). Its "point structure" is very important, as we will see later!

In our schools, we were studying the so-called Euclidian geometry. Geometry is confined to the study of one-dimensional lines, two-dimensional forms and three-dimensional objects' properties. Geometry was founded by the Greek Thale (Thales) some 600 years before Christ. Some 300 years later, another Greek, Euclid (Eucleides) collected all the mathematical knowledge of his day and published it in 13 books. Since that time, all over the world, geometry has been thought of as "Euclidian geometry," honoring his effort for his publications. Until very recently, we thought that only one geometry was possible, which is the Euclidian geometry.

Some 150 years ago, the Hungarian Bolyai, the Russian Lobachewsky and the German Riemann developed a new, previously unknown geometry, which we now call non-Euclidian geometry. This new geometry is confined to different curved surfaces. In general, we can say that the difference between the Euclidian and the non-Euclidian geometries can be characterized as follows: In the Euclidian geometry, the triangles always contain 180 degrees, on the contrary in non-Euclidian geometry, the triangles are either less or more than 180 degrees, depending on the surface characteristics, which are either concave or convex or a mixture of both. But one thing is common to both geometries, and this is very important: THEY ARE STATIC, IMMOVABLE, DEAD GEOMETRIES!

The next section of mathematics is the NUMBERS SYSTEMS. There is the very common base 10 number system with which we are very familiar in our everyday lives. There are the base 12, base 24 and base 60 number systems with which we are familiar through our time measurement, such as 12 a.m. or 12 p.m., 24 hours in a day, 60 minutes, and 60 seconds. There are the base 60 and 360 numbers systems which we use with angular measurements. There are other numbers systems, but the most important is the base 2 number system which was discovered by the German Leibniz some 300 years ago. Only now has it become very important due to computer applications.

In general, we can say that our numbers systems are based on the COUNTING NUMBERS SYSTEM, reflecting "point structure," counting the individuals. One thing is also very important, that the numbers alone are dimensionless! It represents only togetherness. But the numbers are "dimensionless."

For example, we say "53." We realize the "53" is a number. Fifty-three what? Because it is dimensionless. We can say 53 apples, 53 large apples, 53 large red apples, 53 large red sweet apples, 53 large red sweet Jonathan apples. So, we see that if we add more and more identifications or characters to the numbers, we finally know what we are talking about. These identifications or characters are known in mathematics as "dimensions." But the numbers alone are DIMENSIONLESS. This is very important, as we will see later.

The third section of our mathematics is the TOOLS. There are the four basic processes: addition, subtraction, multiplication and division. There are also the powers, roots, the different tables, logarithm, vectors, trigonometry, algebra, integral and differential calculus and other elements — and the computer. This summary is not complete, but gives us an overview of the subject's characteristics.

In general, we know that everything is TRUE here on earth, just as well as in the whole universe and it is part of our mathematics, through different "equations" and "formulas." We believe that other truths are nonexistent. Truth can only be ONE. There cannot be two truths at once. It is a contradiction.

In summary, our present-day mathematics is built on a "STATIC GEOMETRY" and on a "COUNTING NUMBERS SYSTEMS," where the numbers are "DIMENSIONLESS," and the whole mathematical system represents an atomic structure and a point structure, which in turn expresses a "ONE TRUTH WORLD" idea. There is nothing wrong with it; after all, it has served us well for over 4,500 years.

The great realization of my study in 1958 was, if our mathematics is built on a "static geometry" and a "dimensionless numbers system," why can we not have a "DYNAMIC GEOMETRY"? The "Dynamic Geometry" is missing, it is nonexistent!

This was my great realization in 1958. The question of why we do not have a "Dynamic Geometry" was a major discovery for me.

The answer to this question was because it is too complicated. It is so complicated that it became unimaginable to the human mind. It is left to God's domain. After all, we have a finite mind, and we cannot digest the infinite that is too complex. This is the reason we have to have religion, to have and answer to the unanswerable, for our finite minds. This is the final answer to all our complex problems. Without this safety valve we might become insane.

For me, the question of "Dynamic Geometry" had become an obsession. It was a worldly task to spend some time of my life to try to find answers to this important question, to try to discover Dynamic Geometry.

In this respect, 1958 was both the decision and starting time. Because I was a mechanical engineer and not a mathematician, this task to develop a new basic idea in mathematics was gigantic. It seemed impossible to invent and develop a new mathematics. The best that I could do was to go back to the library and find a book on the subject of "how to invent mathematics." Of course, I could not find one. Then a good idea came to me: suppose I studied the great masters' lives and works, and I might find the way they came to their discoveries.

After almost a year of study, I found that every one of them made their discoveries in their own way. So, if I wanted to develop Dynamic Geometry, I had to do it alone, without help. To do it the best way possible, with the logical events of a step-by-step approach.

My task is now to show you how I developed my original idea from the very beginning, step by step, so later generations will have an insight into my thinking. Perhaps they can use this method of thinking with their work, regardless of whether or not they have a complete knowledge or experience in the field of their interests.

What are the foundations of Dynamic Geometry?

What are the building blocks of Dynamic Geometry?

All during the year of 1959, I looked for the basic ideas of Dynamic Geometry and for the building blocks of the system. By the end of the year, I had developed the basic ideas that looked reasonable enough for me to start my task.

The basic ideas were: space, time, energy, movement, growth and direction — I thought. The question then arrived: Now what can I do with these basic ideas?

In engineering, when we invent some gadget, we patent it and try to sell it to someone. To do this, we have to have a prototype of the invention to help explain its properties and its functions to the prospective buyers. It looked very reasonable to me, to do the same for my mathematics. I simply had to find a prototype or a model which contained all the basic properties of Dynamic Geometry.

After some time, at the end of 1960, it occurred to me that an expanding globe or an expanding balloon would be the model because each contains all the properties of Dynamic Geometry. The rest is history.

The discovery, development and application of Dynamic Geometry is a new starting point in human history. We have discovered a New World of Truth which is totally different from the present-day mathematics. It is different, because it is built on a new foundation. It is built on "continuity" instead of "point structure." Continuity is the foundation of this New World, which contains everything that is moving and changing around us. It is new, and yet it is old, because we realize that everything is moving and changing around us. Everything is in flux. Nothing stays, standing as immovable, or as static as our present-day geometry!

We have arrived at a New World of understanding based on "World Reality"!

This World Reality is expressed through Dynamic Geometry. This new Dynamic Geometry is a multidimensional world, because it contains the basic ideas of space, time, energy, movement, growth and direction, and that is just the beginning. All these basic ideas manifest themselves all at once. Always together. It is very complex, yet it expresses itself in the most simple way imaginable. It is a major breakthrough in science in general, and in mathematics in particular.

The following discussions will bring to light how this wonderful world of mathematics came alive, step by step. It is ironic that this important discovery came through an outsider whose persistent curiosity found the key to this totally unknown, untouchable world of reality.

The most important aspect of this discovery is the "new dynamic field" (the expanding globe or balloon) that we are familiar with by now. This is the basis for my three published books.

The first positive result of the study of the new energy field brought to life *The Measuring Numbers System*, published in 1975. It was followed by the book entitled *Fundamentals of Dynamic Geometry; the Fejer Vector System* (1981), which is a new vector system, that is multidimensional and has a memory. This new vector system is used to decipher the dynamic field's properties. The third book, entitled *Time in Dynamic Geometry*, is also based on the new energy field's property. As such, it can be viewed as the Universal Time Theory because it contains all previous time theories which are based on the propagation of light (speed of light) and are hopelessly deadlocked and contradictory. With the use of the new energy field, everything is now falling into the proper place.

To use the new dynamic field for later investigations, it seems to be possible to unlock many other secrets of nature which are presently baffling our minds. This is one of the important aspects of the discovery of "Dynamic Geometry."

To make my point very clear to you, it is necessary to review the history of mathematics. Mathematics started some 2,500 years before Christ in the Valley called Mesopotamia at the Tigris and Euphrates rivers in the City of Ur. By 600 B.C., the foundations of geometry were established by the Greek Thale (Thales). The counting numbers system was developed in 500 B.C. by another Greek, the philosopher and mathematician Pythagoras. Everything we know today in mathematics is based on these foundations.

The discovery, development and application of Dynamic Geometry represents a turning point in mathematical history. The present-day static geometry, the dimensionless counting numbers system and the "One Truth World" idea with its atomic structure has come to an end. The 4,500 years of present day mathematics up to 1975 has ended. A new world of mathematics based on World Reality has come alive. This New World is the Dynamic World. Everything is moving, everything is changing, always in flux. This is the "Dynamic Geometry's World," a new multidimensional world, a new beginning. This is the essence and meaning of my work. If you cannot grasp the meaning of this fundamental departure from present day mathematics, you will never be able to digest any of my books.

The following is a short review of the history of our civilization. Scientists think that human existence here on our Earth started 3,000,000 years before Christ. Nothing significant happened until 7000 B.C., when agriculture was invented. This age is the Old Stone Age, from the very beginning until 7000 B.C. The next turning point was the emergence of civilization around 3500 B.C., the age referred to as the New Stone Age. Human beings used stone tools as extensions of their mental and physical beings. Given the significance of tools as technology - and all that technology has come to mean for human growth and power - it is fitting that the past "ages" of man have been identified and classified according to the development of "tools."

So we started with the Old Stone Age from 3,000,000 B.C. to 7000 B.C., and then the New Stone Age from 7000 B.C. to 3500 B.C. The discovery of bronze made drastic improvements to tools, making them sharper and more wear resistant. From 3500 B.C. to 1000 B.C. was the Bronze Age. The great discovery came with the use of iron around 1000 B.C. The Iron Age lasted from 1000 B.C. until the turn of the century, 1900 A.D. From 1900 A.D. until now we have been in the "Steel Age."

Just a reminder that the New Stone Age and early Bronze Age men had a very comfortable life and highly developed culture. They built the pyramids in Egypt and showed remarkable human developments in many scientific fields.

The discovery of iron is credited to the speeding up of human development all across the human spectrum. The iron tools became refined through the blacksmiths' hard work. Wrought iron was the main reason for advanced iron tools and weapons development. Through the burning and hammering of the heated iron pieces by the blacksmiths, the iron lost it high carbon content and virtually become a steel piece.

The Iron Age lasted until the invention of steel manufacturing by the Englishman Henry Bessemer, who invented and discovered steel manufacturing in 1856 and built his first furnace in 1860. The historians fixed the Steel Age starting time at 1900 A.D. The Steel Age's arrival caused drastic consequences in human history. The light, flexible and very strong steel, which by now can be produced in large quantities, made it possible for the development of new, powerful, light engines. These engines were placed in cars and airplanes and freed man from simple hard work. The machine age came alive in full blast.

Since 1900 the advancements have been phenomenal in all fields of science. Discovery of radioactivity by the French Becquerel in 1896, and radioactivity in thorium discovered by the Curies in 1898, started the modern atomic age. In the USA, Henry Ford built his first automobile in 1892 and started the automobile age. The Italian Marconi invented the wireless telegraph in 1901, and later the radio, starting the electronic age. In the same year the Wright brothers made their first powered flight, starting the avaition age. Einstein's relativity theory gave a new start to physics. World War I enhanced the developments in many new fields of science. In 1936, the television came alive. During World War II, in 1942, the Italian Fermi made the first uranium pile with its self-sustaining fission reaction and started the atomic reactor age. In 1945, the atomic bomb was developed, and in 1950, the hydrogen bomb was introduced by Dr. Edward Teller. In 1945, the first computer was built, and in the late 1940's, the transistor was invented, starting the computer age.

In 1957, the Russian space capsule orbited the Earth, starting the space age. In 1969, the USA landed on the moon's surface, signifying the beginning of the space flight age.

Finally, in 1975, Dynamic Geometry was discovered and introduced by Paul Haralyi Fejer, closing the present day mathematics, which is built on "point structure," static geometry and the counting numbers system, and lasted from 2500 B.C. until 1975 A.D.

The new age in mathematics is the Dynamic Geometry Age, which is built on a new energy field and a multidimensional system, expressing space, time, energy, movement, growth and direction, — everything at once. This new mathematical system consists of Dynamic Geometry, the measuring numbers systems, and as a tool, the Fejer vector system to evaluate the energy field's properties. A new, universal time theory based on the new energy field's properties has also been developed.

All these new developments came from the new point of view called "World Reality." This new World Reality consists on one side of the present day philosophy with the atomic structure, point structure and the present day mathematics, with the "One Truth World" idea, and now on the other side with the discovery of Dynamic Geometry, a new multi-dimensional "World of Truth."

This is the meaning and the importance of the discovery of Dynamic Geometry. It signals the end of the present day mathematics and gives a new start for the totally new world of mathematics with the introduction of Dynamic Geometry, which is based on a new World Reality.

Paul Haralyi Fejér

Mary Harris

Mathematics and Textiles

This lecture proposal is written in three parts, an introduction followed by comments based on the three questions asked for the essay answers for "Symmetry in a Kaleidoscope."

Introduction

My field is mathematics education. By definition this draws on a blend of disciplines that involve at a minimum mathematical, psychological, social and philosophical aspects of how people, particularly children, learn mathematics. For the past ten years I have been working on a project that tries to encourage closer links between the sort of mathematics that goes on inside school and that which goes on outside it. My work is more concerned with realism for students than vocationalism for employers.

In the past I have been involved in research into the mathematics done in the workplace by "unskilled" school leavers and have come to reject the narrow definition of mathematics used in such research. As a way of counter-acting such narrowness I have researched and published school learning materials that exploit mathematically some of the practical problem-solving work that happens on factory floors but tends not to be recognised as such by employers.

Mathematics itself is very much a man's subject. Most histories of mathematics never mention a woman mathematician. Most discussions of the history of the development of mathematics within the social context of the time stress male-oriented activities - engineering, astronomy, architecture or ballistics. Until very recently school mathematics textbooks were written with boys in mind, often explicitly. Girls doing mathematics have always been treated as oddities or worse. As a corollory to this masculine interest, traditional girls' and women's activities tend to be judged as trivial and are normally perceived as non-mathematical and non-scientific.

When it comes to textiles, this attitude becomes more obviously irrational. Even clear geometric designs woven by a woman into a tribal rug (Fig 1) tend to be dismissed as being non-mathematical activity. My challenge to this attitude rarely produces rationally defensible responses, normally only a further defensive, narrowing of definition of mathematics.

Thus I work the union of several sets, some of which do not wish to join the union at all.

1. The methodological and heuristic role played by symmetry (and asymmetry) in your field of study and cultural circle.

In English primary school mathematical symmetry is studied as heuristic. The emphasis is on learning about shapes and their orientation in space and that the shape of a toy or other familair object is not changed though its position in space may be.

Children are encouraged to explore shapes with mirrors and repeat shapes to make patterns. Almost the whole mode of mathematics education in primary schools is practical and exploratory. (Fig 2)

In secondary schools the study of symmetry tends to be more abstract and formal. Children are introduced to the different symbol systems and formal methods of mathematics and practical work on symmetry tends to be done with symbols, diagrams and shapes that are already mathematical, for example in secondary school it is likely that a pupil's exploration of reflection and rotation will do so with a triangle or some other abstract asymmetric shape. (Fig 3) Symmetry in secondary schools tends to be taught as an end in itelf, a part of mathematics leading to more mathematics. A pupil going on to do mathematics at university would become aware of the great unifying power of symmetry in mathematics. A pupil leaving school at 16, if s/he remembers anything about formal symmetry at all, will be unlikely to see its relevance to the whole of life.

In textile work the study or use of symmetry can fill an heuristic or methodological role. For example it is not possible to design or make a garment without some notice being taken of the fact that the human body is symmetrical, that a properly fitting right sleeve or trouser leg is the mirror image of the left one. When it comes to the decoration of cloth or or garments, there is no limit to the explorations and imagination of weavers, knitters and dyers. (Fig 4 and Zazlavsky, Gerdes, Harris etc)

2. The interdisciplinary impact of symmetry - used in your field - on other scientific and/or cultural spheres.

In mathematics and mathematics education there is a tendency to study symmetry in isolation. Weyl, Coxeter and Budden are unusual. There is also a snobbery within mathematics itself that endows pure mathematics with a moral superiority over its practical application. The study of pure symmetry represents the humanistic side of mathematics, the use of symmetry in controlling the flow of water round the piers of a bridge represents it practical, slightly regrettable side. This snobbery filters down into school from the universities and adversely affects the school curriculum.

Within schools there are now initiatives to encourage the teaching of mathematics across the curriculum but there are not enough of them and they are chronically under-resourced. A possible advance could be made in the field of home economics. There is now an increasing amount of mathematics software for studying pattern generation and so on. One such program, called "Freize" being developed at Homerton College Cambridge encourages children to construct and analyse such patterns. The children can print out their designs either on paper or on a knitting machine. Thus there is some advance in looking at a topic, previously the preserve of home economics through mathematical eyes. The mathematicians however seem unwilling as yet to consult the home economists on what the constraints of their discipline are in this case. In other words, they enjoy printing out their designs on cloth but are not interested in adapting them to the use to which the cloth will be put. They do not respoind to the criteria of another discipline. The potential for cross disciplinary work remains nonetheless and it is only just beginning to be exploited to the benefit of both disciplines

3. The special meaning of symmetry influenced by your cultural background (external artifacts.)

England is now a very multicultural society but only recently has she begun to exploit her immigrant cultures for what they can offer instead of regarding them simply as defective in English culture. Most people of all cultures wear clothes. Most, though not by any means all of these clothes and the cloth from which they are made, are made by women. Many women are mothers - and it is safe to say that all mothers are women. The making of cloth and of clothes is thus a very familiar part of the background of all the children in our schools.

Teaching materials and an exhibition of textiles and mathematics produced by the Maths in Work Project both exploit the richness of traditional women's activities as a mathematics resource. A research question that it has not yet been possible to pursue is the extent to which women, exhausted after a hard day in the house or the fields turn their minds to the more intellectual pursuits or exploring symmetry when they do their household needlework. Traditional thinking views these activities as trivial and purely decorative. My work challenges this thinking and brings the designs of a Fair Isle sweater, of a Botswana basket, of printed Adinkra patterns from Ghana, of woven patterns from Bangladesh and of a Turkish rug into school as part of a pack of teaching materials for mathematics. Children are learning about more formal mathematical symmetry from their own garments and from textiles from their own cultures and from each others. Issues of gender and culture as problems in mathematics education have been reversed. The issue is now the narrow mindedness of school mathematics and how to reverse that.

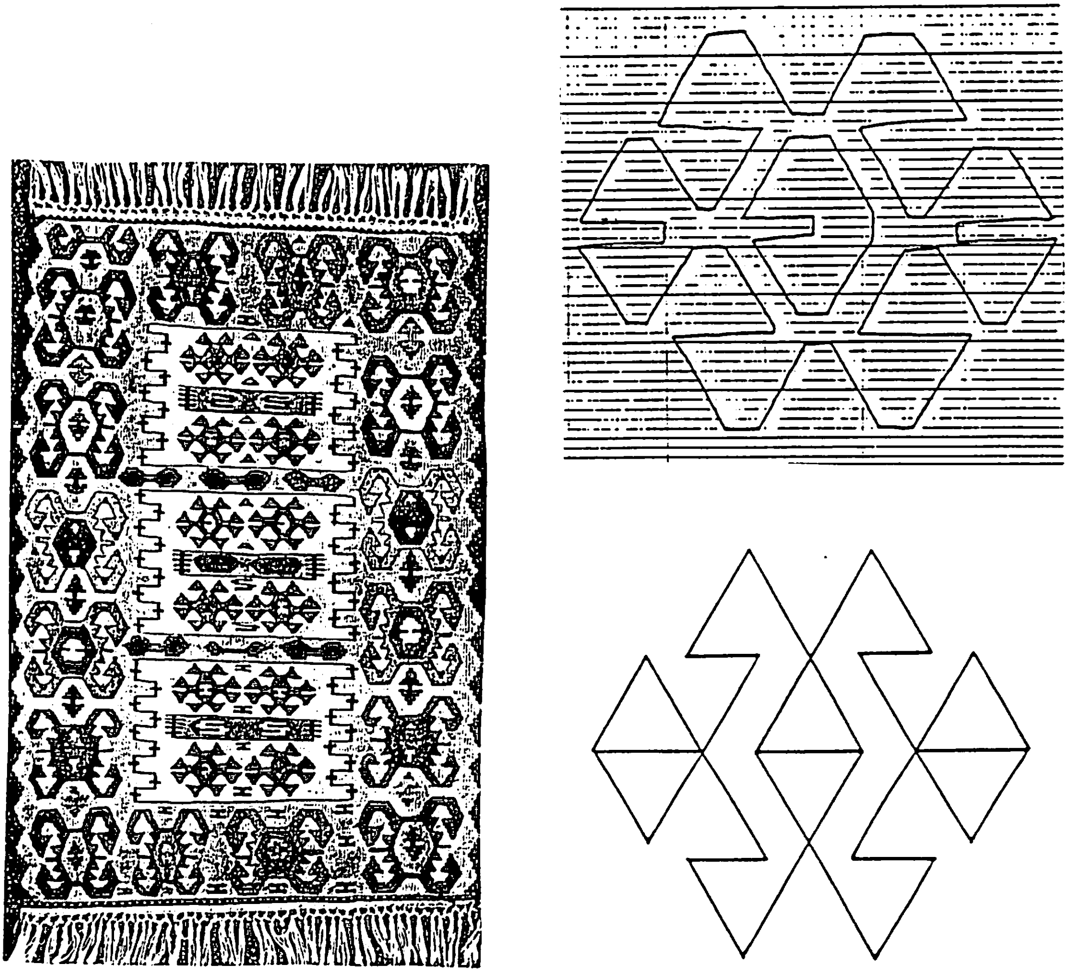

Fig.1

iii) Using a folded piece of squared paper, half of a given shape is copied. The squares can be counted to give accuracy. The other half is obtained by pricking through. This method brings out clearly that in a symmetrical shape each point on one side is matched by a point on the other side which is the same distance from the fold (Figure 5:31).

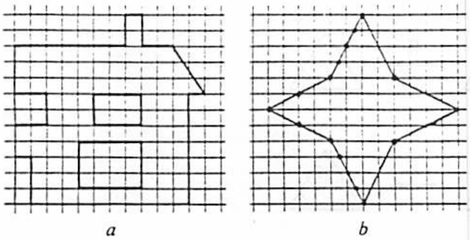

Figure 5:31

Up to the age of about seven children do not distinguish left from right with any certainty. This is understandable because it depends on the point of view of the observer which of two points is seen as left and which as right (Figure 5:32).

Figure 5:32

Making symmetrical shapes helps children to know left from right. Shoes and gloves and double swing doors have this symmetrical relationship and one of the pair must be distinguished as left. Turning a pair of shoes upside down makes it difficult for a child to identify which is his left one.

Folding a sheet into quarters introduces symmetry about two axes, and children can make interesting patterns by tearing or cutting round the edge and cutting out holes.

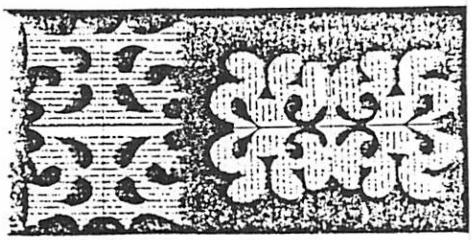

Figure 5:33

It can be seen that Figure 5:31(*b*) also has symmetry about two other axes, and this pattern could have been made by folding the paper into eighths. As children's dexterity grows they will invent ways of making very attractive paper mats using four-fold symmetry.

It is now possible to look again at squares, rectangles, circles, etc. and see which of them can be folded to make two halves which fit (*see page 50*). Any shape can be folded but the two parts are not, of course, usually the same shape. But even the most irregular flat shape, a leaf or a jagged piece of paper, can be folded flat so that the fold is a straight line. Stiff paper folded in this way makes a good edge for ruling a straight line. A further fold, keeping the parts of the first fold together, will make a square corner or right angle. Notice the irregularity of the paper used in Figure 5:34; this makes the right angle stand out clearly. A circle folded in this way is a useful alternative to the jagged paper. The intersection of the folds gives the centre; the folds divide the circle into quarters.

