"THE SYMMETRY OF FINITE-DIFFERENCE EQUATIONS"

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Attempts of adaptation of the group analisis of differential equations to finite-difference equations were failured because of nonlocal nature of finite-difference operators. Therefore some authors applied preliminary localisation replacing the finite-difference equations by various differential approximations.

The first steps towards the group analisis of the finite-difference equations are made in the present work.

The formal transformations groups in the space of differential and mesh variables are considered. It is shown that the conservation of the difference derivatives sence tends nesessaraly to the Lie-Backlund groups. One of them - the Taylor group is used obtaining the formulas of transformations of mesh variables. The criterion of invariance and uniform conservation of the difference mesh is stated. The criterion of invariance for difference equations is applied for obtaining the finite-difference equations allouring the group isomorfic to the natural differential model group.

The group isomorfic to the Taylor group is constructed by means of formal Newton series. This group is applied for factorization of Lie-Backlund operators on uniform mesh. The discrete Noeter identity for some classes of group transformations is obtained, and the conservation criterion of invariance equations is settled.

THE BIRTH OF SYMMETRIES IN THEORETICAL PHYSICS : LAZARE CARNOT'S MECHANICS

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Lazare Carnot wrote a first book on mechanics in 1783[1] and a second book in 1803[2]. In both books the core of his theoretical thinking is the same and highly original with respect the other authors of his times, Lagrange included. From a theoretical point of view, his basic notion is that of geometric motion. The definition is the following: Any motion will be called geometric if the opposite motion is possible. He offered many examples of such motions. For us it is interesting the fact that the translatory motions and rigid rotatory motions of a system as a whole constitute geometric motions.

Geometric motions do not equate virtual motions, contrarely to what Gillispie seems to hold[3]. Rather, they play a crucial rôle in the development of L. Carnot's theory. They allow to obtain the laws of motion from a fundamental equation L. Carnot states (unfortunately in an obscure way) for the shock problem of a system of particles:

$$\sum m_i \vec{U}_i \cdot \vec{u}_i = 0$$

where m_i is the mass of the i-th particle, $\vec{U}_i$ the lost velocity, $\vec{u}_i$ the superimposed geometric motion, possibly in a specific way for any single particle.

Let us remark that since one may assign an infinte number of geometric motions, the $\vec{u}_i$'s velocities play the rôle of indeterminate quantities; then, any specification of $\vec{u}_i$'s provides a determinate equation which gives information about the system motion.

For example, when we assign to u_i's the same velovity , $u \neq 0$, we consider a translatory,uniform motion of the whole system; then, we obtain:

$$0 = \sum m_i \vec{U}_i \cdot \vec{u}_i = \vec{u} \sum m_i \vec{U}_i \ ,$$

being $\vec{u}$ an arbitrary quantity; it follows that

$$\sum m_i \vec{U}_i = 0$$

But $\vec{U}_i$, the lost velocity, is the same that $\vec{W}_i - \vec{V}_i$, i.e. the difference between starting velocity and final velocity; thus,

$$\sum m_i \vec{W}_i = \sum m_i \vec{V}_i$$

which is the law of conservation of momentum for an (isolated) system of material points.

Then, let us assign a new geometric motion which rotates rigidly the whole system about a fixed axis, with angular velocity $\vec{a}$. Then, $\vec{u}_i = \vec{a} \times \vec{r}_i$ and we obtain

$$0 = \sum m_i \vec{U}_i \cdot \vec{a} \times \vec{r}_i = \sum m_i \vec{a} \cdot \vec{U}_i \times \vec{r}_i = \vec{a} \sum m_i \vec{U}_i \times \vec{r}_i$$

Again, $\vec{a}$ is an arbitrary quantity, thus

$$\sum m_i \vec{U}_i \times \vec{r}_i = 0$$

or, what is the same

$$\sum m_i \vec{W}_i \times \vec{r}_i = \sum m_i \vec{V}_i \times \vec{r}_i$$

i.e. the conservation of the momentum of momentum.

Surprisingly enough, until 1970 L. Carnot's mechanics passed almost unnoticed , if not by some French authors[4]. The above derivations have been noticed by C.C.Gillispie in his analysis of L.Carnot's scientific work[5]; however he did not recognized in them the symmetry method of deriving laws.

Actually, L. Carnot was pride of having introduce the notion of geometric motion. He even foresaw an entirely new science, intermediate between geometry and mechanics[6]. The next development of science disregarded such design. However, it is well-known that Sadi Carnot produced almost the whole theory of thermodynamics by developing the main ideas of his father, Lazare. In particular its reversibility notion constitutes a filiation of the geometric motion[7]. Furthermore, H.Callen showed that thermodynamics may be reformulated as "a science of symmetry" since conserved and brocken symmetries offer its coordinates[8].

Therefore, in the historical development of science there exists a branch of theoretical physics (i.e. two basic theories) that presents an alternative mathematics to that of infinitesimal analysis. One may suppose that in past times the above branch has been under-evaluated because his mathematical techniques are not congruent with the dominant one, i.e. calculus.

To have identified the birth of symmetries in physics lead us to recognize the intellectual origins of such notion. Lazare Carnot had as main intellectual teachers D'Alembert and Leibniz. As an example, L. Carnot followed the Leibnizian principle of continuity till to adopt it as his main maxim for both scientific and political lifes(the"deplacement by insensible degrees")[9]

Now,it was a Leibniz' design to add to calculus a new mathematical technique , to be specific for geometry, in order to formulate a "characteristica universalis"[10]. After three centuries of development of theoretical science, there is no better candidate for such addition than symmetry mathematical techniques, that started by Lazare Carnot's theoretical mechanics.

When we ask for a representation in logical terms of such new way of reasoning, we find a possible answer in thermodynamics. S.Carnot did not believe in caloric theory as well as in mechanical theory of heat; as a consequence he did not stated what today is the statement of the first principle of thermodynamics. This one may be synthetized by saying that "heat and work are equivalent"; however an exact statement is "it is not true that heat is not work", provided that one neither can state "heat is work" nor "heat is not work". Since in classical logic a statement is logically equivalent to its double negation, the above statement of the first principle does not belong to classical logic. Really, it put a problem, i.e. to know when and in what terms heat is work.

In fact, Sadi Carnot's thermodynamic theory is aimed to this target. In other words such theory as a whole represents a logical cycle (which is far more important than the operation cycle S.Carnot introduced for representing the fonctioning of an heat engine); i.e. by starting from a doubly negated statement the theory introduces explications which lead us to an affirmative part of the starting statement. It is very interesting that one finds the same logical scheme in Lazare Carnot's books on calculus[11], geometry[12], mechanics[1].

In the last theory , the double negated statement is implicit in his version of the inertia principle, whose core we know to be the equivalence of both states (v=0) and (v=const.); or, more precisely, for the same reasons as above in the case of the first principle of thermodynamics, "it is not true that (v=0) is not equal to (v=const.)"(It is trivial to verify the falsity of the affirmative, corresponding statement as well as that of the simply negated statement). This one is not the version of the inertia principle by Lazare Carnot; however,he offered a new version of it, strictly opertive in nature, compatible with constructive mathematics and more adequate than the Mach's version[13]; furthermore the same notion of geometric motion is drawn from such equivalence principle, since one more definition L. Carnot gave of geometric motion is that of a motion which may be superimposed to a physical system without affecting its mechanical state.

As a consequence, symmetry technique in physics appears to have substantiated a new way of reasoning in science.

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