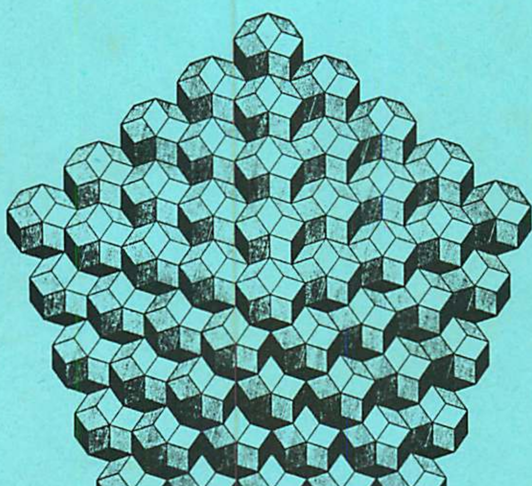
Symmetry
of
STRUCTURE

an interdisciplinary Symposium

Abstracts

I.



Edited by Gy. Darvas and D. Nagy

Buda
pest

August 13-19, 1989

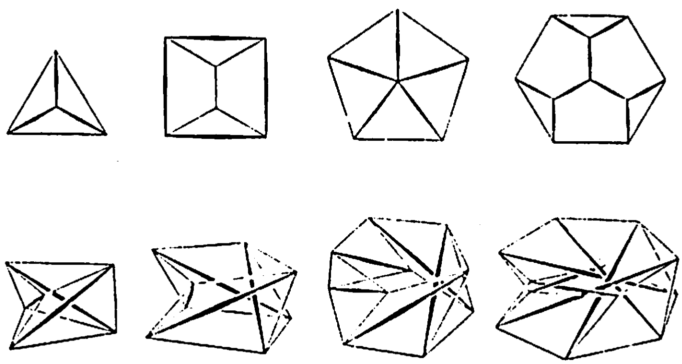
hungary

SYMMETRICAL STABLE SIMPLEX
INTRODUCTION TO THE RESISTANCE OF FORMS.

EMMERICH. Pr. David Georges Ing. Arch.
Ecole d'Architecture de Paris-la Villette
37, Avenue de Breteuil
75007 Paris
FRANCE Phone (1) 47 83 34 24

Symmetry is the generator of equilibrium, omnipresent forever in architecture, it also defines the great monumental orders as the elementary structural forms - lintels, arches, vaults - the stable simplex, which repeat themselves rythmically, being the characteristic motifs of the building.

In an industrialised space structure,based on the standardisation of elements and on the regularity of their assemblage, the symmetry becomes even more important : the employed simplex thus being polygons and polyhedra. The number of arbitrarily chosen forms being infinite, the invention and the systematic inventory of these new simplex stables, which are multidirectional and even multidimensional, would be impossible, as would be the study of their rigidity and the calculation of their resistance, without the introduction of the criterion of regularities - that is to say, symmetry.



The stable simplex is an autonomous equilibrium which maintains itself as an indeformable entity in a non-monolithic system, thus composed essentially of compression or tension bars, assembled by movable joints.

In ancient construction one can discern certain elementary configurations recognised and used as stabilising or bracing elements. These are the triangular brackets of the wooden frame, the St.Andrews cross in skeletons, the tensioned crossbracing in a metal framework. These stable simplex are nearly always planar, they have been very few in the number of types, and the art of building has been satisfied with them until now.

At present with the knowledge that construction is becoming lighter and more spatial, there is place to re-examine then enlarge the list of stable simplex which can enter into the composition of rigid bearing systems.

In an old article, appearing under the title "Principes Edifiants" in the "Techniques et Architecture" no.2-1966, I have already had opportunity to criticise established ideas and at the same time the validity of equations currently used by engineers to verify the indeformability of a configuration. Notably the formula A=3S-6, defining the relationship between the number of edges (A) and the number of articulated vertex (S), which is only valid for the convex compositions with continuous triangulations, thus without polygonal hiatus.

It is therefore suitable to replace the "Euler" formula with the "Ky-Fan Frechet" formula taking into consideration these hiatus. This gives a general controlling formula usable in the case of continuous triangulation : A=3S-6K, K being the number of connections of the enveloping surface of the configuration, even if the surface is torus-like or spongieuse, thus with one or several holes.