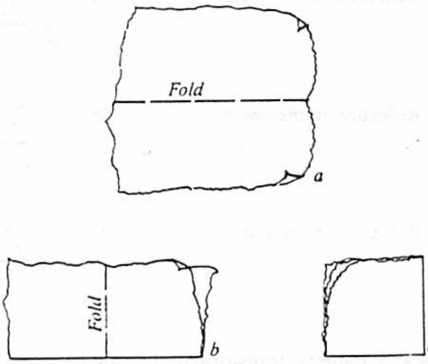

Figure 5:34

If the paper is unfolded the creases show the fitting together of four right angles (Figure 5:35). A right angle made in this way will serve as a home-made set square, and a child can use it to check right angles which occur in the classroom, and in particular to discover the way in

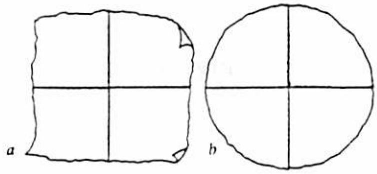

Figure 5:35

(*b*) Transformations which map a shape onto itself are called symmetry transformations. The effects of the six symmetry transformations of an equilateral triangle are shown in Figure 4.

The identity transformation	I
Rotation through 120°	R
Rotation through 240°	S
Reflection in the line *a*	A
Reflection in the line *b*	B
Reflection in the line *c*	C

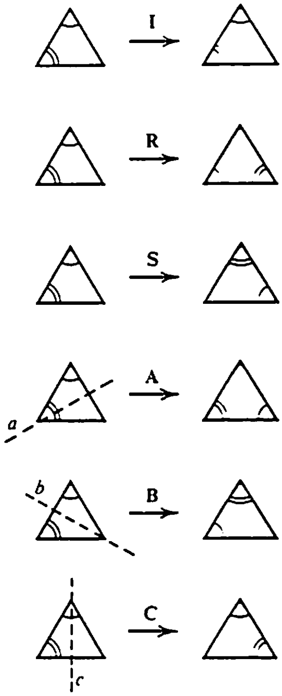

Fig. 4

These symmetry transformations can be combined in the usual way. For example, Figure 5 shows that SC has the same effect as A and we can write

$$SC = A.$$

Remember that SC is the combined transformation 'first C and then S'.

Fig. 3

Figures and References.

Figures

Fig 1. A Turkish kelim in the possession of the author. One motif has been taken out for analysis and comparison with a more formal mathematical exercise of building a symmetrical design based on isometric transformations of an isosceles triangle.

Fig 2. A page from "Primary Mathematics Today" Williams E and Shuard H. Longman 1982 (third edition.)

Fig 3. A page from the "Schools Mathematics Project" Book Z, Cambridge University Press 1976.

Fig 4 Photographs of Aran and Fair Isle sweaters from the "Common Threads" exhibition. (see also the Common Threads Catalogue)

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THE PECULIAR STABILITY BEHAVIOUR OF NON-SYMMETRICALLY LOADED SYMMETRICAL STRUCTURES

by István Hegedűs and László P. Kollár
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Let us consider the simple symmetrical plane structure shown in Fig. 1a which consists of rigid bars and elastic springs, connected by hinges. The degree of freedom of movements is

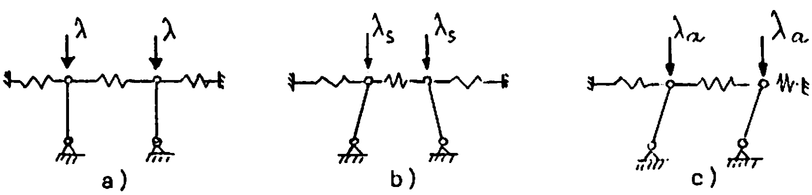


Fig.1

two. The symmetrical load can cause symmetrical (Fig. 1b) or antisymmetrical (Fig. 1c) buckling. The critical loads which belong to these buckling modes are denoted by λ_s and λ_a respectively.

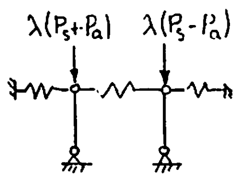
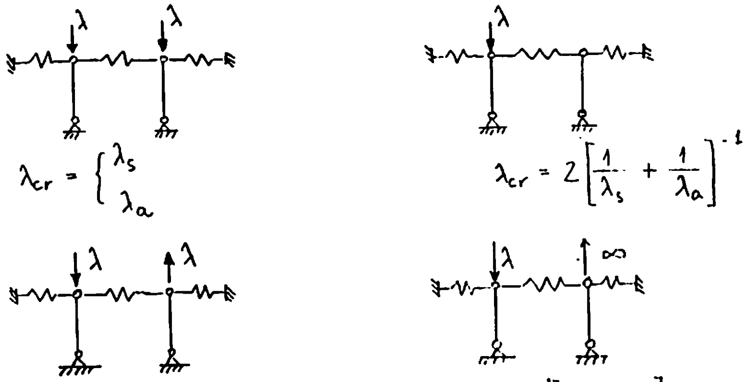


Fig. 2

Assuming different loads acting on the left and right bar (Fig.2), it can be shown that the critical loads (λ_{cr}) are the roots of the following equation of the second degree:

$$\lambda_{cr}^2 (P_s^2 - P_a^2) - \lambda_{cr} P_s (\lambda_s + \lambda_a) + \lambda_s \lambda_a = 0 \,. \tag{1}$$

Consequently, in the case of special non-symmetrical load arrangements shown in Fig.3, the critical load can be calculated from the geometric, arithmetic and harmonic means of λ_s and λ_a. By investigating some similar structures we have obtain the same result, namely that the critical load



$$\lambda_{cr} = \begin{cases} \lambda_s \\ \lambda_a \end{cases} \qquad \lambda_{cr} = 2\left[\frac{1}{\lambda_s} + \frac{1}{\lambda_a}\right]^{-1}$$

$$\lambda_{cr} = \sqrt{\lambda_s \lambda_a} \qquad \lambda_{cr} = \frac{1}{2}\left[\lambda_s + \lambda_a\right]$$

Fig.3

of a non-symmetrically loaded symmetrical structure can be obtained from the critical loads belonging to the symmetrical and antisymmetrical buckling modes of the symmetrically loaded structure on the basis of (1). Now the question arises whether this strange behaviour is a general feature of "symmetric" eigenvalue problems. The answer is no, but we can give the conditions with the aid of which it can be decided when this behaviour occurs.

Let us introduce the following bilinear functional:

$$\Pi = \frac{1}{2}\langle A_1\varphi_s, A_2\varphi_s\rangle + \frac{1}{2}\langle B_1\varphi_a, B_2\varphi_a\rangle + \frac{1}{2}\langle D_1\hat{\varphi}_s, D_2\hat{\varphi}_s\rangle + \frac{1}{2}\langle E_1\hat{\varphi}_a, E_2\hat{\varphi}_a\rangle + \frac{1}{2}\langle F_1\hat{\varphi}_s, F_2\varphi_s\rangle +$$
$$+ \frac{1}{2}\langle G_1\hat{\varphi}_a, G_2\varphi_a\rangle - \frac{1}{2}\lambda\left\{P_s\left[\langle C_1\varphi_s, C_2\varphi_s\rangle + \langle C_1\varphi_a, C_2\varphi_a\rangle\right] + 2P_a\langle C_1\varphi_s, C_2\varphi_a\rangle\right\}$$

where φ_s, φ_a, $\hat{\varphi}_s$, $\hat{\varphi}_a$ are the elements of the real inner product (pre-Hilbert) space $\mathcal{H}$, which form the vector :

$$\underline{\varphi} = \begin{bmatrix} \varphi_s \\ \hat{\varphi}_s \\ \varphi_a \\ \hat{\varphi}_a \end{bmatrix}$$

and A_1 , B_1, ...,A_2, B_2 etc. are linear operators of $\mathcal{H}$. We deal with the

$$\Pi \quad = \quad \text{stationary} \quad !$$

eigenvalue problem. Assuming symmetrical loading, i.e. $P_s = 1$ and $P_a = 0$, we obtain the eigenvalue parameters λ_s and λ_a which belong to the eigenfunctions

$$\Phi_s = \begin{bmatrix} \phi_s \\ \hat{\phi}_s \\ 0 \\ 0 \end{bmatrix} \qquad \Phi_a = \begin{bmatrix} 0 \\ 0 \\ \phi_a \\ \hat{\phi}_a \end{bmatrix}$$

respectively. In the case of non-symmetrical load, the critical load can be calculated on the basis of (1) only if

$$C_1^* \ C_2 \ \phi_s = \alpha \ C_1^* \ C_2 \ \phi_a \quad ,$$

where α is a scalar, and C_1^* is the adjoint operator of C_1, see [Hegedűs and Kollár, 1989a].

The results can be used for the investigation of discrete and continouos problems as well , among others for the flexural wrinkling of sandwich panels [Hegedűs and Kollár, 1989a,b].

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ON THE ASYMMETRY OF THE BIRCH LEAF ROLLER'S INCISIONS

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Previously it has been stated that the leaf cone construction of the birch leaf roller (Deporaus betulae) is determined by the principle of optimal cost, which would result symmetrical incisions cut by the beetle. However the real incisions are asymmetrical. This contradiction and the reason of the asymmetry are explained.

Birch leaf roller rolls a regular, well-closed leaf cone for its offsprings (Fig. 1); it cuts two special S-shaped incisions into the leaf blade to make the leaf twist easier (Fig. 2). It can be seen in Fig. 2 that the birch leaf roller's incisions are asymmetrical.

A widely quoted general view is that the optimal shape of the incisions is determined by the principle of optimal cost: the work needed to roll the leaf halves is as minimal as possible at a given leaf mass rolled into the leaf cone [1,2]. If this variational principle was valid for the twists of both leaf halves, it would result symmetrical incisions. Therefore the question is raised: 'Why are asymmetrical the Deporaus betulae's incisions ?'.

The applicability of the above mentioned principle is refuted in [3], and a new bionical explanation and biomathematical description was presented in [3,4]. Let us sketch out this explanation.

After the cutting of the incisions the beetle begins to roll leaf half 1 into a leaf cone and incision 1 helps in this cone construction. Deporaus betulae first makes a small cone from the leaf lamina on the leaf border and then rolls the whole of leaf half 1 around this core; a regular, slender cone is formed. A suitable microclimate can be insured for the grubs if the peak of the cone is well closed: there may not be any gap on this peak. Figure 3(b) shows the situation of the leaf cone near its last stages, and Figure 3(a) shows the situation of leaf half 1 when it is uncoiled. The external layer of the leaf cone allows the internal core to rise out easily and to close the peak of the leaf cone only if leaf half 1 forms a slanting cone section in the uncoiled stage of Figure 3(a). Therefore Deporaus betulae cuts incision 1 so that after the twist of leaf half 1 the external leaf layer constitutes a slanting cone section. This can be realised if the uncoiled leaf half 1 forms a slanting cone section.

On the basis of these the theoretical incision 1 is the curve of a slanting cone section laid out in the plane. The expression of this curve can be determined [3,4]:

$$\rho(\delta) = y(x_0)\left[1+1/y'^2(x_0)\right]^{1/2}$$

$$\times \frac{1 + \dfrac{\tan\beta \arctan y'(x_0) \cos\gamma}{\left\{4\pi^2 - \left[\arctan y'(x_0)\right]^2\right\}^{1/2}}}{1 + \dfrac{\tan\beta \arctan y'(x_0) \cos\left[\gamma + 2\pi\delta/\arctan y'(x_0)\right]}{\left\{4\pi^2 - \left[\arctan y'(x_0)\right]^2\right\}^{1/2}}} \qquad (1)$$

where the definition of the parameters $y(x_0)$, $y'(x_0)$, β and γ can be found in [3,4].

After the rolling of leaf half 1 into a leaf cone the beetle twists leaf half 2 around this cone. Incision 2 helps this rolling and prevents the uncoiling of the twisted leaf. The flexibility of the leaf lamina plays a primary role in the physics of the leaf twist, therefore consider the torque needed to roll a leaf blade around a cone with half aperture angle α. The thickness of the leaf lamina is a; the width of the rolled leaf blade along the generatrix of the cone is b. The nearer edge of the rolled leaf lamina is at distance x along the generatrix from the peak P of the cone. If E is the Young's modulus of the leaf blade, the torque needed to roll the leaf cone is

$$M = \frac{Ea^3}{12\,tg\,\alpha} Ln(1+b/x) \qquad (2)$$

Cutting the midrib of the leaf causes the leaf tissue to wilt, that is, its cells lose their normal turgor. This is a crucial in the strategy of the birch leaf roller, because the mechanical properties of the flaccid, wilted leaf lamina are more advantageous for the leaf twist than those of the normal, turgid lamina. The Young's modulus E of a wilted leaf blade is much smaller than that of a turgid blade, so on the basis of (2) the torque M needed to flex a flaccid lamina is much smaller than that of a turgid one.

Furthermore it can be observed that the wilted leaf blade cutted by Deporaus betulae is a little twisted in ourself, and this makes also easier the leaf twist. It is important for the suitable microclimate of the grubs that the leaf tissue of the leaf cigar does not dry totally after the leaf cone is constructed, but the tissue does not regain its normal turgor after the cone construction. The upper part of the rolled leaf remains sound and turgid; and a smaller flow of the tissue fluid is possible through the gnawed midrib.

We can see from (2) that Deporaus betulae must cut incision 2 in such a way that during the roll the distance $\overline{PB_2}$ is not too small, because the torque needed to roll leaf half 2 would then be very great, or too large, because then little leaf mass would roll into the leaf cone. The beetle must choose a small distance $\overline{PB_2}$, then it cuts incision 2 so that the edge of the leaf moves away quickly from the point P during the twist, so x increases rapidly, M decreases rapidly, and the part of leaf half 2 near the point P can be rolled.

When the beetle is ready with the twist of the leaf halves, it fastens the leaf layers of the cone together with its proboscis; thus the leaf cone cannot uncoil. The last leaf layer must be tongue-shaped so that it can be fastened easily by the beetle, i.e., the torque M must be small. We see from (2) that M is small if b/x is small. Consequently the last, tongue-shaped layer must be narrow and its edges must be distant from the point P. Therefore Deporaus betulae cuts incision 2 in the leaf lamina so that the last leaf layer is a relatively narrow, long tongue far from the peak point P of the leaf cone.

Since the leaf cone nourishes the grubs, it is very important to have enough leaf mass in it; the beetle must roll as much leaf mass as possible into the leaf cigar.

Consider the angles between the borders of the lower part of leaf half 2 rolled and the generatrix of the cone. If these angles differ very much from each other during the twist, then the last, leaf layer will be suddenly very wide or narrow; either one would contravene the reqiurement of the narrow, long, tongue-shaped last leaf layer far from the peak of the leaf cone. Therefore incision 2 must be cut in such a way that the angles between the edges of the lower part of leaf half 2 rolled and the generatrix are equal.

Refering to Figure 2, assign polar coordinates with origin at P, and angle measured from the midrib AQ of the leaf, using the function $R(\varphi)$ of the leaf border the theoretical curve of incision 2 according to the above twist method is the following [3,4]:

$$r(\varphi) = \frac{\overline{PQ}\cdot\overline{PB}_2}{R(\varphi)} \qquad (3)$$

Expressions (1) and (3) describe well the shape of the Deporaus betulae's incisions [3,4]. The shape of the birch leaf roller's incisions cannot be explained by an only variational principle, this is the reason of their asymmetry.

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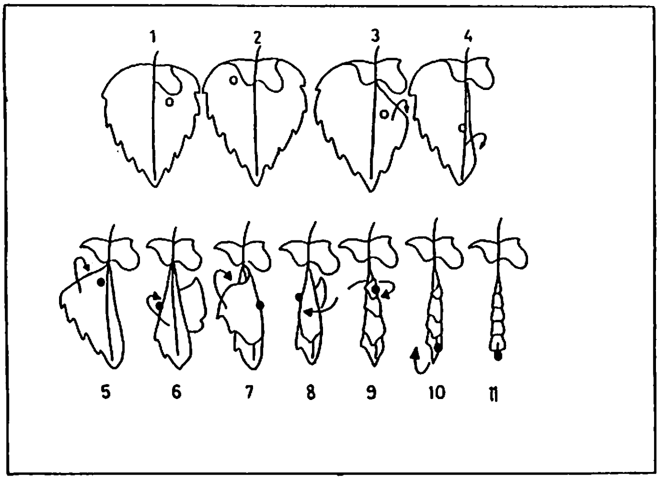

Fig. 1

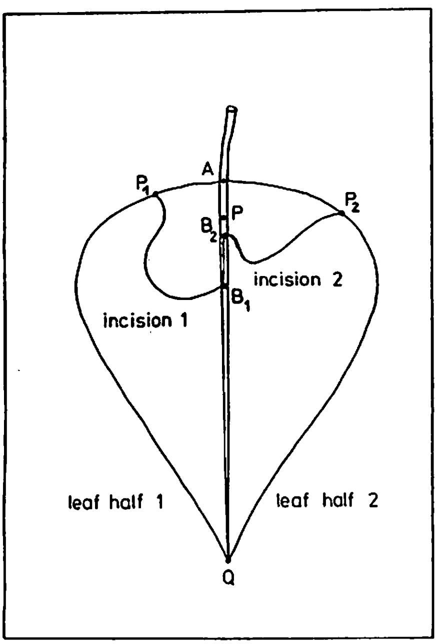

Fig. 2

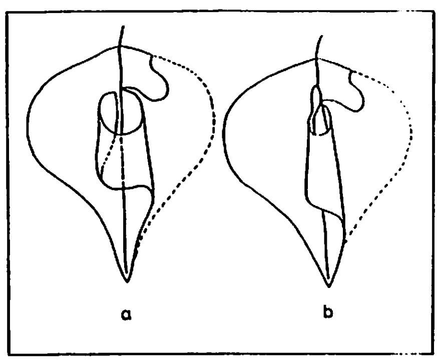

Fig. 3

THAT UNORDINARY MIRROR-ROTATION SYMMETRY

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The history of *mirror-rotation* (or *rotary-reflection*) symmetry begins in the 19th century with crystallographers; but manifestations in its prehistory can be traced to the advent of civilization, when man began to record observations of astronomical bodies.

The mystery of the solar eclipse surely awed man in distant times; and, for those who functioned beyond dismay, there were wonders to observe. Aristotle recorded what many had certainly noticed before him: the holes of a sieve cast round images of the sun, even if they were square or triangular. The leafy canopy of a woodland does the same; and, during an eclipse, a multitude of crescents appear. Strangely, these crescents are inverted in relation to the crescent of the partially eclipsed sun.

Those not located in the total eclipse's corridor will see that, during the event, the position of the sun's crescent will appear to rotate either clockwise or negative clockwise. Whichever way it seems to go, the images will seem to rotate the other way. An observant person would not only be puzzled by the inverted image, but by its perverted rotation. Up becomes down; clockwise becomes negative clockwise.

The sun's images are caught in rooms from holes in roofs. Dangerous to stare at with the naked eye, the eclipse can thus be safely observed. Other apparitions materialize from chinks in window shutters. Whole exterior scenes are projected onto interior walls: figures, colors, movements, all that is outside. The projections are, however, upside down; handedness is reversed.

Hearsay history credits da Vinci, Alberti, Roger Bacon, and other Renaissance notables with the invention of the camera obscura. In fact, they were making use of a device with a remote origin. The earliest known record of it comes from the tenth century Arabian scholar Alhazen, whose writings indicate that he was describing something already known.

A true mirror-rotation configuration is isometric. Though the sun's image can be set to the same size that the sun appears from earth, the sun and its image are a "similitude," not an isometry. Contained apparatuses, such as the boxes of Athanasius Kircher, can be rigged to precise isometric relationships. An image on one wall of a chamber is lighted by candles; a shutter with a hole is set at the center; the wall of the darkened chamber on the other side catches a perverted image.

For all his scientific formalities, Brother Kircher was careless. Embarrassing errors are immortalized in his tomes. The images in Kircher's illustrations of camera obscuras are all duly inverted. In one case, handedness is not reversed; in two others, though letters and words are reversed, the positions of the words are clumsily scrambled. Observation was inconclusive; bungled reversals of handedness evidences defective witness. Kircher, for one, cannot be credited with a faithful presentation of the mirror-rotation principle.

Lenses were introduced in more sophisticated camera obscuras at the shutters' apertures in order to effect clearer images. Those dealing with lenses also saw upside-down, reverse-handed worlds through the honed glasses. Aristophanes is the first to have written about a "burning glass"; and it is reported that Roman emperors viewed gladiatorial games though jewels. Yet, the great innovators of lensed instruments from Galileo on can no more than Kircher be credited with actually specifying the conditions of mirror-rotation symmetry; and modern text books on optics, alas, often present illustrations that repeat faults found in Kircher's works.

The necessity to derive the concept of mirror-rotation arose with the crystallographer. *Inversion*, rather than the "coupled" *mirror-rotation* operation, could stand as definition for two Crystal Classes which were later given the notations S_2 and S_2 by Schönflies. The elusive and critical four-fold mirror-rotation operation, S_4 could not be described as a straightforward inversion.

Frankenheim, using analytical geometry, is given credit for first identifying the 32 Classes (1826), including, of course, S_4, which spontaneously appeared among the permutations of his algebraic expressions. Hessel's confirmation, presented by a peculiar and tedious terminology, soon followed (1830). Gadolin's enduring contribution was his stereographic projections (1867) -- Frankenheim having produced no illustrative aids and Hessel, less than artful diagrams in a supplemental publication (1862).

Logic suggests that, even without Frankenheim or Hessel, S_4 would have shown up for Gadolin through the simple exercise of mapping all the combinatory possibilities that his projections allow. Schönflies, whose notations remain in use today, did, of course, handle S_2, S_6, and S_4 with group theory and applied the term *Drehspiegelung*.

Federov saw clearly the comprehensive nature of mirror-rotation. He submitted that, though S_2 can also be described as the inversion C^i and S_6 as C^i_3, inversion is an unnecessary operation in a succinct theory of symmetry:

> *I have repeatedly pointed out that the concept of the inversion center does not belong to symmetry theory at all, but to the theory of similitude, and is immeasurably older than symmetry theory.*

The mirror-rotation operation (Federov's "compound symmetry") covers the three Classes, S_2, S_4, and S_6, and an infinite array of members, S_8, S_{10}, S_{12} . . . S_{2n} (and S_{4n}), beyond the narrow range of crystallographic possibilities.

Failing an ability to read Russian, I take many clues from the Federov papers translated into English by David Harker. *Gadolin*, Federov declared, *did not yet know anything about compound symmetry and was connecting only one special case* (four-fold mirror-rotation) *with the word sphenoidal.*

Federov, as mentioned, was critical of Schönflies for retaining inversion. Interestingly, Federov's later papers indicate that he himself had not envisioned the comprehensive nature of a mirror-rotation operation at the time of his 1883 treatises: *I have been saying, beginning with 1889, not "sphenoidal symmetry," but "compound symmetry."*

Though four-fold mirror-rotation, rather than two-fold, created consternation (even some discredit for Bravais) and was the element that forced the rigor of a Federov upon the theory, except for Gadolin's often reproduced diagram, there were few early illustrations of it. Curie did provide an utterly straightforward one. Shubnikov and Koptsik resurrected an elegant example attributed to G. Wulff. In their 1956 *Symmetrie*, Wolf and Wolff, that unsymmetrical pair, not to be confused with the yet other Wulff, did not present a proper representation of four-fold mirror-rotation; they resorted to a regular tetrahedron with all its additional mirror planes.

The chemists were slow to adopt symmetry theory in their study of molecules. Of course, they had first to work out general concepts of how molecules are structured. Before the employment of X-ray, both the theories of crystallographic symmetry and of molecular structure were abstract exercises, albeit, compelling ones. Van 't Hoff and Le Bel made a major breakthrough with their brilliantly deduced "asymmetric carbon atom." An error was made, however, which again indicated unfamiliarity with mirror-rotation symmetry and its properties.

Van 't Hoff (and those who followed) depicted the meso-tartaric molecule as a mirror figure. It is now known that in solution there is "free rotation" of the meso-tartaric molecule's two tetrahedral components at the molecule's mid-point. Since like atoms repel one another and effect antipodal positions, the favored alignment, then, of the two components, out of all random possibilites, is not a mirror configuration but a mirror-rotation one.

Chemists, applying crystallographic symmetry theory, sought the hypothetical. While two-fold and six-fold mirror-rotation molecules were soon identified, it wasn't until 1955 that an actual four-fold mirror-rotation molecule was found.

At 1930's international conference in Zürich, Hermann and Mauguin proposed *rotary-inversion* to replace Federov's hard argued *rotary-reflection.* As early as 1897 Federov considered a system with the alternative "compound symmetry" of inversion-rotation; but he stayed with mirror-rotation.

If not from Federov, from where did Hermann and Mauguin's rotary-inversion, as a comprehensive operation, come? Only two years before the Zürich conference, Hermann wrote an article for the *Zeitschrift für Kristallographie* that cited *Drehspiegelung*, but not *Drehinversion*. Wykcoff's text of 1924/1931 presents both *rotary-reflection* and *rotary-inversion* as alternate ways to describe S_2 and S_6; it described S_4, however, only as rotary-reflection. Many agree with an esteemed scholar that rotary-inversion is "one of the worst concepts ever perpetrated on the scientific public."

Aside from the embodiment of pure isometric, two-fold mirror-rotation in certain optical instruments the only other familiar, utilitarian, man-made object with solitary mirror-rotation symmetry is the double-ended dental instrument. (A sphenoidal wedge with four-fold mirror-rotation may be used to stop a door, but it is not *pure*; it is encumbered with two additional mirror planes). The doubled-ended curette, having perfect balance, is easily twirled in the fingers, allowing the dentist, without changing instruments, to pick at both sides of the same tooth with right and left hooks.

Though instances of mirror-rotation are rare in natural and utilitarian objects, abstract objects with these properties can be made at will. Some sculptors have, perhaps knowingly, perhaps accidentally, produced pieces with this unordinary structure. There has been such a one, for example, before the Bahnhof in Zweibrücken.

Innumerable mirror-rotation objects have been made in my instructional design studio at various schools; small though they are, many evoke monumentality. Our first studies involved only two-fold mirror-rotation. Recently, four- and six-fold examples have been developed. Few, if any entities, larger than molecules, have solitary, four-fold mirror-rotation other than, perhaps, scaled-up models of molecules and idealized crystals. It is possible, then, that never in the whole of history have there been so many singular examples of four-fold mirror-rotation in one place as were in my classroom at the University of Hong Kong in 1986.

The symmetry of preference in architecture is, by no coincidence, bilateral symmetry -- its imprint lodged in our own bodies. Formal buildings are often given a second, transverse plane of symmetry. The facade of Palladio's Villa Rotunda proclaims the symmetry of the square, the combination of four mirror planes and a four-fold rotor. There are buildings in the shapes of hexagons, octagons, and circle, but the orders of symmetry of their essential bodies are often offset by openings that do not match to the same degrees.

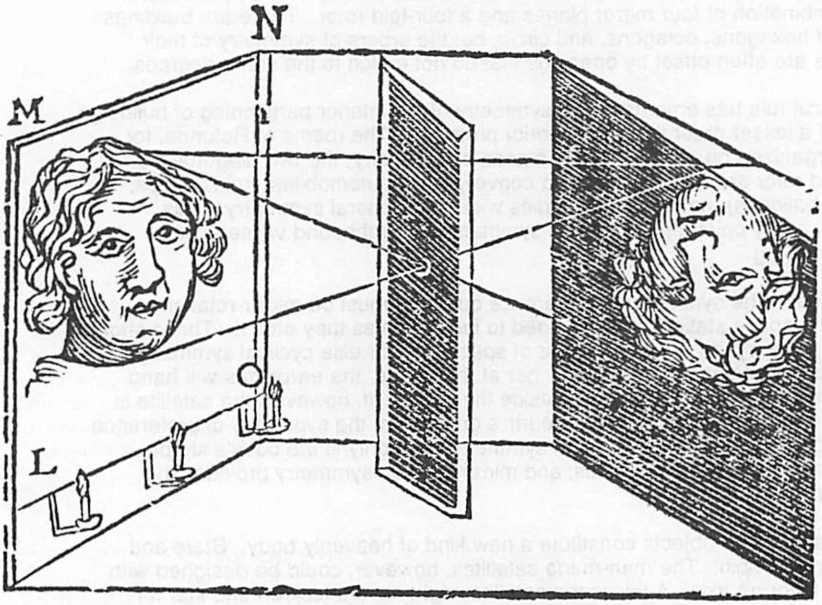
A sort of empirical rule has emerged: the symmetry of the interior partitioning of buildings is predictably of a lesser order than the exterior promises. The rooms of Rotunda, for example, are organized on only two cross planes of symmetry; the two diagonal planes and the four-fold rotor are missing. Moving conveyances, automobiles, trains, ships, aircraft (except balloons), have exterior bodies with only bilateral symmetry. This summary substantially covers the extent of symmetry for earthbound vessels, both stationary and moving.

But what of space? The symmetry of preference out there must be mirror-rotation symmetry. Some space stations are designed to face earth as they orbit it. These should have bilateral symmetry, as abides in a pair of spectacles (or else cyclical symmetry). When spectacles are balanced on one's finger at the bridge, the earpieces will hang downward, since the center of gravity is outside the bridge. If, however, the satellite is meant to rotate freely without the effect of earth's gravitation, the symmetry of preference would, without question, be mirror-rotation symmetry. Not only is the body's shape critical, but so is the distribution of mass; and mirror-rotation symmetry provides the blueprint for that distribution.

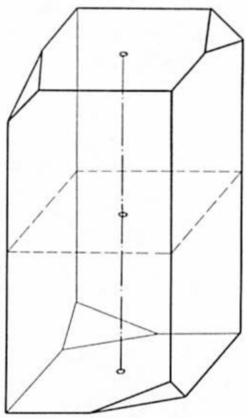
Certain man-made space objects constitute a new kind of heavenly body. Stars and planets have axes of spin. The man-made satellites, however, could be designed with good reason to favor no axis. A telescope satellite might have a lock on one star for a time and, then, by command, swing to a lock on another. This would be accomplished with ease, if the satellite has a center of symmetry. The sphere, cylinder, and icosahedron all have large arrays of mirror planes and multi-fold rotors. They also have centers of symmetry and, therefore, must be classified among mirror-rotation objects. Because of their high orders of symmetry, however, a consistent distribution of massing, in order to preserve equilibrium, would be highly demanding.

Less regular bodies, retaining a center of symmetry, would reduce the redundant matching of mass for mass. Two-fold and four-fold mirror-rotation are prime candidates for the new sedentary vessels of space. Such satellites could have spherical exteriors; and, taking a cue from architecture's mode of reduced interior symmetry, their interior arrangements could be stripped-down mirror-rotation symmetry. [An alternative body with centered equilibrium is one with three, mutually perpendicular, two-fold axes (*V*).]

Mirror-rotation, observable, yet not identified for over six millennia of man's past, has a future in the stars.



1.
camera obscura: Athanasius Kircher; *Ars Magna Lucis et Umbrae*; 1671



2.
prism, modified to exhibit characteristics of four-fold mirror-rotation, after Curie: Pierre Curie, *Œuvres*, 1908

UNIFORM POLYHEDRA FOR BUILDING STRUCTURES

by

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General introduction

Many building structures are in their overall shape or in their internal configuration based on the geometry of one or more kinds of the so-called 'uniform' polyhedra. The cube and the prisms are very popular in this respect, but these are in fact members of a large family of forms that have great potentials for application in building. It is therefore interesting not only but highly relevant also to undertake a systematic study after the properties of this group of figures in order to find out what their characteristics are, what their binding factors and what their differences.

Definition of the uniform polyhedra [1]

1. They are composed of one or more kinds of plane, regular polygons with 3, 4, 5, 6, 8 or (maximally) 10 edges.
2. The polygons meet in pairs at a common edge.
3. The dihedral angle at such an edge is always convex (= less than 180°, if seen from the interior).
4. All vertices of a polyhedron lie on one circumscribed sphere.
5. All these vertices are identical, which means that around each vertex of a particular polyhedron the polygons are grouped in the same number, kind and order of sequence.

Different kinds of polyhedra

It is easy to understand that within these limits the minimum total number of polygons around a vertex is 3, the maximum number 5 and it is also simple to prove, that not more than 5 totally regular polyhedra can exist. These regular solids are composed out of one kind of faces each.
Polyhedra are called semi-regular if more than one kind of polygons is used for their construction. Thus a group of 30 polyhedra in total, answers the definition of uniformity (see Fig. 1):
- 5 regular or Platonic solids, consisting of 8 or 20 triangles, of 6 squares or of 12 pentagons
- 15 semi-regular or Archimedean solids (including the *enantiomorphic* or left-handed versions of the snub cube and the snub dodecahedron)
- 5 prisms, having two parallel congruent regular polygon faces with squares as the interconnecting sides (the square prism is identical to the cube)
- 5 antiprisms with two twisted parallel polygons and with triangles as sides (the triangular antiprism is identical to the octahedron)

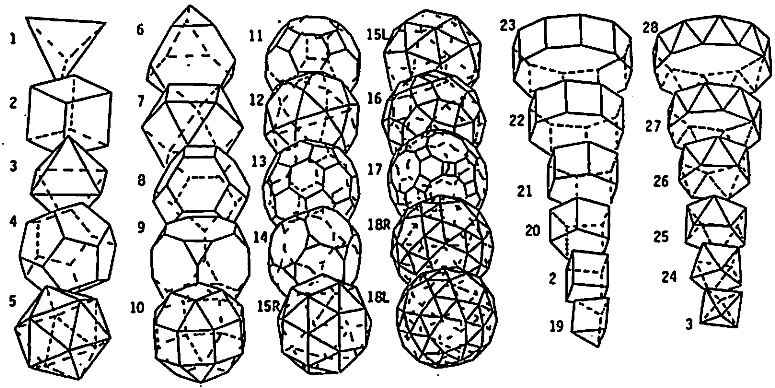

Fig. 1. *Review of the regular and semi-regular uniform polyhedra:*
A. Platonic: tetrahedron(1), cube(2), octahedron(3), dodecahedron(4), icosahedron(5)
B. Archimedean: truncated tetrahedron(6), cuboctahedron (7), truncated octahedron(8), truncated cube(9), rhombicuboctahedron(10), truncated cuboctahedron (11), icosidodecahedron(12), truncated icosahedron(13), truncated dodecahedron(14), righthanded snub cube(15R), lefthanded snub cube(15L), rhombicosidodecahedron(16), truncated icosidodecahedron(17), righthanded snub dodecahedron(18R), lefthanded snub dodecahedron(18L)
D. Prisms: trigonal(19), square(2), pentagonal(20), hexagonal(21), octagonal(22), decagonal(23)
E. Antiprisms: trigonal(3), square(24), pentagonal(25), hexagonal(26), octagonal(27), decagonal(28)

Computer technique for visual representation [3]

The coordinates and other geometric aspects of uniform polyhedra can be generated automatically with the help of computers by firstly forming each of the different kinds of polygons out of which the polyhedron consists and then reproducing it by combined translation and rotation in space as many times as it occurs in the considered polyhedron. Fig. 2. shows the principle of such a procedure as developed for study purposes on personal computers.

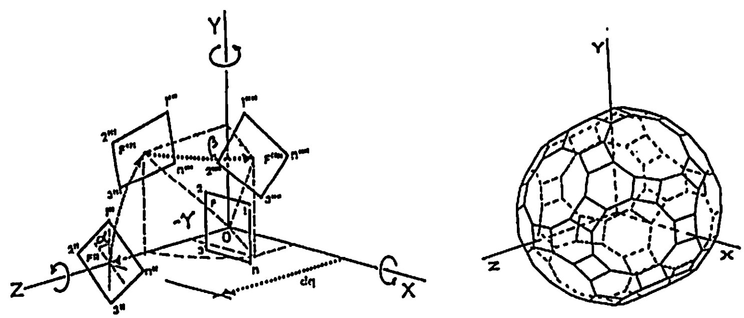

Fig. 2 *Combined translation and rotation of polygons for the formation of polyhedra*

The use of polyhedral forms in building [3]

The role of these polyhedra for the formgiving possibilities in building is very important, although this is not always recognized.

- all trivial uses of cubic and prismatic shapes (vertical or horizontal, pure or distorted versions)
- many solitary applications with macroforms, derived from one of the more complex polyhedra or of their reciprocal (dual) forms
- prismatic and antiprismatic folding structures
- close-packings of one or more kinds of solids in conglomerates or in space structures
- pyramidized or polar versions of solids in order to reduce the relative size of larger polygons
- basic geometry of geodesically subdivided dome structures - mainly on the basis of octahedron, icosahedron, or triacontahedron (reciprocal of icosidodecahedron)

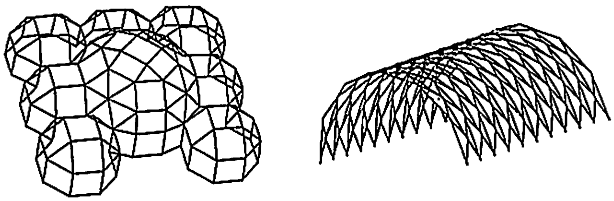

Fig. 3. *Polyhedral building and antiprismatic structure*

Tri-axial ellipsoids [2]

Polyhedra can be considered as subdivision patterns of spherical shapes. A higher break-down frequency provides a closer approximation to the sphere. If to such a dome structure different radii are given in X-, Y- and Z-direction of the coordinate system, then a tri-axial ellipsoid is obtained. A horizontal cross-section according the XY-plane (see Fig. 4) has the form of an ellipse. Any vertical cross-section through the Z-axis is an ellipse also, having its foot points on the horizontal ellipse. The equation of the ellipse is of the second power, but the main form of this ellipse can be influenced considerably by raising or lowering this power.(squared or flattened if respectively larger or smaller than 2, straight or hollow if respectively equal to or smaller than 1).

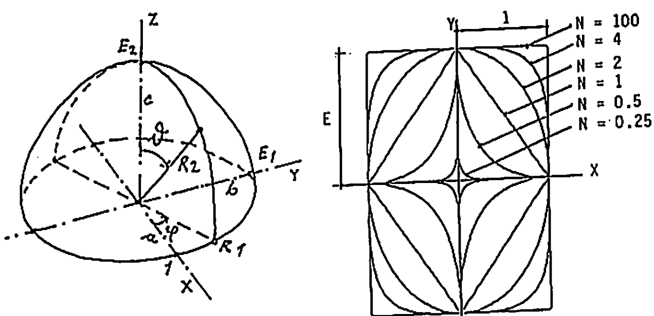

Fig. 4. *Characteristics of tri-axial ellipsoids*

them squarely on the road to progress. The revolutionary significance of this new organon lies in the fact that it has overstepped the original limits set for it. However, the idea of combining concepts and ideograms into a single coherent schema, basically corresponding to the structure of the world and of our knowledge of it, has been less successful. It is founded on perception of the unity of human knowledge and reality. Such a schema is a projected unity, established as a point of reference and correlation, but does not state whether anything is "actual" or "existing". The semantologic multiplicity and arbitrary nature of any language provide many possibilities for communication between conceptual and visual languages. The basic conviction that a feasible "canon" must exist for bridging them, has its roots in the fixation that a rational constructivistic relationship overlies the arbitrary level of a metalanguage.

The organon is an illuminating instrument of science, exposing unknown truths and establishing a common method which contributes to science, philosophy, the arts and architecture. All historical, cultural and scientific development might be viewed as harmonization of vital images, formal structures and diverse ideas. The result of this process of human knowledge and consciousness lies in the realm of essences, which find their formal expression in the "conceptual and ideographic organon". The fundamental principles of the organon impose order and pattern on nature and on human knowledge. If it successful it might yield possible principles of morphogenesis.

On the other hand according to the structure of the Canon, the organon possesses - as a central tool of human knowledge - the ability to analyse all information associated with aquisition or expression of knowledge. There is a duality between information or knowledge already possessed or acquired, and the senses or interpretations attached to it. Scientific and non-scientific inquiry is traditionally over time, despite revolutionary changes in the theoretical models and systems describing its subject matter. Analytical or dialectical use of the organon according to the different subjects of the canon means that its continuity and development are of hermeneutic nature. Such application of hermeneutical elements and methods means in turn that, through continuous reinterpretation of data, the real essences will gradually be exposed to human knowledge. The attendant identification and isolation of the same set of referents requires consistency with all possible changes in the theoretical models or systems. The importance of the hermeneutical approach lies in the ability to explain scientific observations, in finding links with alternative points of view and even in showing the way to new scientific and non-scientific perspectives.

In the attempt to construct our schema of concepts, we concluded that it could be expressed on a new formal basis. Each concept is assigned a visual object, a so-called "element house". The first element house used in this project refers to the concept of space, which - conceived as the foundation stone of human knowledge - has developed over the ages alongside its conceptual, symbolic and architectonic evaluation.

nondegenerate regular eight-member cycle P(2), which is equivalent to the whole previous cyclic closing [P(1)], but also two simplest possible additional degenerate two-member cycles p (the initial, p', and final, p", cycles): $\iota(2) = \varsigma(2) = \varsigma_{[P(2)]} = \varsigma_{p' + P(2) + p''} = \varsigma_{p'} + \varsigma_{P(2)} + \varsigma_{p''} = 2 + \varsigma_{[P(1)]} + 2 = 2 + 8 + 2 = 12$. The longitudinal skeletons of the blocks are of four types (allowing for the feature of the proline residual) and may be described by the proper spinal characteristic S with four pairwise-anti-symmetric half-integer eigenvalues $S(s) = s = \pm\frac{1}{2}, \pm\frac{3}{2}$.

In psychology ($g = 3$), the complete system of typical homo sapiens with their 13 characteristic inverse values of critical levels of intellectual potentialities $1/x(i)$ or, in other words, with the appropriate successive whole numerical eigenvalues $I(i) = 12/x(i) = i = 0,1,\dots,\iota(3) - 1$ ($\iota(3) = 13$) with the particularly singled-out common initial zero eigenvalue $I(0) = D(0) = 0$, which is inherent to the divinely-omnipotent genuises, at the center of symmetry of twelve remaining cyclicly-closed eigenvalues $D(d)$ ($d = 1, \dots, \varsigma(3) - 1$; $\varsigma(3) = 13$) constituting the complete closed nondegenerate regular twelve-member cycle $[P(3)] = P(3)$ which is characteristic of the proper mentality classification of typical homo sapiens and equivalent just to the previous cyclic closing [P(2)], namely, $\iota(3) - 1 = \varsigma(3) - 1 = \varsigma_{[P(3)]} = \varsigma_{P(3)} = \varsigma_{[P(2)]} = 12$. The individuals with opposite traits (extraverts and intraverts) are of sign-opposite half-integer eigenvalues of the proper spinal characteristic S, namely, $S(s) = s = \pm\frac{1}{2}$.

The above mentined systems of the fundamental structural elements of matter at four successive basic levels of its natural self-organization are equivalent to the systems of uniformly-quantized eigenvalues of the respective universal characteristics I, D, and S which are of the same type of symmetry and are defined deductively from an appropriate mathematical induction. In each of the cases ($g = 0,1,2,3$) the total number of so called standard elements ($\sum$*) coincides identically with the total number $\sum = \iota + \varsigma + \sigma$ of all the possible eigenvalues $I(i)$, $D(d)$, and $S(s)$ realized by the standard elements:

$$\Sigma^*(g) = \Sigma(g) = \iota(g) + \delta(g) + \sigma(g) = $$

$$= \begin{cases} 7 + 7 + 2 = 16 & \text{at } g = 0, \\ 8 + 8 + 0 = 16 & \text{at } g = 1, \\ 12 + 12 + 4 = 28 & \text{at } g = 2, \\ 13 + 13 + 2 = 28 & \text{at } g = 3. \end{cases}$$

In principle, all these symmetrical periodic systems not only are determined by the proper dynamic laws of interaction, but also determine them themselves [1 - 8].

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SOME EXAMPLES OF THREEORTHOGONAL OBJECTS OF NONEUCLIDIAN SYMMETRY

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Objects of three dimensions and local similarity, especially threeorthogonal ones, are of great interest in biomechanics, crystallography, architecture etc. (Petukhov, 1987). Analytical description of such objects is very difficult because of the lack the third coordinate, which distinguish oneself by Nature of its numbers from the first (real numbers) and from the second (imaginary numbers). It is well known that first and second coordinates are usually used in obtaining 2-dimensional objects of local similarity, that is in usual conformal mapping. There are given in the Fig.1 (S,Y,M,M',E,T,R,Y) some examples of threeorthogonal new objects of noneuclidian symmetry: a "Mathematical naturmort". For instance, the "COCHLEA", Fig.1E, and its analytical apparatus was described earlier (Bunin, 1985). The curved ellipsoid-"CUCUMBER" may be chosen by equations:

$$X=B\xi\eta/\mathcal{D}, \qquad Y=-\left[B\sqrt{(\xi^2-1)(1-\eta^2)}\,\mathrm{Cos}\,\zeta+2\right]/\mathcal{D}$$

$$Z=-6\sqrt{(\xi^2-1)(1-\eta^2)}\,\mathrm{Sin}\,\zeta/\mathcal{D}, \quad B=\sqrt{3}$$

$$\mathcal{D}=(B\xi\eta)^2+B^2(\xi^2-1)(1-\eta^2)+4B\sqrt{(\xi^2-1)(1-\eta^2)}\,\mathrm{Cos}\,\zeta+4.$$

To obtain coordinates X, Y, Z we assume reasonable values of ξ, η, ζ. Resulting 3-dimensional coordinates are presented in the Fig.2. Calculation of such objects based on a system of numbers with three units of different Nature i_1, i_2, i_3 (Balakshin, Bunin, 1989; Bunin, Chudinov, 1976). A circle means the importance of the Nature i.e. a "mathematical dimension" of a number, for example: a number is reel, imaginary or other one. If it is necessary to describe moving, growing etc. objects like those in the Fig.1, obviosly we need in multidimensional coordinates with n numbers of different Nature. Real numbers $i_1=1$ are created by inverse operations of the 1-st and 2-nd step, imaginary numbers $i_2=\sqrt{-1}$ are created by inverse operations of the 3-d step (root), "superimaginary" numbers i_3 are created by inverse operations of the 4-th step ("superroot") etc. It must be noted, that the "Fundamental theorem of algebra" sais nothing about operations of 4-th, 5-th etc. steps. Consequently thise theorem must not be a ban for creation of new numbers, as take place for a long time; and roots of some numbers are not polygons but polyhedrons.