Otherwise, beyond this amelioration, there exists some stable compositions which are neither triangulated nor constructed as a continuum of rigid elements. And however, analytic studies are prolifering with heavy algebraic symbolism, it is not by such an abstract and non geometrical way that one can best approach the problem of rigidity of complex systems. The rigidity is not quantifiable as easily as those continuous and monolithic elements which constitute the study of the resistance of materials, one should now speak of the resistance of forms.

This resistance of forms - which is not concerning the opposition to the efforts because it depends on the intrinsic ownership of some geometric figures to resist mechanical deformations - brought to light in reality from the domain of kinetics, or more exactly from the field of anti- or non- kinetic, or better still, from all that is just non-kinetic or isostatic.

It is in fact easier to take the graphs one after another, beginning with the triangle which is altogether the most simple constructive subset, thus the most primitive simplex, and examine the polygons and the polyhedra having a more and more rich articulation towards the most complex spatial configurations ; the aim being to transform them by the addition of supplementary and a strictly necessary number of elements to obtain rigid or "instantaneously rigid" isostatic systems or following certain experts : critical or surcritical systems. Remarking that the addition of overabundant or redundant elements would produce hypocritical structures.

Instead of being content with the creation of a terminology which only classifies very different configurations under the same etiquette, one proposes thus a combinatory method which enumerates them systematically. This method is at the same time an experimental one, the figures can be easily modelised avoiding heavy and pedantic algebraic codes which makes the actual studies on the rigidity rather stern for the people engaged in structural research and even more for the practising constructors.

One thus takes a graph, preferably a symmetrical one, having a shape originally with a variable geometry with a more or less regulated liberty of movement, then by the addition of new elements and by the manipulation of the nature of the members, one transforms it by provoking any kind of conflicts in the relative displacement of the members until blocking his mechanism ; somehow like putting a stick, or a string, in the wheel-work.

The open figures - segments, stellations, aborescences- cannot be associated in a stable structure without being rigidly fitted together. They are thus composed of consoles or cantilevers, where a monolithism is established by the continuous embodding of their members, which excludes such structures from these studies, related uniquely to articulated ones.

Our objective is therefore limited to the study of closed figures like a loop, having a polygonal perimeter or composed of polygons. The membrures themselves can be struts or rigid bars working as well in traction as under compression, or slings or tensional members working only in traction.

The regular polyhedra, the solids, represent for three dimensional space the same elementary repetitive components as the regular polygons do for two dimensional space. Naturally, as in the space packing patterns, then in the planar regular tessellations one uses symmetrical basic geometrical figures. Otherwise, the polyhedra, whose edges are materialised by rigid bars and with articulated nodes, are not all solid.

After the theorem of Cauchy (1812) only the polyhedra with indeformable faces are indeformable. Thus among the articulated systems only the triangulated volumes are stable : among the regular polyhedra - the tetra, octa, and icosahedra - and among the semiregulars only the snub cube and the snub dodecahedron having continuous triangulated rings ; and all deltohedra generally as bipyramids, antiprims, etc...

Following this theorem other polyhedra can be rigidified theoretically by an indeformabilisation of the polygonal faces by triangulation by the use of one or another planar simplex. But practically, this method is not always successfull, the planar simplex being subject to warping most of the time. In principal, those with internal prestressed configuration having no cleavage-line or hinge are not warping ; thus, though planar, they remain stable even in space. Some others keeping their rigidity in an assembly of uneven order around a vertice ; disparallelity helps too, etc... But instead of making spatial items with planar ones, it would be more logical to follow again the morphological approach starting from the most simple volumes progressing systematically to the more complex ones and at the same time looking for their stabilisation, if possible, by stereometrical means.

The result is astonishing enough : over the volumes entirely triangulated or stabilised by planar simplex faces, there exists elementery solids with articulated membrures, having not only rigid bars, but even flexible tensional membrures. These bodies are the stable spatial simplex which are at the same time practically empty and thus light volumes, ready to be used as building blocks, as was in the past the stone block in the hand of the mason.

These configurations, having generally screwed, left or right rotating, symmetry, present themselves as very surprising stable compositions, where the rigid bars - each one isolated in a continuous tensional network - appear to be floating. These self stressed solids are the most remarkable among the stable space simplex because of their extreme simplicity and their ability to enter in any kind of structural pattern - linear, planar, polyhedric, cristallographic - by their combinatorial facilities, their shape keeping a high degree of symmetry.