Fig.1 3-DIMENSIONAL OBJECTS OF NONEUCLIDIAN SYMMETRY ("MATHEMATICAL NATURMORT")

S- TWISTED TORUS ("EIGHT")
Y- CURVED ELLIPSOID ("CUCUMBER")
M- THORUS OF VARIABLE CROSS-SECTION ("EAR-RING")
M'-DEFORMATED SPHERE ("PEAR")
E- MATHEMATICAL MODEL ("COCHLEA")
T- CURVED WING ("WING")
R- MULTIPLEXED SPIRAL ("FLOWER")
Y'-CURVED CONE ("HORN OF ABUNDANCE")

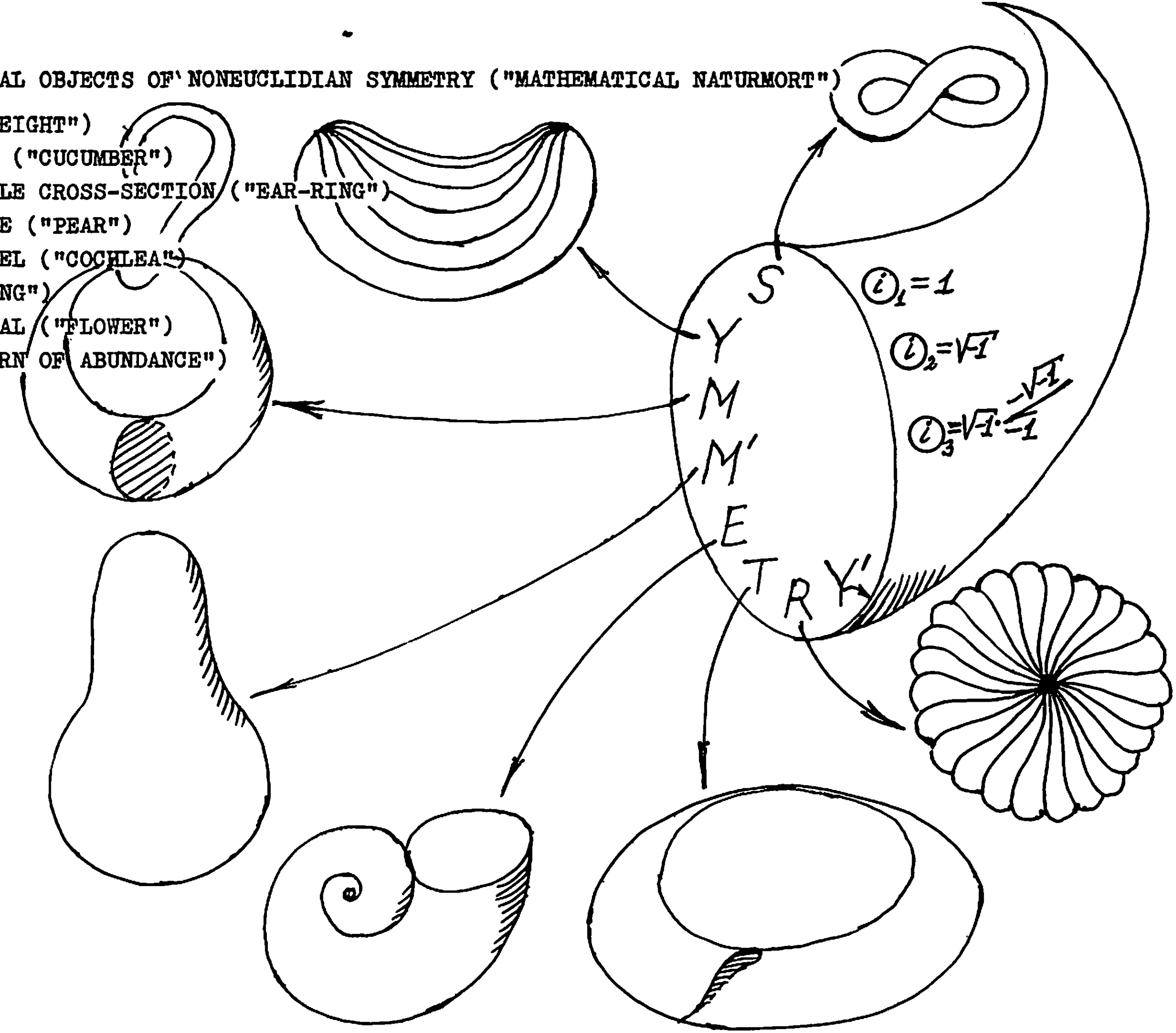
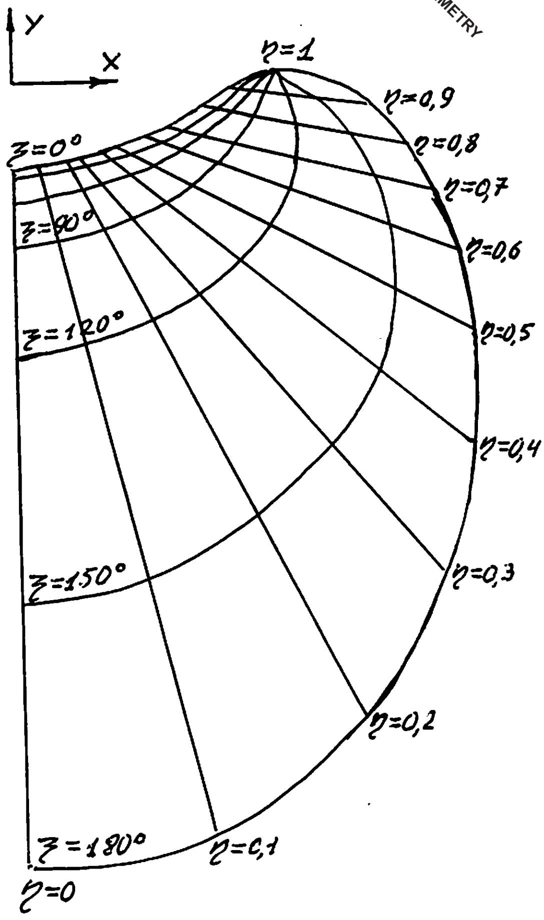


Fig.2 CALCULATED 3-ORTHOGONAL SYSTEM OF COORDINATES OF NONEUCLIDIAN SYMMETRY: CURVED ELLIPSOID ("CUCUMBER")



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Interference Fringes on 2D Diffraction Pattern of Radially Symmetric Markers for Determination of its 3D Relative Positions

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INTRODUCTION

Structures are correlated to functions, and structural changes occur in response to stimuli in the cells and tissues. The structure and its change in the cell and tissue should then not only be detected more precisely but also more dynamically to solve bio-mechanisms. Recently, laser confocal scanning microscopes have been used to observe the 3D static structures in the real image. But, to observe the dynamic living structure, its scanning speed is too slow. A higher scanning speed is requested.

It is well known that the diffraction pattern is made on the back focal plane of the objective lens, concurrent to the real image on the back focal plane of the eye piece in the microscope (Zernike, 1946). The information contained in the diffraction light is equivalent to that in the real image. Moreover, the noise in the diffraction light is lower than in the image light.

In contrast to the real image, the diffraction pattern is fixed in spite of any translational movement of the object. Deformation such as the ciliary beat of the protozoa, rotation and spinning of the cell body are exhibited on the diffraction pattern, separate from the translation of the cell(Ishizaka, 1981 and 1982). The diffraction light is mixed with a reference light to make a hologram. The hologram reveals not only light amplitude but also the light phase (Gabor, 1948 & 1949). But, the experimental conditions are delicate. If two similar apertures are placed parallel to each other, in 2D the diffraction pattern is the same as that of each separately, crossed by Young's fringes; the fringes are perpendicular to the separation of the apertures, and have spacings inversely proportional to the distance between them (Lipson, 1972).

To record the 3D light phase, radially symmetric markers of the same size were put on the specific sites of a structure. Analysis of interference fringes on 2D diffraction pattern assisted 3D dynamic morphometry will be reported in this paper.

DIFFRACTION PATTERN OF MICROSPHERES ON CONFOCAL PLANE

To observe the 3D distribution of a certain kind of active sites $\vec{x}_i$ in a cell or tissue, microspheres $f(\vec{x}-\vec{x}_i)$ of the same size and high index of refraction were used as the radially symmetrical markers. As in Fig. 1a, a lymphocyte was marked with microspheres coated by antibody against the active sites. The markers were illuminated with a plane laser wave $\vec{k}_0$. It is well known that incident light is Fourier-transfered by scattering on the markers.

$$F(\Delta\vec{k}) \exp i\Delta\vec{k}\cdot\vec{x}_i = \iiint f(\vec{x}-\vec{x}_i) \exp i\,\Delta\vec{k}\cdot\vec{x}\,d\vec{x} \qquad (\Delta\vec{k} = \vec{k} - \vec{k}_0)$$

The scattered light $\vec{k}$ with phase differences between the microspheres interferes with each other and superimposes upon a diffraction pattern to produce a series of interference fringes.

$$\left|\sum^{n} F(\Delta\vec{k}) \exp i\Delta\vec{k}\cdot\vec{x}_i\right|^2 = |F(\Delta\vec{k})|^2 \left(n + 2\sum_{i,j}^{n}\sum \cos \Delta\vec{k}\,(\vec{x}_i - \vec{x}_j)\right)$$

Several series of interference fringes $\cos\Delta\vec{k}\,(\vec{x}_i - \vec{x}_j)$ were distinctively produced as shown in Fig.1b.

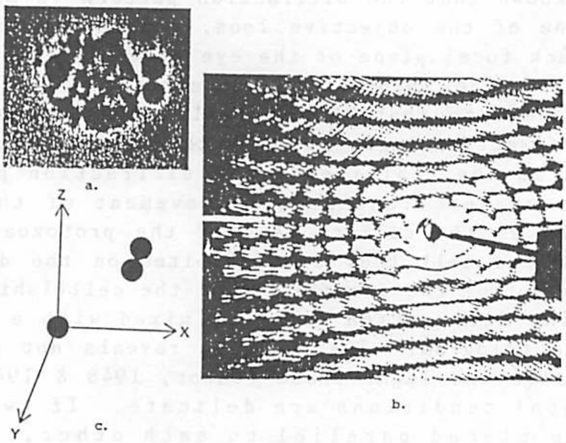

Fig. 1. Diffraction pattern of lymphocytes marked with microspheres. (a)A lymphocyte marked with microspheres. (b)Diffraction pattern with three series of interference fringes. (c)Relative position of the markers.

The position of each series of interference fringes generated on the diffraction pattern is determined by the phase difference,

INTERFERENCE FRINGES ON DIFFRACTION PATTERN

In almost all series of interference fringes

$$\Delta\vec{k}(\vec{x}_i - \vec{x}_j) = \pi m \quad \text{even: Bright, odd: Dark},$$

an eye (sink or source point) was seen, as shown in Fig. 2. The interference fringes are correlated to the relative positions of two microspheres in the three dimensional distribution. The orientation of the two microspheres is represented by the zenith. The direction to the eye from the center of the lens is the zenith. The line to the eye from the optical center of the diffraction plane is the azimuth. The azimuth is perpendicular to the series of interference fringes. The density of the fringes is proportional to the distance between the two microspheres. The 3D distribution of the microspheres could be picturized from the 2D diffraction pattern.

In the case where the zenith is perpendicular to the optical axis, and the eye can not be seen in the view, the azimuth can be determined from the line perpendicular to the interference fringes as shown in Fig. 3.

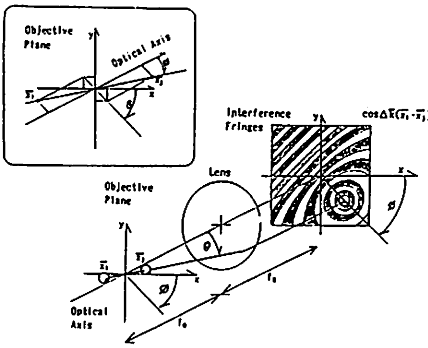

Fig. 2. An eye of the interference fringes.
θ: Zenith angle and ϕ: Azimuth angle which indicates the relative position of two microspheres at $\vec{x}_i$ and $\vec{x}_j$.

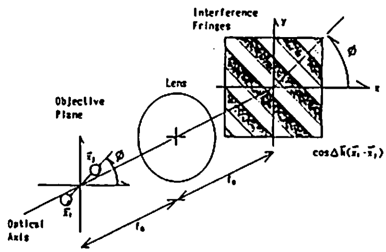

Fig. 3. The case where the zenith is perpendicular to the optical axis and the eye can't be seen. ϕ : Azimuth of the relative positions of two microspheres.

DISCUSSION AND CONCLUSION

Several series of interference fringes were generated on the diffraction pattern by the microspheres immunologically marked on certain active sites in a cell structure. Each combination of two markers generates a series of the fringes from which their relative positions could be determined. The zenith and azimuth between two sites were determined from the eye (sink or source) of the interference fringes and from the line connecting the eye and the optical center on the diffraction pattern, respectively. And the distance between two sites was proportional to the density of the fringes across the azimuth. Thus, from the 2D diffraction pattern, the 3D relative positions of the active sites could easily be determined with the assistance of a computer.

This determination speeds up imaging in confocal scanning microscope. When preliminary scanning catches one of the positions of the active sites in a coarse pixell, the other positions of the sites can be expected and can thus be precisely observed in a shorter time. Concerning the structural changes of the cell, the response time and spatial spectra must be detected to observe the changes to a given spectra applied as stimuli (Ishizaka, 1982; Ishizaka *et al.*, 1983; Ishizaka *et al.*, 1984).

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SIMPLE AND MULTIPLE ANTISYMMETRY

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As an intuitive concept, antisymmetry is present from the very beginnings of ornamental art, appearing with Neolithic "black-white" ceramics (Figure 1). As a scientific concept, it is an acquirement of the 20th century mathematics (Heesch, 1929, 1930).

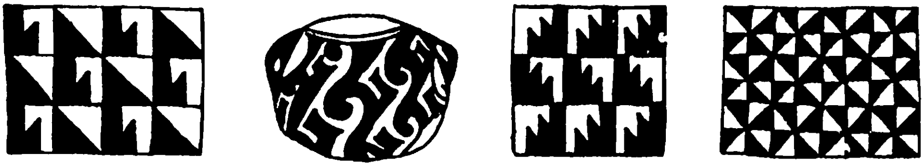

Figure 1. Antisymmetry groups of ornaments in Neolithic art: (a) p2/p1, Hacilar, ≈5200 B.C.; (b) p2/p1, Rahmani, ≈4000 B.C.; (c) pmg/pg, Hacilar, (d) p4m/p4g, Hajji Mohamad, ≈5000 B.C.

Let a symmetry group G and the permutation group $P=C_2$ generated by the antiidentity transformation $e_1=(0\ 1)$ satisfying the relation $e_1{}^2=E$ and commuting with all the elements of the group G, be given. If $S\in G$, $S^1=e_1S=Se_1$ is the antisymmetry transformation derived from S. Every group G^1 derived from G, which contains at least one antisymmetry transformation is called the antisymmetry group, and the group G is called its generating group. All the antisymmetry groups derived from G, consisting of a family, can be divided into the two types: senior groups of the form $G\times C_2$ and junior groups $G^1\cong G$. Every junior antisymmetry group G^1 is uniquely defined by its group/subgroup symbol G/H, where H is the symmetry subgroup of G^1, $G/H\cong C_2$ and $[G:H]=2$ (Shubnikov, Koptsik, 1972).

For denoting symmetry group categories, the Bohm symbols G_r... are used (Bohm, Dornberger-Schiff, 1966). Every category of symmetry groups of the space E^r is defined by the sequence r... of maximal invariant (sub)spaces inserted into one another in succession.

The antiidentity transformation introduced gives different possibilities for its interpretation. The first and most natural was a color-change "black"-"white", introducing in ornamental art a space component: visual representation of 3-D symmetry groups (e.g. symmetry groups of bands G_{321} or layers G_{32}) in a 2-D plane using black-white diagrams (Weber diagrams) of antisymmetry groups of friezes $G_{21}{}^1$ and ornaments $G_2{}^1$. Namely such a interpretation was the origin of the theory of antisymmetry. Its mathematical generalization, the established relation between the antisymmetry groups $G_{r\ldots}{}^1$ and symmetry groups $G_{(r+1)r\ldots}$ of the (r+1)-dimensional space was introduced by H.Heesch for the derivation of four-dimensional symmetry groups G_{430} and for an approximate valuation of the number of symmetry groups G_{43}.

In a general sense, the antiidentity transformation e_1 can be interpreted as a change of any bivalent geometric or non-geometric property commuting with symmetries of the generating symmetry group G (e.g. (+ -), (S N), (yes no), (convex concave)...).

The natural extension of the (simple) antisymmetry is the multiple antisymmetry, introduced by A.M.Zamorzaev in 1957, where besides a generating symmetry group G we have the permutation group $P=C_2{}^l$ generated by l antiidentity transformations e_i (i=1,2,...,l) satisfying the relations $e_i{}^2=E$, commuting between themselves and with all elements of G. In a similar way, we have the senior (S^k-), middle (S^kM^m-) and junior (M^m-type) multiple antisymmetry groups, where only the last ones, isomorphic to G, are non-trivial in the sense of derivation.

During the 30 years, mostly by contribution of the Kishinev school, the theory of multiple antisymmetry has become an integral part of mathematical crystallography and acquired the status of a complete theory extended to all categories of isometric symmetry groups of the space E^r ($r\leq3$), different kinds of non-isometric symmetry groups (of similarity symmetry, conformal symmetry...) and P-symmetry groups ((p)-, (p')-, ($p2$)-symmetry groups) (Zamorzaev, 1976; Zamorzaev, Galyarskij, Palistrant, 1978; Zamorzaev, Palistrant, 1980; Zamorzaev, Karpova, Lungu, Palistrant, 1986). The most important results from that period are: the derivation of the 1191 junior $G_3{}^1$, 9511 $G_3{}^2$, $G_2{}^l$ and $G_{32}{}^l$ (Zamorzaev, 1976). However, some problems (e.g. the derivation of $G_3{}^l$ at $l\geq3$), because of a large number of the multiple antisymmetry groups exceeding even possibilities of computers, remain unsolved.

This and many other problems are solved by the use of antisymmetric characteristic (AC) of a discrete symmetry group G (Jablan, 1986). Let a discrete symmetry group G be given by its presentation (Coxeter, Moser, 1980). The groups of simple and multiple antisymmetry can be derived by applying the general method of Shubnikov-Zamorzaev, i.e. by replacing the generators of the group G with antigenerators of one or several independent kinds of antisymmetry.

Definition 1: Let all products of generators of a group G, within which every generator participates once at the most, be formed and then subsets of transformations equivalent with regard to symmetry, be separated. The resulting system is called the antisymmetric characteristic of the group G ($AC(G)$).

A majority of AC permit the reduction, i.e. a transformation into the simplest form. The method for obtaining *AC* and reduced *AC* can be illustrated by example of symmetry group of ornaments pm, given by the set of generators $\{a,b\}(m)$, with the $AC(\mathrm{pm})=\{m,ma\}\{b\}\{mb,mab\}\{a\}\{ab\}$ and reduced $AC(\mathrm{pm})=\{m,ma\}\{b\}$.

Theorem 1: Two groups of simple or multiple antisymmetry G' and G'' of the M^m-type for fixed m, with common generating group G, are equal iff they possess equal *AC*.

Every $AC(G)$ completely defines the series $N_m(G)$, where by $N_m(G)$ is denoted the number of groups of the M^m-type derived from G, at fixed m ($1\leq m\leq l$). For example, $N_1(\mathrm{pm})=5$, $N_2(\mathrm{pm})=24$, $N_3(\mathrm{pm})=84$.

Theorem 2: Symmetry groups possessing isomorphic *AC* generate the same number of simple and multiple antisymmetry groups of the M^m-type for every fixed m ($1\leq m\leq l$), which correspond to each other with regard to structure.

Corollary: The derivation of all simple and multiple antisymmetry groups can be completely reduced to the construction of all non-isomorphic *AC* and derivation of simple and multiple antisymmetry groups of the M^m-type from these *AC*.

By the use of the *AC*-method and notion of the *AC*-type, the 109139 G_3^3, 1640955 G_3^4, 28331520 G_3^5 and 419973120 G_3^6 multiple antisymmetry groups of the M^m-type, are derived (Jablan, 1987).

The *AC*-method can be also used for a derivation of (P,l)-symmetry groups from *P*-symmetry groups. Let G^P be a junior group of *P*-symmetry derived from G (Zamorzaev, Galyarskij, Palistrant, 1978; Zamorzaev, Karpova, Lungu, Palistrant, 1986). By replacing in Definition 1 the term "transformations equivalent with regard to symmetry" with a more general notion "transformations equivalent with regard to *P*-symmetry", the transition from G to G^P induces the transition from $AC(G)$ to $AC(G^P)$, making possible the derivation of groups of (P,l)-symmetry of the M^m-type by the use of the *AC*-method.

The derivation of (P,l)-symmetry groups of the M^m-type from *P*-symmetry groups by use of the *AC*-method can be reduced to a series of successive transitions

$$G\longrightarrow G^P\longrightarrow G^{P,1}\longrightarrow\ldots\longrightarrow G^{P,l}$$

and induced transitions

$$AC(G)\longrightarrow AC(G^P)\longrightarrow AC(G^{P,1})\longrightarrow\ldots\longrightarrow AC(G^{P,l}).$$

Every induced *AC* consists of the same number of generators. Since every transition $G^{P,k-1}\longrightarrow G^{P,k}$, ($1\leq k\leq l$), is a derivation of simple antisymmetry groups using $AC(G^{P,k-1})$, for the derivation of all multiple antisymmetry groups the catalogue of all non-isomorphic *AC* formed by l generators and simple antisymmetry groups derived from these *AC* is completely sufficient.

One of the most important results, obtained jointly with A.F.Palistrant, is the derivation of the junior M^m-type groups $G_3^{l,p}$ from the groups G_3^p (p=3,4,6): 4840(4134) $G_3^{1,p}$, 40996(29731) $G_3^{2,p}$, 453881(260114) $G_3^{3,p}$, 5706960(2048760) $G_3^{4,p}$ and 59996160(1249920) $G_3^{5,p}$, where the numbers of complete (p,l)-symmetry are given in parentheses.

Finaly, the use of such a generalized AC makes possible the reduction of the theory of multiple antisymmetry to the theory of simple antisymmetry. The basis of this reduction is the transition $G \longrightarrow G^p$ and induced transition $AC(G) \longrightarrow AC(G^p)$, where $AC(G)$ and $AC(G^p)$ consist of the same number of generators. This means that every step in the derivation of multiple antisymmetry groups: $G \longrightarrow G^1 \longrightarrow G^2 \longrightarrow \ldots \longrightarrow G^{k-1} \longrightarrow G^k \longrightarrow \ldots \longrightarrow G^l$, i.e. the transition $G^{k-1} \longrightarrow G^k$, $(1 \le k \le l)$, is a derivation of simple antisymmetry groups by the use of $AC(G^{k-1})$, followed by the induced transition $AC(G^{k-1}) \longrightarrow AC(G^k)$, $(1 \le k \le l-1)$. All the AC of the induced series consist of the same number of generators.

The said can be illustrated by example of the derivation of multiple antisymmetry groups from the symmetry group of ornaments pm: $\{a,b\}(m)$ with the $AC{:}\{m,ma\}\{b\} \cong \{A,B\}\{C\}$. At $m=1$, the five junior simple antisymmetry groups, are obtained:

$\{A,B\}\{C\}$ $\quad \{E,E\}\{e_1\} \longrightarrow \{A,B\}\{C\}$
$\{e_1,e_1\}\{E\} \longrightarrow \{A,B\}\{C\}$
$\{e_1,e_1\}\{e_1\} \longrightarrow \{A,B\}\{C\}$
$\{E,e_1\}\{E\} \longrightarrow \{A\}\{B\}\{C\}$
$\{E,e_1\}\{e_1\} \longrightarrow \{A\}\{B\}\{C\}$.

In the first three cases AC remains unchanged, but in two other cases AC is transformed into the new $AC{:}\{A\}\{B\}\{C\}$. To continue the derivation of multiple antisymmetry groups of the M^m-type from the symmetry group pm, only the derivation of simple antisymmetry groups from the $AC{:}\{A\}\{B\}\{C\}$ is indispensable. This AC is trivial and gives the seven groups of simple antisymmetry. If the $AC{:}\{A,B\}\{C\}$ is denoted by 3.2 and $AC{:}\{A\}\{B\}\{C\}$ by 3.1, then the result obtained can be denoted in a symbolic form by $3.2 \longrightarrow 2(3.1)+3(3.2)$. Knowing that $N_1(\mathrm{pm})=N_1(3.2)=5$, $N_1(3.1)=7$, we can simply conclude that $N_2(\mathrm{pm})=24$ and $N_3(\mathrm{pm})=84$ (Yablan, 1988).

So, after the 30 years we are coming back to the roots of the theory of multiple antisymmetry – to the simple antisymmetry, but knowing today some more about the first.

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A MATHEMATICAL STUDY OF SYMMETRIES ON PLANTS

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1. INTRODUCTION

Primordia, the rudiments of leaves on stems, of florets on daisies, of scales on cones, etc, arise on the plant apex in a definite and precise pattern, i.e. with phyllotaxis. A leafy stem may show whorled symmetry, the leaves arising in bunches of n = 2, 3, 4, ... at the same level of the stem, with consecutive whorls either alternating of superposed. The majority of plants exhibits spiral symmetry, and wherever spiral phyllotaxis is evident in leaf production, it is also found in floral development. In the capitulum of the chrysanthemum two families of n and m spirals can be observed winding around a common pole or center of symmetry. In the plant kingdom, n and m are generally two consecutive terms of the Fibonacci series <1, 1, 2, 3, 5, 8, 13, ...>, corresponding to a divergence angle between consecutively born primordia of about 137.51^o or $\emptyset^{-2}$ where $\emptyset$ is the golden ratio. This constitutes the problem of phyllotaxis (study of patterns of plant organs).

This paper does not deal first with pattern generation, but with pattern recognition. It exhibits symmetry properties inherent to one of the author's mathematical model meant to describe the patterns on plants, so as to be useful to botanists involved in practical pattern assessment. The model is expressed by the formula $r = p(\tau)\ (m+n)^{-2}$, the exponent representing the fractal dimension of the set of phyllotactic patterns. One of the properties presented here, involving crucial phyllotactic ratios coined into the single formula $5^{1/2}\log\emptyset$, recently served to define a minimality criterion and a new predictive model for pattern generation where the possible divergence angles are ordered according, apparently, to their relative frequencies. The heuristic role played by symmetry and the interdisciplinary potential of the methods in phyllotaxis will be briefly discussed.

2. HEURISTIC AND INTERDISCIPLINARITY

The symmetries on plants considered in this paper are easily observable at the surface of the especially beautiful and colorful spiral arrangement of the florets in the capitulum of the sunflower, or of the daisy, and in the cross-sections of buds or shoots (see 1984). The aesthetic aspect of these regular, rhythmic, and ordered arrangements or patterns, together with the astonishment created by the overwhelming presence of the Fibonacci sequence and of the golden number in these patterns, generated a strong desire to look into the depths of the phenomenon. Astonishment, enhanced by the richness of the presence of beauty, is at the beginning of creative thinking, and is certainly the best incentive the scientific mind can have.

This intellectual and emotional stimulant acted as an heuristic, considered here as something that helps to discover. It is based on the metaphysical faith in regularities. From the heuristic, a first model emerged, able to generate spiral symmetries; it was later on refined (1988a), and soon extended (1988b), given the unavoidable presence of asymmetrical patterns and the belief that the exceptional cases are not bits of sand in a mechanism but rather the hiding place for deeper secrets.

Then one comes to realize that other fields of research (e.g. the study of micro-organisms, proteins (1985), medusae and even quasi-crystals) show the same kind of symmetries. Meaningful conclusions can then be drawn (1989a). Also, the methods of analysis which proved to be fruitful in one case, are seen to be applicable to the analysis of the symmetries in these latter cases. In phyllotaxis the central concern is believed to be the building of models that can make predictions (e.g. 1988a), that can be concretely used by biologists (e.g. 1987, 1988b) and that can be taught in classrooms (e.g. 1989b). Finally, the treatment of mathematical notions brought forward by a specific biological problem, is bound to produce new mathematical results (e.g. 1988c).

3. A MATHEMATICAL MODEL FOR BIO-SYMMETRIES

It has been shown that

$$\ln R = 2\pi p(\tau)(m+n)^{-2} \tag{1}$$

where τ is the angle of intersection of any two opposed spirals in the pair (m, n), the spirals in each family being evenly spaced logarithmic spirals, $R > 1$ is the ratio of the distances to the common pole of the spirals of two consecutively born florets in the capitulum, or primordia in the cross-section, and

$$p(\tau) = \phi^3(5^{1/2} \cot \tau + (5 \cot^2 \tau + 4)^{1/2})/2 \cdot 5^{1/2}, \tag{2}$$

where $\phi = (5^{1/2} + 1)/2$. Function $p(\tau)$ decreases monotonically from ∞ to 0 as τ increases from 0 to 180°. By the transformation

$$R = e^{2\pi r} \tag{3}$$

formula (1) becomes

$$r = p(\tau)(m+n)^{-2} \tag{4}$$

where r is the ordinate of primordium 1 in the regular lattice of points expressing for example the arrangement of scales on a cone, or on a pineapple, considered as a cylinder unfolded in the plane. By (3) the spirals in (1) become evenly spaced parallel straight lines in (4). In the former case, we speak about the centric representation of phyllotaxis, and in the latter case about the cylindrical representation.

Formula (1) has shown recently to be a very accurate tool, much more easy to apply, thus an improvement over earlier methods meant for the practical assessment of phyllotactic patterns. From formula (1) we can deduce formulae involving other phyllotactic parameters (1987), but we will concentrate on the parameters τ, r, m+n, and the divergence angle d, on which the symmetry of the systems depend. The model shows an explicit relation between the first three parameters, and an implicit relation between the last two.

4. ANALYTIC, GEOMETRICAL AND ARITHMETICAL SYMMETRIES

Formulae (1) and (4) express the global symmetries of all the systems. Considering (1) for example, if one observes any spiral pattern (m, n) with ratio R, then the point (log R, m+n) is on the line with slope - 2 in the log-log grid. For each given value of τ this remarkable symmetry property is true, whatever be the spiral pattern (m, n) observed. Thus formula (1) is an allometry-type expression of differential growth. Concerning formula (4) the straight line of slope -2 intersects the vertical axis at (0, log p(τ)), the horizontal axis at ((logp(τ))/2, 0), and for $\tau \approx 121.43^o$ the line goes through the origin of coordinates.

Formulae (2) and (4) involve symmetries expressed by the following relations, for any pair (m, n) and for two symmetrical values of τ around 90^o, that is 90 $\pm$x, where x is in degrees:

$$p(90-x)\ p(90+x) = \phi^6/5 = p(90)^2, \qquad (5)$$
$$r(90-x)\ r(90+x) = r(90)^2. \qquad (6)$$

They show the importance of the orthogonality of the opposed spirals or straight lines in the respective representations of phyllotactic patterns. In fact, the practical assessment of these patterns is a search for the pair(s) (m, n) for which τ is closest to 90^o. The symmetry in formulae (5) and (6) allows one to deduce the value for 90 + x from the value for 90-x. Also for τ fixed we have

$$J^2 r(Jm, Jn) = r(m, n) \qquad (7)$$

where J is a positive integer specifying the number of genetic spirals in the natural system. For each fixed value of r, when τ goes from 0 to 180^o, m+n decreases monotonically, and the value of m+n for which τ is closest to 90^o gives the conspicuous pair (a, b) of the system such that a+b = m+n, and such that a, b, a+b, 2b+a, 3b+2a, ... is the series corresponding to the observed value of d.

On each straight line of slope -2 in the log-log grid, τ fixed, the distance from the point (r_1, m+n) to the point (r_2, 2m+n), to the point (r_3, $3m+2n$), ... rapidly stabilizes around the value $5^{1/2} \log\phi$. For example, one goes from the orthogonal system (m, n) = (5, 3) to the orthogonal system (m, m+n) = (5, 8), by a jump of $5^{1/2}\log\phi \approx 0.46731$ on the allometric line $r = 1.8944\,(m+n)^{-2}$ representing a special case of (4). The number $5^{1/2} \log\phi$ is <u>a condensed expression for the symmetry</u> ratios involved in phyllotaxis.

There are systems for which (m, n) does not change during a period of time while τ varies. In that case, the point (r, cot τ) <u>moves on an hyperbola</u> given by the relation

$$br \cot \tau = r^2 - a, \tag{8}$$

where $a = \phi^6/5\,(m+n)^4$ and $b = (5a)^{1/2}$, with center (0, 0), and asymptotes $b \cot \tau = r$ and $r = 0$. Given that $r > 0$ for the phenomenon concerned, the right-hand side branch of the hyperbola is concerned, underneath the asymptote $b \cot \tau = r$.

Arithmetically, these arrangements (m, n) show symmetries expressed by formulae of the type

$$| \, m(nd) - n(md) \, | = 1 \tag{9}$$

where d is the divergence angle (as a fraction of the circle in the centric (spiral symmetry) representation, or as the abscissa of point 1 in the cylindrical (orchard-like symmetry) representation, between two consecutively born primordia, and (xd) is the integer nearest to xd. In fact, (9) is a necessary and sufficient condition for a pair (m, n) to be visible.

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New Models of Synergetics Topology

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All the unstable Archimedean polyhedral systems can be symmetrically transformed into platonic polyhedral systems while still maintaing the same distances between the adjacent two vertices. Furthermore, these platonic polyhedral systems can always ultimately be transformed into at least one of the three possible cases of fundamental omnitriangulated structural systems, viz. <u>the tetrahedron, the octahedron and the icosahedron.</u> Finally, the periodic relations inherent in these rational transformations can be reduced to "Structural Quanta".

Symmetrical Contraction of Cube with Axial Spinnability of Two Poles: Tetrahedral Progression

The axis transfixes the cube through the opposing north and south poles (1). If each pole rotates about the axis in the opposite direction by the angular closing of adjacent edges (1), the cube will contract symmetrically to bring together the two opposed vertices which lie on the diagonal of the square (2) until it becomes first the incomplete octahedral phase (3). In this case the two sets of double edges suggest polarization. Next, as the two poles approach each other on the axis to come together the other pairs of opposing vertices (4) (5), the cube folds into two congruent tetrahedra (6). The cube consists of a positive and a negative tetrahedron and is an indivisible unity. In other words, since the total number of edges and vertices of the cube is exactly twice that of the tetrahedron, we can refer to the cube as "two quanta" in our tetrahedral system. Ranking the models in terms of the number of edges, column 5 of the table shows how 18 types of platonic and Archimedean polyhedra are all, without exception, composed of edges whose numbers are multiples of six.

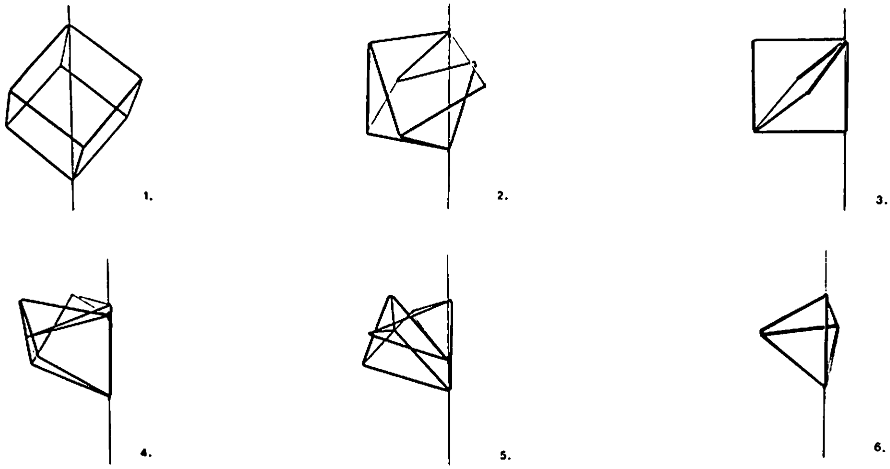

Fig. 1. *Cube can be Transformable into Two Tetrahedra*

Basic Frame Models of Synergetics Topology

We can thread a nylon string through each of the 12 equal length tubes twice to make a loop and fasten them together with three tubes joined at each of 8 corners to make the cube, which proves to be structurally unstalble. The tubular frame models of all platonic and Archimedean polyhedra can be constructed by using this loop-ligature technique.

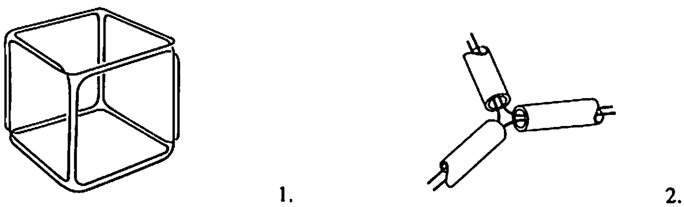

Fig. 2. *Synergetics Path: A Loop Formed by Stringing Twice Through Each Tube*
1. A loop of the cube. 2. Detail of the loop joint connected with three tubes.

Periodic Table of Synergetics Topology: Structural Quanta

1	2	3	4	5	6												7	8
	3V − 6 = E V = Vertices E = Edges	3V − 6 > E	Name of polyhedron	The number of vector-struts	The number of congruent polyhedra: Structural Quanta												Topological synergetics hierarchy	Quantum leap of first four prime numbers
					2 Archimedian polyhedra		5 Doubling Platonic polyhedra					5 Platonic polyhedra		3 Ground States				
1			TETRAHEDRON	6												1	1	1
2			OCTAHEDRON	12											1 →	(2)	2	
3			HEXAHEDRON	12									1 →			2	2	1
4			TRUNCATED TETRAHEDRON	18												3	3	
5			CUBOCTAHEDRON	24		1 →										4	2^2	1
															2 →	4		
6			ICOSAHEDRON	30										1 →		(5)	5	1
7			DODECAHEDRON	30								1 →				5	5	
8			TRUNCATED OCTAHEDRON	36									3 →			6	2 × 3	1
											3 →		3 →			6		
															3 →	6		
9			TRUNCATED CUBE	36											3 →	6	2 × 3	2
10			RHOMBI-CUBOCTAHEDRON	48		2 →									4 →	8	2^3	
																8		
															4	8		
11			SNUB CUBE	60											5	10	2 × 5	2
															(5) →	(10)		
12			ICOSI-DODECAHEDRON	60	1 →										5 →	(10)	2 × 5	
														(2) →		(10)		
13			TRUNCATED CUBOCTAHEDRON	72		3 →									6 →	12	$2^2 \times 3$	2
								3 →								12		
															6			
											6 →					12		
										3 →	6 →		6 →			12		
															6			
								3 RHOMBIC DODECAHEDRA								12		
													6 →			12		
															6	12		3
14			TRUNCATED DODECAHEDRON	90										3		15	3 × 5	
														3 →		(15)		
15			TRUNCATED DODECAHEDRON	90								3 →				15	3 × 5	
														3		15		
														3 →		(15)		5
16			RHOMBICOSI-DODECAHEDRON	120	2 →											20	$2^2 \times 5$	
															10 →	20		
														4 →	10 →	20		
17			SNUB DODECAHEDRON	150												25	5^2	5
														(5) →		(25)		
18			TRUNCATED ICOSI-DODECAHEDRON	180	3 →									6 →	15 →	30	2×3×5	5
															15			
							3 →				15 →		15 →			30		
														6				
																30		
										3 →	15 →		15 →			30		
						3 RHOMBIC TRIACONTAHEDRA												
													15 →			30		
														6 →	15 →	30		

Each of the 5 Doubling Platonic polyhedra binds two struts at each of the midpoints of its edges. Some of the transformations of Nos. 2, 6, 11, 12 and 17 require the detaching and rejoining of the vertexial connections. They are represented by means of ( ), i.e. irreversibility.

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DISSYMMETRY OF LIVING SYSTEMS

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At the present time, it may be considered as an established fact that chirality substances are preferable for constructing the most important elements of living systems and that homochirality is advantageous since it provides a great kinetic efficiency of stereoselective reactions [Kizel, 1985].

However, the question of a definite sign of chirality observed in nature is up to now the point at issue. It should be remarked that some substances used in living organisms appear to be exclusively right handed, others (e.g. amino acids) are exclusively lefthanded. At any rate, in vitally important structural formations the combination of different signs seems justified but the reason for a distinct choice of one of the alternative combinations remains unclear.

As is known, in the system of chiral molecules of different signs in thermodynamically nonequilibrium systems (living systems belong to them) in the presence of metabolism with the environment, two nonequilibrium but stationary states with an excess of antipodes of this or that kind occur; with an increase of metabolism the excess increases up to the total optical purity. Bifurcation takes place when a certain threshold value of metabolism in achieved.

It is also shown that at the initial stages of originating metabolism and in the region of the bifurcation of a system, a slight excess of one of the antipodes caused by fluctuations or an outside influence is sufficient for orienting the system along one of the routes [Morozov et al. 1983, Fajszi, 1982] . Thus, three mains suppositions appeared:

1) An accidental fluctuation in an initially racemic substance at the moment of formation of the first living systems and of the origination of metabolism with the environment, i.e., in the region of bifurcation, and a further development of the systems of only one sign;

2) The origination of living systems of different signs in various areas with a subsequent "victory" of one of them when the areas come into contact with each other.

3) The influence, especially in the region of bifurcation, of a permanent external dissymmetric factor.

Assumption 1) accounts for dissymmetry most simply and uncontradictorily, provided the living systems originate during one event in a definite small area at a definite time. The calculations of the necessary and permissible range of fluctuations [Morozov et al., 1983] confirm the acceptability of this mechanism though inherently local independent fluctuations of different signs at adjacent points are possible.

However, a great number of such events, which is, apparently, unquestionable, will inevitably result in the areas of right-handed and lefthanded systems (substrates of different signs, if any, may, in principle, be used).

Assumption 2) is, in essence, a development of the first one. It is shown in [Zeldovitch et al. , 1986] that in the presence of the contact of two areas of different signs, utilization of an achiral substrate in both of them, possible mutual diffusion and mutual inhibition, of crucial importance is the geometry of areas - the curvature radius of the interface (i.e., indirectly, the relative dimensions of the areas). If the curvature radius is finite ($\sim 10^4 - 10^{14}$ cm), then there will be the victory of one of the forms the expansion of one area and the contraction of the other.

The velocity of motion of the interface of the areas depends on two independent factors: the ratio of a diffusion coefficient to the radius of curvature and a greater efficiency of self replication of one of the forms as a result of the possible, in principle, influence of an external dissymmetry factor.

Assumption 3. The discussion about the possible influence of a dissymmetrical external factor on the dissymmetry of living systems continues up to now. The search for such a factor in the chiral combinations of electric, magnetic and gravitational fields has failed. Nowadays, the only possible factor is taken to be the effect of weak interactions where "righthanded" and "lefthanded" sides are distinguished.

There are two probabilities: 1) the effect of neutral currents in atoms and 2) the asymmetric radiolysis produced by polarized electrons.

The first one results in the difference of the energy of enantiomers 10^{-17} eV, only for the atoms at the end of the Mendeleev table it can be of the order 10^{-12} ($\sim Z^5$); for the molecules consisting of light atoms the recent estimates give 10^{-18} eV [Tranter, 1985] . This quantity is extremely small, especially, as compared to the influence of the curvature of interface and of fluctuations. Besides, a very large resource of primary biomass is necessary for producing appreciable influence - such a great, that the fluctuations dispersion could reduce or exclude their influence. However, recent estimates [Morozov et al., 1984] show, that the supposed resource is still sufficient for the effect of weak interactions to manifest itself, and this factor cannot be disregarded.

The possibility of asymmetric radiolysis has been investigated experimentally for a long time (see, e.g.,[Bonner et al.,1984])
But all the results were negative; radioratcemization was even observed. According to the estimates the asymmetry of radiolysis must not exceed 10^{-9} - 10^{-11}; the latter value is more probable. Besides, the electron depolarises gradually.

The advocates of the hypotheses concerning the influence of weak interactions put forward assumptions about the presence of the local powerful sources of polarized electrons - the deposits of ^{235}U , its spontaneous fission, natural "fission reactors", and the availability of ^{26}Al in nature. The assumptions about the role which can play the emission of novae and supernovae were even put forward. It was also pointed out, that, according to Eigen, life could originate at low (up to -20°C) temperatures when fluctuations were essentially suppressed, as well, as the racemisation [Hegström et al, 1985].

We see the problem is not solved yet, it requires the interpretation of some possible events and certainly merits attention.

The final result of discussion disregarding, we must agree, that only living systems, because of their physico-chemical peculiarities, can essentially amplify the initially arised (or so far arising) an very insignificant excess of one of chiral forms of them.

So far as the presence of chirality striking manifest itself at all steps of biological hierarchy, the question is of great interest.

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ON 3-PERIODIC MINIMAL SURFACES. I. SYMMETRY AND DERIVATION.

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A **minimal surface** in 3-dimensional space R^3 is defined as a surface with mean curvature zero at each of its points, i.e. the two extreme values of curvature (main curvatures) are equal in magnitude but opposite in sign for each point of the surface. Thus all points of a minimal surface are saddle points.

In crystallography, especially those minimal surfaces have attracted attention that are periodic in three independent directions and, therefore, may be related to crystal structures. In this connection mainly those surfaces that are free of self-intersections seem to be of interest. Such a surface subdivides R^3 into two regions or labyrinths such that each labyrinth is connected but not simply connected. If the two labyrinths are congruent the intersection-free, 3-periodic minimal surface is called a **minimal balance surface** (Fischer & Koch, 1987).

The symmetry of a minimal balance surface is best characterized by a pair of space groups G-H: G describes the full symmetry of the (non-oriented) surface, and H is that subgroup of G with index 2 which consists of all symmetry operations that do not interchange the two sides of the surface and the two labyrinths. Obviously, the pairs G-H correspond uniquely to the proper black-white space groups (cf. also Mackay & Klinowski, 1986).

Let us consider a symmetry operation s of G that does not belong to H. Then s interchanges the two sides of each minimal balance surface with symmetry G-H, and all fixed points of s must lie on the surface. This property, however, is inconsistent with the absence of self-intersections for minimal balance surfaces if s is a 3-, 4- or 6-fold rotation, a reflection or a 6-fold rotoinversion. As a consequence, certain space-group pairs G-H are incompatible with minimal balance surfaces. A detailed examination of the 1156 types of group-subgroup pairs with index 2 shows that - for the reasons described above - only 547 of them are not incompatible with minimal balance surfaces.

For these 547 types of space-group pairs all 2-fold rotation axes and all (roto)inversion centres $\bar{1}$, $\bar{3}$ and $\bar{4}$ have been tabulated that must be located on each minimal balance surface with that symmetry (Koch & Fischer, 1988). This knowledge gives an aid for the derivation of new families of minimal balance surfaces. Especially useful are 2-fold rotation axes which exist for 352 out of the 547 types. Considering only the sets of all 2-fold axes belonging to G but not to H, 52 different configurations of straight lines on minimal balance surfaces result. In 18 of these cases all 2-fold axes are 3-dimensionally connected, in 12 cases they form infinite sets of parallel plane nets. Both situations are favourable for the derivation of minimal balance surfaces:

(1) In a 3-dimensional connected set of 2-fold axes skew polygons are formed that may be spanned by **disk-like surface patches.** Such a surface patch may be continued with the aid of those 2-fold rotations that correspond to its straight edges. If the original skew polygon has been adequately chosen the resulting infinite surface is free of self-intersections, i.e. it is a minimal balance surface. An adequately chosen skew polygon has to fulfill the following conditions: (i) All its vertex angles must be chosen as small as possible; in particular, no angles larger than 90° are allowed. (ii) The skew polygon must not be penetrated by a further 2-fold axis belonging to the same set.

The 18 configurations of 3-dimensionally connected 2-fold axes give rise to 15 families of minimal balance surfaces that may be generated from disk-like spanned skew polygons (cf. Schoen, 1970; Fischer & Koch, 1987; Koch & Fischer, 1988). Eight of these families had not been known before.

(2) The 12 configurations of 2-fold axes that disintegrate into parallel plane nets are compatible with different kinds of surface patches. Again an original surface patch may be continued with the aid of the 2-fold rotations referring to its boundaries.

(i) If all plane nets are congruent and if at least half the polygon centres for a pair of adjacent nets lie directly above each other, **catenoid-like surface patches** may be spanned between neighbouring polygons from adjacent nets. Such catenoids give rise to seven families of minimal balance surfaces (cf. Schoen, 1970; Koch & Fischer, 1988), one of which had not been described before.

(ii) If plane nets of two different kinds are stacked alternately upon each other surface patches may be spanned that have been called **branched catenoids.** A branched catenoid is bounded by a convex polygon at one end and by a concave polygon with one point of self-contact at its other end. The convex polygon stems from one of the more wide-meshed nets, whereas the concave polygon is formed by two, three or four polygons with a common vertex of an adjacent close-meshed net. Branched catenoids refer to three new families of minimal balance surfaces (Fischer & Koch, 1989a).

(iii) Congruent parallel plane nets stacked directly upon each other allow surface patches that have been called **multiple catenoids.** A multiple catenoid may be imagined as resulting from fusion of two, three, four or six neighbouring catenoids. It is bounded by two congruent concave polygons with one point of self-contact each. Multiple catenoids give rise to eight new families of minimal balance surfaces (cf. Karcher, 1988; Koch & Fischer, 1989a).

(iv) Configurations of 2-fold axes that disintegrate into parallel plane quadrangular nets are compatible with 1-dimensionally infinite surface patches, called **infinite strips.** Such an infinite strip is bounded by two infinite (zigzag or meander) lines. The strips may be regarded as resulting from fusion of an infinite row of neighbouring catenoids. In most cases infinite strips constructed in this way produce minimal surfaces that may be also built up from finite surface patches as described above. In two cases, however, minimal surfaces of new families are formed (Fischer & Koch, 1989b).

(v) For configurations of 2-fold axes that disintegrate into congruent plane parallel nets stacked directly upon each other the catenoid-like surface patches [cf. (i)] may be replaced by more complicated ones, called **catenoids with spout-like attachments**. For this, spouts are attached to the "faces" of the catenoids resulting in surface patches with two, three or four additional ends that are not bounded by straight lines. Spouts of neighbouring catenoids are united to handles or to three-armed or four-armed handles, respectively. Six families of minimal balance surfaces correspond to such surface patches (Koch & Fischer, 1989b); only one of these families had been described before (Schoen, 1970).

In addition, two families of minimal balance surfaces have been derived which contain skew 2-fold axes in three independent directions (Fischer & Koch, 1987, 1989c; Koch & Fischer, 1988).

Two families of minimal balance surfaces without 2-fold axes are known so far, the gyroid surfaces (cf. Schoen, 1970; Fischer & Koch, 1987) and orthorhombically distorted P surfaces (cf. Karcher, 1988; Fischer & Koch, 1989c).

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SUPERSYMMETRY PROBLEM IN BIOMORPHOLOGY

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The symmetry of proportions of human body as well as the algorithmic order of leaves on a plant (philotaxis) are known from ancient times. Arcy d'Thompson in the beginning of this century (1917) undertook an attempt to systematize the volume of empirical knowledge of bioforms of growth on the basis of similarity symmetry. After that D.V.Nalivkin put forward an idea of curve-linear symmetry.

It is common knowledge that isometric symmetry of bioforms on submolecular level is quite low: just bilateral (mirror) symmetry in the world of animals endowed with freedom of movement and pentagonal ones (in general) in flowers. Such scarcity is not correspond to the high level of organization of biological objects - philotaxis, cyclomery, proportionality of their structure and other types of algorithmic orderness. This contradiction brought V.I.Vernadsky to formulate the problem (1931): "It is necessary to develop the science of symmetry in close relation with morphology of life. Such is the new and enormous problem which is now on the agenda". V.I.Vernadsky suggested that to the drastic difference between living and non-living matter there correspond the non-Euclidean space or to be more precise the space-time of the living matter.

In 1940 A.V.Shubnikov in his book "Symmetry" (see also A.V.Shubnikov, V.A.Koptsik, 1972) summarized our knowledges in this field as follows: "As to living organisms we do not have any theory for them which could answer what kinds of symmetry are or are not compatible with the existence of living matter." Having introduced into the symmetry theory a notion of relative equality A.V.Shubnikov discovered in 1960 the point groups of similarity symmetry modelling cyclomeries for a number of geometric schemes. A.M.Zamerzaev extrapolated this model to the space similarity groups. Biologically important groups of Möbius, projective and tangent transformations, containing the subgroup of similarity, were discovered recently by S.V.Petukhov (1981, 1988) in cyclomeric biological objects and in kinematics of movements. Realization of these biosymmetries is connected with iteratively-algorithmic peculiarities of biological self-organization and information inheritance.

Does the discovery of non-Euclidean symmetry groups in biomorphology of submolecular level of organization verify Vernadsky's first hypothesis of non-Euclidean character of "the geometric space of living organisms"? The answer to this question should be polysystemic. According to the principle of relativity of symmetries revealed by A.V.Shubnikov, there are different groups of symmetry which act on different structural levels of the same material object. This statement also holds true for biological objects in which the hierarchy of structural levels and, hence, symmetries is most fully developed. "The principle of superposition of structures and the principle of symmetry for composite systems connected with the previous one acts in specific forms at every of the biological levels." (A.V.Shubnikov, V.A.Koptsik, 1972). It turns out that in biomorphology as well as in physics or crystallography the broken symmetries (isometry, similarity, affinity) do not disappear but are retained in the structure in the modified form on the level of generalized (so called coloured) positional groups (V.A.Koptsik, 1988). By choosing the proper local compensate transformations, each of the cyclomeries studied by S.V.Petukhov can be alternatively described not as global groups of Möbius or projective transformations, but as coloured affine groups. Such an alternative in description of biocyclomeries makes the problem of non-Euclidean character of biospace quite clear: such statements are only true for a given considered model and fixed level of its structure.

The study of laws and algorithms of organic biomorphology is one of the prominent directions of research in modern biology, which in accordance with general expectations should bring about fundamental discoveries and some practical outcome in the nearest future. In contrast with crystallography the modern mathematical biology does not have a generally accepted formalized theory of morphogenesis, although numerous attempts are known to develop this theory on the basis of different primary models - cybernetic, reaction-diffusion, etc. The difficulty of developing such a theory lies primarly in the lack of sufficient knowledge about general biological properties of morphogenesis which could be mathematically formalized and theoretically explained. The level of the mathematical biology of evolution requires yet a long way to go, accumulating knowledge of the key morphological qualities and adequate geometric classifications till it can arrive at the desired theory - a common way, already passed by crystallography and other natural sciences, which deals with far simpler objects than biology. Revealing non-Euclidean biosymmetries gives valuable material for development of formal theory of the morfogenesis, whereas the ability to account for the existence of those biosymmetries is an important criterion of adequacy of such a theory.

The authors of the present report are physicists, specialising in crystallography and biology who in their research are oriented by the tendency of "geometrization of biology", as well as on the experience of crystallography. They discuss fundamental problems of application of group-theoretical approach to mathematical biology, biomorphology and biomechanics.

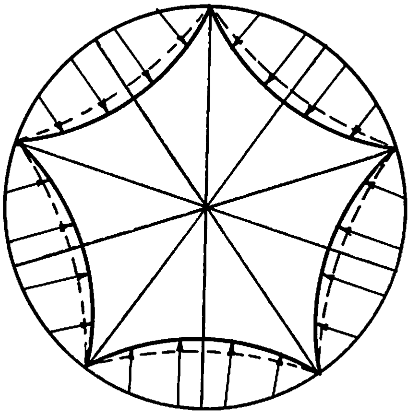

Fig.1. Scheme of polyblock structure of Ludvig perennis fruit and its orthogonal (G = 5 mm), conformal (spheric-reflectional) and coloured affine symmetry (field of arrows).

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ASYMMETRY OF MOVEMENT REACTIONS UNDER CONDITIONS OF VIBRATION AND IMPULSIVE FORCE INFLUENCES ON MAN-OPERATOR

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Industrial activity of man as a link of control of the most of Man-Machine Systems is constantly accompanied by the vibration, noise, impulsive forces and sign inversionable rectilinear accelerations. In practice, the movement reactions realization accuracy under such conditions decreases significantly. The urgency of the carried out researches is conditioned by the necessity of the rise of man-operator working capacity in many kinds of Man-Machine Systems.

In the research process the sign inversionable linear accelerations (the maximum amplitude of the deviation in frontal direction with the acceleration effect of about 0,1 g was 25 cm.), vibration (f=23 Hz, n=0,3 g) and the combination of the vibration and impulsive force (2,5 g during 20 msec.) were created.

15 healthy volunteers from 25 to 37 years old without special operator training were investigated. The program of force loads included the realization of prescribed muscular efforts left and right legs, left and right hands in turn under the consistent influence of the conditions we have already mentioned.

The value of the muscular effort separately for left and right leg and also for left and right hand in each of the problems was averaged not less than 30 realizations. Data array of 15 10^3 realization parameters of muscular efforts of upper and lower extremitirs was analyzed.

As a result of our investigations was found a statisticaly realiable influence of above-mentioned factors on the examined movement reactions. The statistic analysis showed that linear or impulsive stimulation as well as their combination with vibration led to the intensification of the background values of muscular efforts of the operator. However the phenomenology of this intensification was different. At linear vestilular stimulation appeared an asymmetry between right and left lower extremities muscular efforts:

the homolateral leg efforts were reliably more intensive than those of controlateral. This phenomen was observed in 11 cases from 15. Other cases showed no statistically reliable asymmetry. The degree of asymmetry was calculated by means of a special coefficient introduction:

$$AK = \frac{M_R - M_L}{M_R + M_L} \cdot 100\,\%$$

AK - asymmetry coefficient of muscular efforts;
M_R - mean value of right leg (or hand) muscular efforts;
M_L - mean value of left leg (or hand) muscular efforts.

Individual and averaged values of for lower extremities are given in the Table 1.

Asymmetry coefficient values — Table 1

volont. No.	1	2	3	4	5	6	7	8
AK	2,13	2,34	-3,19	22,76	-1,69	1,35	3,63	0,33
AK_L	6,82	9,54	4,85	1,16	3,64	1,82	16,3	9,76
AK_{L+V}	7,91	2,0	-1,02	1,27	-1,021	7,86	5,89	8,16

volont. No.	9	10	11	12	13	14	15	Mean
AK	-0,29	-1,76	-3,16	2,42	-2,88	-1,65	0,87	-0,53
AK_L	2,19	8,08	-1,11	6,51	5,79	12,19	4,59	6,06
AK_{L+V}	6,02	7,22	5,58	8,36	11,21	6,39	7,41	5,55

AK - asymmetry coefficient control data, %;
AK_L - asymmetry coefficient under conditions of rectilinear stimulation, %;
AK_{L+V} - item, in combination with the superimposed vibration, %.

A statistically reliable increasing of muscular efforts of all volunteers was observed at impulsive stimulation. Simultaneously, a significant asymmetry of movement reactions has been found in all cases too. Still according to reaction time of each volunteer the direction of instantaneous acceleration, acting on him in the point of command realization, was different. That is why the sign of AK, reflecting the direction of asymmetry between left and right extremities efforts,was also different.

As compared with the isolated rectilinear and impulsive stimulation a simultaneous vibration stimulation didn't provoke any significant difference in muscular efforts values. It was noted that the moderate decreasing of asymmetry coefficient value appeared in some cases of vibration superposition, particularly in the rectilinear stimulation experiments. (Table 1). The muscular efforts realization accuracy for upper extremities, just as it was expected, turned out to be considerably higher than for lower ones (average - 5,2 times higher, and for 6 volunteers from 15 - more than 10 times higher).

In conclusion in should be noted that stimulations with the frontal directed vector provoke a non-symmetric movement reactions alternation for the majority of the healthy volunteers: muscular efforts of homolateral extremity are usually more intensive that those of controlateral. Such biophenomenon must be taken into consideration when designing force elements control in Man-Machine Systems to neutralize its negative influences on acuracy and quality of man-operator's work.

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ON THE HISTORY AND ARRANGEMENT OF THE CLASSES OF CRYSTAL SYMMETRY

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In most of the contemporary crystallographic works the 32 symmetry classes are usually arranged in a tabular manner fitting the crystal systems and type of symmetry. From a historical point of view it is interesting to trace the main trends of their arrangement.

The 32 crystallographic point groups have been described by Frankenheim in 1826 (Burckhardt, 1984) and confirmed partly or fully later on independently by Hessel, Bravais, Möbius, Gadolin, Curie, Fedorov, Minnegerode, Schoenflies and Wulff. The first table of the classes of crystal symmetry is considered to be that of Tschermak in 1905, accepted by Becke in 1926 and other authors (Yushkin et al., 1987).

A tabular arrangement, as a sequence only of the crystal systems has been suggested by Gadolin in 1867 and by Fedorov in 1891. The combination of crystal systems with type of symmetry arranged in a tabular way has bean applied by Schoenflies (1891) and later in a supplement of the Russian translation of Groth (1897). This procedure branched in the next years through using both sequence of crystal systems and type of symmetry. Still later differentiation followed chronologicaly. Trends in Figure 1 show symmetrical branching according to the crystal systems (A - triclinic, M - monoclinic, O - orthorhombic, T - trigonal, Q - tetragonal, H - hexagonal and C - cubic). On the left hand side there are A $\rightarrow$ C transitions and on the right hand side C $\rightarrow$ A transitions with an exception in the center. It is obvious that the A $\rightarrow$ C transition has been prefered by most crystallographers. In this branch an interesting diversion has been suggested by Shubnikov et al. (1940), viz. cubic system vertical fitting the columns of types of symmetry.

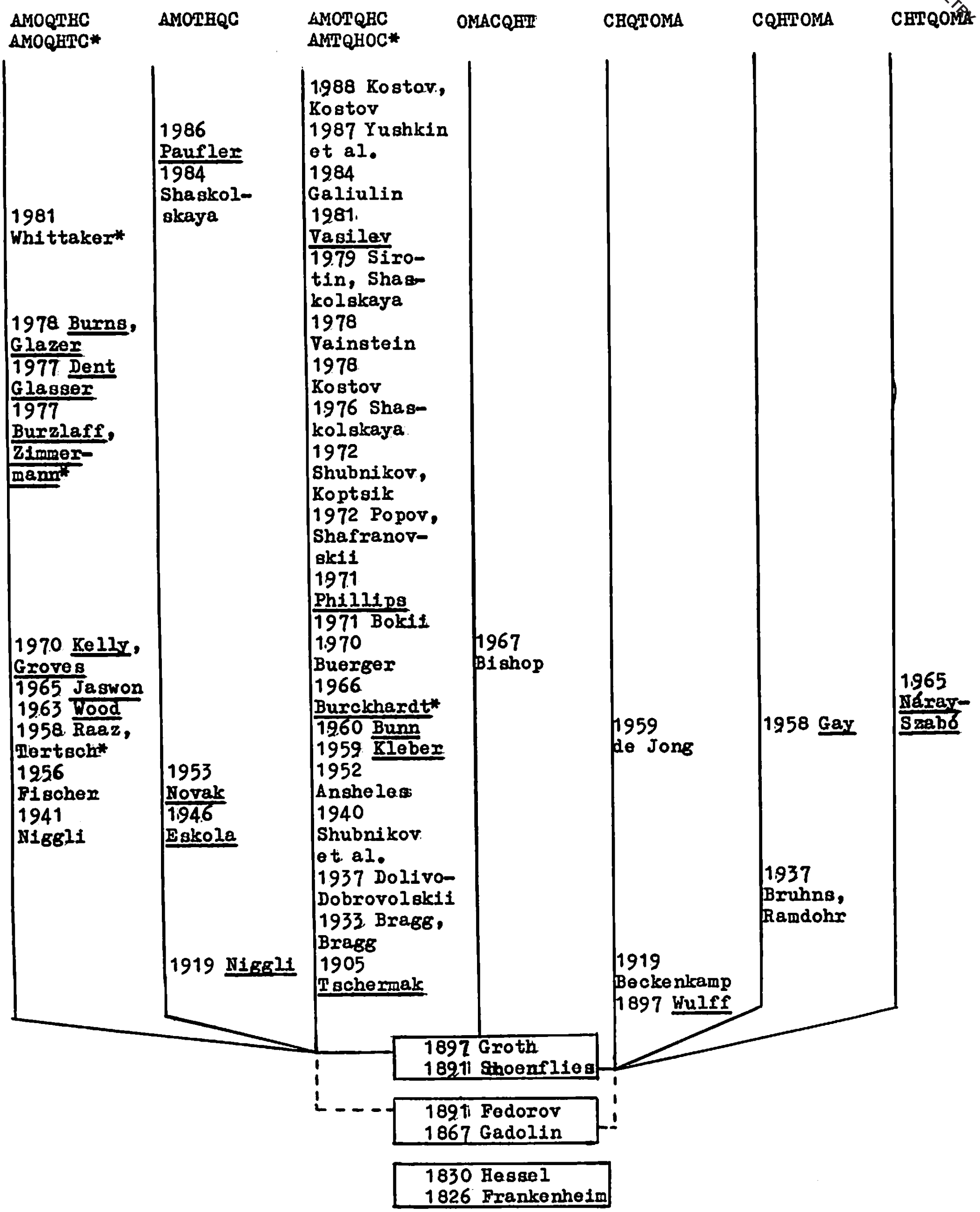

Fig. 1. Trends of the sequences according to crystal systems.

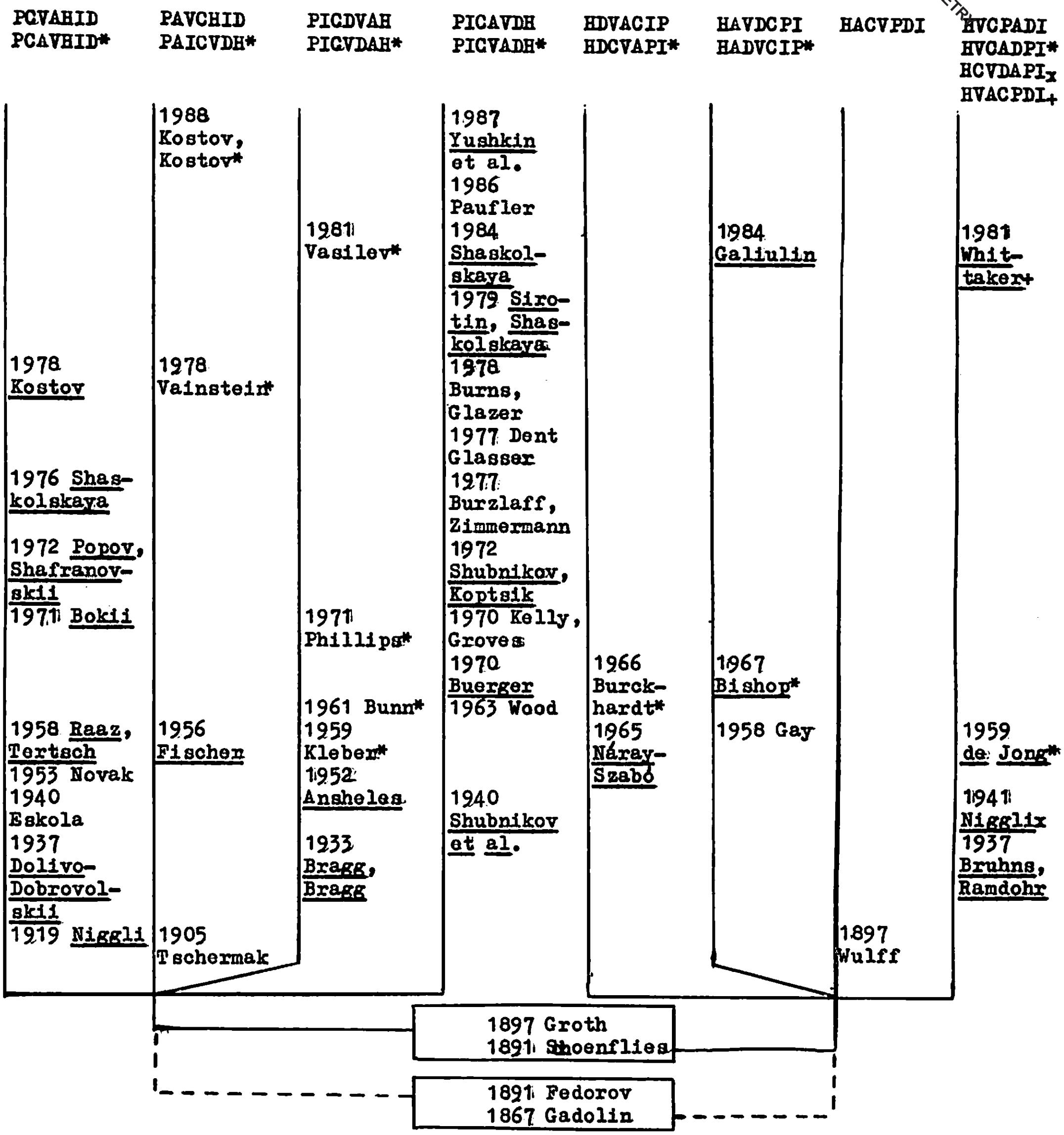

Fig. 2. Trends of the sequences according to type of symmetry.

The trends in Figure 2 show similar symmetrical branching along four lines each of the two sets according to the type of symmetry (P - polar, A - axial, I - inverse-primitive, C - central, V - planar, D - inverse-planar and H - planar-axial). The sequence in the tetragonal system has mainly been used. On the left hand side are the P → other transitions and on the right hand side the H → other transitions. The variety of arrangements in this case is greater, but most authors prefer the P → other transition type.

In both figures along each separate line underlined are the authors who use vertical and not horizontal manner of presentation. Asterixes mark small variations in the arrangement.

Crystallography is a fundamental science and the table of alignement of the crystal systems and symmetries like the Periodic Table of Mendeleev has its logical meaning and significance. Further development of this table including icosahedral, formal pentagonal and infinite groups of symmetry has also been made (Kostov, Kostov, 1988).

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THE AESTHETICS OF STRUCTURAL SYMMETRY IN PAINTING, MUSIC AND POETRY: APPLICATION TO EDUCATION

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The lecture will develop according to the following guidelines:

Introduction: a. Legitimacy of Discussion
b. Definitions

Painting : a. Liniar symmetry
b. Transfer symmetry
c. All-round symmetry
d. Spatial symmetry

Music : a. Tonal symmetry (vertical and horizontal)
b. Dodecaphonic symmetry (vertical and horizontal)

Poetry : a. Echo symmetry
b. Symmetry of meaning
c. "Anti-symmetry"

Conclusion : a. Modern vs. modernistic
b. Enlargement of concept
c. Application to other Art-media
d. Application to Education
e. Learners' activities -- A model proposal (see fig. 4)

At the end of the lecture the following questions will be raised for discussion:

A.

In light of the ideas offered above, what patterns of symmetry -- reflection, rotation, translation and their combinations(Shubnikov, 1974; Senechal, 1977) -- can be discerned in the following works-of-art:

1. Paul Klee's "Running-away from Oneself" (see fig. 1).

2. Arnold Schoenberg's Dodecaphonic Series from <u>String Quartet no. 4</u> and the opening melody based on it (see fig. 2).

3. Timm Ulrichs' Concrete Poem (see fig. 3).

In the above works-of-art, which are the aspects of transformation (change) and which of conservation (invariance) (Shubnikov, 1974) ?

Does the enquiry of symmetry operations in one art-medium -- e.g. the visual -- contribute to the understanding of symmetry operations in another art-medium -- e.g. the verbal?

Should the discussion on the Level-of-Form -- namely of structural symmetry --aspire to reveal truths on the Level-of-Content as well as on the Level-of-Meaning?

B.

Jerome Bruner introduced the concept of "The Structure of Knowledge" in relation to Curriculum Planning (Bruner, 1960). He advocated that instead of memorizing multitudes of details, the learner should become acquainted with the principles of a discipline. Harry Broudy proposed to apply Bruner's ideas to Art Education (Broudy, 1966). According to Weyl, "all a priori statements in physics have their origin in symmetry" (Weyl, 1952); is this true also of the art-media? If so, can symmetry be regarded as one of the depth-patterns at the basis of not only one art-discipline, but of The art-disciplines as a whole?

C.

"Quarendo Invenietis" -- Seek and Ye Shall Find -- wrote J.S.Bach on one of the three "puzzle canons" in his Musical Offering: the composer provided a theme and the task was to determine by what symmetry operations he intended the theme to be repeated (Senechal, 1977). Indeed, through symmetry we can discover a norm for many things, thus unifying large bodies of knowledge. Therefore we keep on Seeking and hope to Find. Yet, should not the warning implied in the following questions/statements be re-heeded: when does this search for symmetry -- in the creation of works-of-art, as well as in their interpretation -- "begin to tend toward excess, toward a need for proportion and repetition and regularity so acute that it becomes obsessive and life-denying? How do we know when symmetry-seeking is still human?" (G. Fayen, "Ambiguities in Symmetry-Seeking: Borges and Others", in Senechal, 1977). Where is the limit?

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Fig. 1: Paul Klee. "Running-away from Oneself" (Phase I), 1931; Paul Klee Foundation, inv. 778, K5(25).

Fig. 2: Arnold Schoenberg. The Series from String Quartet No. 4 and the opening melody based on it; in S. Sadie and A. Lathan (ed.), The Cambridge Music Guide, Cambridge University Press, 1985, p. 62, ex. III.2.

Fig. 3: Timm Ulrichs. A concrete poem; in P. Garnier, Spatialisme et poésie concrète, Gallimard, 1968, p. 99.

Fig. 4: "Symmetry in Art Education: A Three-Dimetional Model":

X-axis: Learners' Activities: (Xa)--Assimilation; (Xb)--Reproduction; (Xc)--Creation; (Xd)--Interpretation.

Y-axis: Modes of Symmetry: (Ya)--A-Symmetry; (Yb)--Symmetry; (Yc)--Anti-Symmetry; (Yd)--Re-Symmetry.The modes correspond to phases in the historical development of the use of symmetry in The Arts.

Z-axis: Art-Media: (Za)--Painting; (Zb)--Music; (Zc)--Poetry;

fig. 1

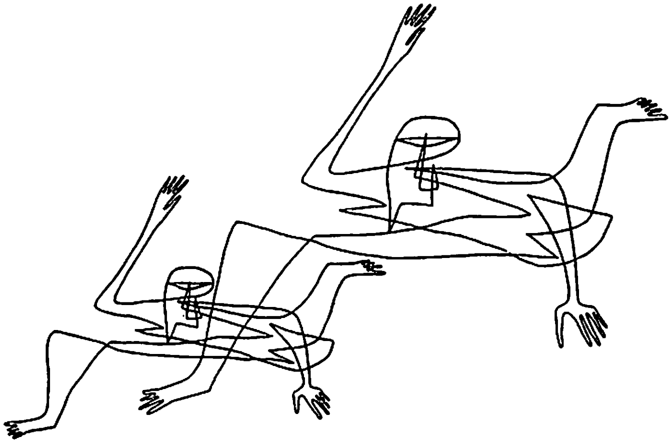
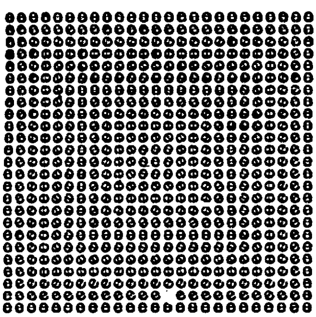
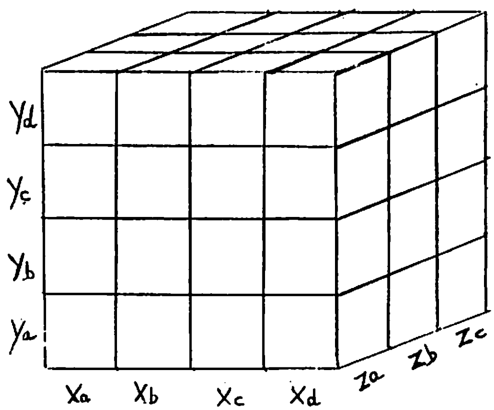


fig. 2

fig. 4

fig. 3





Quasiperiodic Patterns and Golden Sections.

Prof.Dr. Peter Kramer, Institut für Theoretische Physik der Universität Tübingen, FRG.

In this lecture we review the transition from periodic to quasiperiodic patterns, describe the new features of quasiperiodic structure and its links to the golden section, and illustrate the appearance of these patterns in the new physics of quasicrystals.

1. The appearance of periodicity:
The most familiar roots for the notion of periods is our experience of time. Time is experienced in terms of units like days, months, years. These units or periods follow each other in a sequential repetition pattern. With the expansion of experience and science, we have learned about the astronomical origin of these and other much longer periods, and we have also found extremely short periods of time on the atomic and subatomic scale. Among the arts, music is a prominent field where a pattern in time is created, very often with a background of periodic rhythmic units.
Periods in space were created, a long time before science, in the sequential order of band ornaments, and the additional dimensions of space allowed for the extension of periodic structures to ornaments in the plane. The inclusion of new dimensions opened the way not only to doubly periodic patterns, but also brought the new repetion patterns of rotational, mirror and color type. The richness of ornamentics is obtained from the variation of motives and from their combination. In the structure of crystals one finds up to triply periodic structures from the macroscopic down to the atomic level. The motives of this periodic structure are formed from atoms and have as their typical length scale multiples of an Angstrom.
The systematic description of these periodic patterns in science has a long history up to present-day crystallography.

2. The structure of periodic patterns:
A simple way of representing sequential periodic patterns is given by forming artificial words, that is, sequences from an alphabet of letters a,b,c,.... As an example consider the words cde, cdecde, cdecdecde,....
which are generated recursively from the motive cde. In the limit of an infinite number of repetitions we get a periodic sequence which is very similar to a periodic decimal fraction.
In mathematics we describe the periodic property of this infinite sequence by a shift map. This shift map moves each motive cde by three letters to the right, so that the unshifted and shifted patterns are
...cdecdecde...... ,
......cdecdecde... .
Since the points are meant to represent the infinite sequence, the unshifted and shifted sequences represent the same pattern, and so the shift map generates a symmetry map of the sequence.
The elaboration of the possible repetition patterns for multiply periodic patterns in two- and three-space is the subject of crystallography.

The notion of various types of symmetries, of maps and motives and their interplay in art and science will be illustrated by selected examples.

3. The structure of quasiperiodic patterns:
To introduce the concept of quasiperiodic patterns, we take as an example the Fibonacci words formed recursively from two letters a, b:
a, b, ba, bab, babba, babbabab, ...
To continue this sequence recursively, one simply links to the right of the last word its precessor. The infinite sequence obtained in this fashion is called the Fibonacci sequence. If one counts the frequency of the letters a and b in the sequence, one finds subsequent pairs of the well known Fibonacci numbers
1, 1, 2, 3, 5, ...
From the relation of these numbers to the golden section it is easy to demonstrate that the Fibonacci sequence cannot be a periodic sequence based on a finite motive formed from the letters a and b. So this sequence is an example of a so-called quasiperiodic object generated by a strict recursive rule. Patterns formed in a similar fashion by words are studied in the new field of mathematics called Combinatorics on Words, compare Lothaire (1983).
Now let us replace the letters a,b by two general motives to obtain the prototype of a quasiperiodic structure. Quasiperiodic patterns are not restricted to one-dimensional sequences, and richer patterns are obtained in two and three dimensions. Moreover, these quasiperiodic patterns may display rotational symmetries which are incompatible with all the well-known periodic structures. Patterns in the plane of this type with fascinating properties were invented by Penrose (1974). In three dimensions,similar patterns were proposed by Mackay (1982) and analyzed by Kramer and Neri (1984).
Properties and principles for the generation of these and similar patterns will be discussed and illustrated. The new interpretation of quasiperiodic patterns as sections through periodic patterns in hyperspace will be elaborated. Recently it has been realized that Kepler (1611) anticipated some important concepts of quasiperiodic structure. The significance of Keplers work was recognized already by Kowalewski (1938).

4. The golden section:
From the numerous ramifications of the golden section in the arts, in mathematics and science, as described for example by Huntley (1970) and by Richter and Scholz (1987), we select and discuss its relation to geometry and in particular to pentagonal and icosahedral symmetry. These symmetries necessarily are in conflict with periodic symmetry and hence were excluded from traditional crystallography, as will be illustrated by examples. The unexpected new result is that the golden section can be linked to quasiperiodic patterns rather than to periodic ones.

The connection between the golden section and quasiperiodic symmetry will be discussed in detail, starting from the Fibonacci sequence which is described above. As a result, quasiperiodic patterns with pentagonal and icosahedral symmetry will be interpreted as golden sections through periodic patterns in hyperspace. Periodic patterns formed from hypercubes as described by Haase et al. (198) will serve as a main example. Sections through these hypercubes generate families of cells in three-dimensional space known in part as zonohedra. These cells fit together in a subtle way to form quasiperiodic fillings of three-space.

5. Quasicrystals:

Experimental evidence for the existence of quasiperiodic structures formed by metal alloys was given first by Shechtman et al.(1984). Not only the study of these materials by X-ray and electron diffraction, but also the outer morphology of these quasicrystals given by Dubost et al. (1986) shows that icosahedral symmetry and hence the golden section is manifest in these alloys. Only part of the physics of quasicrystals has been explored and understood up to now. A non-technical survey of the present state of this field will be given, with emphasis on the evidence for a quasiperiodic cell structure.

For a long time, the golden section has played its main role in the non-scientific fields of culture. With the new role of the golden section played in quasicrystals, there is a much better chance of fruitful explorations from science and other fields of culture into the fascinating new landscape of quasiperiodicity.

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SYMMETRY AND ASYMMETRY IN THE ACTIVE VIBRATION CONTROL

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Attention to active vibration control problems has increased in recent years. The proposed applications are many complex technical projects, such as robotics, aerospace structures, track/vehicle systems and others. Active vibration control systems using the optimal feedback can reduce the destructive vibration and noise effects on technical and operating characteristics and on the health of people [1,2]. This paper is concerned with important role of symmetry and asymmetry in active vibration control systems design.

1. It is well known that in symmetric, many-degrees-of-freedom linear control systems it is possible to provide the autonomous vibration control of every mode [2]. In this case the modal decomposition simplifies the stability and performance analysis, because every mode control problem can be solved separately, like for a single-degree-of-freedom system. For example, let us consider the vibrating beam with two active vibration isolators (Fig.1) at the beam ends [4].

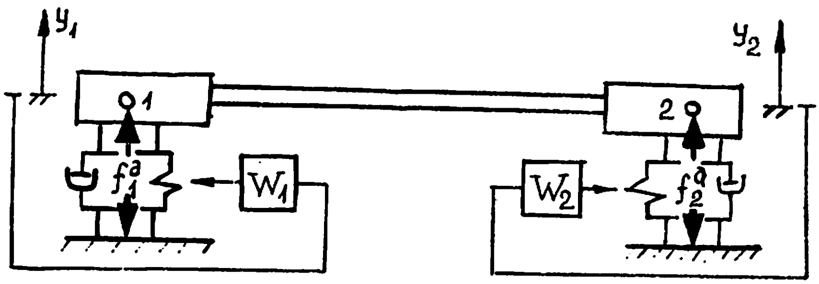

Fig.1

Taking into consideration the n first beam normal modes, the equations of motion are written in control theory terms:

$$(s^2 + \lambda_k\omega_k s + \omega_k^2)q_k = \frac{1}{m_k}\Sigma\varphi_{kj}f_j^a, \quad k=1,n$$

$$f_j^a = W_j(s)y_j, \quad j=1,2 \qquad (1)$$

$$y_j = \sum_{i=1}^{n} q_i\varphi_{ij}, \quad j=1,2$$

where q_k, ω_k, λ_k, m_k are the generalized coordinate, natural frequency, damping and generalized mass of the k -th mode; y_j are the displacements of the beam ends, where the isolators are attached, φ_{kj} is the k -th normal mode, $W_j(s)$ are the trasfer functions for each loop, s - the differentiation operator.

If it is assumed that the beam and the control loops are symmetric after the normalisation ($\overline{m}_k$ is the new generalized mass) one can find all the complex natural frequencies of the system (1) by solving characteristic equations of the form :

$$D_k(s) = s^2 + \lambda_k\omega_k s + \omega_k^2 - \frac{2}{\overline{m}_k} W(s), \qquad k = 1,n \qquad (2)$$

Thus the property of symmetry helps to estimate the critical frequences and feedback gains of the rigid object multi-degrees-of-freedom control systems, by considering single low-order subsystems, which can be described by simple characteristic equations.

2 The advantages of the symmetric structures are well known in the vibration control theory [1,5]. Only in symmetric structures it is possible to uncouple motions, and solve the problem of vibration control for each type of motion seperately. In this case there are used the principles of mutual compensation, etc. [1,5]. Vibration isolation problem for symmetric structures very often can be solved by the passive vibration isolation. But in practice for some constructions it is very difficult to keep symmetry of the structures under all operation conditions. For example, the mass or inertial parametres of the structure may be modificated during the operation. In this case it may be more advantageous to use the active vibration control not to reduce the vibration, but to make the real structure vibration to be like the vibration of the absolutelly symmetric system. This formulation implies some special methods of control system design,which can be explained using the previous example (Fig.1). Assume that the rigid body is asymmetric. Then for the dynamic symmetry of points 1 and 2 vibration one can find the needed transfer functions ratio $W_1(s)/W_2(s)$ by substitution the expression $|y_1| = |y_2|$ in (1).

In this problem the principles of mechanical vibration parametres control [1,5] are used for the assigning the property of dynamic symmetry to the vibration of real asymmetric structures.

3. The inverse problem can be also solved using the active vibration control i.e. it is possible to introduce the optimal asymmetry into the symmetric structures vibration. This fact can be used for the vibration reduction of the self-excited nonconservative mechancal systems (such as rigid unbalanced rotor shaft, structures subject to flatter, etc.) to increase the dissipation of some low-damped modes by strengthening their coupling with the heavily damped modes.

Consider two neighbouring normal modes in (1). Assume that the first mode ω_1 has the dissipation much lower than the second one ω_2 ($\lambda_1 \ll \lambda_2$). Let $W_1 = k_1$, $W_2 = k_2$,where k_1 and k_2 are some feedback gains. If it is assumed that the original mechanical structure (e.g. beam)is symmetric, one can write the characteristic equation of two coupled neighbouring modes, as follows:

$$[s^2+\lambda_1\omega_1 s+1-a_1(1+\alpha)]\,[s^2+\lambda_2\omega_2 s+\omega_2^2-a_2(1+\alpha)]-a_1a_2(1-\alpha)^2 = 0 \qquad (3)$$

where $a_1 = k_1/m_1$, $a_2 = k_1/m_2$, $\alpha = k_2/k_1$ is parameter, which describes asymmetric control. The equation (3) can be analityically treated by the root loci method. Fig.2 shows the root loci of the equation (3),when the parameter α is varied.In the case of symmetric control $\alpha = 1$ ($k_1 = k_2$) the coupling between two modes is very low - the starting points are marked by stars (Fig.2). When α increases, root loci approach each other,i.e. rich damping mode ω_2 gives some dissipation to the poor one ω_1 . There is an optimal value of parameter α^* (Fig.2), when the coupling between modes reaches maximum, and at the same time there is the greatest value of the damping δ_{max} equal for both modes:

$$\delta_{max} = -0.25(\lambda_1\omega_1 + \lambda_2\omega_2)$$

Equation (3) gives the analitical expression of the optimal parameter α^* , which helps to determine the best control points which provide the maximum coupling connection between the modes.

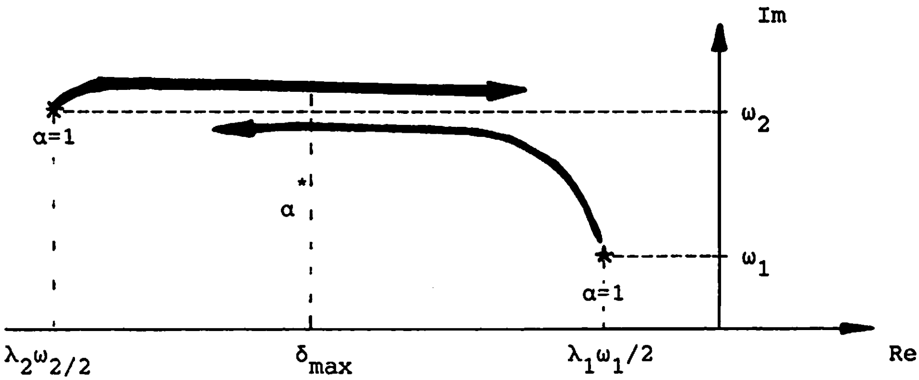

FIG.2

CONCLUSION

The examples show, that it is very useful to understand the role of symmetry and asymmetry when designing the active vibration control systems.

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PARTIALLY DETERMINISTIC STRUCTURES

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Abstract

This presents the basic agreements on randomness as accepted in mathematics and theoretical physics (randomness as the intrinsic probabilistic measure in the axiomatized theory of probabilities; randomness as an algorithmic complexity in the algorithmic theory of probabilities) and in experimental physics (randomnesss as the decreasing correlation, as the absence of a discrete spectrum, as the presence of a local instability or fractal structures, and at last as nonreproducibility, noncontrollability, unpredictability, etc.).

Based on the agreement about randomness as unpredictability, the relations between the observed , Y, and model (predicted), Z, structures were formalized. The formalization leads to the idea of partially deterministic, i.e. partially predictable, structures which is similar to the previously introduced ideas of partially deterministic process and fields [1,2].

The degree of determinism $D(Y|Z)$ of the observation Y relative to the model Z that is defined as a "share" of the model Z in the observation Y, is proposed as a quantitative measure of the prediction quality. Formally, the quantity $D(Y|Z)$ can be expressed as a generalized scalar product $\{Y,Z\}$ which, in a sense, characterizes the projection of Y onto Z:

$$D(Y|Z) = \{Y,Z\} / \{Y,Y\}^{1/2} \{Z,Z\}^{1/2}.$$

The equality of $D(Y|Z)$ to unity means the perfect determinism (perfect predictability) of Y , using the model Z; a small value of $D(Y|Z)$, as compared with unity, corresponds to randomness (perfect unpredictability) of Y on the basis of the model Z, while the intermediate values of D between zero and unity are interpreted as partial determinism.

In the specific case, when we speak of a system of spatial points, whose position is characterized by the radius vector $\vec{Y}_j$, the operation $\{Y,Z\}$ can be defined as the number of cases where the distance $|Y_j - Z_j|$ is not larger than ε , i.e.

$$\{Y,Z\} = \sum_{j=1}^{N} \theta\left(\varepsilon - |\vec{Y}_j - \vec{Z}_j|\right),$$

where $\theta(x)$ is the unity function : $\theta(x) = 1$ for $x \geqslant 0$ and $\theta(x) = 0$ for $x < 0$.

Then the quantity $D(Y|Z)$ acquires the meaning of the relative number of coincidences between $\vec{Y}_j$ and $\vec{Z}_j$ within the sphere of radius ε . If $\vec{Z}_j$ is a regular (hypothetic) lattice and $\vec{Y}_j$ are coordinates of atoms in the real lattice, $D(Y|Z)$ can be used as the characteristic of regularity of the lattice. If, however, $\vec{Z}_j$ is not a fully regular (not perfectly periodical) structure, $D(Y|Z)$ characterises the level of mismatch between the structures Y and Z.

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RESONANT INFLUENCE, SYMMETRY AND SPECTRUM OF STRUCTURES IN NONLINEAR MEDIA

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The formation of structures in nonlinear media is one of the important problems in synergetics. Determination of spectrum of structures enables efficient control of processes in such media.

Symmetry and groups of transformations sometimes play important role for the study of structures. First, the concept of "structure" implies conservation of some properties in time, the existence of some invariants. Sometimes structures can be described by self-similar (group invariant) solutions, that depend on combination of x and t: $\xi(x,t)$. Second, invariance of structure under a finite group of transformation also provides valuable information about solution.

One of the models that we investigated has been proposed in course of study of fast processes in plasma physics. It describes the medium with nonlinear coefficient of heat conductivity and bulk heat source. The distribution of temperature satisfies the equation

$$T_t = \mathrm{div}(T^{\sigma}\nabla T) + T^{\beta} \tag{1}$$

The process of burning (heating of the medium) goes on in regime with peacking, when $T \to \infty$ as $t \to t_f$ at least in one point. Such regimes are intermediate asymptotics of some real processes. Burning in regime with peacking gives rise to the localization of heat in certain region, outside of it T being equal to zero or limited. Temperature growth and localization result in the formation of structures – regions of intensive heating, where the form of temperature profile does not change (fig.1). For initial data of general type the process tends to burning as one or more simple structures with one temperature maximum. It is possible to create more complicated structures with several maxima. They can be considered as unification of a number of simple structures, that are interacting with others and attracting to the common center. The decrease of scales goes on synchronous in the whole structure, its form being described by the solution $T=g(t)\cdot y^{\alpha}(\xi)$, $\xi=x/(t_f-t)^{\gamma}$. Thus, if the initial temperature profile is set in accordance with the function $y(\xi)$ the development of process differs from that in the case of an arbitrary data. Such influence on a medium sometimes is called resonant.

All the concepts mentioned above appeared during the study of one-dimensional problem. In the multi-dimensional case new properties appear – form of the localization region and configuration of maxima. If one uses the concept of complex structure as the set of simple ones then symmetry is important for its existence.

In multi-dimensional case $y(\xi)$ satisfies the equation

$$\alpha\Delta y - 0.5(\beta-\sigma-1)(\xi,\nabla y^{\alpha})+y^{\alpha\beta}-y^{\alpha}=0 \qquad \alpha=(\sigma+1)^{-1} \qquad (2)$$

In our days there are no general methods of its investigation, but the hypothesis of symmetry has enabled us to propose approximate method of investigation. It is based upon linearization of (2) and matching the solution of linearized equation with asymptotics on a number of rays. With the help of symmetry it is possible to choose the directions of the rays and find a proper solution of linearized equation. Then we made an estimation of the number of structures and found some of the solutions $y(\xi)$ numerically. In two-dimensional case there are two classes of structures with maxima situated on the concentric circles and in the sites of the rectangular lattice.

This is not the only problem where symmetry enables efficient investigation of structures. The second example is the two-component medium with trigger properties. The processes in it are described by the system of parabolic equations with cubic nonlinearity

$$W_t = D\Delta W + AW - |W|^2 \cdot BW \qquad (3)$$

Under certain conditions on matrices D,A,B there are two stable solutions W_0 and $-W_0$ (that can be denoted by black and white colors). In one-dimensional case system from an arbitrary initial data evolves to the stationary structure that consists of elementary ones - narrow regions of transition from W_0 to $-W_0$ and vice versa. Two-dimensional case differs substantially. From an arbitrary initial data system usually evolves to homogeneous background - black or white - and no structures appear. Like in the case of heat structures the symmetry of initial data is necessary for the formation of stable stationary structures. The symmetry must be colored - the invariance under reflection (or turning) and mutual change black ↔ white. This approach has enabled to use a number of approximate methods mentioned above and to find several two-dimensional structures.

Two of simple two-dimensional structures are shown in fig. 3. It is interesting that they can be combined to form "parquets", e.g. like in fig. 4.

Thus, the symmetry of initial perturbation is of great importance for the resonant properties, formation of structures of different complexity in a nonlinear medium.

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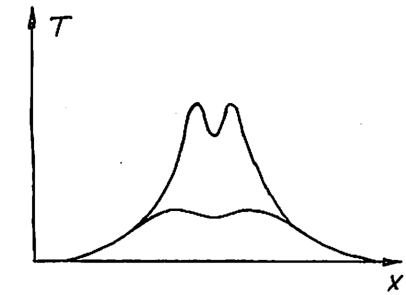

Fig.1 Complex heat structure with two maxima.

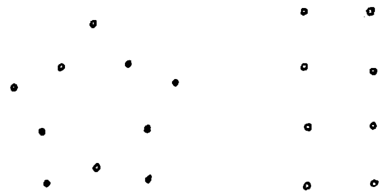

Fig. 2. Examples of configurations of maxima in 2-D complex heat structures.

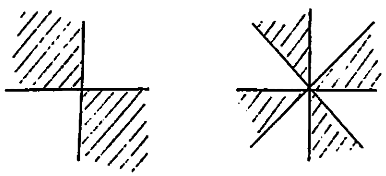

Fig. 3

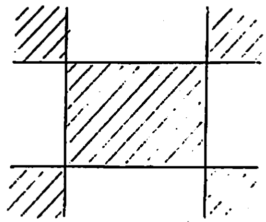

Fig.4

STRUCTURES AND META-STRUCTURES

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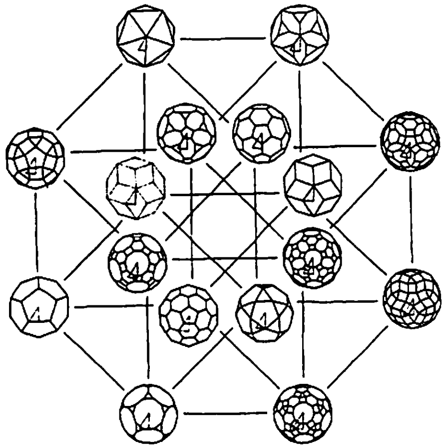
This presentation deals with spatial patterns termed *space structures* , and the patterns underlying these patterns. The patterns-underlying-patterns are termed *meta -structures* (or meta-patterns). This work is based on author's previous work [1,2] which suggests that just as patterns have a structure, as determined by the organising principles of symmetry, topology or geometry, meta-patterns also have a structure which is determined by the same ordering principles. That is to say, meta-patterns have a symmetry, topology, and a geometry, and in this sense both patterns and meta-patterns are self-similar.

Among the meta-structures, the more interesting and useful cases are the higher-dimensional structures, principally n-dimensional cubes or *n -cubes* . In author's previous work, n-cubes have been shown to serve as organising and transformational diagrams for families of regular-faced structures and some of its derivatives, namely, polyhedra, plane tesselations, polyhedral packings, and more recently, infinite polyhedra and non-periodic packings of icosahedral polyhedra. In all cases, the structures and their transformations are characterised by the underlying Boolean logic of the n-cubic space and are governed by higher-dimensional DeMorgan's laws relating unions, intersections and their complements. The structures are correspondingly indexed in binary combinations of 0's and 1's.

In a different field, that of movement studies, this concept has been used by the author to organise Rudolf Laban's movement "efforts" in a hyper-cubic space [3]; we know of this dance-theoretician's work from the better known Labanotation system for dance. In a broader sense, meta-structural ideas hold potential for organising concepts both *within* and *between* a wide variety of disciplines, and form the basis of a new science [4]. Goranson suggests applications in artificial intelligence [5] and Zellweger has used similar ideas for Logic [6].

The concept is illustrated with the example of plane tesselations and polyhedra, though the idea can be extended to higher-dimesnional space structures. The tesselations and polyhedra are organised according to their symmetry into a 2-dimensional lattice where each distinct symmetry occupies a distinct vertex of this lattice and is determined by the complimentary pair of angles of a right triangular fundamental region. Inter-symmetry transformations take place in this lattice plane. If we restrict to mirror-symmetric structures, a family of 16 topological structures are possible for each symmetry from four basic types of transformations within the fundamental region. These 16 are arranged on the vertices of a 4-cube (Fig.1; example shown for icosahedral symmetry), and each symmetry has its own 4-cube associated with it. This extends the meta-space into a hyper-cubic lattice within which the structures transform continuously to one another both within and between symmetries. The edges of the meta-lattice provide direct transformation paths between structures, and higher-level transformation paths are along the face, 3-cell and 4-cell diagonals of each 4-cube.

Through the addition of three other transformations based on special subdivisions of the



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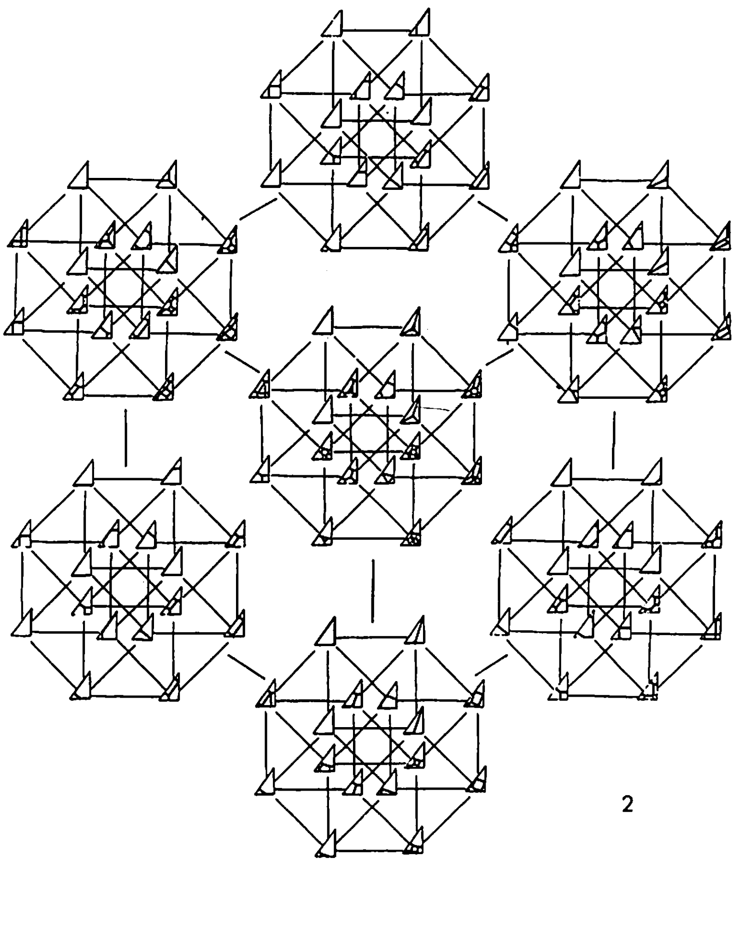

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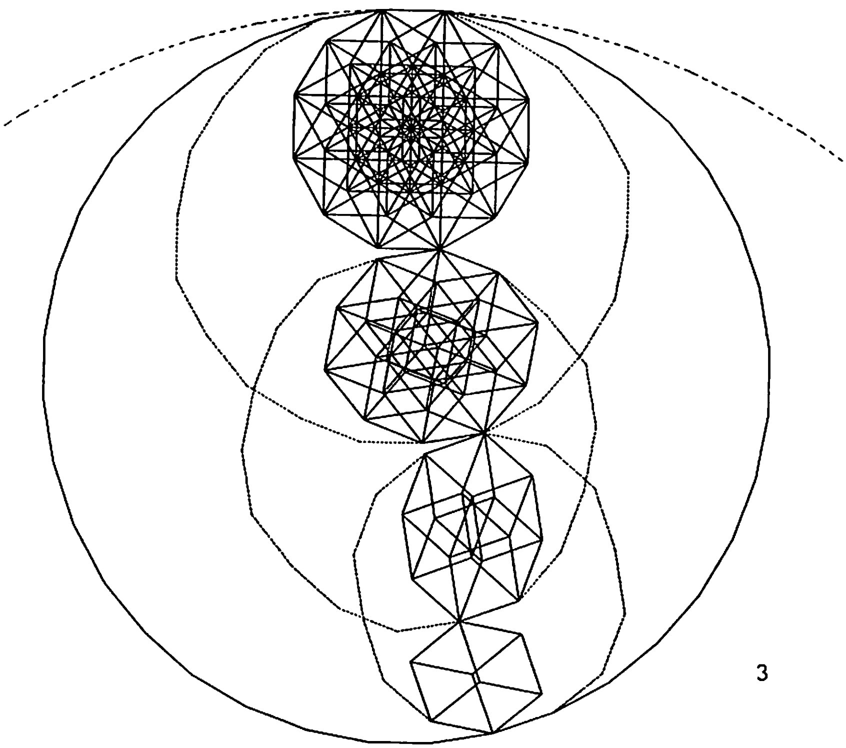
fundamental regions, the 16 structures within each family can be converted into 128 structures which can be organised in a 7-cube space. This space decomposes into sixteen 3-cubes on the vertices of a tesseract, or eight 4-cubes on the vertices of a 3-cube (Fig.2; fundamental regions shown). New transformations (e.g. dualisation, frequency, handedness, etc.) can be continually added, extending the meta-structural space correspondingly into an increasingly encompassing meta-space for defining, generating, and transforming complex space structures. Such a system is open-ended and provides a simultaneous way of classification, generation and transformations of space structures. The open-ended nature of the system makes it more complex than the I-Ching, the Chinese system of changes, which is restricted to 64 hexagrams.

A possible generalised model (Fig.3) for a system of changes with multiple parameters and hierchichies is a recursive n-cube (or n-cubic lattice) which can be decomposed *downwards* into i-cubes (i=<n) on the vertices of an (n-i)-cube, where each i-cube itself can be decomposed into j-cubes (j=<i) on the vertices of (i-j)-cubes, and so on. In a similar way, the n-cube can grow *upwards* into larger and larger super hyper-cubes. Though the application of such a model to space structures remains to be fully established, aspects of the present work point in that direction. A promising application is in the area of "shape grammar" as supported by our present studies in architectural form-generation.



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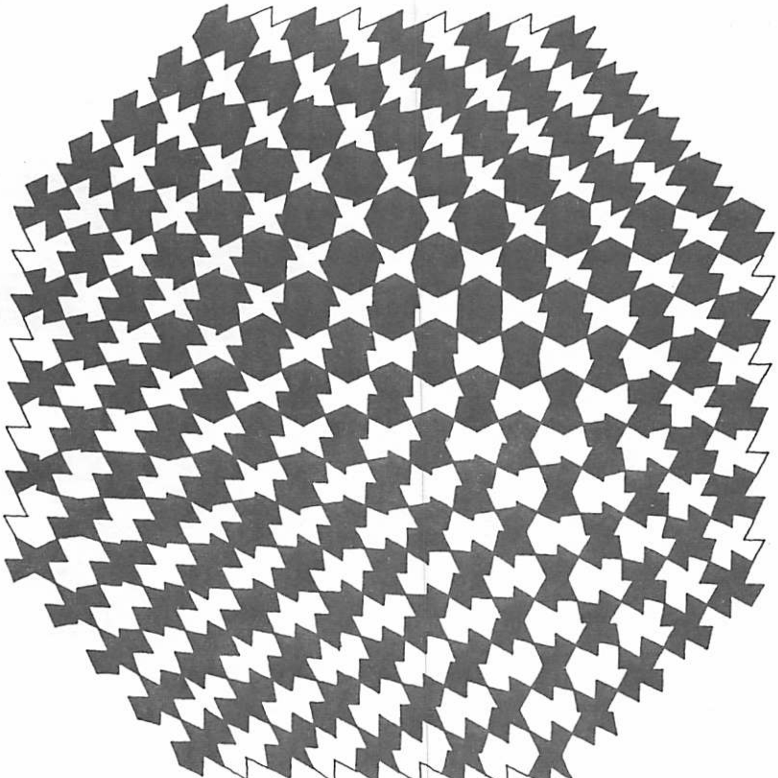
The transformations between structures can be *discontinuous* (discrete or digital mode) or *continuous* (analog mode) , providing two fundamental models for transforming information. The discrete transformations change a structure to another by changing 0's *to* 1's or vice versa in an on-off manner. The analog transformations change a structure in a graded manner, from 0 *through* 1. The latter are gradual, continuous and incremental changes. Both types of transformations provide a natural basis for computer animation of structures. Both provide alternative modes of meta-structural thinking. The discrete 0-1 changes are well-known from the binary system used in computational sciences. The continuous 0-thru-1 changes provide a basis for a new transformational logic.

The concepts presented lend themsleves naturally to computer-animation. Computer-animated continuous polyhedral transformations can be seen in a collaborative project with R.McDermott and Patrick Hanrahan [7]. Continuous transformation of the Penrose tiling can be seen in a joint project with David Sturman [8]. An interesting fallout of such a system of a continuum of transformations is the new class of *hyper-Escher* patterns (Fig.4). These are higher-dimensional analogs of Escher's metamorphic patterns.



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Credits: Computer image in Fig.3 is executed by Neil Katz.

SYMMETRY IN THE ANALYSIS OF STRUCTURES

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The existing symmetries influence the numerical analysis of structures remarkably. They reduce the mass of mathematical operations which should be done. Four different levels of this "facility" can be found in the analysis of structures.

1. Discrete Symmetries in the Body

The discrete symmetries of analysed body are the mirror symmetry, the cyclical symmetry and the discrete shifting symmetry (periodical skeleton construction).

In the case of mirror symmetry the order of algebraic problem which should be solved is divided into two (roughly) equal parts decomposing the unknown quantities into symmetrical and asymmetrical parts. In the case of existing of double or triple mirror symmetry the order of algebraic problem becomes a quarter the value or eighth one. Of course two, four or eighth (independent) algebraic problems must be solved accordingly to the number of symmetry.

In the case of cyclical symmetry or periodical skeleton construction the structure of matrix of the algebraic equations simplifies. In the case of skeleton construction this matrix is a symmetrical, uniformly continuant one (its elements themselvs are matrices too), while at the case of cyclical symmetry the system "closes", i.e. an element, standing in the neighbour to the principal diagonal line, gets into the left lower and rigth upper corner.

If the cyclical symmetry and/or periodical skeleton structure exists in two or three directions too, then the blocks of the matrix also have the structure described above.

The existing of discrete symmetry can give a help in the last step of the numerical analysis of structures, i.e. in the solving of the algebraic equations.

2. Continuous Symmetry in the Body and in the Mechanical State

In the Eucledian space there are two (independent) motions: the shift and the rotation. The first is the continuous case of the skeleton structure, while the second is the continuous case of the cyclical symmetry. The combination of them is the translation along a helical curve (heliodromic). Accordingly to these three operations three symmetry names are applied: shifting, rotational and helical symmetry. These three symmetries generate three body: prismatic one, body of rotation and the helio-symmetrical body.

The mathematical state of the body having continuous symmetry also can be fulfilled a continuous symmetry. Then the mechanical state - plane, cylinder-symmetrical problems and helio-symmetrical state - reduces to the twoo-variable problem.

Of course the double and triple continuous symmetry can be interpreted both in the case of bodies and mechanical states, the last ones are one-variable, or zero-variable (the real coefficiens are the unknown qualities) respectively.

Finally it must be mentioned, that the cylinder-symmetrical and plane problems can be divided into symmetrical and asymmetrical (which is the torsional one) problems.

The continuous symmetry reduces the number of variables from three to two (separetely to one or zero), that implies the reduction of the order of algebraic problem (which should be set), too.

3. The Symmetry Group of a System of Equations

The solution of the partial differential equation of the setting problem (independently of the exterior geometrical symmetry and symmetry in boundary conditions) has the symmetry property. Namely (usually) there is a symmetry group of the solutions of the equation.

The symmetry group plays an important role in the solution of partial differential equations. On the one hand the symmetry group indicates those coordinate systems in which the method of separation of variables can be used to solve the given differential equations, on the other hand the symmetry group makes possible to determine those special functions, which determine the function space of the solution for the separated one dimensional space. Usually these functions turn to be the characteristic (eigen-)functions using which the (initial and/or) boundary value problem can be solved "easely".

4. Symmetry in the System of Equations

The analysis of structure - solid, deformable bodies - is based upon the consequences of the continuum mechanics. In this system of equations the symmetry is observable at several points.

Firstly the symmetry of constitutive equations is mentioned. It is in need, because both the strain and the stress tensor are symmetrical. But the cause of this symmetry is deeper, it has a thermodynamic explanation.

Secondly let us see the symmetry of strain and stress tensors. Both are consequences of the fact that continuum mechanics is discussed in a Euclidean space, the metric tensor of which is symmetric. And both mechanical tensors are in close connaction with the metric tensor.

Finally the symmetry in a system of equations must be touched upon. This is also the consequence of the fact that the mathematical theory of the physical phenomenon is constructed in a Euclidean space, identifying the physical phenomenon with it.

If the (geometric) space in the mathematical theory is generalized or the continuous description is already irrelevant, then the above mentioned symmetry property "is lost" partially: asymmetrical tensor functions and differential expressions appear in the system beside the symmetrical ones.

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THE SYMMETRICALLY IRREVERSIBLE UNIVERSE

Ervin Laszlo

THE PROBLEM

Since the advent of thermodynamics in the 19th century, dynamic change in the physical universe has been known to be irreversible. According to the famous Second Law, heat can only flow from a hotter to a colder body and never in reverse; thus energies eventually dissipate and become unavailable for performing work. In time, any isolated system in the universe is bound to run down. The universe as a whole may be a system that does not receive energy from the outside; thus it is likewise bound to run down. That for a time it can--and apparently does--evolve is ascribed to the local concentration of free energy--and hence increase of negative entropy--which is always and precisely balanced by the global dissipation of free energy and thus the corresponding increase in entropy. This introduces a major asymmetry into our concept of reality. Order and complexity, while for a time building up in limited regions of space and time, tend on the whole to disappear, ultimately to be replaced by randomness and disorder.

Since the 19th century statement of the laws of thermodynamics, much more has become known about physical processes and the mechanistic conception of the universe, which views it as a closed thermodynamic system, has been questioned. But the global irreversibility of evolution in the cosmos has not been challenged Even in the new cosmologies, order and complexity are expected ultimately to break down, although under varying conditions--either in the infinite reaches of an eternally expanding open universe, or in the superdense mass of a re-contracting closed universe. This study suggests that the conception of the irreversible loss of order and complexity is incomplete. It ignores the inverse process of the irreversible build-up, not of order and complexity in space and time but of the information on which such build-up is based in the spectral domain. Attention to this factor can reestablish symmetry in our concept of reality. This study shall explore the reasons why the factor is required, outline what it is, and how it can bring back symmetry into our concept of the evolving universe.

THE MISSING FACTOR

Still missing from the standard accounts of evolution in the sciences is the factor that would introduce the necessary guidance or direction into the otherwise random processes of universal evolution. The missing factor, in the view put forward here, needs to be conceived as a combination of <u>field</u> and <u>memory</u>, in other words, as a universally extended memory-field. Theories of this kind are actively explored today--we need only to refer to the earlier infamous but now already famous writings of David Bohm and Rupert Sheldrake. Fields and memory, the crucial components of the new conceptions, have been understood for some time. "Before Clerk Maxwell", wrote Einstein already in 1934, "people conceived of physical reality--insofar as it is supposed to represent events in nature--as material points, whose changes consist exclusively of motions...After Maxwell they conceived physical reality as represented by continuous fields, not mechanically explicable." This change in our conception of reality, he added, is the most profound and fruitful that has come to physics since Newton. [2]

In the so-called "new physics," developed in the last decades, the field concept is basic. In microphysics each particle is viewed as a singularity in a field that extends simultaneously to every part of the configuration-space. In field physics the metric of the spacetime field defines the trajectories and hence the behavior of the particles. Although there is no need of a material substrate such as "ether," without the assumption of continuous fields underlying and embedding the observed mass-points the new physics would be left with a world that resembled the grin of the Cheshire Cat: it would have observations, but nothing that the observations would be observations <u>of</u>.

The fields now known to physics cannot be reduced either to matter or to energy, although they may be generated by the one and may carry the latter. There are at least three major kinds of fields: matter-dependent energy fields; matter-independent energy fields; and energy- as well as matter-independent probability fields. Matter-dependent fields include the flow field of a moving fluid, the electric and magnetic fields surrounding bodies, the temperature field of the atmosphere, and the stress field within a compressed solid. Matter-independent fields comprise the gravitational field, the metric field of general relativity, radiation fields, the electromagnetic field in Maxwell's formulation (in which it does not reduce to Coulomb's law according to which the electric charges associated with matter particles generate the field), and the various nuclear fields. Energy- and matter-independent probability fields may underlie nonlocal interaction among quantum phenomena as well as the esoteric domains of human experience. The best understood instance of such a field is the state-vector or state-function field of quantum field theory. It consists of the probability distribution designated by the square of the state function 0 (phi). The thus designated field is independent of matter: the function 0(x) refers to a point in space without implying that any matter exists at x. The probability field is also independent of energy: there is no implication that at x there is any work performed by an energy vector.

If contemporary physics provides a basically sound description of physical reality (a realist assumption contested only by the Copenhagen school of quantum physics), the reality of the universe is rooted in various kinds of fields. Fields are even more fundamental in physics' world picture than matter and energy, since fields may exist without either matter or energy, but neither matter nor energy can exist without some field.

The field concept required as the missing element in contemporary theories of evolution is more than a mechanical juxtaposition of the currently known physical fields. Physics, after all, is primarily concerned with the physical universe, and that universe constitutes but one stage in the evolution of the cosmos. If the trans-physical --that is, the organismic, ecologic, psychologic and sociocultural--domains are to be explained by the same basic laws of evolution as phenomena in the physical universe, the concepts currently emerging in the physical sciences need to acquire a wider interpretation. This brings us to the second element of the required concept, namely memory.

Memory is required to overcome the specious alternatives of chance and determinism in evolution. In theories of biological evolution metaphors such as "blind watchmaker" (Richard Dawkins) and "bricoleur" (François Jacob) have become popular; indeed, they are useful in countering teleological theories relying on final causes and preconceived designs. But metaphors of chance and randomness have limited applicability confronted with the overall spectacle of evolution not only in the biological, but also in the physical and in the human and social spheres. To gain a complete explanation, chance must be tempered with memory. The reason is that nature, not being guided by a final cause and not following a preconceived design, must be able to recall what has already evolved. Without memory the process of building systems and configuration from basic units and materials would have to start anew each time a chance disturbance destroyed what has already been assembled. This fallback, as Herbert Simon has shown, would call for infinitely more time than would be required if stable assemblies were created to be used as parts in the construction of more complex units. Since in nature any assembled system is subject to dissolution, evolution cannot build on the stability of assemblies that have already been produced. However, it can build on a model or blueprint of it. If we are to account for the evolution of order and complexity, we must assume that the pattern of already created assemblies is conserved and used over again. This does not convert nature as bricoleur into purposeful craftsman, nor does it make the blind watchmaker into fully sighted designer. But it provides the kind of information that enables nature to build systems upon systems in energy flows that constantly shake, and frequently destroy, the systems that have already been evolved.

The conservation of form, pattern, or "blueprint" in evolution suggests the presence of memory, but does not call for consciousness and intelligence. Memory, after all, is not uniquely associated with mind. The computer that processes the text now being written has memory and is unlikely to have mind, not to mention consciousness. Even a simple pendulum has a kind of memory: in each of its swings it "remembers" its initial displacement. The exposed film remembers the pattern of light of various intensity that reached it through the lens of the camera, and the film exposed in a hologram remembers the interference pattern of two coherent beams of light in the reproduction of a 3-D virtual image. And even the simplest of living organisms conserves the relevant impressions of its environment and displays that minimal flexibility that is connoted by the term "learning."

Memory in nature is not likely to be localized. For evolution to have taken the observed course, the conservation of the form or pattern of the already achieved configurations must be available at all points in space at all subsequent instances of time. This kind of memory is not supernatural: it is holographic. Holographic storage is entirely distributed, and it has vast storage capacity. Holograms store the inference pattern produced by intersecting beams of coherent light, one of which is scattered off the object reproduced. Since light from all parts of the object's surface is spread across the entire holographic film, all parts of the film receive information from all surface points of the object.

Consequently the recorded pattern can be retrieved from any and all points of the film: under proper viewing conditions the image appears even if only a small part of the film is viewed. Moreover, holographic media have staggering storage capacity; John Caulfield calculated that the entire contents of the U.S. Library of Congress could be stored in a medium the size of a cube of sugar. The distributed nature of the stored information, and the storage capacity of holographic media, suggests that the missing factor is a holographic kind of memory field. This does not mean that nature functions as a holographic apparatus, nor that the pattern conserved is a hologram. It also does not mean that the holographic medium in question is a film. The hypothesis is only that the conservation of pattern occurs in an extended field where the information is distributed, and that it occurs not in the domain where objects are extended in space and endure in time but in the spectral domain of multiply superposed waves.

Evolution in nature is no longer ascribed to the chance mixing of matter in blindly iterative interactions. There is now an explanation of the indeterminate but significantly probable emergence of consistent order and complexity in all spheres of observation. From the substructure of continuous fields emerge the quantized units known as matter; and from matter come the more complex configurations that surround and include us. The memory field transforms a blindly groping universe into an evolving cosmos without doing away with probabilities, violating the laws of physics, or assuming cosmic designs and other teleologies. The feedback of achieved pattern into incipient formative processes makes the cosmos self-evolving and our account of it conformant to experience.

In the memory-field universe symmetry is reestablished. Energy, as we know, is conserved in the universe and so is matter (baryon number). These symmetries of the standard conception remain valid. But in the standard account organization is irreversibly destroyed as global entropy irreversibly grows. The overall increase of entropy and decrease of order is not compensated: it is asymmetrical. According to the here presented hypothesis there is a compensatory process: the complexification of the fine-structure of the energy potential field. As the sum of ordered complexity in the matter-component of the universe decreases, so the sum of the information conserved in the memory field increases. The two processes precisely complement and compensate for one another. Symmetry is assured, even though the manifest course of evolution is irreversible.

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